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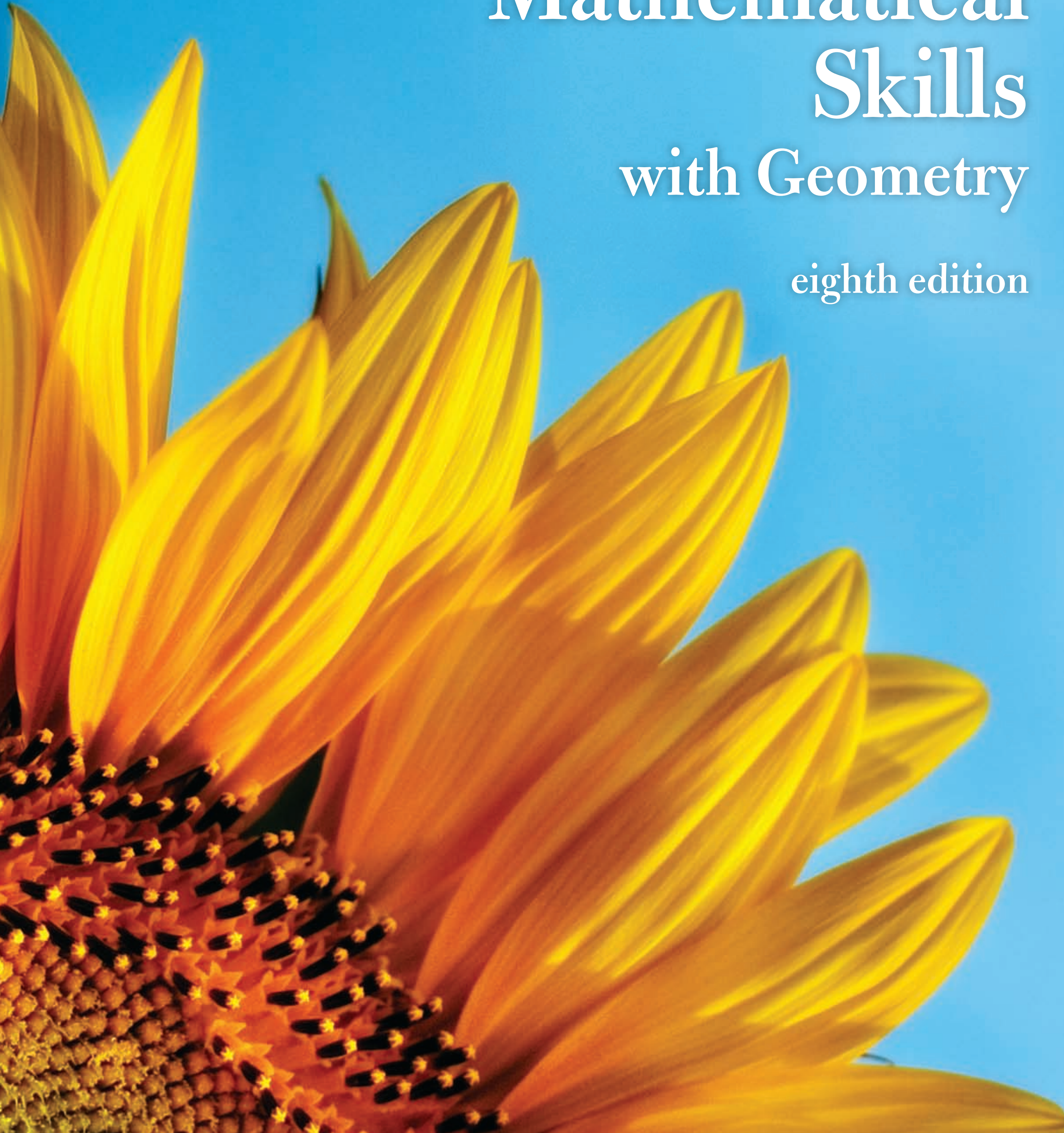
THE HUTCHISON SERIES
IN MATHEMATICS

BARATTO ∞ BERGMAN

Basic Mathematical Skills with Geometry

eighth edition



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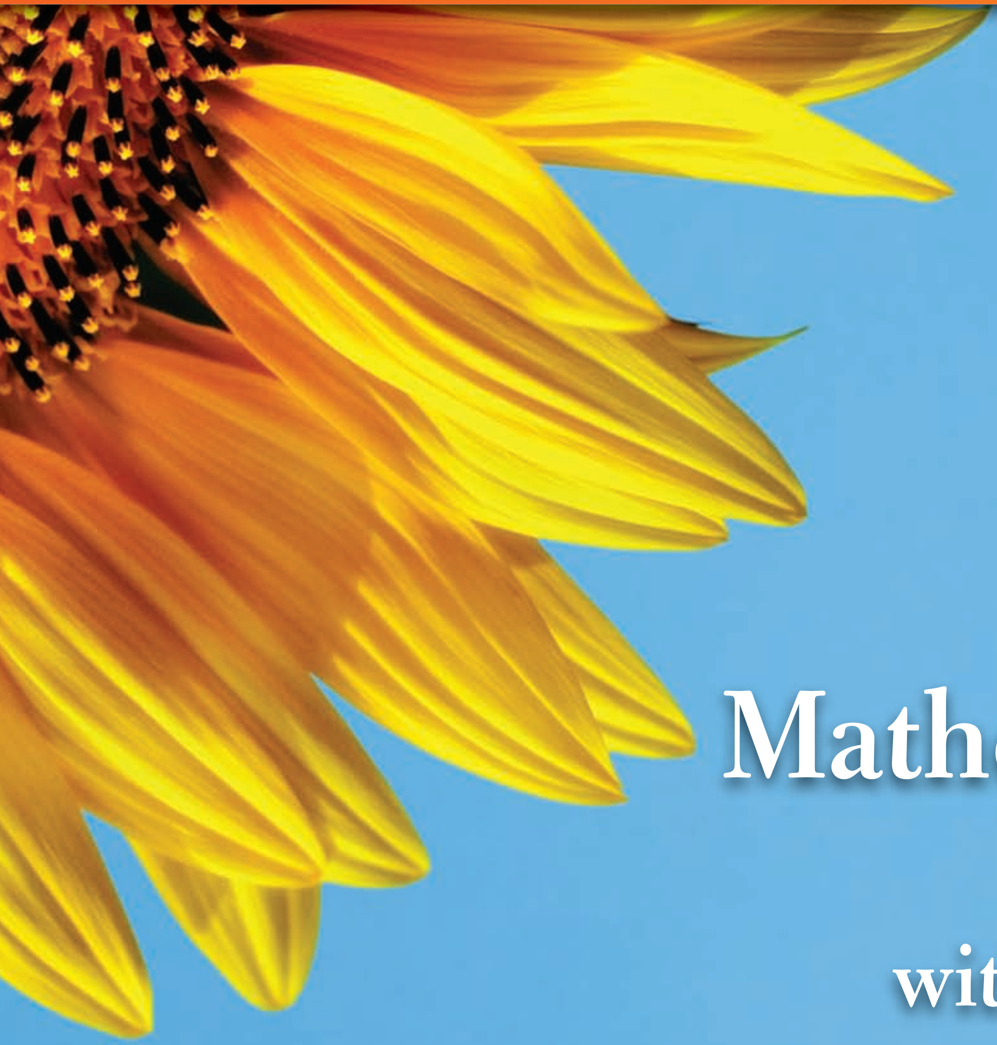

Basic Mathematical Skills
with Geometry

eighth edition

Stefan Baratto
Clackamas Community College

Barry Bergman
Clackamas Community College

Don Hutchison
Clackamas Community College

Connect Learn Succeed™

HUTCHISON'S BASIC MATHEMATICAL SKILLS WITH GEOMETRY, EIGHTH EDITION

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about the authors/dedication

Stefan Baratto

Stefan began teaching math and science in New York City middle schools. He also taught math at the University of Oregon, Southeast Missouri State University, and York County Technical College. Currently, Stefan is a member of the mathematics faculty at Clackamas Community College where he has found a niche, delighting in the CCC faculty, staff, and students. Stefan's own education includes the University of Michigan (BGS, 1988), Brooklyn College (CUNY), and the University of Oregon (MS, 1996).

Stefan is currently serving on the AMATYC Executive Board as the organization's Northwest Vice President. He has also been involved with ORMATYC, NEMATYC, NCTM, and the State of Oregon Math Chairs group, as well as other local organizations. He has applied his knowledge of math to various fields, using statistics, technology, and web design. More personally, Stefan and his wife, Peggy, try to spend time enjoying wonders of Oregon and the Pacific Northwest. Their activities include scuba diving and hiking.

Barry Bergman

Barry has enjoyed teaching mathematics to a wide variety of students over the years. He began in the field of adult basic education and moved into the teaching of high school mathematics in 1977. He taught high school math for 11 years, at which point he served as a K-12 mathematics specialist for his county. This work allowed him the opportunity to help promote the emerging NCTM standards in his region.

In 1990, Barry began the next portion of his career, having been hired to teach at Clackamas Community College. He maintains a strong interest in the appropriate use of technology and visual models in the learning of mathematics.

Throughout the past 32 years, Barry has played an active role in professional organizations. As a member of OCTM, he contributed several articles and activities to the group's journal. He has presented at AMATYC, OCTM, NCTM, ORMATYC, and ICTCM conferences. Barry also served 4 years as an officer of ORMATYC and participated on an AMATYC committee to provide feedback to revisions of NCTM's standards.

Don Hutchison

Don began teaching in a preschool while he was an undergraduate. He subsequently taught children with disabilities, adults with disabilities, high school mathematics, and college mathematics. Although each position offered different challenges, it was always breaking a challenging lesson into teachable components that he most enjoyed.

It was at Clackamas Community College that he found his professional niche. The community college allowed him to focus on teaching within a department that constantly challenged faculty and students to expect more. Under the guidance of Jim Streeter, Don learned to present his approach to teaching in the form of a textbook. Don has also been an active member of many professional organizations. He has been president of ORMATYC, AMATYC committee chair, and ACM curriculum committee member. He has presented at AMATYC, ORMATYC, AACC, MAA, ICTCM, and a variety of other conferences.

Above all, he encourages you to be involved, whether as a teacher or as a learner. Whether discussing curricula at a professional meeting or homework in a cafeteria, it is the process of communicating an idea that helps one to clarify it.

Dedication

We dedicate this text to the thousands of students who have helped us become better teachers, better communicators, better writers, and even better people. We read and respond to every suggestion we get—every one is invaluable. If you have any thoughts or suggestions, please contact us at

Stefan Baratto: sbaratto@clackamas.edu
Barry Bergman: bfbergman@gmail.com
Don Hutchison: donh@collegemathtext.com

Thank you all.

Letter from the Authors

Dear Colleagues,

We believe the key to learning mathematics, at any level, is active participation! We have revised our textbook series to specifically emphasize *GROWING MATH SKILLS* through active learning. Students who are active participants in the learning process have a greater opportunity to construct their own mathematical ideas and make stronger connections to concepts covered in their course. This participation leads to better understanding, retention, success, and confidence.

In order to grow student math skills, we have integrated features throughout our textbook series that reflect our philosophy. Specifically, our *chapter opening vignettes* and an array of section exercises relate to a singular topic or theme to engage students while identifying the relevance of mathematics.

The *Check Yourself* exercises, which include optional calculator references, are designed to keep students actively engaged in the learning process. Our exercise sets include application problems as well as challenging and collaborative writing exercises to give students more opportunity to sharpen their skills.

Originally formatted as a work-text, this textbook allows students to make use of the margins where exercise answer space is available to further facilitate active learning. This makes the textbook more than just a reference. Many of these exercises are designed for insight to generate mathematical thought while reinforcing continual practice and mastery of topics being learned. Our hope is that students who uses our textbook will grow their mathematical skills and become better mathematical thinkers as a result.

As we developed our series, we recognized that the use of technology should not be simply a supplement, but should be an essential element in learning mathematics. We understand that these “millennial students” are learning in different modes than just a few short years ago. Attending course lectures is not the only demand these students face—their daily schedules are pulling them in more directions than ever before. To meet the needs of these students, we have developed videos to better explain key mathematical concepts throughout the textbook. The goal of these videos is to provide students with a better framework—showing them how to solve a specific mathematical topic, regardless of their classroom environment (online or traditional lecture). The videos serve as refreshers or preparatory tools for classroom lecture and are available in several formats, including iPOD/MP3 format, to accommodate the different ways students access information.

Finally, with our series focus on ***growing math skills,*** we strongly believe that ALEKS® software can truly help students to remediate and grow their math skills given its adaptiveness. ALEKS is available to accompany our textbooks to help build proficiency. ALEKS has helped our own students to identify mathematical skills they have mastered and skills where remediation is required.

Thank you for using our textbook! We look forward to learning of your success!

Stefan Baratto
Barry Bergman
Donald Hutchison

Grow Your Mathematical Skills

About the Cover

A flower symbolizes transformation and growth—a change from the ordinary to the spectacular! Similarly, students in an arithmetic/basic math course have the potential to grow their math skills with resources to help them to become stronger math students. Authors Stefan Baratto, Barry Bergman, and Don Hutchison seek to help students to *grow their mathematical skills*—guiding them through the different stages to mathematical success!

Helping students develop study skills is critical for student success. With over 80 years in the classroom, Stefan Baratto, Barry Bergman, and Don Hutchison have helped students sharpen their mathematical skills and learn how to use their mathematical knowledge in everyday life! The Hutchison Series helps ***grow mathematical skills*** to motivate students to learn!

> *"This is a good book. The best feature, in my opinion is the readability of this text. It teaches through example and has students immediately check their own skills. This breaks up long text into small bits easier for students to digest."*
>
> *– Robin Anderson, Southwestern Illinois College*

With Baratto/Bergman/Hutchison!

Grow Your Mathematical Skills Through Better Conceptual Tools!

Stefan Baratto, Barry Bergman, and Don Hutchison know that students succeed once they have built a strong conceptual understanding of mathematics. ***"Make the Connection"*** chapter-opening vignettes help students to better understand mathematical concepts through everyday examples. Further reinforcing real-world mathematics, each vignette is accompanied by activities and exercises in the chapter to help students focus on the mathematical skills required for mastery.

Make the Connection
Chapter-Opening Vignettes
Activities
Reading Your Text
Learning Objectives
Self-Tests
Cumulative Reviews
Group Activities

Grow Your Mathematical Skills Through Better Exercises, Examples, and Applications!

A wealth of exercise sets is available for students at every level to actively involve them through the learning process in an effort to grow mathematical skills, including:

Prerequisite Tests
Check Yourself Exercises
Career Application Exercises
End-of-Section Exercises
Summary Exercises

Grow Your Mathematical Study Skills Through Better Active Learning Tools!

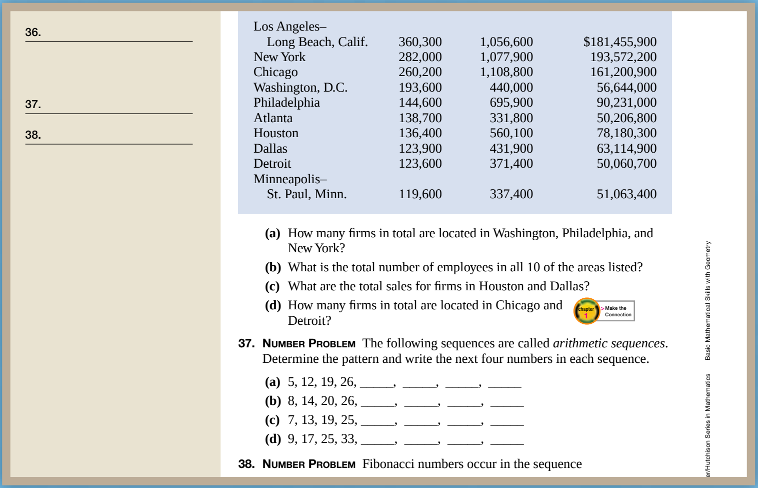
In an effort to meet the needs the "millennial student," we have made active-learning tools available to sharpen mathematical skills and build proficiency.

ALEKS
MathZone
Conceptual Videos
Lecture Videos

> *"The Baratto/Bergman/Hutchison textbook gives the student a well-rounded foundation into many concepts of algebra, taking the student from prior knowledge, to guided practice, to independent practice, and then to assessment. Each chapter builds upon concepts learned in other chapters. Items such as Check Yourself exercises and Activities at the end of most chapters help the student to be more successful in many of the concepts taught."*
>
> *– Karen Day, Elizabethtown Technical & Community College*

Grow Your Mathematical Skills

***"Make the Connection"*—Chapter-Opening Vignettes** provide interesting, relevant scenarios that will capture students' attention and engage them in the upcoming material. Exercises and *Activities* related to the *Opening Vignette* are available to utilize the theme most effectively for better mathematical comprehension (marked with an icon).



36. ______

37. ______

38. ______

Los Angeles– Long Beach, Calif.	360,300	1,056,600	$181,455,900
New York	282,000	1,077,900	193,572,200
Chicago	260,200	1,108,800	161,200,900
Washington, D.C.	193,600	440,000	56,644,000
Philadelphia	144,600	695,900	90,231,000
Atlanta	138,700	331,800	50,206,800
Houston	136,400	560,100	78,180,300
Dallas	123,900	431,900	63,114,900
Detroit	123,600	371,400	50,060,700
Minneapolis– St. Paul, Minn.	119,600	337,400	51,063,400

(a) How many firms in total are located in Washington, Philadelphia, and New York?

(b) What is the total number of employees in all 10 of the areas listed?

(c) What are the total sales for firms in Houston and Dallas?

(d) How many firms in total are located in Chicago and Detroit?

37. NUMBER PROBLEM The following sequences are called *arithmetic sequences*. Determine the pattern and write the next four numbers in each sequence.

(a) 5, 12, 19, 26, _____, _____, _____, _____

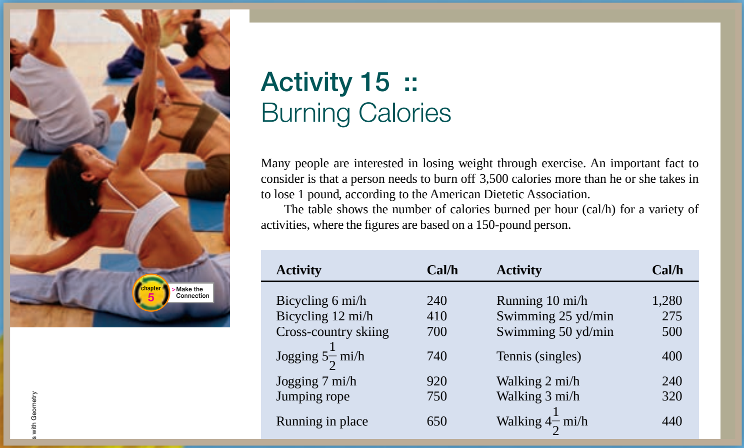
(b) 8, 14, 20, 26, _____, _____, _____, _____

(c) 7, 13, 19, 25, _____, _____, _____, _____

(d) 9, 17, 25, 33, _____, _____, _____, _____

38. NUMBER PROBLEM Fibonacci numbers occur in the sequence

Activities are incorporated to promote *active learning* by requiring students to find, interpret, and manipulate real-world data. The activity seen in the chapter-opening vignette ties the chapter together by way of questions to sharpen student mathematical and conceptual understanding highlighting the cohesiveness of the chapter. Students can complete the activities on their own, but they are best worked on in small groups.



Activity 15 ::

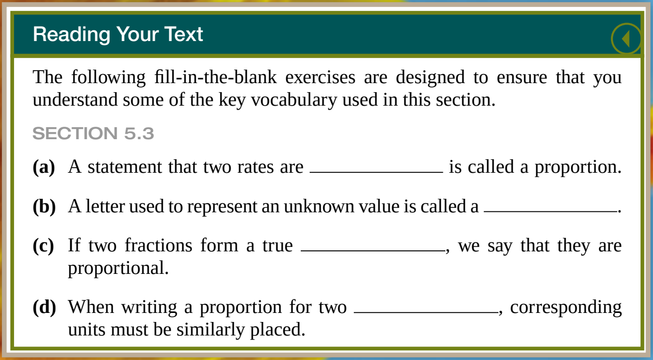
Burning Calories

Many people are interested in losing weight through exercise. An important fact to consider is that a person needs to burn off 3,500 calories more than he or she takes in to lose 1 pound, according to the American Dietetic Association.

The table shows the number of calories burned per hour (cal/h) for a variety of activities, where the figures are based on a 150-pound person.

Activity	Cal/h	Activity	Cal/h
Bicycling 6 mi/h	240	Running 10 mi/h	1,280
Bicycling 12 mi/h	410	Swimming 25 yd/min	275
Cross-country skiing	700	Swimming 50 yd/min	500
Jogging $5\frac{1}{2}$ mi/h	740	Tennis (singles)	400
Jogging 7 mi/h	920	Walking 2 mi/h	240
Jumping rope	750	Walking 3 mi/h	320
Running in place	650	Walking $4\frac{1}{2}$ mi/h	440

NEW! Reading Your Text offers a brief set of exercises at the end of each section to assess students' knowledge of key vocabulary terms. These exercises are designed to encourage careful reading for greater conceptual understanding. *Reading Your Text* exercises address vocabulary issues, which students often struggle in learning to understand core mathematical concepts. Answers to these exercises are provided at the end of the book.



Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 5.3

(a) A statement that two rates are ______________ is called a proportion.

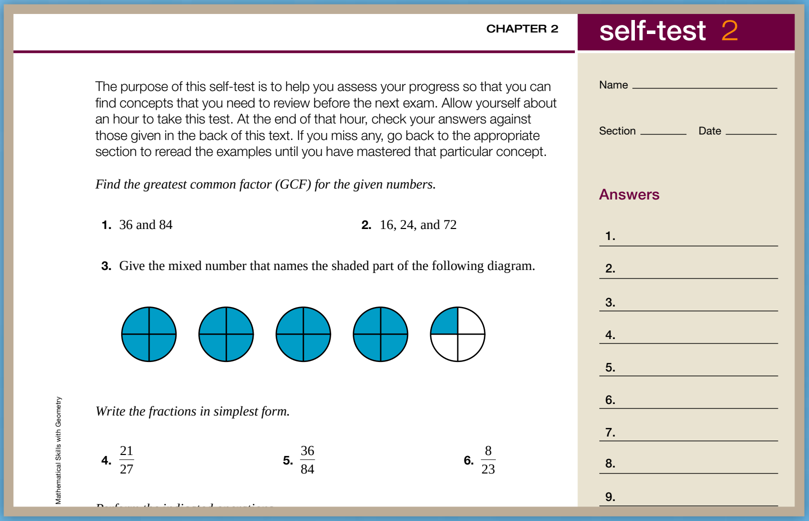
(b) A letter used to represent an unknown value is called a ______________.

(c) If two fractions form a true ______________, we say that they are proportional.

(d) When writing a proportion for two ______________, corresponding units must be similarly placed.

Through Better Conceptual Tools!

Self-Tests appear in each chapter to provide students with an opportunity to check their progress and to review important concepts, as well as provide confidence and guidance in preparing for exams. The answers to the *Self-Test* exercises are given at the end of the book. Section references are given with the answers to help the student.



CHAPTER 2

self-test 2

The purpose of this self-test is to help you assess your progress so that you can find concepts that you need to review before the next exam. Allow yourself about an hour to take this test. At the end of that hour, check your answers against those given in the back of this text. If you miss any, go back to the appropriate section to reread the examples until you have mastered that particular concept.

Find the greatest common factor (GCF) for the given numbers.

1. 36 and 84

2. 16, 24, and 72

3. Give the mixed number that names the shaded part of the following diagram.

Write the fractions in simplest form.

4. $\frac{21}{27}$

5. $\frac{36}{84}$

6. $\frac{8}{23}$

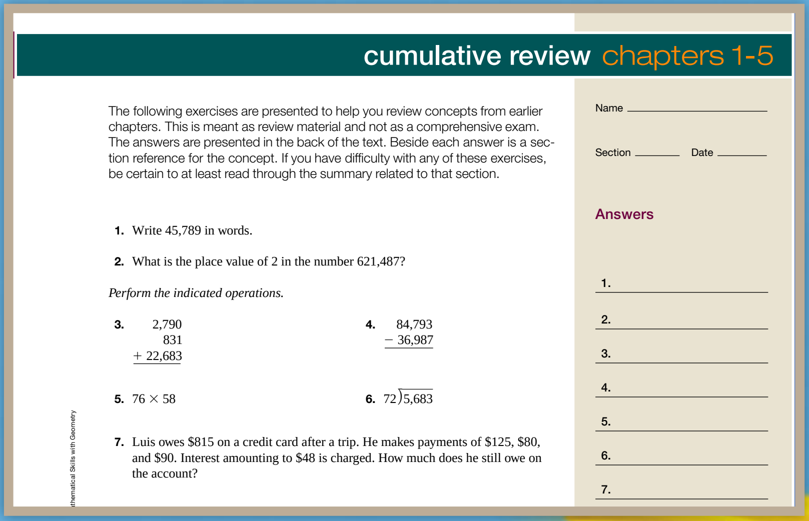
Name ______________

Section ________ Date ________

Answers

1. ______________
2. ______________
3. ______________
4. ______________
5. ______________
6. ______________
7. ______________
8. ______________
9. ______________

Cumulative Reviews are included starting with Chapter 2, following the *self-tests.* These reviews help students build on previously covered material and give them an opportunity to reinforce the skills necessary in preparing for midterm and final exams. These reviews assist students with the retention of knowledge throughout the course. The answers to these exercises are also given at the end of the book, along with section references.



cumulative review chapters 1-5

The following exercises are presented to help you review concepts from earlier chapters. This is meant as review material and not as a comprehensive exam. The answers are presented in the back of the text. Beside each answer is a section reference for the concept. If you have difficulty with any of these exercises, be certain to at least read through the summary related to that section.

1. Write 45,789 in words.

2. What is the place value of 2 in the number 621,487?

Perform the indicated operations.

3. $\begin{array}{r} 2{,}790 \\ 831 \\ +\ 22{,}683 \\ \hline \end{array}$

4. $\begin{array}{r} 84{,}793 \\ -\ 36{,}987 \\ \hline \end{array}$

5. 76×58

6. $72\overline{)5{,}683}$

7. Luis owes $815 on a credit card after a trip. He makes payments of $125, $80, and $90. Interest amounting to $48 is charged. How much does he still owe on the account?

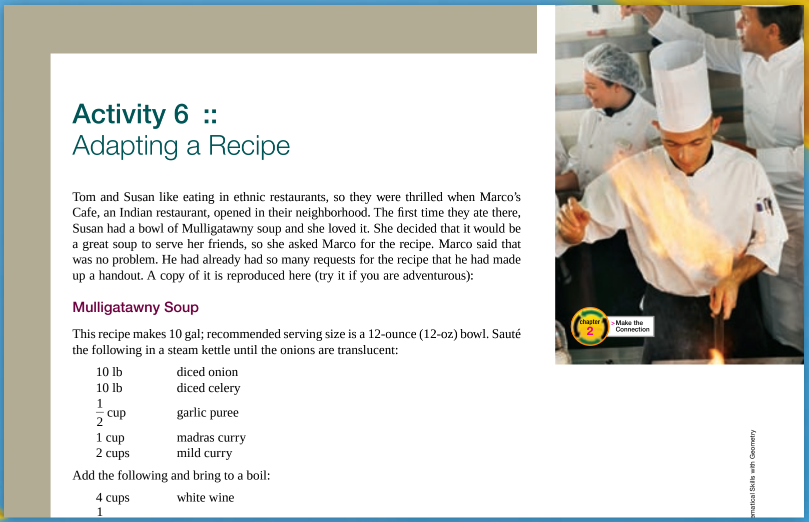
Name ______________

Section ________ Date ________

Answers

1. ______________
2. ______________
3. ______________
4. ______________
5. ______________
6. ______________
7. ______________

Group Activities offer practical exercises specifically designed to grow student comprehension through group work. Group activities are great for instructors and adjuncts—bringing a more interactive approach to teaching mathematics!



Activity 6 ::
Adapting a Recipe

Tom and Susan like eating in ethnic restaurants, so they were thrilled when Marco's Cafe, an Indian restaurant, opened in their neighborhood. The first time they ate there, Susan had a bowl of Mulligatawny soup and she loved it. She decided that it would be a great soup to serve her friends, so she asked Marco for the recipe. Marco said that was no problem. He had already had so many requests for the recipe that he had made up a handout. A copy of it is reproduced here (try it if you are adventurous):

Mulligatawny Soup

This recipe makes 10 gal; recommended serving size is a 12-ounce (12-oz) bowl. Sauté the following in a steam kettle until the onions are translucent:

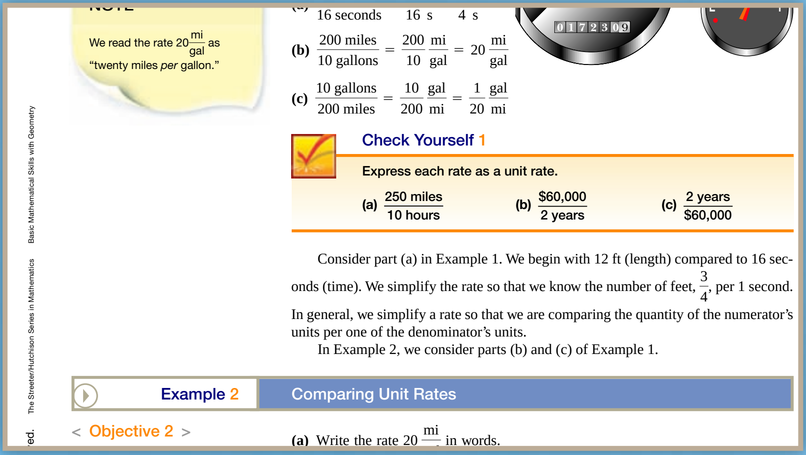
10 lb	diced onion
10 lb	diced celery
$\frac{1}{2}$ cup	garlic puree
1 cup	madras curry
2 cups	mild curry

Add the following and bring to a boil:

4 cups	white wine

Grow Your Mathematical Skills with Better Worked Examples, Exercises, and Applications!

"Check Yourself" Exercises are a hallmark of the Hutchison series; they are designed to actively involve students in the learning process. Every example is followed by an exercise that encourages students to solve a problem similar to the one just presented and check, through practice, what they have just learned. Answers are provided at the end of the section for immediate feedback.



The Streeter/Hutchison Series in Mathematics Basic Mathematical Skills with Geometry

We read the rate $20\frac{\text{mi}}{\text{gal}}$ as "twenty miles *per* gallon."

16 seconds 16 s 4 s

$$\text{(b)}\quad \frac{200\text{ miles}}{10\text{ gallons}} = \frac{200}{10}\,\frac{\text{mi}}{\text{gal}} = 20\,\frac{\text{mi}}{\text{gal}}$$

$$\text{(c)}\quad \frac{10\text{ gallons}}{200\text{ miles}} = \frac{10}{200}\,\frac{\text{gal}}{\text{mi}} = \frac{1}{20}\,\frac{\text{gal}}{\text{mi}}$$

Check Yourself 1

Express each rate as a unit rate.

(a) $\frac{\text{250 miles}}{\text{10 hours}}$ (b) $\frac{\$60{,}000}{\text{2 years}}$ (c) $\frac{\text{2 years}}{\$60{,}000}$

Consider part (a) in Example 1. We begin with 12 ft (length) compared to 16 seconds (time). We simplify the rate so that we know the number of feet, $\frac{3}{4}$, per 1 second. In general, we simplify a rate so that we are comparing the quantity of the numerator's units per one of the denominator's units.

In Example 2, we consider parts (b) and (c) of Example 1.

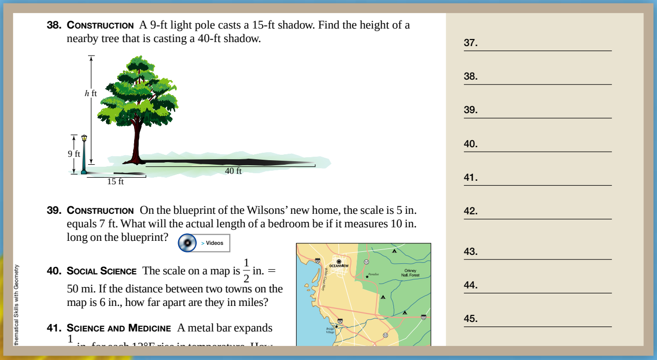
Example 2 Comparing Unit Rates

< Objective 2 >

(a) Write the rate $20\frac{\text{mi}}{}$ in words.

> *"I like the placement of the 'check yourself's.' Students are confronted with thought provoking questions to answer without mindlessly proceeding through the text."*
>
> *– Byron D. Hunter, College of Lake County*

End-of-Section Exercises enable students to evaluate their conceptual mastery through practice as they conclude each section. These comprehensive exercise sets are structured to highlight the progression in level not only providing clarity for the student, but also making it easier for instructors to determine exercises for assignments. The *application exercises* that are now integrated into every section are a crucial component of this organization.



38. **CONSTRUCTION** A 9-ft light pole casts a 15-ft shadow. Find the height of a nearby tree that is casting a 40-ft shadow.

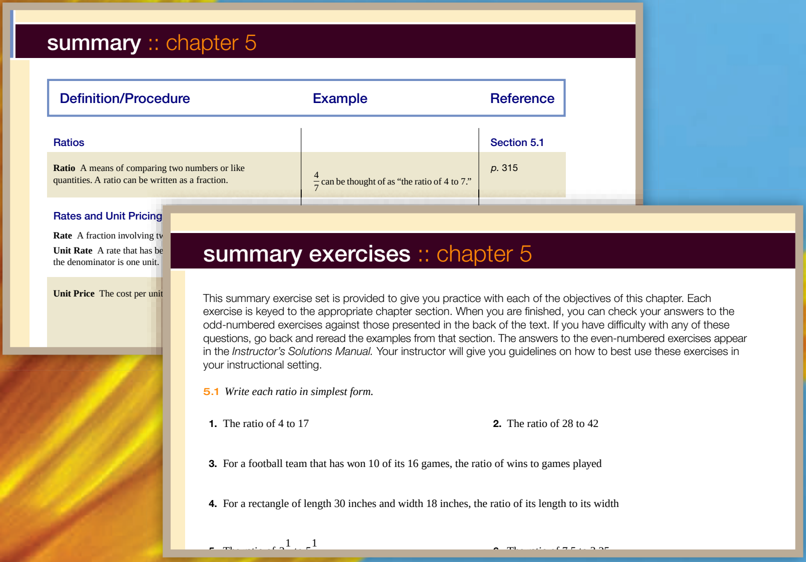
39. **CONSTRUCTION** On the blueprint of the Wilsons' new home, the scale is 5 in. equals 7 ft. What will the actual length of a bedroom be if it measures 10 in. long on the blueprint?

40. **SOCIAL SCIENCE** The scale on a map is $\frac{1}{2}$ in. = 50 mi. If the distance between two towns on the map is 6 in., how far apart are they in miles?

41. **SCIENCE AND MEDICINE** A metal bar expands $\frac{1}{}$

37. ____________
38. ____________
39. ____________
40. ____________
41. ____________
42. ____________
43. ____________
44. ____________
45. ____________

Summary and Summary Exercises at the end of each chapter allow students to review important concepts. The *Summary Exercises* provide an opportunity for the student to practice these important concepts. The answers to odd-numbered exercises are provided in the answers appendix.



summary :: chapter 5

Definition/Procedure	Example	Reference
Ratios		**Section 5.1**
Ratio A means of comparing two numbers or like quantities. A ratio can be written as a fraction.	$\frac{4}{7}$ can be thought of as "the ratio of 4 to 7."	*p.* 315
Rates and Unit Pricing		
Rate A fraction involving tw **Unit Rate** A rate that has be the denominator is one unit.		
Unit Price The cost per unit		

summary exercises :: chapter 5

This summary exercise set is provided to give you practice with each of the objectives of this chapter. Each exercise is keyed to the appropriate chapter section. When you are finished, you can check your answers to the odd-numbered exercises against those presented in the back of the text. If you have difficulty with any of these questions, go back and reread the examples from that section. The answers to the even-numbered exercises appear in the *Instructor's Solutions Manual.* Your instructor will give you guidelines on how to best use these exercises in your instructional setting.

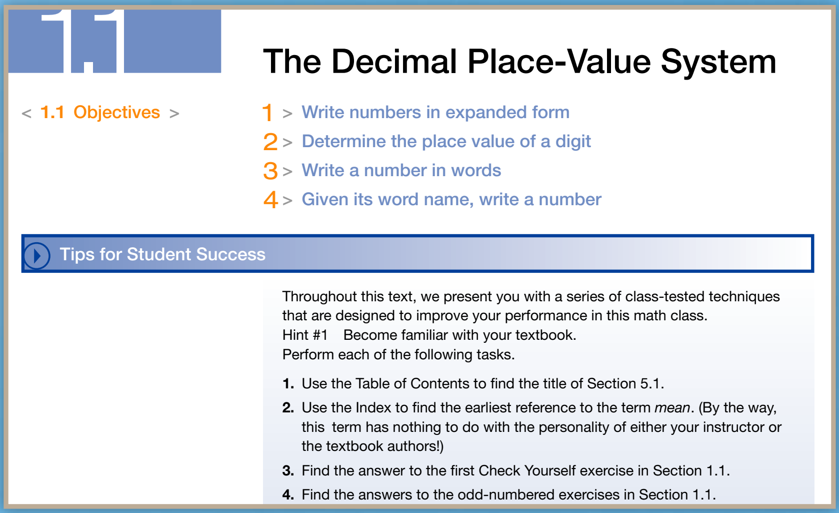
5.1 *Write each ratio in simplest form.*

1. The ratio of 4 to 17
2. The ratio of 28 to 42
3. For a football team that has won 10 of its 16 games, the ratio of wins to games played
4. For a rectangle of length 30 inches and width 18 inches, the ratio of its length to its width

The Streeter/Hutchison Series in Mathematics Basic Mathematical Skills with Geometry

Grow Your Mathematical Study Skills Through Better Active Learning Tools!

Tips for Student Success offers a resource to help students learn how to study, which is a problem many new students face, especially when taking their first exam in college mathematics. For this reason, Baratto/Bergman/Hutchison has incorporated *Tips for Student Success* boxes in the beginning of this textbook. The same suggestions made by great teachers in the classroom are now available to students outside of the classroom, offering extra direction to help improve understanding and further insight.



1.1

The Decimal Place-Value System

< 1.1 Objectives >

1 > Write numbers in expanded form
2 > Determine the place value of a digit
3 > Write a number in words
4 > Given its word name, write a number

Tips for Student Success

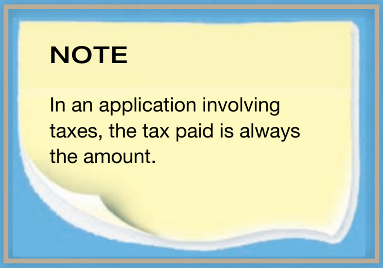
Throughout this text, we present you with a series of class-tested techniques that are designed to improve your performance in this math class.

Hint #1 Become familiar with your textbook.

Perform each of the following tasks.

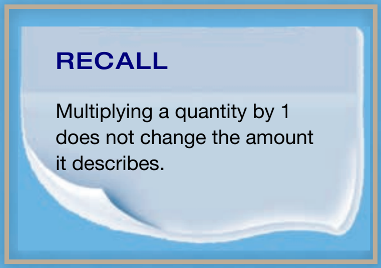
1. Use the Table of Contents to find the title of Section 5.1.
2. Use the Index to find the earliest reference to the term *mean*. (By the way, this term has nothing to do with the personality of either your instructor or the textbook authors!)
3. Find the answer to the first Check Yourself exercise in Section 1.1.
4. Find the answers to the odd-numbered exercises in Section 1.1.

Notes and Recalls accompany the step-by-step worked examples helping students **focus on information critical to their success.** ***Recall Notes*** give students a ***just-in-time*** reminder, reinforcing previously learned material through references.



NOTE

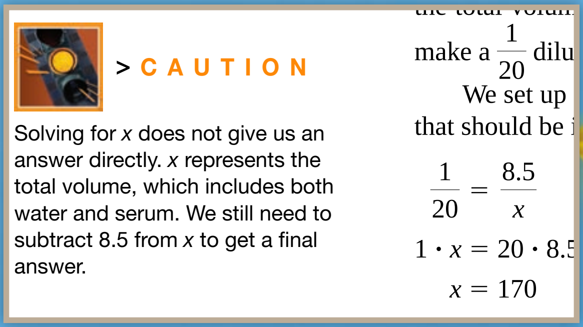
In an application involving taxes, the tax paid is always the amount.



RECALL

Multiplying a quantity by 1 does not change the amount it describes.

Cautions are integrated throughout the textbook to alert students to common mistakes and how to avoid them.



> CAUTION

Solving for x does not give us an answer directly. x represents the total volume, which includes both water and serum. We still need to subtract 8.5 from x to get a final answer.

make a $\frac{1}{20}$ dilu

We set up

that should be

$$\frac{1}{20} = \frac{8.5}{x}$$

$$1 \cdot x = 20 \cdot 8.5$$

$$x = 170$$

grow your math skills with ALEKS®

Experience Student Success!

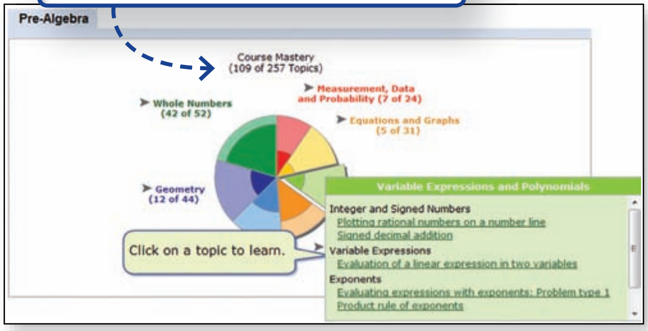
ALEKS® ALEKS is a unique online math tool that uses adaptive questioning and artificial intelligence to correctly place, prepare, and remediate students . . . all in one product! Institutional case studies have shown that **ALEKS has improved pass rates by over 20% versus traditional online homework, and by over 30% compared to using a text alone.**

By offering each student an individualized learning path, ALEKS directs students to work on the math topics that they are ready to learn. Also, to help students keep pace in their course, instructors can correlate ALEKS to their textbook or syllabus in seconds.

To learn more about how ALEKS can be used to boost student performance, please visit www.aleks.com/highered/math or contact your McGraw-Hill representative.

ALEKS Pie

Each student is given her or his own individualized learning path.


Pre-Algebra
Course Mastery (109 of 257 Topics)
Measurement, Data and Probability (7 of 24)
Whole Numbers (42 of 52)
Equations and Graphs (5 of 31)
Geometry (12 of 44)
Click on a topic to learn.

Variable Expressions and Polynomials

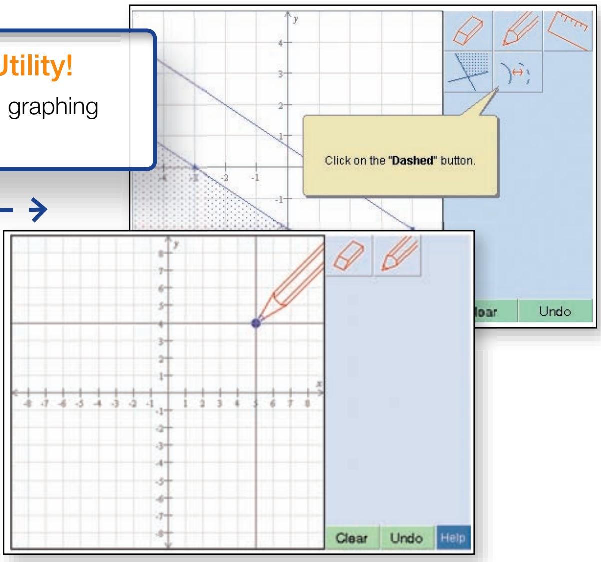
Integer and Signed Numbers
- Plotting rational numbers on a number line
- Signed decimal addition

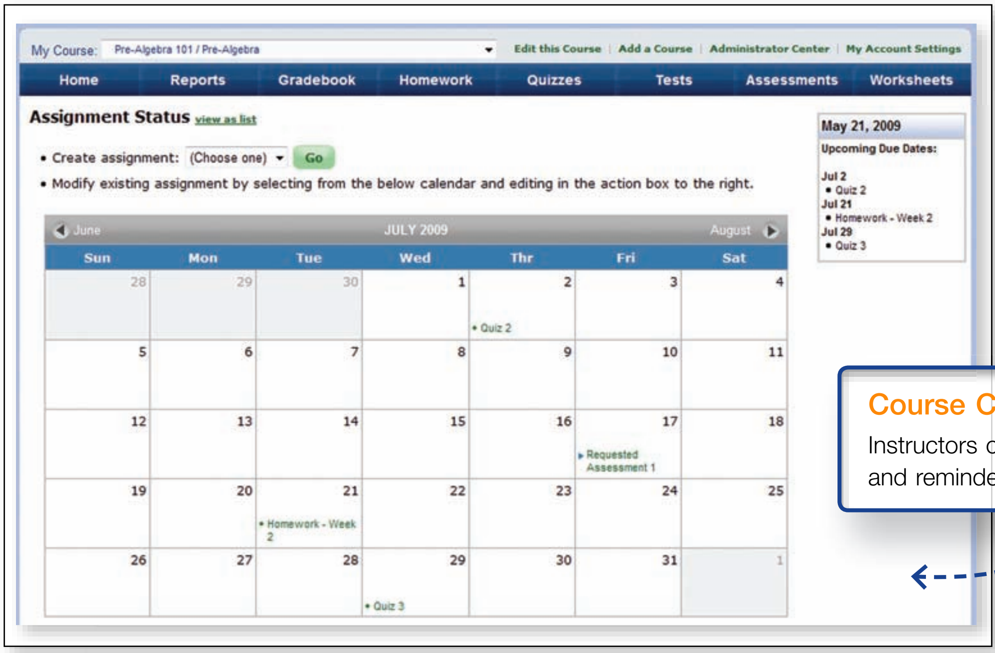
Variable Expressions
- Evaluation of a linear expression in two variables

Exponents
- Evaluating expressions with exponents: Problem type 1
- Product rule of exponents

Easy Graphing Utility!

Students can answer graphing problems with ease!


Click on the "Dashed" button.
Clear
Undo
Help


My Course: Pre-Algebra 101 / Pre-Algebra
Edit this Course | Add a Course | Administrator Center | My Account Settings
Home
Reports
Gradebook
Homework
Quizzes
Tests
Assessments
Worksheets
Go

Assignment Status view as list

- Create assignment: (Choose one)
- Modify existing assignment by selecting from the below calendar and editing in the action box to the right.

June JULY 2009 August

Sun	Mon	Tue	Wed	Thr	Fri	Sat
28	29	30	1	2 • Quiz 2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17 ▸ Requested Assessment 1	18
19	20	21 • Homework - Week 2	22	23	24	25
26	27	28	29 • Quiz 3	30	31	1

May 21, 2009

Upcoming Due Dates:

Jul 2
- Quiz 2

Jul 21
- Homework - Week 2

Jul 29
- Quiz 3

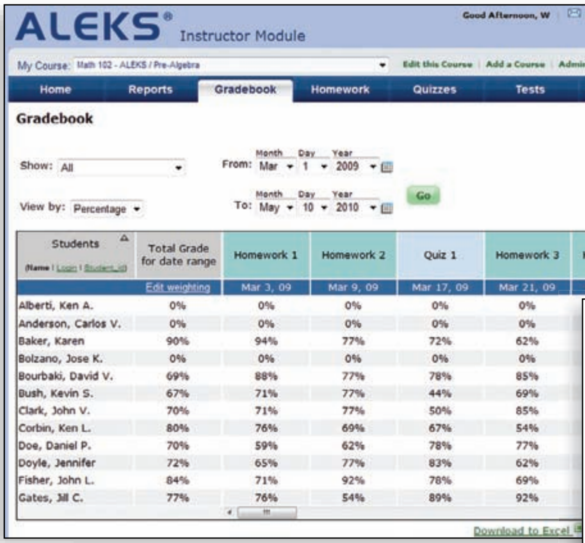
Course Calendar

Instructors can schedule assignments and reminders for students.

New ALEKS Instructor Module

Enhanced Functionality and Streamlined Interface Help to Save Instructor Time

ALEKS® The new ALEKS Instructor Module features enhanced functionality and a streamlined interface based on research with ALEKS instructors and homework management instructors. Paired with powerful assignment-driven features, textbook integration, and extensive content flexibility, the new ALEKS Instructor Module simplifies administrative tasks and makes ALEKS more powerful than ever.


ALEKS® Instructor Module
Good Afternoon, W | Inbox | Use Advanced IM-2 | Sign Out
My Course: Math 102 - ALEKS / Pre-Algebra
Edit this Course | Add a Course | Administrator Center | My Account Settings
Home
Reports
Gradebook
Homework
Quizzes
Tests
Assessments
Worksheets
Show: All
View by: Percentage
Go
Download to Excel

Gradebook

From: Month Mar Day 1 Year 2009

To: Month May Day 10 Year 2010

Students (Name | Login | Student_id)	Total Grade for date range	Homework 1	Homework 2	Quiz 1	Homework 3
	Edit weighting	Mar 3, 09	Mar 9, 09	Mar 17, 09	Mar 21, 09
Alberti, Ken A.	0%	0%	0%	0%	0%
Anderson, Carlos V.	0%	0%	0%	0%	0%
Baker, Karen	90%	94%	77%	72%	62%
Bolzano, Jose K.	0%	0%	0%	0%	0%
Bourbaki, David V.	69%	88%	77%	78%	85%
Bush, Kevin S.	67%	71%	77%	44%	69%
Clark, John V.	70%	71%	77%	50%	85%
Corbin, Ken L.	80%	76%	69%	67%	54%
Doe, Daniel P.	70%	59%	62%	78%	77%
Doyle, Jennifer	72%	65%	77%	83%	62%
Fisher, John L.	84%	71%	92%	78%	69%
Gates, Jill C.	77%	76%	54%	89%	92%

May 22, 2009
Upcoming Due Dates:
May 24
- Homework 8

May 29
- Homework 9

Jun 2
- Quiz 5

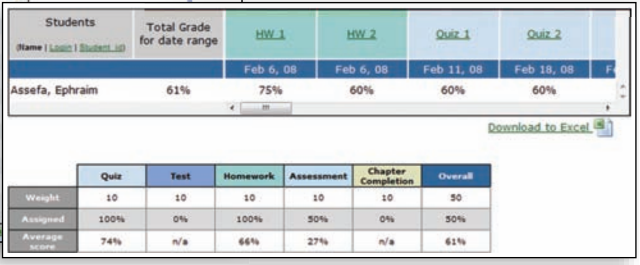
Jun 12
- Homework 10

Course Calendar

Gradebook view for all students

New Gradebook!

Instructors can seamlessly track student scores on automatically graded assignments. They can also easily adjust the weighting and grading scale of each assignment.


Download to Excel

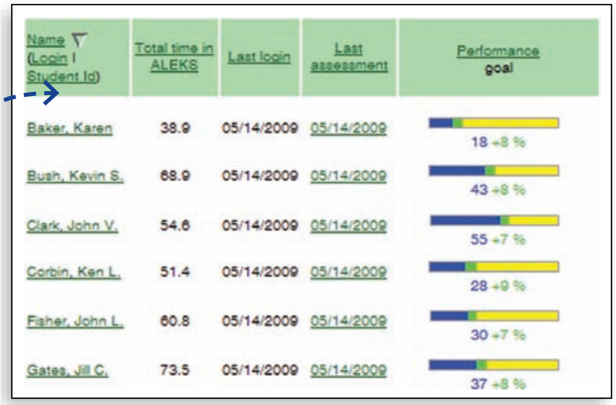
Students (Name | Login | Student_id)	Total Grade for date range	HW 1	HW 2	Quiz 1	Quiz 2
		Feb 6, 08	Feb 6, 08	Feb 11, 08	Feb 18, 08
Assefa, Ephraim	61%	75%	60%	60%	60%

	Quiz	Test	Homework	Assessment	Chapter Completion	Overall
Weight	10	10	10	10	10	50
Assigned	100%	0%	100%	50%	0%	50%
Average score	74%	n/a	66%	27%	n/a	61%

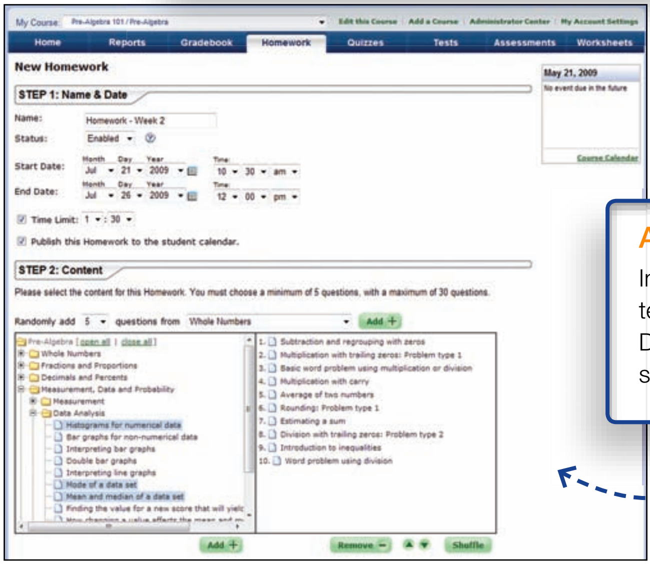
Gradebook view for an individual student

Track Student Progress Through Detailed Reporting

Instructors can track student progress through automated reports and robust reporting features.



Name (Login | Student Id)	Total time in ALEKS	Last login	Last assessment	Performance goal
Baker, Karen	38.9	05/14/2009	05/14/2009	18 +8 %
Bush, Kevin S.	68.9	05/14/2009	05/14/2009	43 +8 %
Clark, John V.	54.6	05/14/2009	05/14/2009	55 +7 %
Corbin, Ken L.	51.4	05/14/2009	05/14/2009	28 +9 %
Fisher, John L.	60.8	05/14/2009	05/14/2009	30 +7 %
Gates, Jill C.	73.5	05/14/2009	05/14/2009	37 +8 %


My Course: Pre-Algebra 101 / Pre-Algebra
Edit this Course | Add a Course | Administrator Center | My Account Settings
Home
Reports
Gradebook
Homework
Quizzes
Tests
Assessments
Worksheets
Add +
Remove −
Shuffle

New Homework

STEP 1: Name & Date

Name: Homework - Week 2

Status: Enabled

Start Date: Month Jul Day 21 Year 2009 Time: 10 : 30 am

End Date: Month Jul Day 26 Year 2009 Time: 12 : 00 pm

☑ Time Limit: 1 : 30

☑ Publish this Homework to the student calendar.

STEP 2: Content

Please select the content for this Homework. You must choose a minimum of 5 questions, with a maximum of 30 questions.

Randomly add 5 questions from Whole Numbers

Pre-Algebra [open all | close all]
- Whole Numbers
- Fractions and Proportions
- Decimals and Percents
- Measurement, Data and Probability
  - Measurement
  - Data Analysis
    - Histograms for numerical data
    - Bar graphs for non-numerical data
    - Interpreting bar graphs
    - Double bar graphs
    - Interpreting line graphs
    - Mode of a data set
    - Mean and median of a data set
    - Finding the value for a new score that will yiel

1. Subtraction and regrouping with zeros
2. Multiplication with trailing zeros: Problem type 1
3. Basic word problem using multiplication or division
4. Multiplication with carry
5. Average of two numbers
6. Rounding: Problem type 1
7. Estimating a sum
8. Division with trailing zeros: Problem type 2
9. Introduction to inequalities
10. Word problem using division

May 21, 2009
No event due in the future
Course Calendar

Select topics for each assignment

Automatically Graded Assignments

Instructors can easily assign homework, quizzes, tests, and assessments to all or select students. Deadline extensions can also be created for select students.

Learn more about ALEKS by visiting www.aleks.com/highered/math or contact your McGraw-Hill representative.

360° Development Process

McGraw-Hill's 360° Development Process is an ongoing, never-ending, market-oriented approach to building accurate and innovative print and digital products. It is dedicated to continual large-scale and incremental improvement driven by multiple customer feedback loops and checkpoints. This is initiated during the early planning stages of our new products, and intensifies during the development and production stages, then begins again upon publication, in anticipation of the next edition.

A key principle in the development of any mathematics text is its ability to adapt to teaching specifications in a universal way. The only way to do so is by contacting those universal voices—and learning from their suggestions. We are confident that our book has the most current content the industry has to offer, thus pushing our desire for accuracy to the highest standard possible. In order to accomplish this, we have moved through an arduous road to production. Extensive and open-minded advice is critical in the production of a superior text.

Listening to you...

This textbook has been reviewed by over 300 teachers across the country. Our textbook is a commitment to your students, providing clear explanations, concise writing style, step-by-step learning tools, and the best exercises and applications in developmental mathematics. How do we know? You told us so!

Teachers *just like you* are saying great things about the Baratto/Bergman/Hutchison developmental mathematics series:

"Well written and organized. It tends to get directly into what the students actually are required to do without lengthy preamble. Nice division of exercises into basic/advanced suitable for our students and personal finance exercises they can relate to."

– Jonathan Cornick, Queensborough Community College

"This text is clearly written with developmental students in mind, including numerous examples and exercises to reinforce the concepts presented. The examples are thorough, including step-by-step guidance to students."

– Shelly Hansen, Mesa State College

"A very readable and comprehensive textbook that does a great job of presenting and describing the basic and more advanced concepts. "Practice" and "learning by doing" are themes that penetrate throughout the text, for there is a large volume of (homework) problems from which both teachers and students can choose. Each problem section builds upon the concepts learned previously: a very sound pedagogical approach."

– Bob Rhea, J. Sargeant Reynolds Community College

Acknowledgments and Reviewers

The development of this textbook series would never have been possible without the creative ideas and feedback offered by many reviewers. We are especially thankful to the following instructors for their careful review of the manuscript.

Symposia

Every year McGraw-Hill conducts a general mathematics symposium, which is attended by instructors from across the country. These events are an opportunity for editors from McGraw-Hill to gather information about the needs and challenges of instructors teaching these courses. This information helped to create the book plan for *Basic Mathematical Skills with Geometry*. They also offer a forum for the attendees to exchange ideas and experiences with colleagues they might have not otherwise met.

Napa Valley Symposium

Antonio Alfonso, *Miami Dade College*
Lynn Beckett-Lemus, *El Camino College*
Kristin Chatas, *Washtenaw Community College*
Maria DeLucia, *Middlesex College*
Nancy Forrest, *Grand Rapids Community College*
Michael Gibson, *John Tyler Community College*
Linda Horner, *Columbia State College*
Matthew Hudock, *St. Phillips College*
Judith Langer, *Westchester Community College*
Kathryn Lavelle, *Westchester Community College*
Scott McDaniel, *Middle Tennessee State University*
Adelaida Quesada, *Miami Dade College*

Susan Schulman, *Middlesex College*

Stephen Toner, *Victor Valley College*

Chariklia Vassiliadis, *Middlesex County College*

Melanie Walker, *Bergen Community College*

Myrtle Beach Symposium

Patty Bonesteel, *Wayne State University*

Zhixiong Chen, *New Jersey City University*

Latonya Ellis, *Bishop State Community College*

Bonnie Filer-Tubaugh, *University of Akron*

Catherine Gong, *Citrus College*

Marcia Lambert, *Pitt Community College*

Katrina Nichols, *Delta College*

Karen Stein, *University of Akron*

Walter Wang, *Baruch College*

La Jolla Symposium

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Yvonne Aucoin, *Tidewater Community College*

Sylvia Carr, *Missouri State University*

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John Jerome, *Suffolk County Community College*

Sandra Jovicic, *Akron University*

Carolyn Robinson, *Mt. San Antonio College*

Carolyn Shand-Hawkins, *Missouri State*

Manuscript Review Panels

Over 150 teachers and academics from across the country reviewed the various drafts of the manuscript to give feedback on content, design, pedagogy, and organization. This feedback was summarized by the book team and used to guide the direction of the text.

Reviewers of the Hutchison/Baratto/Bergman Developmental Mathematics Series

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Latrica Williams, *St. Petersburg College*
Paul Wozniak, *El Camino College*
Christopher Yarrish, *Harrisburg Area Community College*
Steve Zuro, *Joliet Junior College*

Finally, we are forever grateful to the many people behind the scenes at McGraw-Hill without whom we would still be on page 1. Most important, we give special thanks to all the students and instructors who will *grow* their *Basic Math Skills!*

Supplements for the Student

MathZone www.mathzone.com

McGraw-Hill's **MathZone** is a powerful Web-based tutorial for homework, quizzing, testing, and multimedia instruction. Also available in CD-ROM format, MathZone offers:

- **Practice exercises** based on the text and generated in an unlimited quantity for as much practice as needed to master any objective
- **Video** clips of classroom instructors showing how to solve exercises from the text, step by step
- **e-Professor** animations that take the student through step-by-step instructions, delivered on-screen and narrated by a teacher on audio, for solving exercises from the textbook; the user controls the pace of the explanations and can review as needed
- **NetTutor,** which offers personalized instruction by live tutors familiar with the textbook's objectives and problem-solving methods

Every assignment, exercise, video lecture, and e-Professor is derived from the textbook.

ALEKS Prep for Developmental Mathematics

ALEKS Prep for Beginning Algebra and Prep for Intermediate Algebra focus on prerequisite and introductory material for Beginning Algebra and Intermediate Algebra. These prep products can be used during the first 3 weeks of a course to prepare students for future success in the course and to increase retention and pass rates. Backed by two decades of National Science Foundation funded research, ALEKS interacts with students much like a human tutor, with the ability to precisely assess a student's preparedness and provide instruction on the topics the student is most likely to learn.

ALEKS Prep Course Products Feature:

- Artificial Intelligence Targets Gaps in Individual Students Knowledge
- Assessment and Learning Directed Toward Individual Students Needs
- Open Response Environment with Realistic Input Tools
- Unlimited Online Access-PC & Mac Compatible

Free trial at www.aleks.com/free_trial/instructor

Student's Solutions Manual

The *Student's Solutions Manual* provides comprehensive, worked-out solutions to the odd-numbered exercises in the Pre-Test, Section Exercises, Summary Exercises, Self-Test and the Cumulative Review. The steps shown in the solutions match the style of solved examples in the textbook.

New connect2Developmental Mathematics video Series!

Available on DVD and the MathZone website, these innovative videos bring essential Developmental Mathematics concepts to life! The videos take the concepts and place them in a real world setting so that students make the connection from what they learn in the classroom to real world experiences outside the classroom. Making use of 3D animations and lectures, Connect2Developmental Mathematics video series answers the age-old questions “Why is this important?” and “When will I ever use it?” The videos cover topics from Arithmetic and Basic Mathematics through the Algebra sequence, mixing student-oriented themes and settings with basic theory.

Video Lectures on Digital Video Disk

The video series is based on exercises from the textbook. Each presenter works through selected problems, following the solution methodology employed in the text. The video series is available on DVD or online as part of MathZone. The DVDs are closed-captioned for the hearing impaired, are subtitled in Spanish, and meet the Americans with Disabilities Act Standards for Accessible Design.

NetTutor

Available through MathZone, NetTutor is a revolutionary system that enables students to interact with a live tutor over the web. NetTutor’s Web-based, graphical chat capabilities enable students and tutors to use mathematical notation and even to draw graphs as they work through a problem together. Students can also submit questions and receive answers, browse previously answered questions, and view previous sessions. Tutors are familiar with the textbook’s objectives and problemsolving styles.

Supplements for the Instructor

MathZone **www.mathzone.com**

McGraw-Hill's **MathZone** is a complete online tutorial and course management system for mathematics and statistics, designed for greater ease of use than any other system available. Available with selected McGraw-Hill textbooks, the system enables instructors to **create and share courses and assignments** with colleagues and adjuncts with only a few clicks of the mouse. All assignments, questions, e-Professors, online tutoring, and video lectures are directly tied to **text-specific** materials.

MathZone courses are customized to your textbook, but you can edit questions and algorithms, import your own content, and **create** announcements and due dates for assignments.

MathZone has **automatic grading** and reporting of easy-to-assign, algorithmically generated homework, quizzing, and testing. All student activity within **MathZone** is automatically recorded and available to you through a **fully integrated grade book** that can be downloaded to Excel.

MathZone offers:

- **Practice exercises** based on the textbook and generated in an unlimited number for as much practice as needed to master any topic you study.
- **Videos** of classroom instructors giving lectures and showing you how to solve exercises from the textbook.
- **e-Professors** to take you through animated, step-by-step instructions (delivered via on-screen text and synchronized audio) for solving problems in the book, allowing you to digest each step at your own pace.
- **NetTutor,** which offers live, personalized tutoring via the Internet.

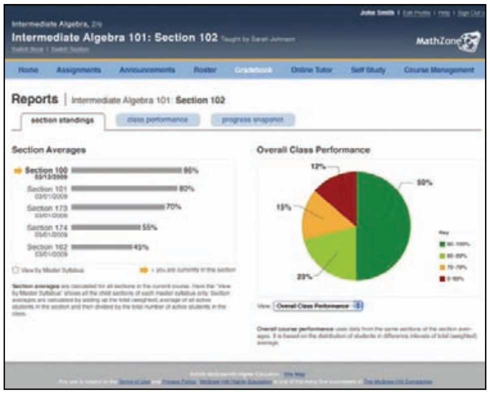
Instructor's Testing and Resource Online

Provides a wealth of resources for the instructor. Among the supplements is a **computerized test bank** utilizing Brownstone Diploma® algorithm-based testing software to create customized exams quickly. This user-friendly program enables instructors to search for questions by topic, format, or difficulty level; to edit existing questions or to add new ones; and to scramble questions and answer keys for multiple versions of a single test. Hundreds of text-specific, open-ended, and multiple-choice questions are included in the question bank. Sample chapter tests are also provided. CD available upon request.

Grow Your Knowledge with MathZone Reporting

Visual Reporting

The new dashboard-like reports will provide the progress snapshot instructors are looking for to help them make informed decisions about their students.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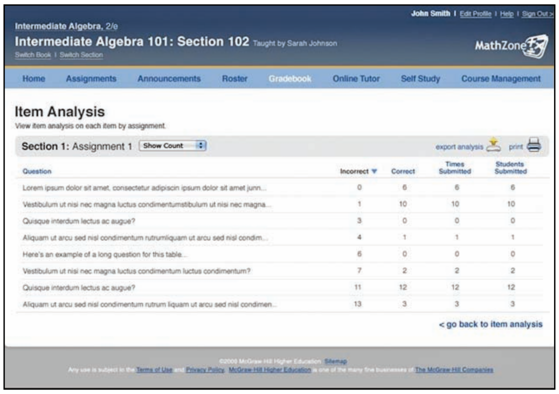
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Intermediate Algebra 101: Section 102 Taught by Sarah Johnson

Item Analysis

View item analysis on each item by assignment.

Section 1: Assignment 1

Question	Incorrect	Correct	Times Submitted	Students Submitted
Lorem ipsum dolor sit amet, consectetur adipiscin ipsum dolor sit amet junn...	0	6	6	6
Vestibulum ut nisi nec magna luctus condimentumstibulum ut nisi nec magna...	1	10	10	10
Quisque interdum lectus ac augue?	3	0	0	0
Aliquam ut arcu sed nisl condimentum rutrumliquam ut arcu sed nisl condim...	4	1	1	1
Here's an example of a long question for this table...	6	0	0	0
Vestibulum ut nisi nec magna luctus condimentum luctus condimentum?	7	2	2	2
Quisque interdum lectus ac augue?	11	12	12	12
Aliquam ut arcu sed nisl condimentum rutrum liquam ut arcu sed nisl condimen...	13	3	3	3

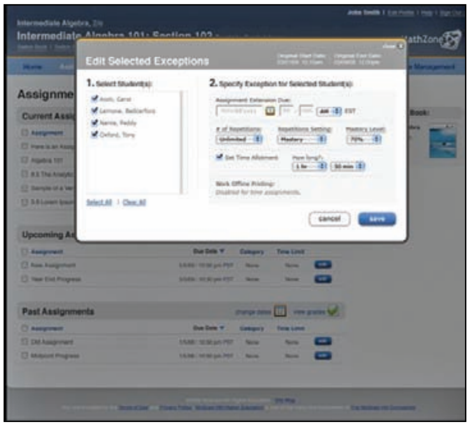
< go back to item analysis

Item Analysis

Instructors can view detailed statistics on student performance at a learning objective level to understand what students have mastered and where they need additional help.

Managing Assignments for Individual Students

Instructors have greater control over creating individualized assignment parameters for individual students, special populations and groups of students, and for managing specific or ad hoc course events.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Edit Selected Exceptions
1. Select Student(s):
2. Specify Exception for Selected Student(s):
Select All | Clear All
cancel
save
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Current Assignments
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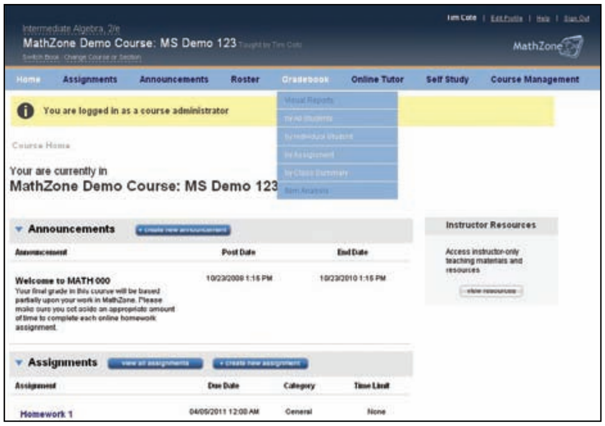
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You are logged in as a course administrator

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MathZone Demo Course: MS Demo 123

Announcements

Announcement	Post Date	End Date
Welcome to MATH 000 Your final grade in this course will be based partially upon your work in MathZone. Please make sure you set aside an appropriate amount of time to complete each online homework assignment.	10/23/2009 1:15 PM	10/23/2010 1:15 PM

Access instructor-only teaching materials and resources

Assignments

Assignment	Due Date	Category	Time Limit
Homework 1	04/09/2011 12:00 AM	General	None

New User Interface

Designed by You! Instructors and students will experience a modern, more intuitive layout. Items used most commonly are easily accessible through the menu bar such as assignments, visual reports, and course management options.

New ALEKS Instructor Module

The new ALEKS Instructor Module features enhanced functionality and a streamlined interface based on research with ALEKS instructors and homework management instructors. Paired with powerful assignment driven features, textbook integration, and extensive content flexibility, the new ALEKS Instructor Module simplifies administrative tasks and makes ALEKS more powerful than ever. Features include:

Gradebook Instructors can seamlessly track student scores on automatically graded assignments. They can also easily adjust the weighting and grading scale of each assignment.

Course Calendar Instructors can schedule assignments and reminders for students.

Automatically Graded Assignments Instructors can easily assign homework, quizzes, tests, and assessments to all or select students. Deadline extensions can also be created for select students.

Set-Up Wizards Instructors can use wizards to easily set up assignments, course content, textbook integration, etc.

Message Center Instructors can use the redesigned Message Center to send, receive, and archive messages; input tools are available to convey mathematical expressions via email.

Baratto/Bergman/Hutchison Video Lectures on Digital Video Disk (DVD)

In the videos, qualified instructors work through selected problems from the textbook, following the solution methodology employed in the text. The video series is available on DVD or online as an assignable element of MathZone. The DVDs are closed-captioned for the hearing-impaired, are subtitled in Spanish, and meet the Americans with Disabilities Act Standards for Accessible Design. Instructors may use them as resources in a learning center, for online courses, and to provide extra help for students who require extra practice.

Annotated Instructor's Edition

In the *Annotated Instructor's Edition (AIE),* **answers to exercises and tests appear adjacent to each exercise set,** in a color used *only* for annotations. Also found in the *AIE* are icons within the Practice Exercises that serve to guide instructors in their preparation of homework assignments and lessons.

Instructor's Solutions Manual

The *Instructor's Solutions Manual* provides comprehensive, worked-out solutions to all exercises in the Pre-Test, Section Exercises, Summary Exercises, Self-Test, and the Cumulative Review. The methods used to solve the problems in the manual are the same as those used to solve the examples in the textbook.

A commitment to accuracy

You have a right to expect an accurate textbook, and McGraw-Hill invests considerable time and effort to make sure that we deliver one. Listed below are the many steps we take to make sure this happens.

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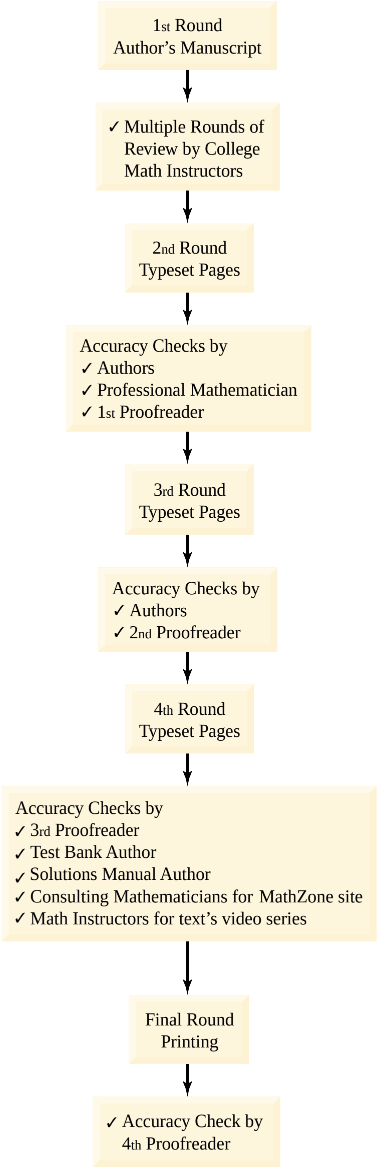
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✓ Test Bank Author
✓ Solutions Manual Author
✓ Consulting Mathematicians for MathZone site
✓ Math Instructors for text's video series
Final Round
Printing
✓ Accuracy Check by 4th Proofreader

First Round

Step 1: Numerous **college math instructors** review the manuscript and report on any errors that they may find. Then the authors make these corrections in their final manuscript.

Second Round

Step 2: Once the manuscript has been typeset, the **authors** check their manuscript against the first page proofs to ensure that all illustrations, graphs, examples, exercises, solutions, and answers have been correctly laid out on the pages, and that all notation is correctly used.

Step 3: An outside, **professional, mathematician** works through every example and exercise in the page proofs to verify the accuracy of the answers.

Step 4: A **proofreader** adds a triple layer of accuracy assurance in the first pages by hunting for errors, then a second, corrected round of page proofs is produced.

Third Round

Step 5: The **author team** reviews the second round of page proofs for two reasons: (1) to make certain that any previous corrections were properly made, and (2) to look for any errors they might have missed on the first round.

Step 6: A **second proofreader** is added to the project to examine the new round of page proofs to double check the author team's work and to lend a fresh, critical eye to the book before the third round of paging.

Fourth Round

Step 7: A **third proofreader** inspects the third round of page proofs to verify that all previous corrections have been properly made and that there are no new or remaining errors.

Step 8: Meanwhile, in partnership with **independent mathematicians,** the text accuracy is verified from a variety of fresh perspectives:

- The **test bank author** checks for consistency and accuracy as he/she prepares the computerized test item file.
- The **solutions manual author** works every exercise and verifies his/her answers, reporting any errors to the publisher.
- A **consulting group of mathematicians,** who write material for the text's MathZone site, notifies the publisher of any errors they encounter in the page proofs.
- A video production company employing **expert math instructors** for the text's videos will alert the publisher of any errors they might find in the page proofs.

Final Round

Step 9: The **project manager,** who has overseen the book from the beginning, performs a **fourth proofread** of the textbook during the printing process, providing a final accuracy review.

⇒ What results is a mathematics textbook that is as accurate and error-free as is humanly possible. Our authors and publishing staff are confident that our many layers of quality assurance have produced textbooks that are the leaders in the industry for their integrity and correctness.

brief contents

Preface v

Applications Index xxvii

Chapter 1
Operations on Whole Numbers 1

Chapter 2
Multiplying and Dividing Fractions 97

Chapter 3
Adding and Subtracting Fractions 181

Chapter 4
Decimals 247

Chapter 5
Ratios and Proportions 313

Chapter 6
Percents 365

Chapter 7
Measurement 423

Chapter 8
Geometry 481

Chapter 9
Data Analysis and Statistics 565

Chapter 10
The Real Number System 643

Chapter 11
An Introduction to Algebra 701

Answers to Prerequisite Tests, Reading Your Text, Summary Exercises, Self-Tests, and Cumulative Reviews A-1

Index I-1

contents

Preface v

Chapter 1

Operations on Whole Numbers

1.1 **The Decimal Place-Value System** 2
1.2 **Addition** 10
1.3 **Subtraction** 26
Activity 1 :: Population Changes 38
1.4 **Rounding, Estimation, and Order** 39
1.5 **Multiplication** 48
1.6 **Division** 65
Activity 2 :: Restaurant Management 78
1.7 **Exponential Notation and the Order of Operations** 79
Activity 3 :: Package Delivery 89
Chapter 1 :: Summary 90
Chapter 1 :: Summary Exercises 92
Chapter 1 :: Self-Test 95

Chapter 2

Multiplying and Dividing Fractions

Chapter 2 :: Prerequisite Test 98
2.1 **Prime Numbers and Divisibility** 99
2.2 **Factoring Whole Numbers** 108
2.3 **Fraction Basics** 117
2.4 **Simplifying Fractions** 128
Activity 4 :: Daily Reference Values 138
2.5 **Multiplying Fractions** 139
Activity 5 :: Overriding a Presidential Veto 152
2.6 **Dividing Fractions** 153
Activity 6 :: Adapting a Recipe 166
Chapter 2 :: Summary 167
Chapter 2 :: Summary Exercises 171
Chapter 2 :: Self-Test 175
Cumulative Review :: Chapters 1-2 177

Chapter 3

Adding and Subtracting Fractions

Chapter 3 :: Prerequisite Test 182
3.1 **Adding and Subtracting Fractions with Like Denominators** 183
3.2 **Common Multiples** 191
3.3 **Adding and Subtracting Fractions with Unlike Denominators** 200
Activity 7 :: Kitchen Subflooring 212
3.4 **Adding and Subtracting Mixed Numbers** 213
Activity 8 :: Sharing Costs 224
3.5 **Order of Operations with Fractions** 225
3.6 **Estimation Applications** 231
Activity 9 :: Aerobic Exercise 236
Chapter 3 :: Summary 237
Chapter 3 :: Summary Exercises 239
Chapter 3 :: Self-Test 242
Cumulative Review :: Chapters 1-3 244

Chapter 4

Decimals

Chapter 4 :: Prerequisite Test 248
4.1 **Place Value and Rounding** 249
4.2 **Converting Between Fractions and Decimals** 258
Activity 10 :: Terminate or Repeat? 267
4.3 **Adding and Subtracting Decimals** 268
4.4 **Multiplying Decimals** 281
Activity 11 :: Safe Dosages? 290
4.5 **Dividing Decimals** 291

Activity 12 :: The Tour de France 303

Chapter 4 :: Summary 304

Chapter 4 :: Summary Exercises 306

Chapter 4 :: Self-Test 309

Cumulative Review :: Chapters 1-4 311

Chapter 5

Ratios and Proportions

Chapter 5 :: Prerequisite Test 314

5.1 Ratios 315

Activity 13 :: Working with Ratios Visually 323

5.2 Rates and Unit Pricing 324

Activity 14 :: Baseball Statistics 334

5.3 Proportions 335

5.4 Solving Proportions 342

Activity 15 :: Burning Calories 355

Chapter 5 :: Summary 356

Chapter 5 :: Summary Exercises 358

Chapter 5 :: Self-Test 361

Cumulative Review :: Chapters 1-5 363

Chapter 6

Percents

Chapter 6 :: Prerequisite Test 366

6.1 Writing Percents as Fractions and Decimals 367

6.2 Writing Decimals and Fractions as Percents 377

Activity 16 :: M&M's 388

6.3 The Three Types of Percent Problems 389

Activity 17 :: A Matter of Interest 399

6.4 Applications of Percent Problems 400

Activity 18 :: Population Changes Revisited 413

Chapter 6 :: Summary 414

Chapter 6 :: Summary Exercises 416

Chapter 6 :: Self-Test 419

Cumulative Review :: Chapters 1-6 421

Chapter 7

Measurement

Chapter 7 :: Prerequisite Test 424

7.1 The U.S. Customary System of Measurement 425

7.2 Metric Units of Length 439

7.3 Metric Units of Weight and Volume 449

7.4 Converting Between the U.S. Customary and Metric Systems of Measurement 459

Activity 19 :: Tool Sizes 470

Chapter 7 :: Summary 471

Chapter 7 :: Summary Exercises 475

Chapter 7 :: Self-Test 478

Cumulative Review :: Chapters 1-7 479

Chapter 8

Geometry

Chapter 8 :: Prerequisite Test 482

8.1 Lines and Angles 483

Activity 20 :: Know the Angles 497

8.2 Perimeter and Area 498

8.3 Circles and Composite Figures 515

Activity 21 :: Exploring Circles 525

8.4 Triangles 526

Activity 22 :: Composite Geometric Figures 539

8.5 Square Roots and the Pythagorean Theorem 540

Activity 23 :: The Pythagorean Theorem 549

Chapter 8 :: Summary 550

Chapter 8 :: Summary Exercises 554
Chapter 8 :: Self-Test 558
Cumulative Review :: Chapters 1-8 561

Chapter 9

Data Analysis and Statistics

Chapter 9 :: Prerequisite Test 566
9.1 **Means, Medians, and Modes** 567
Activity 24 :: Car Color Preferences 582
9.2 **Tables, Pictographs, and Bar Graphs** 583
9.3 **Line Graphs and Predictions** 596
9.4 **Creating Bar Graphs and Pie Charts** 604
Activity 25 :: Graphing Car Color Data 615
9.5 **Describing and Summarizing Data Sets** 616
Activity 26 :: Outliers in Scientific Data 628
Chapter 9 :: Summary 629
Chapter 9 :: Summary Exercises 632
Chapter 9 :: Self-Test 637
Cumulative Review :: Chapters 1-9 641

Chapter 10

The Real Number System

Chapter 10 :: Prerequisite Test 644
10.1 **Real Numbers and Order** 645
10.2 **Adding Real Numbers** 654
Activity 27 :: Hometown Weather 663
10.3 **Subtracting Real Numbers** 664
Activity 28 :: Plus/Minus Ratings in Hockey 671
10.4 **Multiplying Real Numbers** 672
10.5 **Dividing Real Numbers and the Order of Operations** 682
Activity 29 :: Building Molecules 692
Chapter 10 :: Summary 694
Chapter 10 :: Summary Exercises 696
Chapter 10 :: Self-Test 698
Cumulative Review :: Chapters 1-10 699

Chapter 11

An Introduction to Algebra

Chapter 11 :: Prerequisite Test 702
11.1 **From Arithmetic to Algebra** 703
11.2 **Evaluating Algebraic Expressions** 711
Activity 30 :: Evaluating Net Pay 720
11.3 **Simplifying Algebraic Expressions** 721
Activity 31 :: Writing Equations 729
11.4 **Using the Addition Property to Solve an Equation** 730
Activity 32 :: Graphing Solutions 742
11.5 **Using the Multiplication Property to Solve an Equation** 743
11.6 **Combining the Properties to Solve Equations** 754
Chapter 11 :: Summary 764
Chapter 11 :: Summary Exercises 766
Chapter 11 :: Self-Test 769
Cumulative Review :: Chapters 1-11 770
Answers to Prerequisite Tests, Reading Your Text, Summary Exercises, Self-Tests, and Cumulative Reviews A-1
Index I-1

applications index

Business and finance
  advertising costs increase, 418
  advertising expenditures and sales, 602–603
  airline passengers, 92
  alternator sales, 753
  annual budget spent, 375
  apparel company revenues, 578
  art exhibit ticket sales, 739
  assets and debts, 643
  bank branch customers, 23
  bankruptcy filings, 639
  benefits package cost, 641, 771
  break-even point, 769
  budget categories, 611
  budgeting income, 207, 208
  budget remaining, 661, 667
  business trip expenses, 30, 276, 385
  car loans
    interest, 229
    payments, 283
    principal, 420
  car price increase, 417
  car sales, 86, 404, 591, 612
  car shipments, 61
  charity drive donations, 300, 308
  checking account balance, 29, 33, 34, 273, 279, 660, 689
  checking account running balance, 273, 277, 278
  checking account withdrawal, 651
  checks written, 241
  check writing with words, 8
  coin circumferences, 524
  commission
    amount earned, 395, 401–402, 417, 751
    rate of, 402, 408
    sales needed for, 402, 403, 420, 748, 771
  compact cars sold, 368
  company budget categories, 610
  copy machine lease, 289
  cost per item, 293, 297, 300, 308
  credit card balance, 93, 363
  credit card bill, 280
  daily pay increase, 86
  deposit amounts, 276
  dimes as fractions, 134
  discount rate, 403, 417, 420
  Dow-Jones average increase, 651
  dress shop expenses, 21
  earnings
    and education level, 610
    on investment, 345, 350
    monthly, 762
    partial, 173
  employee gender ratio, 319
  employee hiring increase, 405
  employees before layoffs, 422
  exchange rate, 749, 752
  factory payroll, 63
  gasoline sales, 35
  gross pay, 149
  home sales prices, 568, 571, 575, 633
  hotel costs, 224
  hourly pay rate, 294, 312, 345, 353, 422
  hours worked, 144, 188, 221, 223, 246, 435, 475, 762
  income tax, 287, 401
  interest
    earned, 396, 399, 405, 406–407, 418
    on loan, 229
    paid, 391, 405, 408
    simple, 287, 289, 308, 708, 718
  inventory remaining, 727
  inventory sold, 727
  investment amount, 391
  investment earnings, 345, 350
  investment profits, 691
  investment value decrease, 411
  land price per acre, 162
  laser printer purchase, 36
  loans
    balance, 179
    down payment, 408
    interest, 229
    interest rate, 408, 419
    payments, 283, 300, 479
    principal, 406, 420, 748
  markup rate, 403
  McDonald's employees, 78
  monthly deposits, 270
  monthly earnings, 762
  monthly expense account, 34
  monthly loan payments, 300
  mortgage payments, 602
  net pay, 720
  newspaper ad cost, 55
  new staff members, 136
  office purchase order, 288
  overdrawn account, 279
  overtime hours, 243
  package deliveries, 89
  packages total weight, 435
  partial earnings, 173
  pay
    daily increase, 86
    gross, 149
    hourly rate, 294, 312, 345, 353, 422
    hourly wages, typical, 571
    net, 720
    take-home, 33
    weekly, 287, 289, 308, 706
    withheld, 751
  paycheck deductions, 208
  paycheck withholding, 33
  personal computer sales, 634
  phone calls made, 75
  pizzas made in three days, 45
  pocket money division, 77
  positive trade balance, 651
  price per item, 292–293
  produce market delivery, 724
  products sold at a loss, 681
  profit, 726, 740, 741
  project over budget, 648
  property taxes, 350
  quarters as fractions, 134
  quarters to dollars conversion, 122, 126
  rental deposit, 8
  rental fleet cost, 51
  revenue loss, 689
  road inspection, 188
  salaries
    after deductions, 395, 411, 418
    after raise, 411, 418
    before raise, 409
    of camp counselors, 92
    deductions from, 420
    division of, 165
    of group members, 576
    of university graduates, 568, 570
  sales
    of alternators, 753
    of art exhibit tickets, 739
    of bolts, 741
    of carriage bolts, 710
    of cars, 86, 404, 591, 612
    of compact cars, 368
    of gasoline, 35
    of inventory, 727
    at a loss, 681
    of personal computers, 634
    of SUVs, 592
    of tickets, 639, 739
    of used cars, 125
    weekly, 632
  savings account deposit, 651
  shipping methods, 640
  simple interest, 287, 289, 308, 708, 718
  splitting tips, 66
  spreadsheets, 701
  steel mill work schedule, 116
  stock
    closing prices, 619, 624
    price drop, 220
    purchasing, 365
    share prices, 159, 331
    by type, 642
    value decrease, 409, 651
  store profit, 279
  store purchase order, 288
  SUV sales, 592
  take-home pay, 33
  television set cost, 75
  television set financing, 308
  ticket sales, 639, 739
  time for tasks, 188
  tipping percentage, 401, 412
  total cost of items, 308, 310
  truck fleet cost, 96
  trust fund income, 601–602
  typical hourly wages, 571
  U.S. trade with Mexico, 410
  used cars sold, 125
  vendor's weekend profits, 658
  video rentals by categories, 21

Business and finance—*Cont.*
  weekly pay, 287, 289, 308, 706
  weekly sales, 632
  women-owned firms, 22
  work stoppages, 579

Construction and home improvement
  adhesive temperature, 468
  barn stall width, 447
  baseboard length, 513
  bedroom length, 351
  bedroom perimeter, 18
  board remaining, 240, 421
  board shrinkage, 375
  bolt diameters, 197
  bolt length, 220, 246, 771
  bolt sales, 741
  books per shelf, 163, 176
  brick garden path, 65
  brick laying pace, 331
  bridge construction schedule, 649
  building heights, 33
  carpentry, 181
  carpeting
    cost of, 57, 94, 173, 178, 180, 363, 479, 561, 642
    coverage, 173, 506, 512, 518
    remaining, 221
    selection, 77
  carriage bolts sold, 710
  castle wall, 547
  concrete remaining, 204
  construction company schedule, 228
  countertop thickness, 207
  deck board weight, 458
  deck railing length, 435
  desktop area, 56
  dowel diameters, 197
  elementary school site area, 507
  factory floor thickness, 222
  fence post shadow, 351
  fencing
    cost of, 478
    length of, 276
    needed, 17, 270, 276, 512
  floor molding length, 475
  heat from furnace, 763
  hook installation, 210
  house addition area, 522
  jobsite elevations, 652, 669
  kennel dimensions, 64
  kitchen subflooring, 211
  knee wall studs, 437
  laminate covering, 57
  land
    for home lots, 150, 174, 176, 310, 771
    for lots, 294, 308, 312
    for office complex, 203
  light pole shadow, 351
  linoleum cost, 173
  linoleum coverage, 176, 507
  lot area, 512, 513
  lumber costs, 353
  lumber needed, 435, 699
  metal fitting length, 276
  metal remaining, 205
  molding length needed, 517
  mortar needed, 457
  Norman windows, 481
  paint coverage, 360, 512, 562
  paint used, 241
  parking lot capacity, 61, 74
  pen dimensions, 63
  pipe needed, 435
  plank length remaining, 447
  plywood sheets in a stack, 180
  plywood thickness, 197, 241
  primer temperature, 468
  roofing cost, 555
  roofing nails needed, 234
  room area, 56
  room dimension ratio, 319
  room volume, 58
  shelves from lumber, 218
  shelving length needed, 240
  sidewalk concrete, 147, 190
  stair riser height, 164
  street improvement cost, 310
  tabletop refinishing cost, 522
  terrace paving cost, 522
  threaded bolts, 164
  tool sizes, 470
  tracts of land, 158–159
  tubing thickness, 277
  two-by-four prices, 349
  vinyl flooring coverage, 421, 555
  wallpaper coverage, 335
  wallpaper remaining, 240
  wall studs used, 763
  wall thickness, 220
  window area, 145
  windows in building, 49
  wire lengths, 162, 174
  wood remaining, 205
  wrought-iron gate material, 519

Consumer concerns
  airplane load limit, 96
  apple prices, 176
  art exhibit ticket sales, 739
  beef purchases, 146
  book prices, 349
  butterfat in milk, 397, 412
  cake mix purchase, 234
  candy bars per box, 75
  candy purchase, 185
  car color preferences, 582, 615
  car cost after rebate, 93
  car loans
    interest, 229
    payments, 72, 283, 287
    principal, 420
  car price increase, 417
  car purchase balance, 178
  car purchase prices, 21
  cash remaining, 771
  cat food purchase, 233
  circus attendance, 592
  clothes purchases, 45
  concert ticket prices, 567–568
  cost per CD, 300
  credit card balance, 93, 363
  credit card bill, 280
  credit card debt, 277
  dinner bill total, 42
  discount prices, 372, 373
  dog food purchase, 233
  dog food unit price, 327
  dryer price, 736
  electronics sale prices, 241
  food items cost, 42
  frequent flyer miles, 34
  fuel oil cost, 289
  fund drive progress, 409
  grocery bill change, 272
  hamburgers for lunch, 125
  hardware store items cost, 45
  health club membership, 55
  heating bills, 577
  household energy use, 612
  installment plan payments, 283, 422
  loans
    balance, 179
    down payment, 408
    interest, 229
    interest rate, 408, 419
    payments, 283, 300, 479
    principal, 406, 420, 748
    total paid, 308
  luggage weight restrictions, 435
  lunch bill total, 45
  lunch meat budget, 235
  lunch meat purchases, 146
  mailing label printer, 61
  milk price comparison, 327
  M&M color ratio, 322
  M&M colors per package, 388
  money remaining, 28, 307, 310
  movie screen size, 316
  movie theater capacity, 62
  newspaper column dimensions, 318
  newspaper deliveries, 61
  nuts purchase, 207
  oranges price comparison, 328
  oregano prices, 457
  peanut can sizes, 435
  pizza cost, 524
  pocket money division, 77
  postage rates, 598–599
  price after markup, 409, 411
  price before discount, 408
  price before increase, 408
  price before tax, 396, 408, 419
  printer pages per second, 331
  propane cost, 283
  refrigerator capacities, 319
  rental car cost, 287
  restaurant bill, 278
  roast purchase, 287
  sales price after markdown, 411, 412, 417

  sales price after markup, 408
  sales tax paid, 403, 408, 419
  sales tax rate, 402, 412, 422
  saving for computer system, 741
  shirts purchase, 286
  soap prices, 302
  stationery items cost, 45
  stereo equipment cost, 30
  stereo system sales price, 307
  tea bag prices, 349
  television sales price, 277
  television set cost, 75
  television set financing, 308
  theater ticket prices, 360
  trip expenses, 96
  TV ownership, 608
  typical phone bills, 572
  unit prices, 327, 330–331, 361, 364, 436, 478, 699
  utility bills, 580, 601
  video rentals by categories, 21
  walnuts price, 457

Crafts and hobbies
  bones for costume, 739
  drill bit sizes, 197
  estimating recipes, 232
  fabric
    for blouses, 174
    for cloth strips, 164
    for dress, 331
    needed, 220, 241
    remaining, 204, 217, 435
  film developer concentration, 346
  film developer remaining, 435
  film processed, 748–749
  frame molding needed, 435
  French toast batter, 149
  guitar string vibrations, 198
  hamburger remaining, 435
  ingredients totals, 242
  milk remaining, 240
  notepaper area, 145
  nutritional facts labels, 593
  oranges for juice, 164
  paper area, 282, 287
  paper remaining, 220
  peanuts in mixed nuts, 395
  personal chef, 97
  photocopy reduction, 771
  photograph enlargement, 350, 360
  picture album capacity, 75
  picture perimeter, 188
  pitcher capacity, 235
  pizzas made in three days, 45
  plastic for kite, 512
  poster dimensions, 217–218
  pottery clay, 162
  produce market delivery, 724
  recipe adaptation, 166
  roast servings, 163
  roast weight, 220
  rug binding, 522
  soup servings, 457
  spices weight, 203
  string lengths, 164
  sugar for recipe, 149

Education
  age and college education, 639
  biology students, 173
  campus parking spaces, 331
  correct test answers, 125, 134, 419
  dividing students into groups, 106
  enrollment
    in algebra class, 15
    decrease in, 405, 412, 562
    in elementary school, 576
    increase in, 396, 404, 408, 412, 420, 642
    by year, 634
  exam finishing times, 318
  exam questions answered correctly, 400, 751
  exam scores, 575, 581, 636, 679
  first-year and second-year students, 360
  foreign language students, 408
  income levels and education, 602
  injuries during class, 640
  math class final grades, 581
  mathematics students, 62, 66
  questions on test, 408
  quiz scores, 577
  scholarship money spent, 606
  school day activities, 607
  school election votes ratio, 319
  science students, 418
  student average age, 587
  student gender ratio, 319, 350
  students passing class, 400
  students passing exam, 367
  students per major, 586
  students receiving As, 391, 395
  students with jobs, 173
  technology in public schools, 638
  test scores, 33, 45, 577, 632, 637
  test time remaining, 242
  textbook costs, 270, 568, 632
  training program dropout rate, 751
  transportation to school, 605–606
  women students, 118

Electronics
  appliance power consumption, 46, 577
  battery voltage, 279–280, 653, 662
  cable run, 36, 222, 229
  cable unit price, 332
  cable weight, 345
  cell phone repairs, 320
  circuit board cost, 76
  circuit board soldering, 63, 230
  components ordered, 127
  conductive trace, 547
  conductor resistance, 332
  cooling fan capacity, 669
  current rate of change, 715
  electric motor efficiency, 386
  power consumption, 9
  printed circuit board area, 448, 513
  printed circuit board population, 230
  resistance values, 653
  resistor replacement, 210
  resistors purchase, 63, 76
  solder spool, 36
  stereo system savings, 272
  transformer ratios, 321
  voltage output, 354
  voltmeter readings, 496
  wire gauges, 593
  wire lengths, 157

Environment
  amphibian sightings, 572
  Douglas fir cross sections, 524
  emissions
    carbon dioxide, 409
    carbon monoxide, 457
    nitrous oxide, 457
    suspended particulates, 435
    volatile organic compounds, 435
  green space requirements, 397, 412
  precipitation levels, 640
  rainfall, 276, 280, 375, 387
  species remaining, 708
  temperatures
    drop in, 689
    high, 576, 597
    low, 660
    mean, 624–625
    at noon, 660
    range of, 666, 667
  topsoil erosion, 652
  topsoil formation, 652
  tree height, 529–530, 557
  tree shadow length, 347, 537
  weather data, 663
  weather in Philadelphia, 576

Farming and gardening
  area of field, 506
  area of garden, 512
  barley harvest, 753
  corn field yield, 762
  cow distribution, 164
  crops total value, 34
  fertilizer coverage, 72, 331, 341, 522
  fertilizer weight, 458
  harvest by crop, 613
  mulch coverage, 150, 159
  soil temperature, 468
  soil testing, 321
  soybean prices, 333
  tomato planting, 74
  total wheat production, 571
  vegetable garden area, 145
  walking length, 270

Games
  black and white markers ratio, 323
  checker board squares, 49
  gambling losses, 679
  marbles in a bag, 46
  poker hands, 23
  poker winnings, 679
  separating card deck, 75

Geography
  area of Pacific coast states, 35
  elevation changes, 33
  height above trench, 666
  land area, 583–584, 589–590, 591
  length of Nile River, 7
  map distances, 64, 147, 173, 176, 345–346, 350, 351, 360, 364, 422

Geometry
  area
    of circle, 718, 771
    of oddly shaped figure, 57
    of rectangle, 289, 310, 364, 700
    of square, 700
    of triangle, 718
  circumference of circle, 700
  length of square sides, 312
  missing dimensions, 209, 220, 221, 277, 307
  perimeter
    of figure, 276
    of rectangle, 17–18, 188, 307, 311, 364, 642, 718, 726
    of square, 642
    of triangle, 207, 240, 726
  polygon angles, 497
  volume of box, 150

Health and medicine
  alcohol solution preparation, 397, 411
  alveolar minute ventilation, 62
  amniotic fluid levels, 524
  anion gap, 35
  antibiotic prices, 319
  blood alcohol content, 302
  blood concentration, 719
  blood pumped by heart, 437
  blood serum dilution, 126
  body mass index, 727
  body temperature decrease, 648
  breaths in a lifetime, 437
  burning calories, 355
  calories per day, 33
  cardiac index, 301
  cardiac output, 452
  chemotherapy treatment, 190, 209
  clinic patients treated, 752
  coffee consumption, 243
  daily reference values, 138, 373
  endotracheal tube diameter, 266, 763
  eye color, 573, 576
  fetal head circumference, 523
  gestational diabetes, 573–574
  hair color, 637
  health care spending, 598
  heart rate, 437
    maximal, 236
    upper limit, 762
  height, change in, 217, 240, 272
  height of adult female, 468
  height of female, 575
  humidity deficit, 279
  intravenous solution flow rate, 76
  intravenous solution schedule, 649
  intravenous solution volume, 62
  length of newborn, 468
  live births by race, 612
  lung capacity, 24
  medication dilution, 457–458
  medication dosage
    amifostine, 8
    bexarotene, 76
    calculating, 709
    Cerezyme, 229
    chemotherapy, 290
    children's, 190, 209, 226–227
    Dilantin, 256
    laxative, 136
    penicillin, 8–9
    Pepto-Bismol, 136
    Reglan, 256
    sodium pertechnetate, 279
  medication rates, 341
  oxygen intake per day, 438
  radiographic imaging, 24, 35
  radiopharmaceutical specific concentration, 301
  specimen dilution ratio, 320, 347
  stroke volume, 266
  tidal volume, 332
  weight conversion, 467
  weight loss over time, 603
  weight of newborn, 437

Information technology
  computer costs, 62
  computer lab cable, 76
  computer scanning time, 401
  computer screen size, 316
  computer transmission speed, 341, 354
  database field size, 661
  data transmission rate, 302
  disk drive assembly rate, 360
  Ethernet packet overhead, 397, 411
  file compression, 753
  floppy disk shipment, 724
  floppy disk storage, 302
  hard drive capacity, 370, 381, 753
  help desk volume, 709
  indexing computer files, 401
  Internet connection speeds, 321
  lab computers by brand, 573
  laser printer purchase, 36
  mailing label printer, 61
  mixing music tracks, 198
  modem transmission speed, 76
  network cable cost, 62
  network transmission rates, 210
  personal computer sales, 634
  ping readings, 257, 266
  ping response times, 579–580
  printer lines per minute, 75
  printer pages per second, 331
  printers shipment, 75
  servers installed, 127
  server traffic, 136
  technology in public schools, 638
  virus scan duration, 409, 418
  wireless network speeds, 322

Manufacturing
  aluminum sealer remaining, 431
  aluminum weight, 468
  area of stock, 513
  automobile countries of origin, 607–608
  belt length, 227
  bottle diameters, 317
  bottle filling machine, 289
  bottle quality control, 234
  chipping hammer angles, 537
  computer-aided design drawing, 279
  copper-nickel alloy temperature, 468
  crate volume, 58
  cutting material, 151, 229, 513
  defective parts
    number expected, 345, 350, 353, 360, 364
    percentage, 368, 391, 396, 751
    ratio of, 320
  defective product returns, 387
  delivery truck loading, 46
  distance between holes, 63, 151, 163, 229
  drawing of part, 164
  drill sizes, 257
  engine block profit, 36
  factory payroll, 63
  faulty replacement parts, 118
  hole in sheet metal, 512
  liquid petroleum supply, 76
  machine screws inventory, 46
  machines still in operation, 375
  materials cost, 151
  mill bit sizes, 257
  motor vehicle production, 591–592, 632–633, 634–636
  MP3 player assembly, 349
  muffler installation rate, 361
  nonstick coating coverage, 524
  O-ring area, 519–520
  packaging machines in operation, 127, 136
  packing boxes, 199
  parts dimensions, 210
  parts thickness, 210, 580

- parts weight, 76, 437
- pencil quality control, 302
- production order fulfillment, 23
- productivity losses, 676
- products in specific color, 375
- pulley belt length, 87
- rate of steel cut, 432
- round stock needed, 222
- sealer coverage, 513
- sealer remaining, 458
- shaft key dimensions, 524
- shipping box choices, 234
- sled manufacturing, 62
- steel in stock, 431
- steel inventories, 669
- steel mill work schedule, 116
- steel plate weight, 513
- steel price per pound, 337
- steel remaining, 443
- steel round stock inventory, 23
- steel shearing, 436
- steel shipment, 190
- steel stock density, 337
- steel stock remaining, 447, 458, 468
- tin can production rate, 346
- tire patch area, 513
- truss height, 537
- tubing weight, 360
- water pipe length, 158
- weight removed with holes, 354
- weld tensile strength, 386, 580
- wire cut in pieces, 444
- wood-burning stove manufacturing, 61
- worker absence and defects, 639

Measurement
- fractional hours, 134
- fractional kilometers, 134
- fractional meters, 134
- grams to kilograms conversion, 288
- half-gallons to gallons conversion, 122, 126
- kilograms to grams conversion, 285
- kilometers to meters conversion, 284
- meters to centimeters conversion, 288
- metric system conversion, 423
- miles per hour to feet per second conversion, 431
- millimeters to meters conversion, 296
- pounds to kilograms conversion, 282
- width as fraction of length, 136

Motion and transportation
- airline passengers, 92
- airplane fuel efficiency, 467
- airplane load limit, 96
- automobiles in total travel, 372
- bus passengers, 637
- car air filter dimensions, 513
- carpooling, 351
- car thermostat, 464
- departure time, 234
- distance
  - driven, 146, 173, 350, 364, 576
  - flown, 150
  - run, 202, 276, 307, 522, 561
  - walked, 185, 207
- driving hours remaining, 229, 246, 435
- driving time, 234, 346, 436
- fuel efficiency, 385, 386, 436
  - airplane, 467
  - kilometers per gallon, 462–463
- gas mileage, 72, 94, 145, 149, 272, 295, 302, 308, 331, 350, 361
- gas mileage by model, 613
- gasoline consumption, 409, 642
- gasoline prices, 580, 590
- gasoline purchased, 283, 310
- gasoline remaining in tank, 232
- gasoline sales, 35
- gasoline usage, 269, 275, 759
- gas tank capacities, 319
- highway exits, 198
- highway mileage ratings, 576
- motor oil used, 351
- petroleum consumption, 409
- single commuters, 351
- speed, 437
  - of airplane, 72
  - average, 157, 162, 174, 364, 421, 561, 642
  - driving, 331
  - gas mileage and, 603
  - of greyhound, 436
  - running, 229
  - of train, 75
- speedometer reading, 467
- tire revolutions, 524
- tire tread wear, 354
- trucks in total travel, 372
- vehicle registrations, 409

Politics and public policy
- city council election, 15
- petition signatures, 61
- presidential veto override, 152
- school board election, 736
- Senate members with military service, 611
- state wildlife preserve, 145
- vote ratio, 349
- votes received, 759, 762

Science and engineering
- acid in solution, 408, 417, 751
- bending moment, 681
- buoy research data, 565, 628
- cast iron melting point, 464
- compressive stress of steel, 513
- covalent bonding, 692–693
- crankshaft turns, 496
- cylinder stroke length, 706, 741
- electrical power, 710
- engine block weight, 437
- engine cylinders firing, 381
- engine displacement, 458
- engine oil level, 661
- focal length, 715
- fouled spark plugs, 134
- gear pitches, 332, 341
- gear ratio, 320
- kinetic energy of objects, 87
- kinetic energy of particle, 708, 719
- landfill contents, 229
- LP gas used, 691
- metal bar expansion, 351
- metal densities, 594, 613
- metal melting points, 594, 613
- metal shrinkage, 375
- moment of inertia, 727
- motor efficiency, 370
- motor rpms, 669
- nuclear energy reliance, 374
- origin of universe, 7
- pipe flow rate, 436
- piston weight, 437
- planetary alignment, 116
- plant food concentrate, 361
- pneumatic actuator pressure, 661
- power of circuit, 87
- primary beam load, 727
- radio wave speed, 448
- resistors total resistance, 87
- satellite altitude, 448
- satellite propagation delay, 266
- shadow length, 347, 530, 537
- speed of sound, 353
- stress calculation, 333
- temperature conversion, 718
- temperature decrease, 651
- temperature protection, 594
- test tube capacities, 689
- thermometer reading, 232
- trailer load limit, 463, 468
- volume of oil, 468
- water pump gallons per hour, 331
- weight of oil, 458, 478
- weight of water, 287

Social sciences and demographics
- age and college education, 639
- auditorium seat rows, 106
- car registrations, 585
- chairs for concert, 65
- daily activities, 207
  - inverses, 77
- education and income levels, 602
- foreign-born U.S. residents, 611
- grouping words, 63
- larceny theft cases, 599
- left-handed people, 408
- morning activities, 63
- national debt payment, 127
- play attendance, 178
- population
  - by age, 127, 610
  - changes in, 413

- Social sciences and demographics—*Cont.*
  - decrease in, 404, 651
  - female, 127
  - increase in, 409, 411
  - state, 127, 413
  - urban, 604–605
  - of U.S., 47, 413, 638
  - of U.S. cities, 7–8, 605
  - of U.S. states, 36, 38
  - world, 47, 583–584, 589–590, 591, 708
- robberies per month, 601
- school lockers opened, 107
- Social Security beneficiaries, 597–598
- survey respondents, 408
- tax return rounding, 46
- unemployment in U.S., 579
- unemployment rate, 408
- workers employed as farm workers, 585
- zero, in Hindu-Arabic number system, 9

Sports
- baseball
  - batting averages, 134, 264, 312, 331, 334
  - distance from home to second base, 547
  - earned run average, 334
  - losing streak, 651
  - strikeout per inning rate, 326
  - time in batting cage, 188
  - winning percentages, 264, 310
- basketball
  - free throws, 264–265
  - game attendance, 96
  - game receipts, 62
  - team starters, 125
  - wins to losses ratio, 316
- bicycling for fundraising, 247
- football
  - final score, 660
  - Steelers winning seasons, 596
  - winning percentages, 264
  - wins to losses ratio, 319
- goals per game rate, 326
- golf putting distances, 317
- hockey plus/minus statistics, 671
- personal trainers, 313
- points per game rate, 326
- scuba diving time, 623
- softball final score, 658
- spectator sport popularity, 586
- Tour de France, 303
- track and field
  - area of track, 519
  - distance run, 202, 276, 307, 522, 561
  - jogging distances, 318
  - length of track, 519
  - running speed, 229
- U.S. Open winnings, 7

chapter
1
> Make the Connection

CHAPTER 1

Operations on Whole Numbers

INTRODUCTION

To supplement her income while attending college, Nadia answered an ad in a local paper to become a census worker. She got the job at a census bureau in her community. Because she was juggling school, studying, and raising a 4-year-old daughter, she was able to work only part-time. Nadia thought this would be a good way to earn some extra money.

Eventually, Nadia began to realize how important her work was. She spent time studying data pertaining to people and businesses in her county. She saw that by analyzing facts such as age, education, income, and family size, she could formulate certain ideas about her environment. She began working with city planners to help make projections pertaining to population growth. This enabled them to sketch out a blueprint to accommodate new housing, office buildings, roads, and a new school, based on certain predictions. Until this new job, Nadia never realized the impact math has on the lives of ordinary people. She decided to change her major to something that would allow her to continue her work with strategic planning. She also realized that she would need to take more mathematics and statistics classes to continue on this track.

To learn more about the U.S. Census Bureau data, do Activity 1 on page 38.

CHAPTER 1 OUTLINE

1.1 The Decimal Place-Value System 2
1.2 Addition 10
1.3 Subtraction 26
1.4 Rounding, Estimation, and Order 39
1.5 Multiplication 48
1.6 Division 65
1.7 Exponential Notation and the Order of Operations 79
Chapter 1 :: Summary / Summary Exercises / Self-Test 90

1.1

The Decimal Place-Value System

< **1.1 Objectives** >

1 > Write numbers in expanded form
2 > Determine the place value of a digit
3 > Write a number in words
4 > Given its word name, write a number

Tips for Student Success

Throughout this text, we present you with a series of class-tested techniques that are designed to improve your performance in this math class.

Hint #1 Become familiar with your textbook.

Perform each of the following tasks.

1. Use the Table of Contents to find the title of Section 5.1.
2. Use the Index to find the earliest reference to the term *mean*. (By the way, this term has nothing to do with the personality of either your instructor or the textbook authors!)
3. Find the answer to the first Check Yourself exercise in Section 1.1.
4. Find the answers to the odd-numbered exercises in Section 1.1.
5. In the margin notes for Section 1.1, find the origin of the term *digit*.

Now you know where some of the most important features of the text are. When you have a moment of confusion, think about using one of these features to help you clear up that confusion. You should also find out whether a solutions manual is available for your text. Many students find these to be helpful.

Number systems have been developed throughout human history. Starting with simple tally systems used to count and keep track of possessions, more and more complex systems were developed. The Egyptians used a set of picturelike symbols called **hieroglyphics** to represent numbers. The Romans and Greeks had their own systems of numeration. We see the Roman system today in the form of Roman numerals. Some examples of these systems are shown in the following table.

Numerals	Egyptian	Greek	Roman
1	I	I	I
10	∩	Δ	X
100	𓍢	H	C

Any number system provides a way of naming numbers. The system we use is described as a **decimal place-value system.** This system is based on the number 10 and uses symbols called **digits.** (Other numbers have also been used as bases. The Mayans used 20, and the Babylonians used 60.)

The basic symbols of our system are the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

These basic symbols, or digits, were first used in India and then adopted by the Arabs. For this reason, our system is called the Hindu-Arabic numeration system.

Numbers may consist of one or more *digits*.

NOTES

The prefix *deci* means 10. Our word *digit* comes from the Latin word *digitus,* which means finger.

Any number, no matter how large, can be represented using the 10 digits of our system.

The numbers 3, 45, 567, and 2,359 are examples of the **standard form** for numbers. We say that 45 is a two-digit number, 567 is a three-digit number, and so on.

As we said, our decimal system uses a *place-value* concept based on the number 10. Understanding how this system works will help you see the reasons for the rules and methods of arithmetic that we will be introducing.

Example 1 Writing a Number in Expanded Form

< Objective 1 >

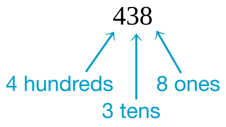
NOTES

Each digit in a number has its own place value.

Here the parentheses are used for emphasis. (4×100) means 4 is multiplied by 100. (3×10) means 3 is multiplied by 10. (8×1) means 8 is multiplied by 1.

Look at the number 438.

We call 8 the *ones digit*. As we move to the left, the digit 3 is the *tens digit*. Again as we move to the left, 4 is the *hundreds digit*.


438
4 hundreds
8 ones
3 tens

If we rewrite a number such that each digit is written with its units, we have used the **expanded form** for the number.

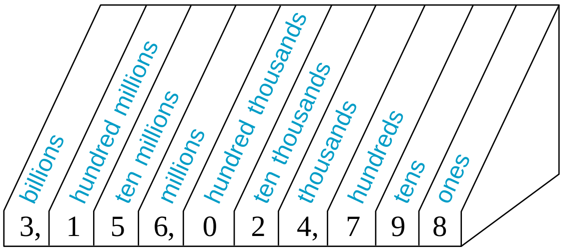
In expanded form, we write 438 as

$$400 + 30 + 8 \quad \text{or} \quad (4 \times 100) + (3 \times 10) + (8 \times 1)$$

Check Yourself 1

Write 593 in expanded form.

The following place-value diagram shows the place value of digits as we write larger numbers. For the number 3,156,024,798, we have


billions
hundred millions
ten millions
millions
hundred thousands
ten thousands
thousands
hundreds
tens
ones
3,
1
5
6,
0
2
4,
7
9
8

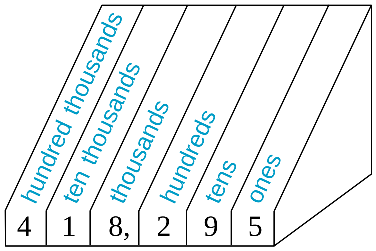
Of course, the naming of place values continues for larger numbers beyond the chart.

For the number 3,156,024,798, the place value of the digit 4 is thousands. As we move to the left, each place value is 10 times the value of the previous place. The place value of 2 is ten thousands, the place value of 0 is hundred thousands, and so on.

Example 2 Identifying Place Value

< Objective 2 >

Identify the place value of each digit in the number 418,295.


hundred thousands
ten thousands
thousands
hundreds
tens
ones
4
1
8,
2
9
5

Check Yourself 2

Use a place-value diagram to answer the following questions for the number 6,831,425,097.

(a) **What is the place value of 2?**
(b) **What is the place value of 4?**
(c) **What is the place value of 3?**
(d) **What is the place value of 6?**

NOTE

A four-digit number, such as 3,456, can be written with or without a comma. We have chosen to write them with a comma in this text.

Understanding place value will help you read or write numbers in word form. Look at the number

$$\underbrace{7\ \ 2}_{\text{Millions}},\ \ \underbrace{3\ \ 5\ \ 8}_{\text{Thousands}},\ \ \underbrace{6\ \ 9\ \ 4}_{\text{Ones}}$$

Commas are used to set off groups of three digits in the number. The name of each group—millions, thousands, ones, and so on—is then used as we write the number in words. To write a word name for a number, we work from left to right, writing the numbers in each group, followed by the group name. The following chart summarizes the group names.

Billions Group			Millions Group			Thousands Group			Ones Group		
Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones

Example 3 Writing Numbers in Words

< Objective 3 >

Write the word name for each of the following.

27,345 is written in words as twenty-seven *thousand*, three hundred forty-five.

2,305,273 is two *million*, three hundred five *thousand*, two hundred seventy-three.

Note: We do *not* write the name of the ones group. Also, the word *and* is not used when a number is written in words. It will have a special meaning later.

NOTE

The commas in the word statements are in the same place as the commas in the number.

Check Yourself 3

Write the word name for each of the following numbers.

(a) 658,942 **(b)** 2,305

We reverse the process to write the standard form for numbers given in word form. Consider the following.

Example 4 Translating Words into Numbers

< Objective 4 >

Forty-eight thousand, five hundred seventy-nine in standard form is

48,579

Five hundred three thousand, two hundred thirty-eight in standard form is

503,238

Note the use of 0 as a placeholder in writing the number.

Check Yourself 4

Write twenty-three thousand, seven hundred nine in standard form.

Check Yourself ANSWERS

1. (5 × 100) + (9 × 10) + (3 × 1) **2.** **(a)** Ten thousands; **(b)** hundred thousands; **(c)** ten millions; **(d)** billions **3.** **(a)** Six hundred fifty-eight thousand, nine hundred forty-two; **(b)** two thousand, three hundred five **4.** 23,709

Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 1.1

(a) The system we use for naming numbers is described as a ____________ place-value system.

(b) We say the number 567 is a three-________________ number.

(c) A four-digit number can be written with or without a ________________.

(d) In words, the number 2,000,000 is written as two ________________.

1.1 exercises

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Name ____________________

Section ________ Date ________

Answers

1. ____________________
2. ____________________
3. ____________________
4. ____________________
5. ____________________
6. ____________________
7. ____________________
8. ____________________
9. __________ 10. __________
11. __________ 12. __________
13. ____________________
14. ____________________
15. ____________________
16. ____________________
17. ____________________
18. ____________________
19. ____________________
20. ____________________

< Objective 1 >

Write each number in expanded form.

1. 456

2. 637

3. 5,073 > Videos

4. 20,721

< Objective 2 >

Give the place values for the indicated digits.

5. 4 in the number 416

6. 3 in the number 38,615

7. 6 in the number 56,489

8. 4 in the number 427,083

9. In the number 43,729

(a) what digit tells the number of thousands?
(b) what digit tells the number of tens?

10. In the number 456,719

(a) what digit tells the number of ten thousands?
(b) what digit tells the number of hundreds?

11. In the number 1,403,602

(a) what digit tells the number of hundred thousands?
(b) what digit tells the number of ones? > Videos

12. In the number 324,678,903

(a) what digit tells the number of millions?
(b) what digit tells the number of ten thousands?

< Objective 3 >

Write the word name of each number.

13. 5,618 > Videos

14. 21,812

15. 200,304 > Videos

16. 103,900

< Objective 4 >

Write each number in standard form.

17. Two hundred fifty-three thousand, four hundred eighty-three

18. Three hundred fifty thousand, three hundred fifty-nine

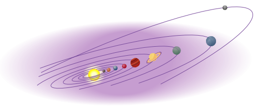
19. Five hundred two million, seventy-eight thousand > Videos

20. Four billion, two hundred thirty million

Write the whole number in each sentence in standard form.

21. **STATISTICS** The first-place finisher in the 2004 U.S. Open won one million one hundred twenty-five thousand dollars.

22. **SCIENCE AND MEDICINE** Scientific speculation is that the universe originated in the explosion of a primordial fireball approximately fourteen billion years ago.



23. **SOCIAL SCIENCE** The population of Kansas City, Missouri, in 2000 was approximately four hundred forty-one thousand, five hundred.

24. **SOCIAL SCIENCE** The Nile river in Egypt is about four thousand, one hundred forty-five miles long.

Basic Skills | **Challenge Yourself** | Calculator/Computer | Career Applications | Above and Beyond

Sometimes numbers found in charts and tables are abbreviated. The following table represents the population of the 10 largest cities in the 2000 U.S. census. Note that the numbers represent thousands. Thus, Detroit had a population of 951 thousand or 951,000.

chapter 1 > Make the Connection

Name	Rank	Population (thousands)
New York City, NY	1	8,008
Los Angeles, CA	2	3,695
Chicago, IL	3	2,896
Houston, TX	4	1,954
Philadelphia, PA	5	1,518
Phoenix, AZ	6	1,321
San Diego, CA	7	1,223
Dallas, TX	8	1,189
San Antonio, TX	9	1,145
Detroit, MI	10	951

Source: U.S. Census Bureau

SOCIAL SCIENCE In exercises 25 to 28, write your answers in standard form, using the preceding table.

25. What was the population of San Diego in 2000?

Answers

21. ____________________

22. ____________________

23. ____________________

24. ____________________

25. ____________________

Answers

26. ______________________

27. ______________________

28. ______________________

29. ______________________

30. ______________________

31. ______________________

32. ______________________

33. ______________________

34. ______________________

35. ______________________

36. ______________________

26. What was the population of Chicago in 2000?

27. What was the population of Philadelphia in 2000?

28. What was the population of Dallas in 2000?

Determine the number represented by the scrambled place values.

29. 4 thousands
1 tens
3 ten thousands
5 ones
2 hundreds

30. 7 hundreds
4 ten thousands
9 ones
8 tens
6 thousands

Assume that you have alphabetized the word names for every number from one to one thousand.

31. Which number would appear first in the list?

32. Which number would appear last?

33. **Business and Finance** Inci had to write a check for $2,565. There is a space on the check to write out the amount of the check in words. What should she write in this space?

0000 000 0000000

34. **Business and Finance** In a rental agreement, the amount of the initial deposit required is two thousand, five hundred forty-five dollars. Write this amount as a number.

Basic Skills | Challenge Yourself | Calculator/Computer | **Career Applications** | Above and Beyond

35. **Allied Health** Doctor Edwards prescribes four hundred eighty thousand units of penicillin G benzathine to treat a 3-year-old child with a streptococcal infection. Write this amount as a number.

36. **Allied Health** Doctor Hill prescribes one thousand, one hundred eighty-three milligrams (mg) of amifostine to be administered together with an adult patient's chemotherapy to reduce the adverse effects of the treatment. Write this amount as a number.

Use this table to complete exercise 37 on the following page.

Child's Weight (lb)	Dose of Penicillin G Potassium (thousands of units)
30	680
35	795
40	910
45	1,020
50	1,135

Answers

37. ____________

38. ____________

39. ____________

40. ____________

41. ____________

42. ____________

37. **ALLIED HEALTH** Write your answers in standard form using the preceding table.

- **(a)** Carla weighs 35 pounds (lb). What dose of penicillin G potassium should her doctor prescribe?
- **(b)** Nelson weighs 50 lb. What dose of penicillin G potassium should his doctor prescribe?
- **(c)** What dose of penicillin G potassium is required for a child weighing 40 lb?

38. **ELECTRONICS** Write your answers in standard form using the following table.

Appliance	Estimated Power Consumption [thousands of watts/hour (W/h)]
Drip coffeemaker	301
Electric blanket	120
Laser printer	466
Personal computer	25
Video game system	49

- **(a)** What is the estimated power consumption of a laser printer?
- **(b)** What is the estimated power consumption of a video game system?
- **(c)** What is the estimated power consumption of an electric blanket?

39. **NUMBER PROBLEM** Write the largest five-digit number that can be made using the digits 6, 3, and 9 if each digit is to be used at least once.

Basic Skills | Challenge Yourself | Calculator/Computer | Career Applications | **Above and Beyond** ▲

40. What are the advantages of a place-value system of numeration?

41. **SOCIAL SCIENCE** The number 0 was not used initially by the Hindus in our number system (about 250 B.C.E.). Go to your library (or "surf the net"), and determine when a symbol for zero was introduced. What do you think is the importance of the role of 0 in a numeration system?

42. A *googol* is a very large number. Do some research to find out how big it is. Also try to find out where the name of this number comes from.

Answers

We provide the answers for the odd-numbered exercises at the end of each exercise set. The answers for the even-numbered exercises are provided in the instructor's resource manual.

1. $(4 \times 100) + (5 \times 10) + (6 \times 1)$ **3.** $(5 \times 1{,}000) + (7 \times 10) + (3 \times 1)$
5. Hundreds **7.** Thousands **9.** **(a)** 3; **(b)** 2 **11.** **(a)** 4; **(b)** 2
13. Five thousand, six hundred eighteen
15. Two hundred thousand, three hundred four
17. 253,483 **19.** 502,078,000 **21.** $1,125,000
23. 441,500 **25.** 1,223,000 **27.** 1,518,000 **29.** 34,215
31. Eight **33.** Two thousand, five hundred sixty-five **35.** 480,000
37. **(a)** 795,000; **(b)** 1,135,000; **(c)** 910,000 **39.** 99,963
41. Above and Beyond

1.2

Addition

< **1.2 Objectives** >

1 > Add single-digit numbers
2 > Identify the properties of addition
3 > Add groups of numbers with no carrying
4 > Use the language of addition
5 > Solve simple applications
6 > Add any group of numbers
7 > Find the perimeter of a figure
8 > Solve applications that involve perimeter

Tips for Student Success

Hint #2 Become familiar with your syllabus.

In the first class meeting, your instructor probably handed out a class syllabus. If you haven't done so already, you need to put important information into your calendar and address book.

1. Write all important dates in your calendar. This includes homework due dates, quiz dates, test dates, and the date and time of the final exam. Never allow yourself to be surprised by any deadline!
2. Write your instructor's name, contact number, and office number in your address book. Also include the office hours and e-mail address. Make it a point to see your instructor early in the term. Although this is not the only person who can help clear up your confusion, he or she is the most important person.
3. Make note of other resources that are made available to you. These include CDs, videotapes, Web pages, and tutoring.

Given all these resources, it is important that you never let confusion or frustration mount. If you can't "get it" from the text, try another resource. All the resources are there specifically for you, so take advantage of them!

NOTE

The series of three dots (. . .) is called an **ellipsis;** they mean that the indicated pattern continues.

The *natural* or *counting numbers* are the numbers we use to count objects.

The natural numbers are 1, 2, 3, . . .

When we include the number 0, we have the set of *whole numbers.*

The whole numbers are 0, 1, 2, 3, . . .

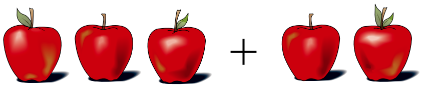
Let's look at the operation of *addition* on the whole numbers.

Definition

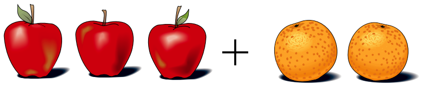
Addition	**Addition** is the combining of two or more groups of the same kind of objects.

This concept is extremely important, as we will see in our later work with fractions. We can only combine or add numbers that represent the same kind of objects.

From your first encounter with arithmetic, you were taught to add "3 apples plus 2 apples."

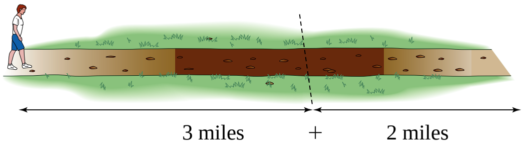


On the other hand, you have probably encountered a phrase such as "that's like combining apples and oranges." That is to say, what do you get when you add 3 apples and 2 oranges?



You could answer "5 fruits," or "5 objects," but you cannot combine the apples and the oranges.

What if you walked 3 miles and then walked 2 more miles? Clearly, you have now walked 3 miles + 2 miles = 5 miles. The addition is possible because the groups are of the same kind.



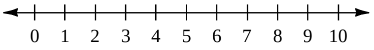
NOTES

The first printed use of the symbol + dates back to 1526 (Smith, *History of Math,* Vol. II.)

The point labeled 0 is called the **origin** of the number line.

Each operation of arithmetic has its own special terms and symbols. The addition symbol + is read **plus.** When we write 3 + 4, 3 and 4 are called the **addends.**

We can use a number line to illustrate the addition process. To construct a number line, we pick a point on the line and label it 0. We then mark off evenly-spaced units to the right, naming each marked point with a successively larger whole number.



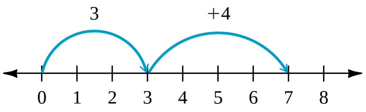
We use arrowheads to show the number line continues.

Example 1 Representing Addition on a Number Line

< Objective 1 >

Represent 3 + 4 on the number line.

To represent an addition, such as 3 + 4, on the number line, start by moving 3 spaces to the right of the origin. Then move 4 more spaces to the right to arrive at 7. The number 7 is called the *sum* of the addends.

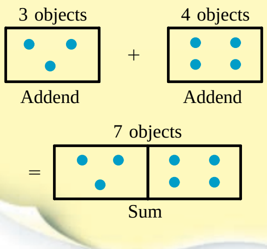


We can write 3 + 4 = 7

Addend Addend Sum

NOTE

Again, addition corresponds to combining groups of the same kind of objects.



Check Yourself 1

Represent 5 + 6 on the number line.

A statement such as 3 + 4 = 7 is one of the **basic addition facts.** These facts include the sum of every possible pair of digits. Before you can add larger numbers correctly and quickly, you must memorize these basic facts.

Basic Addition Facts

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

NOTES

To find the sum 5 + 8, start with the row labeled 5. Move along that row to the column headed 8 to find the sum, 13.

Commute means to move back and forth, as to school or work.

Examining the basic addition facts leads us to several important properties of addition on whole numbers. For instance, we know that the sum 3 + 4 is 7. What about the sum 4 + 3? It is also 7. This is an illustration of the fact that addition is a **commutative** operation.

Property

The Commutative Property of Addition

The order of two numbers around an addition sign *does not* affect the sum.

Example 2 Using the Commutative Property

< Objective 2 >

NOTE

The *order* does not affect the sum.

$8 + 5 = 13 = 5 + 8$

$6 + 9 = 15 = 9 + 6$

Check Yourself 2

Show that the sum on the left equals the sum on the right.

7 + 8 = 8 + 7

If we wish to add *more* than two numbers, we can group them and then add. In mathematics this grouping is indicated by a set of parentheses ( ). This symbol tells us to perform the operation inside the parentheses first.

Example 3 Using the Associative Property

NOTES

We add 3 and 4 as the first step and then add 5.

Here we add 4 and 5 as the first step and then add 3. Again the final sum is 12.

$\overbrace{(3 + 4)} + 5 = 7 + 5 = 12$

We also have

$3 + \overbrace{(4 + 5)} = 3 + 9 = 12$

This example suggests the following property of whole numbers.

Property

The Associative Property of Addition

The way in which several whole numbers are grouped *does not* affect the final sum when they are added.

NOTE

In Example 3, the addend 4 can be "associated" with the 3 or the 5.

Check Yourself 3

Find

(4 + 8) + 3 and 4 + (8 + 3)

The number 0 has a special property in addition.

Property

The Additive Identity Property

The sum of 0 and any whole number is just that whole number.

Because of this property, we call 0 the **identity** for the addition operation.

Example 4 Adding Zero

Find the sum of **(a)** $3 + 0$ and **(b)** $0 + 8$.

(a) $3 + 0 = 3$ **(b)** $0 + 8 = 8$

Check Yourself 4

Find each sum.

(a) 4 + 0 **(b)** 0 + 7

Next, we turn to the process of adding larger numbers. We apply the following rule.

Property

Adding Digits of the Same Place Value

We can add the digits of the same place value because they represent the same types of quantities.

NOTE

Remember that 25 means 2 tens and 5 ones; 34 means 3 tens and 4 ones.

Adding two numbers, such as 25 + 34, can be done in expanded form. Here we write out the place value for each digit.

$$\begin{array}{rl} 25 & = 2 \text{ tens} + 5 \text{ ones} \\ +\ 34 & = 3 \text{ tens} + 4 \text{ ones} \\ \hline & = 5 \text{ tens} + 9 \text{ ones} \\ & = 59 \end{array}$$

↓ Add down.

In actual practice, we use a more convenient short form to perform the addition.

Example 5 Adding Two Numbers

< Objective 3 >

NOTE

In using the short form, be very careful to line up the numbers correctly so that each column contains digits of the same place value.

Add 352 + 546.

Step 1 Add in the ones column.

$$\begin{array}{r} 352 \\ +\ 546 \\ \hline 8 \end{array}$$

Step 2 Add in the tens column.

$$\begin{array}{r} 352 \\ +\ 546 \\ \hline 98 \end{array}$$

Step 3 Add in the hundreds column.

$$\begin{array}{r} 352 \\ +\ 546 \\ \hline 898 \end{array}$$

Check Yourself 5

Add.

$$\begin{array}{r} 245 \\ +\ 632 \\ \hline \end{array}$$

You have already seen that the word *sum* indicates addition. There are other words that also tell you to use the addition operation.

The *total* of 12 and 5 is written as

12 + 5 or 17

8 *more than* 10 is written as

10 + 8 or 18

12 *increased by* 3 is written as

12 + 3 or 15

Example 6 Translating Words That Indicate Addition

< Objective 4 >

Find each of the following.

(a) 36 increased by 12.

36 increased by 12 is written as 36 + 12 = 48.

(b) The total of 18 and 31.

The total of 18 and 31 is written as 18 + 31 = 49.

Check Yourself 6

Find each of the following.

(a) 43 increased by 25

(b) The total of 22 and 73

NOTE

Get into the habit of writing down *all* your work, rather than just an answer.

Now we consider applications, or word problems, that will use the operation of addition. An organized approach is the key to successful problem solving, and we suggest the following strategy.

Step by Step

Solving Addition Applications

Step 1 Read the problem carefully to determine the given information and what you are being asked to find.

Step 2 Decide upon the operation (in this case, addition) to be used.

Step 3 Write down the complete statement necessary to solve the problem and do the calculations.

Step 4 Write your answer as a complete sentence. Check to make sure you have answered the question of the problem and that your answer seems reasonable.

We work through an example, using these steps.

Example 7 Setting Up a Word Problem

< Objective 5 >

Four sections of algebra were offered in the fall quarter, with enrollments of 33, 24, 20, and 22 students. What was the total number of students taking algebra?

Step 1 The given information is the number of students in each section. We want the total number.

Step 2 Since we are looking for a total, we use addition.

Step 3 Write $33 + 24 + 20 + 22 = 99$ students.

Step 4 There were 99 students taking algebra.

NOTE

Remember to attach the proper unit (here "students") to your answer.

Check Yourself 7

Elva Ramos won an election for city council with 3,110 votes. Her two opponents had 1,022 and 1,211 votes. How many votes were cast in that election?

In the previous examples and exercises, the digits in each column added to 9 or less. We now look at the situation in which a column has a two-digit sum. This will involve the process of **carrying.** Look at the process in expanded form.

Example 8 Adding in Expanded Form When Regrouping Is Needed

< Objective 6 >

$$\begin{array}{rl} 67 & = 60 + 7 \\ +\ 28 & = 20 + 8 \\ \hline & \quad 80 + 15 \end{array}$$

We have written 15 ones as 1 ten and 5 ones.

or $\underbrace{80 + \overbrace{10}}\overbrace{{}+ 5}$

The 1 ten is then combined with the 8 tens.

or $90 \quad + 5$

or 95

NOTE

Regrouping in addition is also called *carrying*. Of course, the name makes no difference as long as you understand the process.

The more convenient short form carries the excess units from one column to the next column to the left. Recall that the place value of the next column to the left is 10 times the value of the original column. It is this property of our decimal place-value system that makes carrying work. We work this problem again, this time using the short, or "carrying," form.

NOTE

This is true for any sized number. The place value thousands is 10 times the place value hundreds, and so on.

Step 1 **Step 2**

Carry 1 ten.

$$\begin{array}{r} {}^{1} \\ 67 \\ +\ 28 \\ \hline 5 \end{array} \qquad\qquad \begin{array}{r} {}^{1} \\ 67 \\ +\ 28 \\ \hline 95 \end{array}$$

Step 1: The sum of the digits in the ones column is 15, so write 5 and make the 10 ones **a 1** in the tens column. **Step 2:** Now add in the tens column, being sure to include the carried 1.

Check Yourself 8

Add.

(a) $\begin{array}{r} 58 \\ +\ 36 \\ \hline \end{array}$ **(b)** $\begin{array}{r} 73 \\ +\ 18 \\ \hline \end{array}$ **(c)** $\begin{array}{r} 68 \\ +\ 25 \\ \hline \end{array}$

The addition process often requires more than one regrouping step, as is shown in Example 9.

Example 9 Adding in Short Form When Regrouping Is Needed

Add 285 and 378.

$$\begin{array}{r} 1 \\ 285 \\ +\ 378 \\ \hline 3 \end{array}$$

← Write the 10 as 1 ten.

The sum of the digits in the ones column is 13, so write 3 and carry 1 to the tens column.

Carry ⟶ 1 hundred.

$$\begin{array}{r} 1\,1 \\ 285 \\ +\ 378 \\ \hline 63 \end{array}$$

Now add in the tens column, being sure to include the carry. We have 16 tens, so write 6 in the tens place and carry 1 to the hundreds column.

$$\begin{array}{r} 1\,1 \\ 285 \\ +\ 378 \\ \hline 663 \end{array}$$

Finally, add in the hundreds column.

Check Yourself 9

Add.

(a) $\begin{array}{r} 479 \\ +\ 287 \\ \hline \end{array}$

(b) $\begin{array}{r} 585 \\ +\ 368 \\ \hline \end{array}$

The regrouping process is the same if we want to add more than two numbers.

Example 10 Adding in Short Form with Multiple Regrouping Steps

Add 53, 2,678, 587, and 27,009.

$$\begin{array}{r} 1\,1\ 2\,2 \\ 53 \\ 2{,}678 \\ 587 \\ +\ 27{,}009 \\ \hline 30{,}327 \end{array}$$

← Carries

Add in the ones column: $3 + 8 + 7 + 9 = 27$.
Write 7 in the sum and carry 2 to the tens column.

Now add in the tens column, being sure to include the carry. The sum is 22. Write 2 tens and carry 2 to the hundreds column. Complete the addition by adding in the hundreds column, the thousands column, and the ten thousands column.

Check Yourself 10

Add 46, 365, 7,254, and 24,006.

Finding the *perimeter* of a figure is one application of addition.

Definition

Perimeter — **Perimeter** is the distance around a closed figure.

If the figure has straight sides, the perimeter is the sum of the lengths of its sides.

Example 11 Finding the Perimeter

< Objective 7 >

NOTE

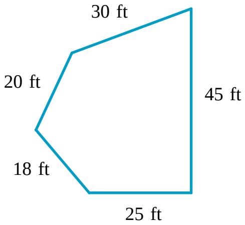
Make sure to include the unit with each number.

We wish to fence in the field shown in the figure. How much fencing, in feet (ft), will be needed?

The fencing needed is the perimeter of (or the distance around) the field. We must add the lengths of the five sides.

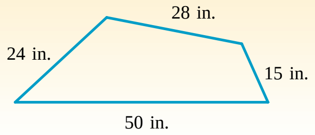
20 ft + 30 ft + 45 ft + 25 ft + 18 ft = 138 ft

So the perimeter is 138 ft.



Check Yourself 11

What is the perimeter of the region shown?



A **rectangle** is a figure, like a sheet of paper, with four equal corners. The perimeter of a rectangle is found by adding the lengths of the four sides.

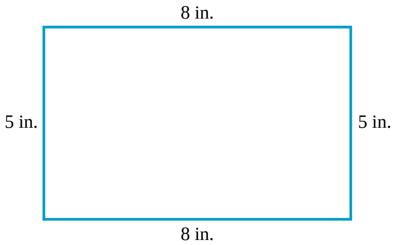
Example 12 Finding the Perimeter of a Rectangle

Find the perimeter in inches (in.) of the rectangle pictured here.

The perimeter is the sum of the lengths 8 in., 5 in., 8 in., and 5 in.

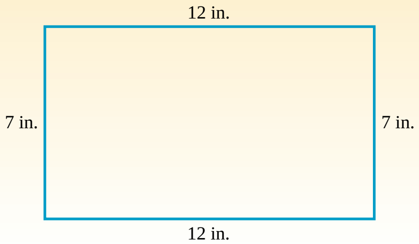
8 in. + 5 in. + 8 in. + 5 in. = 26 in.

The perimeter of the rectangle is 26 in.



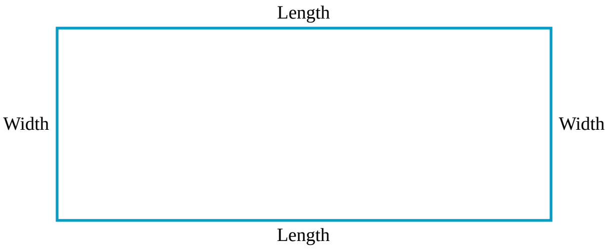
Check Yourself 12

Find the perimeter of the rectangle pictured here.



In general, we can find the perimeter of a rectangle by using a *formula*. A **formula** is a set of symbols that describe a general solution to a problem.

Look at a picture of a rectangle.



The perimeter can be found by adding the distances, so

Perimeter = length + width + length + width

To make this formula a little more readable, we abbreviate each of the words, using just the first letter.

Property

Formula for the Perimeter of a Rectangle

$$P = L + W + L + W \qquad (1)$$

There is another version of this formula that we can use. Because we add the length (L) twice, we could write that as $2 \cdot L$. Because we add the width (W) twice, we could write that as $2 \cdot W$. This gives us another version of the formula.

Property

Formula for the Perimeter of a Rectangle

$$P = 2 \cdot L + 2 \cdot W \qquad (2)$$

In words, we say that the perimeter of a rectangle is twice its length plus twice its width. Example 13 uses formula (1).

Example 13 Finding the Perimeter of a Rectangle

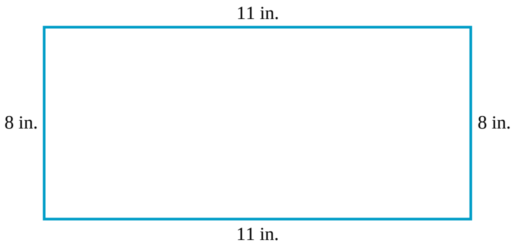
< Objective 8 >

NOTE

We say the rectangle is 8 in. by 11 in.

A rectangle has length 11 in. and width 8 in. What is its perimeter?

Start by drawing a picture of the problem.



Now use formula (1).

$$\begin{aligned} P &= 11 \text{ in.} + 8 \text{ in.} + 11 \text{ in.} + 8 \text{ in.} \\ &= 38 \text{ in.} \end{aligned}$$

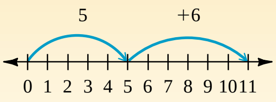
The perimeter is 38 in.

Check Yourself 13

A bedroom is 9 ft by 12 ft. What is its perimeter?

Check Yourself ANSWERS

1.



$5 + 6 = 11$

2. $7 + 8 = 15$ and $8 + 7 = 15$

3. $(4 + 8) + 3 = 12 + 3 = 15$; $4 + (8 + 3) = 4 + 11 = 15$

4. **(a)** 4; **(b)** 7 **5.** 877 **6.** **(a)** 68; **(b)** 95 **7.** 5,343 votes

8. **(a)** 94; **(b)** 91; **(c)** 93 **9.** **(a)** 766; **(b)** 953 **10.** 31,671

11. 117 in. **12.** 38 in. **13.** 42 ft

Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 1.2

(a) The __________ or counting numbers are the numbers used to count objects.

(b) A statement such as $3 + 4 = 7$ is one of the basic __________ facts.

(c) The __________ of two numbers around an addition sign does not affect the sum.

(d) The first step in solving an addition application is to __________ the problem carefully.

1.2 exercises

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Name ____________

Section ________ Date ________

Answers

1. ____________

2. ____________

3. ____________

4. ____________

5. ____________

6. ____________

7. ____________

8. ____________

9. ______ 10. ______

11. ______ 12. ______

13. ______ 14. ______

15. ______ 16. ______

17. ______ 18. ______

19. ______ 20. ______

21. ______ 22. ______

23. ____________

24. ____________

25. ______ 26. ______

27. ______ 28. ______

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< Objective 2 >

Name the property of addition that is illustrated. Explain your choice of property.

1. $5 + 8 = 8 + 5$ > Videos

2. $2 + (7 + 9) = (2 + 7) + 9$

3. $(4 + 5) + 8 = 4 + (5 + 8)$ > Videos

4. $9 + 7 = 7 + 9$

5. $4 + (7 + 6) = 4 + (6 + 7)$ > Videos

6. $5 + 0 = 5$

7. $5 + (2 + 3) = (2 + 3) + 5$

8. $3 + (0 + 6) = (3 + 0) + 6$

< Objectives 3, 6 >

Perform the indicated addition.

9. $$\begin{array}{r} 2,792 \\ +\quad 205 \\ \hline \end{array}$$

10. $$\begin{array}{r} 5,463 \\ +\quad 435 \\ \hline \end{array}$$

11. $$\begin{array}{r} 2,345 \\ +\ 6,053 \\ \hline \end{array}$$

12. $$\begin{array}{r} 3,271 \\ +\ 4,715 \\ \hline \end{array}$$

13. $$\begin{array}{r} 2,531 \\ +\ 5,354 \\ \hline \end{array}$$

14. $$\begin{array}{r} 5,003 \\ +\ 4,205 \\ \hline \end{array}$$

15. $$\begin{array}{r} 21,314 \\ +\ 43,042 \\ \hline \end{array}$$

16. $$\begin{array}{r} 12,325 \\ +\ 35,403 \\ \hline \end{array}$$

17. $$\begin{array}{r} 3,490 \\ 548 \\ +\quad 25 \\ \hline \end{array}$$ > Videos

18. $$\begin{array}{r} 678 \\ 4,533 \\ +\quad 70 \\ \hline \end{array}$$

19. $$\begin{array}{r} 2,289 \\ 38 \\ 578 \\ +\ 3,489 \\ \hline \end{array}$$

20. $$\begin{array}{r} 3,678 \\ 259 \\ 27 \\ +\ 2,356 \\ \hline \end{array}$$

21. $$\begin{array}{r} 23,458 \\ +\ 32,623 \\ \hline \end{array}$$

22. $$\begin{array}{r} 52,591 \\ +\ 59,739 \\ \hline \end{array}$$

< Objective 4 >

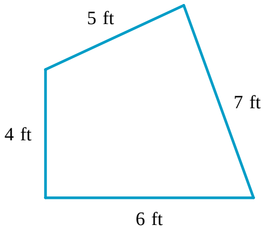
23. In the statement $5 + 4 = 9$
5 is called an ________.
4 is called an ________.
9 is called the ________.

24. In the statement $7 + 8 = 15$
7 is called an ________.
8 is called an ________.
15 is called the ________.

< Objectives 7, 8 >

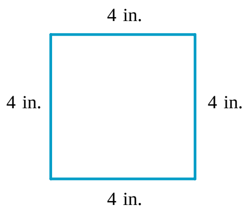
Find the perimeter of each figure.

25.

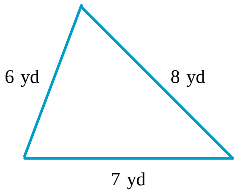


> Videos

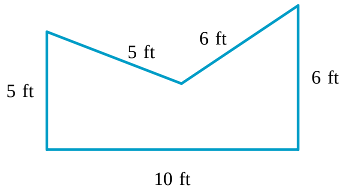
26.



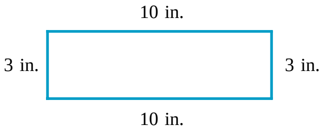
27.



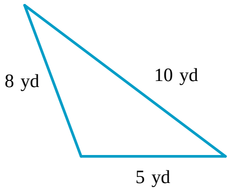
28.



29.



30.



31. In each of the following exercises, find the appropriate sum.

(a) Find the number that is 356 more than 1,213.

(b) Add 23, 2,845, 5, and 589.

(c) What is the total of the five numbers 2,195, 348, 640, 59, and 23,785?

(d) Find the number that is 34 more than 125.

(e) What is 457 increased by 96?

32. In each of the following exercises, find the appropriate sum.

(a) Find the number that is 567 more than 2,322.

(b) Add 5,637, 78, 690, 28, and 35,589.

(c) What is the total of the five numbers 3,295, 9, 427, 56, and 11,100?

(d) Find the number that is 124 more than 2,351.

(e) What is 926 increased by 86?

33. **BUSINESS AND FINANCE** Tral bought a 1931 Model A for $15,200, a 1964 Thunderbird convertible for $17,100, and a 1959 Austin Healy Mark I for $17,450. How much did he invest in the three cars?

34. **BUSINESS AND FINANCE** The following chart shows Family Video's monthly rentals for the first 3 months of 2006 by category of film. Complete the totals.

Category of Film	Jan.	Feb.	Mar.	Category Totals
Comedy	4,568	3,269	2,189	
Drama	5,612	4,129	3,879	
Action/Adventure	2,654	3,178	1,984	
Musical	897	623	528	
Monthly Totals				

35. **BUSINESS AND FINANCE** The following chart shows Regina's Dress Shop's expenses by department for the last 3 months of the year. Complete the totals.

Department	Oct.	Nov.	Dec.	Department Totals
Office	$31,714	$32,512	$30,826	
Production	85,146	87,479	81,234	
Sales	34,568	37,612	33,455	
Warehouse	16,588	11,368	13,567	
Monthly Totals				

Answers

29. ____________

30. ____________

31. ____________

32. ____________

33. ____________

34. ____________

35. ____________

Answers

36. ______

37. ______

38. ______

36. **Social Science** The following table ranks the top 10 areas for women-owned firms in the United States.

Metro Area	Number of Firms	Employment	Sales (in millions)
Los Angeles–Long Beach, Calif.	360,300	1,056,600	$181,455,900
New York	282,000	1,077,900	193,572,200
Chicago	260,200	1,108,800	161,200,900
Washington, D.C.	193,600	440,000	56,644,000
Philadelphia	144,600	695,900	90,231,000
Atlanta	138,700	331,800	50,206,800
Houston	136,400	560,100	78,180,300
Dallas	123,900	431,900	63,114,900
Detroit	123,600	371,400	50,060,700
Minneapolis–St. Paul, Minn.	119,600	337,400	51,063,400

(a) How many firms in total are located in Washington, Philadelphia, and New York?

(b) What is the total number of employees in all 10 of the areas listed?

(c) What are the total sales for firms in Houston and Dallas?

(d) How many firms in total are located in Chicago and Detroit?

chapter 1 > Make the Connection

37. **Number Problem** The following sequences are called *arithmetic sequences*. Determine the pattern and write the next four numbers in each sequence.

(a) 5, 12, 19, 26, ______, ______, ______, ______

(b) 8, 14, 20, 26, ______, ______, ______, ______

(c) 7, 13, 19, 25, ______, ______, ______, ______

(d) 9, 17, 25, 33, ______, ______, ______, ______

38. **Number Problem** Fibonacci numbers occur in the sequence

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, . . .

This sequence begins with the numbers 1 and 1 again, and each subsequent number is obtained by adding the two preceding numbers.

Find the next four numbers in the sequence.

Basic Skills | Challenge Yourself | **Calculator/Computer** | Career Applications | Above and Beyond

Although this text is designed to help you master the basic skills of arithmetic, it is occasionally preferable to perform complex calculations on a calculator. To that end, many of the exercise sets include a short explanation of how to use a calculator to do an operation described in the section. This explanation will be followed by a set of exercises for which the calculator might be the preferred tool. As indicated by the placement of the explanation, you should refrain from using a calculator on the exercises that precede it.

To perform the addition

2,473 + 258 + 35 + 5,823

Step 1 Press the clear key. [C]
Step 2 Enter the first number. 2473
Step 3 Enter the plus key followed by the next number. [+] 258
Step 4 Continue with the addition until the last number is entered. [+] 35
[+] 5823
Step 5 Press the equal-sign or enter key. [=]
The desired sum should now be in the display. 8589

Use your calculator to find each sum.

39. 3,295,153 + 573,128 + 21,257 + 2,586,241 + 5,291

40. 23,563 + 5,638,487 + 385,005 + 27,345

Use your calculator to solve each application.

41. **BUSINESS AND FINANCE** The following table shows the number of customers using three branches of a bank during one week. Complete the table by finding the daily, weekly, and grand totals.

Branch	Mon.	Tues.	Wed.	Thurs.	Fri.	Weekly Totals
Downtown	487	356	429	278	834	______
Suburban	236	255	254	198	423	______
Westside	345	278	323	257	563	______
Daily Totals						______

42. **STATISTICS** The following table lists the number of possible types of poker hand. What is the total number of hands possible?

Royal flush	4
Straight flush	36
Four of a kind	624
Full house	3,744
Flush	5,108
Straight	10,200
Three of a kind	54,912
Two pairs	123,552
One pair	1,098,240
Nothing	1,302,540
Total Possible Hands	

Basic Skills | Challenge Yourself | Calculator/Computer | Career Applications | Above and Beyond

43. **MANUFACTURING TECHNOLOGY** An inventory of steel round stock shows 248 feet (ft) of $\frac{1}{4}$ inch (in.), 124 ft of $\frac{3}{8}$ in., 428 ft of $\frac{1}{2}$ in., and 162 ft of $\frac{5}{8}$ in. How many total feet of steel round stock are in inventory?

44. **MANUFACTURING TECHNOLOGY** B & L Industries produces three different products. Orders for today are for 351 of product A, 187 of product B, and 94 of product C. How many total products need to be produced today to fill the orders?

Answers

39. ______
40. ______
41. ______
42. ______
43. ______
44. ______

Answers

45. ______________

46. ______________

47. ______________

48. ______________

45. **ALLIED HEALTH** The source-image receptor distance (SID) for radiographic images is the sum of the object-film distance (OFD) and the focus-object distance (FOD). Determine the SID if the distance from the object to the film is 8 inches (in.), and the distance from the object to the focus is 48 in.

46. **ALLIED HEALTH** Total lung capacity, measured in milliliters (mL), is the sum of the vital capacity and the residual volume. Determine the total lung capacity for a patient whose vital capacity is 4,500 mL and whose residual volume is 1,800 mL.

Basic Skills | Challenge Yourself | Calculator/Computer | Career Applications | **Above and Beyond** ▲

47. **NUMBER PROBLEM** A magic square is a square in which the sum along any row, column, or diagonal is the same. For example

35	10	15
0	20	40
25	30	5

Use the numbers 1 to 9 to form a magic square.

48. The following puzzle gives you a chance to practice some of your addition skills.

Across

1. 23 + 22
3. 103 + 42
6. 29 + 58 + 19
8. 3 + 3 + 4
9. 1,480 + 1,624
11. 568 + 730
13. 25 + 25
14. 131 + 132
16. The total of 121, 146, 119, and 132
17. The perimeter of a 4 × 6 rug

Down

1. The sum of 224,000, 155, and 186,000
2. 20 + 30
4. 210 + 200
5. 500,000 + 4,730
7. 130 + 509
10. 90 + 92
12. 100 + 101
15. The perimeter of a 15 × 16 room

1	2	■	3	4	5
6		7	■	8	
	■	9	10		
11	12			■	
13		■	14	15	
16			■	17	

Answers

1. Commutative property of addition
3. Associative property of addition
5. Commutative property of addition
7. Commutative property of addition **9.** 2,997 **11.** 8,398
13. 7,885 **15.** 64,356 **17.** 4,063 **19.** 6,394 **21.** 56,081
23. 5 is an addend, 4 is an addend, 9 is the sum **25.** 22 ft **27.** 21 yd
29. 26 in. **31.** **(a)** 1,569; **(b)** 3,462; **(c)** 27,027; **(d)** 159; **(e)** 553
33. $49,750
35.

Department	Oct.	Nov.	Dec.	Department Totals
Office	$31,714	$32,512	$30,826	$95,052
Production	85,146	87,479	81,234	$253,859
Sales	34,568	37,612	33,455	$105,635
Warehouse	16,588	11,368	13,567	$41,523
Monthly Totals	$168,016	$168,971	$159,082	$496,069

37. **(a)** 33, 40, 47, 54; **(b)** 32, 38, 44, 50; **(c)** 31, 37, 43, 49; **(d)** 41, 49, 57, 65
39. 6,481,070
41.

Branch	**Mon.**	**Tues.**	**Wed.**	**Thurs.**	**Fri.**	**Weekly Totals**
Downtown	487	356	429	278	834	2,384
Suburban	236	255	254	198	423	1,366
Westside	345	278	323	257	563	1,766
Daily Totals	1,068	889	1,006	733	1,820	5,516
						Grand Total

43. 962 ft **45.** 56 in.
47.

8	3	4
1	5	9
6	7	2

1.3 Subtraction

< **1.3 Objectives** >

1 > Subtract whole numbers without borrowing

2 > Use the language of subtraction

3 > Solve applications of simple subtraction

4 > Use borrowing in subtracting whole numbers

5 > Solve applications that require borrowing

Tips for Student Success

Hint #3 Don't procrastinate!

1. Do your math homework while you're still fresh. If you wait until too late at night, your mind will be tired and have much more difficulty understanding the concepts.
2. Do your homework the day it is assigned. The more recent the explanation is, the easier it is to recall.
3. When you are finished with your homework, try reading the next section through once. This will give you a sense of direction when you next hear the material. This works whether you are in a lecture or lab setting.

Remember that, in a typical math class, you are expected to do two or three hours of homework for each weekly class hour. This means two or three hours per night. Schedule the time and stay on schedule.

NOTE

By *opposite operation* we mean that subtracting a number "undoes" an addition of that same number. Start with 1. Add 5 and then subtract 5. Where are you?

We are now ready to consider a second operation of arithmetic—subtraction. In Section 1.2, we described addition as the process of combining two or more groups of the same kind of objects. Subtraction can be thought of as the *opposite operation* to addition. Every arithmetic operation has its own notation. The symbol for subtraction, −, is called a **minus sign.**

When we write $8 - 5$, we wish to subtract 5 from 8. We call 5 the **subtrahend.** This is the number being subtracted. And 8 is the **minuend.** This is the number we are subtracting from. The **difference** is the result of the subtraction.

To find the *difference* of two numbers, we will assume that we wish to subtract the smaller number from the larger. Then we look for a number which, when added to the smaller number, will give us the larger number. For example,

$$8 - 5 = 3 \qquad \text{because} \qquad 3 + 5 = 8$$

This special relationship between addition and subtraction provides a method of checking subtraction.

Property

Relationship Between Addition and Subtraction

The sum of the difference and the subtrahend must be equal to the minuend.

Example 1 Subtracting a Single-Digit Number

< Objective 1 >

$12 - 5 = 7$

$$\text{Check: } \underset{\substack{\uparrow\\ \text{Difference}}}{7} + \underset{\substack{\uparrow\\ \text{Subtrahend}}}{5} = \underset{\substack{\uparrow\\ \text{Minuend}}}{12}$$

Our check works because 12 − 5 asks for the number that must be added to 5 to get 12.

Check Yourself 1

Subtract and check your work.

13 − 9

The procedure for subtracting larger whole numbers is similar to the procedure for addition. We subtract digits of the same place value.

Example 2 Subtracting a Larger Number

Step 1 **Step 2** **Step 3**

$$\begin{array}{r} 789 \\ -\ 246 \\ \hline 3 \end{array} \qquad \begin{array}{r} 789 \\ -\ 246 \\ \hline 43 \end{array} \qquad \begin{array}{r} 789 \\ -\ 246 \\ \hline 543 \end{array}$$

We subtract in the ones column, then in the tens column, and finally in the hundreds column.

$$\text{To check: } \begin{array}{r} 789 \\ -\ 246 \\ \hline 543 \end{array} \ \text{Add } 543 + 246 = 789$$

The sum of the difference and the subtrahend must be the minuend.

Check Yourself 2

Subtract and check your work.

(a) $\begin{array}{r} 3{,}468 \\ -\ 2{,}248 \\ \hline \end{array}$ **(b)** $\begin{array}{r} 4{,}984 \\ -\ 1{,}081 \\ \hline \end{array}$

You know that the word *difference* indicates subtraction. There are other words that also tell you to use the subtraction operation. For instance, 5 *less than* 12 is written as

$12 - 5$ or 7

20 *decreased* by 8 is written as

$20 - 8$ or 12

Example 3 Translating Words That Indicate Subtraction

< Objective 2 >

Find each of the following.

(a) 4 less than 11

4 less than 11 is written $11 - 4 = 7$.

(b) 27 decreased by 6

27 decreased by 6 is written $27 - 6 = 21$.

Check Yourself 3

Find each of the following.

(a) **6 less than 19** **(b)** **18 decreased by 3**

Units ANALYSIS

This is the first in a series of essays that are designed to help you solve applications using mathematics. Questions in the exercise sets will require the skills that you build by reading these essays.

A number with a unit attached (such as 7 **feet** or 26 **mi/gal**) is called a denominate number. Any genuine application of mathematics involves denominate numbers.

When adding or subtracting denominate numbers, the units must be identical for both numbers. The sum or difference will have those same units.

EXAMPLES:

\$4 + \$9 = \$13	Notice that, although we write the dollar sign first, we read it after the quantity, as in "four dollars."

7 feet + 9 feet = 16 feet

39 degrees − 12 degrees = 27 degrees

7 feet + 12 degrees yields no meaningful answer!

3 feet + 9 inches yields a meaningful result if 3 feet is converted into 36 inches. We will discuss conversion of units in later essays.

Now we consider subtraction word problems. The strategy is the same one presented in Section 1.2 for addition word problems. It is summarized with the following four basic steps.

Step by Step

Solving Subtraction Applications

Step 1	Read the problem carefully to determine the given information and what you are asked to find.
Step 2	Decide upon the operation (in this case, subtraction) to be used.
Step 3	Write down the complete statement necessary to solve the problem and do the calculations.
Step 4	Check to make sure you have answered the question of the problem and that your answer seems reasonable.

Here is an example using these steps.

Example 4 Setting Up a Subtraction Word Problem

< Objective 3 >

Tory has \$37 in his wallet. He is thinking about buying a \$24 pair of pants and a \$10 shirt. If he buys them both, how much money will he have remaining?

First we must add the cost of the pants and the shirt.

\$24 + \$10 = \$34

Now, that amount must be subtracted from the \$37.

\$37 − \$34 = \$3

He will have \$3 left.

Check Yourself 4

Sonya has $97 left in her checking account. If she writes checks for $12, $32, and $21, how much will she have in the account?

Difficulties can arise in subtraction if one or more of the digits of the subtrahend are larger than the corresponding digits in the minuend. We will solve this problem by using another version of the regrouping process called **borrowing.**

First, we look at an example in expanded form.

Example 5 Subtracting When Regrouping Is Needed

< Objective 4 >

$$\begin{array}{r}52 = 50 + 2\\ -\ 27 = 20 + 7\\ \hline\end{array}$$

Do you see that we cannot subtract in the ones column?

We regroup by borrowing 1 ten in the minuend and writing that ten as 10 ones:

$$\begin{array}{ll} & 50 + 2\\ \text{becomes} & \overbrace{40 + 10}^{50} + \underbrace{2}\\ \text{or} & 40 + 12\end{array}$$

We now have

$$\begin{array}{r}52 = 40 + 12\\ -\ 27 = 20 + \ \ 7\\ \hline 20 + \ \ 5\end{array}$$

We can now subtract as before.

or 25

In practice, we use a more convenient short form for the subtraction.

$$\begin{array}{r}52\\ -\ 27\\ \hline\end{array} \qquad \begin{array}{r}\overset{4}{\cancel{5}}\,\overset{1}{2}\\ -\ 27\\ \hline 25\end{array}$$

We indicate the fact that we have borrowed 1 ten by putting a slash through the 5 and then writing 4 tens. Add 10 ones to the original 2 ones to get 12 ones. We can then subtract.

Check: 25 + 27 = 52

Check Yourself 5

Subtract and check your work.

$$\begin{array}{r}64\\ -\ 38\\ \hline\end{array}$$

In Example 6, we work through a subtraction example that requires a number of regrouping steps. Here, zero appears as a digit in the minuend.

Example 6 Subtracting When Regrouping Is Needed

NOTE

Here we borrow 1 thousand; this is written as 10 hundreds.

Step 1

$$\begin{array}{r}4{,}0\overset{4}{\cancel{5}}\overset{1}{3}\\ -\ 2{,}365\\ \hline 8\end{array}$$

In this first step we regroup by borrowing 1 ten. This is written as 10 ones and combined with the original 3 ones. We can then subtract in the ones column.

Step 2

$$\begin{array}{r}\overset{3}{\cancel{4}},\overset{10}{\cancel{0}}\overset{4}{\cancel{5}}\overset{1}{3}\\ -\ 2{,}365\\ \hline 8\end{array}$$

We must regroup again to subtract in the tens column. There are no hundreds, and so we move to the thousands column.

NOTE

We now borrow 1 hundred; this is written as 10 tens and combined with the remaining 4 tens.

Step 3

$$\begin{array}{r} \overset{3}{\cancel{4}},\overset{9}{\cancel{0}}\overset{14}{\cancel{5}}\overset{1}{3} \\ -\ 2,365 \\ \hline 8 \end{array}$$

The minuend is now renamed as 3 thousands, 9 hundreds, 14 tens, and 13 ones.

Step 4

$$\begin{array}{r} \overset{3}{\cancel{4}},\overset{\overset{9}{\cancel{10}}}{\cancel{0}}\overset{\overset{14}{\cancel{4}}}{\cancel{5}}\overset{1}{3} \\ -\ 2,365 \\ \hline 1,688 \end{array}$$

The subtraction can now be completed.

To check our subtraction: $1,688 + 2,365 = 4,053$

Check Yourself 6

Subtract and check your work.

$$\begin{array}{r} \mathbf{5,024} \\ \mathbf{-\ 1,656} \\ \hline \end{array}$$

You need to use both addition and subtraction to solve some problems, as Example 7 illustrates.

Example 7 Solving a Subtraction Application

< Objective 5 >

Bernard wants to buy a new piece of stereo equipment. He has \$142 and can trade in his old amplifier for \$135. How much more does he need if the new equipment costs \$449?

First we must add to find out how much money Bernard has available. Then we subtract to find out how much more money he needs.

$\$142 + \$135 = \$277$ The money available to Bernard

$\$449 - \$277 = \$172$ The money Bernard still needs

Bernard will need \$172.

Check Yourself 7

Martina spent \$239 in airfare, \$174 for lodging, and \$108 for food on a business trip. Her company allowed her \$375 for the expenses. How much of these expenses will she have to pay herself?

Check Yourself ANSWERS

1. $13 - 9 = 4$ Check: $4 + 9 = 13$ **2.** **(a)** 1,220; **(b)** 3,903

3. **(a)** 13; **(b)** 15 **4.** Sonya will have \$32 left.

5. $\begin{array}{r} \overset{5}{\cancel{6}}\overset{1}{4} \\ -\ 38 \\ \hline 26 \end{array}$ To check: $26 + 38 = 64$ **6.** 3,368 Check: $3,368 + 1,656 = 5,024$

7. $\begin{array}{r} \$239 \\ 174 \\ +\ 108 \\ \hline \$521 \end{array}$ ⟵ Total expenses

$\begin{array}{r} \$521 \\ -\ 375 \\ \hline \$146 \end{array}$

\$521 ⟵ Total expenses

375 ⟵ Amount allowed

\$146 Martina will have to pay \$146.

Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 1.3

(a) The __________ is the result of subtraction.

(b) 5 __________ than 12 is written as $12 - 5$.

(c) The first step in solving a subtraction application is to __________ the problem carefully.

(d) The regrouping process used in subtraction is called __________.

1.3 exercises

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Name ____________________

Section __________ Date __________

Answers

1. ____________ 2. ____________
3. ____________ 4. ____________
5. ____________ 6. ____________
7. ____________ 8. ____________
9. ____________ 10. ____________
11. ____________ 12. ____________
13. ____________ 14. ____________
15. ____________ 16. ____________
17. ____________ 18. ____________
19. ____________ 20. ____________
21. ____________
22. ____________
23. ____________ 24. ____________
25. ____________ 26. ____________
27. ____________ 28. ____________
29. ____________ 30. ____________

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< Objectives 1, 4 >

In exercises 1 to 20, do the indicated subtraction and check your results by addition.

1. $\begin{array}{r} 347 \\ -\ 201 \\ \hline \end{array}$ **2.** $\begin{array}{r} 575 \\ -\ 302 \\ \hline \end{array}$ **3.** $\begin{array}{r} 689 \\ -\ 245 \\ \hline \end{array}$

4. $\begin{array}{r} 598 \\ -\ 278 \\ \hline \end{array}$ **5.** $\begin{array}{r} 3{,}446 \\ -\ 2{,}326 \\ \hline \end{array}$ > Videos **6.** $\begin{array}{r} 5{,}896 \\ -\ 3{,}862 \\ \hline \end{array}$

7. $\begin{array}{r} 64 \\ -\ 27 \\ \hline \end{array}$ **8.** $\begin{array}{r} 73 \\ -\ 36 \\ \hline \end{array}$ **9.** $\begin{array}{r} 627 \\ -\ 358 \\ \hline \end{array}$

10. $\begin{array}{r} 642 \\ -\ 367 \\ \hline \end{array}$ **11.** $\begin{array}{r} 6{,}423 \\ -\ 3{,}678 \\ \hline \end{array}$ **12.** $\begin{array}{r} 5{,}352 \\ -\ 2{,}577 \\ \hline \end{array}$

13. $\begin{array}{r} 6{,}034 \\ -\ 2{,}569 \\ \hline \end{array}$ > Videos **14.** $\begin{array}{r} 5{,}206 \\ -\ 1{,}748 \\ \hline \end{array}$ **15.** $\begin{array}{r} 4{,}000 \\ -\ 2{,}345 \\ \hline \end{array}$

16. $\begin{array}{r} 6{,}000 \\ -\ 4{,}349 \\ \hline \end{array}$ **17.** $\begin{array}{r} 33{,}486 \\ -\ 14{,}047 \\ \hline \end{array}$ **18.** $\begin{array}{r} 53{,}487 \\ -\ 25{,}649 \\ \hline \end{array}$

19. $\begin{array}{r} 29{,}400 \\ -\ 17{,}900 \\ \hline \end{array}$ **20.** $\begin{array}{r} 53{,}500 \\ -\ 28{,}700 \\ \hline \end{array}$

< Objective 2 >

21. In the statement $9 - 6 = 3$
9 is called the ____________.
6 is called the ____________.
3 is called the ____________.
Write the related addition statement.

22. In the statement $7 - 5 = 2$
5 is called the ____________.
2 is called the ____________.
7 is called the ____________.
Write the related addition statement.

< Objective 3 >

23. Find the number that is 25 less than 76. > Videos

24. Find the number that results when 58 is decreased by 23.

25. Find the number that is the difference between 97 and 43.

26. Find the number that is 125 less than 265.

27. Find the number that results when 298 is decreased by 47.

28. Find the number that is the difference between 167 and 57.

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Based on units, determine if each operation produces a meaningful result.

29. 8 miles − 4 miles

30. \$560 + \$314

31. 7 feet + 11 meters

32. 18°F − 6°C

33. 17 yards − 10 yards

34. 4 mi/h + 6 ft/s

In exercises 35 to 38, for various treks by a hiker in a mountainous region, the starting elevations and various changes are given. Determine the final elevation of the hiker in each case.

35. Starting elevation 1,053 feet (ft), increase of 123 ft, decrease of 98 ft, increase of 63 ft.

36. Starting elevation 1,231 ft, increase of 213 ft, decrease of 112 ft, increase of 78 ft.

37. Starting elevation 7,302 ft, decrease of 623 ft, decrease of 123 ft, increase of 307 ft. > Videos

38. Starting elevation 6,907 ft, decrease of 511 ft, decrease of 203 ft, increase of 419 ft.

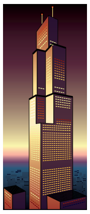
< **Objective 5** >

Solve each application.

39. **Social Science** Shaka's score on a math test was 87, and Tony's score was 23 points less than Shaka's. What was Tony's score on the test?

40. **Business and Finance** Duardo's monthly pay of $879 was decreased by $175 for withholding. What amount of pay did he receive? > Videos

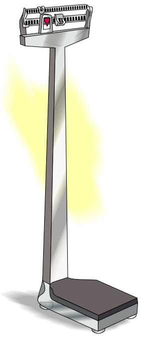
41. **Construction** The Sears Tower in Chicago is 1,454 ft tall. The Empire State Building is 1,250 ft tall. How much taller is the Sears Tower than the Empire State Building?



42. **Business and Finance** In one week, Margaret earned $278 in regular pay and $53 for overtime work, and $49 was deducted from her paycheck for income taxes and $18 for Social Security. What was her take-home pay?

43. **Business and Finance** Rafael opened a checking account and made deposits of $85 and $272. He wrote checks during the month for $35, $27, $89, and $178. What was his balance at the end of the month?

44. **Science and Medicine** Dalila is trying to limit herself to 1,500 calories per day (cal/day). Her breakfast was 270 cal, her lunch was 450 cal, and her dinner was 820 cal. By how much was she *under* or *over* her diet?



Answers

31. ____________

32. ____________

33. ____________

34. ____________

35. ____________

36. ____________

37. ____________

38. ____________

39. ____________

40. ____________

41. ____________

42. ____________

43. ____________

44. ____________

Answers

45. ____________________

46. ____________________

47. ____________________

48. ____________________

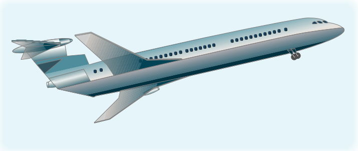
45. Business and Finance Complete the following record of a monthly expense account.

Monthly income	$1,620
Share of rent	343
Balance	
Car payment	183
Balance	
Food	312
Balance	
Clothing	89
Amount remaining	

46. Business and Finance To keep track of a checking account, you must subtract the amount of each check from the current balance. Complete the following statement.

Beginning balance	$351
Check #1	29
Balance	
Check #2	139
Balance	
Check #3	75
Ending balance	

47. Business and Finance Carmen's frequent-flyer program requires 30,000 miles (mi) for a free flight. During 2004 she accumulated 13,850 mi. In 2005 she took three more flights of 2,800, 1,475, and 4,280 mi. How much farther must she fly for her free trip?



48. Business and Finance The value of all crops in the Salinas Valley in 2003 was nearly $3 billion. The top four crops are listed in the following table. **(a)** How much greater is the combined value of both types of lettuce than broccoli? **(b)** How much greater is the value of the lettuce and broccoli combined than that of the strawberries?

Crop	Crop value, in millions
Head lettuce	$361
Leaf lettuce	$277
Strawberries	$298
Broccoli	$259

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Now that you have reviewed the process of subtracting by hand, look at the use of the calculator in performing this operation.

Find

$23 - 13 + 56 - 29$

Enter the numbers and the operation signs exactly as they appear in the expression.

$23 \boxed{-} 13 \boxed{+} 56 \boxed{-} 29 \boxed{\text{ENTER}}$

Display $\boxed{37}$

An alternative approach would be to add 23 and 56 first and then subtract 13 and 29. The result is the same in either case.

Do the indicated operations.

49. $\begin{array}{r} 5{,}830 \\ -\ 3{,}987 \\ \hline \end{array}$

50. $\begin{array}{r} 15{,}280 \\ -\ 7{,}595 \\ \hline \end{array}$

51. Subtract 235 from the sum of 534 and 678.

52. Subtract 476 from the sum of 306 and 572.

Solve each application.

53. BUSINESS AND FINANCE Readings from Fast Service Station's storage tanks were taken at the beginning and end of a month. How much of each type of gas was sold? What was the total sold?

	Diesel	Unleaded	Super Unleaded	Total
Beginning reading	73,255	82,349	81,258	
End reading	28,387	19,653	8,654	
Gallons used				

The land areas, in square miles (mi^2), of three Pacific coast states are California, 155,959 mi^2; Oregon, 95,997 mi^2; and Washington, 66,544 mi^2.

54. SOCIAL SCIENCE How much larger is California than Oregon?

55. SOCIAL SCIENCE How much larger is California than Washington?

56. SOCIAL SCIENCE How much larger is Oregon than Washington?

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57. ALLIED HEALTH A patient's anion gap is used to help evaluate her overall electrolyte balance. The anion gap is equal to the difference between the serum concentration [measured in milliequivalents per liter (mEq/L)] of sodium and the sum of the serum concentrations of chloride and bicarbonate. Determine the patient's anion gap if the concentration of sodium is 140 mEq/L, chloride is 93 mEq/L, and bicarbonate is 24 mEq/L.

58. ALLIED HEALTH To increase the geometric sharpness of a radiographic image, it is easiest to set the focus-object distance (FOD), which is the difference between the source-image receptor distance (SID) and the object-film distance (OFD), to its maximum value. What is the maximum FOD possible if the distance between the object and the film is fixed at 8 inches and the maximum distance possible between the source-image and the receptor is 72 inches?

Answers

49. ______________________

50. ______________________

51. ______________________

52. ______________________

53. ______________________

54. ______________________

55. ______________________

56. ______________________

57. ______________________

58. ______________________

Answers

59. ______________

60. ______________

61. ______________

62. ______________

63. ______________

64. ______________

65. ______________

66. ______________

67. ______________

59. INFORMATION TECHNOLOGY Sally's department needs a new printer. The old printer just died. Sally bought an ink-jet printer at a cost of $150. For a growing department, the ink-jet printer is not appropriate. The problem is she really needs a laser printer at a cost of $500. If she returns the ink-jet printer, how much extra money will she need to buy the laser printer?

60. INFORMATION TECHNOLOGY Max has a 20-foot roll of cable, and he needs to run the cable from a wiring closet to an outlet in a room that is adjacent to the closet. The distance from the wiring closet to the outlet is about 25 feet. How much cable will Max need to buy to be able to run the cable from the wiring closet to the outlet in the adjacent room?

61. ELECTRONICS Solder looks like flexible wire and typically comes wrapped on spools. When heated with a soldering iron or any other heat source, solder melts. It is used to connect an electronic component to wires, other components, or conductive traces. If a certain spool holds 10 pounds (lb) of solder, yet the shipping weight for the spool is 14 lb, how much does the empty spool and shipping materials weigh in pounds?

62. MANUFACTURING TECHNOLOGY Kinetics, Inc., sells an engine block for $168. Production of the block is $72 for materials, $58 for labor, and $19 for shipping and packaging. How much is the profit on the engine block?

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NUMBER PROBLEM Complete the magic squares.

63.

	7	2
	5	
8		

64.

4	3	
	5	
		6

65.

16	3		13
	10	11	
9	6	7	
			1

66.

7			14
2	13	8	11
16			
	6	15	

67. SOCIAL SCIENCE Use the Internet to find the population of Arizona, California, Oregon, and Pennsylvania in each of the last three censuses.

chapter 1 > Make the Connection

(a) Find the total change in each state's population over this period.

(b) Which state shows the greatest change over the past three censuses?

(c) Write a brief essay describing the changes and any trends you see in these data. List any implications that they might have for future planning.

68. Think of any whole number.

Add 5.
Subtract 3.
Subtract 2 less than the original number.
What number do you end up with?
Check with other people. Does everyone have the same answer? Can you explain the results?

Answers

1. 146 **3.** 444 **5.** 1,120 **7.** 37 **9.** 269 **11.** 2,745
13. 3,465 **15.** 1,655 **17.** 19,439 **19.** 11,500
21. 9 is the minuend, 6 is the subtrahend, and 3 is the difference; 3 + 6 = 9
23. 51 **25.** 54 **27.** 251 **29.** Yes **31.** No **33.** Yes
35. 1,141 ft **37.** 6,863 ft **39.** 64 **41.** 204 ft **43.** $28
45. $1,277; $1,094; $782; $693 **47.** 7,595 mi **49.** 1,843 **51.** 977
53. Diesel, 44,868 gal; unleaded, 62,696 gal; super unleaded, 72,604 gal; total, 180,168 gal **55.** 89,415 mi^2 **57.** 23 mEq/L **59.** $350 **61.** 4 lb

63.

6	7	2
1	5	9
8	3	4

65.

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

67. Above and Beyond

Answers

68. ______________________

Activity 1 ::
Population Changes

The following table gives the population for the United States and each of the six largest states from both the 1990 census and the 2000 census. Use this table to answer the questions that follow.

	1990 Population	2000 Population
United States	248,709,873	281,421,906
California	29,760,021	33,871,648
Texas	16,986,510	20,851,820
New York	17,990,455	18,976,457
Florida	12,937,926	15,982,378
Illinois	11,430,602	12,419,293
Pennsylvania	11,881,643	12,281,054

1. Which two pairs of states switched population ranking between the two censuses?

2. By how much did the population of the United States increase between 1990 and 2000?

3. Which state had the greatest increase in population from 1990 to 2000? What was that difference?

4. Which state had the smallest increase in population from 1990 to 2000? What was that difference?

5. What was the total population of the six largest states in 1990?

6. How many people living in the United States did not live in one of the six largest states in 1990?

7. What was the total population of the six largest states in 2000?

8. How many people living in the United States did not live in one of the six largest states in 2000?

9. What regional trends might be true based on what you see in this table?

chapter
1
> Make the Connection

1.4

Rounding, Estimation, and Order

< **1.4 Objectives** >

1 > Round a whole number to a given place value

2 > Estimate sums and differences by rounding

3 > Use the inequality symbols

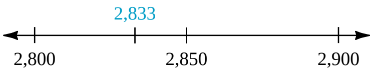
It is a common practice to express numbers to the nearest hundred, thousand, and so on. For instance, the distance from Los Angeles to New York along one route is 2,833 miles (mi). We might say that the distance is 2,800 mi. This is called **rounding,** because we have rounded the distance to the nearest hundred miles.

One way to picture this rounding process is with the use of a number line.

Example 1 Rounding to the Nearest Hundred

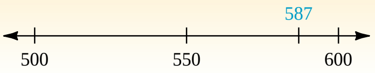
< **Objective 1** >

To round 2,833 to the nearest hundred, we mark notations on a number line counting by hundreds: 2,800 and 2,900. (We only include those that "surround" 2,833.) Then we mark the spot halfway between these: 2,850. When we also mark 2,833 on the line, estimating its location, we must place the mark to the left of 2,850. This makes it clear that 2,833 is closer to 2,800 than it is to 2,900. So we round *down* to 2,800.



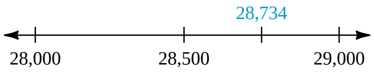
Check Yourself 1

Round 587 to the nearest hundred.



Example 2 Rounding to the Nearest Thousand

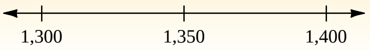
To round 28,734 to the nearest thousand:



Because 28,734 is closer to 29,000 than it is to 28,000, we round *up* to 29,000.

Check Yourself 2

Locate 1,375 and round to the nearest hundred.



Instead of using a number line, we can apply the following rule.

Step by Step

Rounding Whole Numbers

Step 1 Identify the place of the digit to be rounded.

Step 2 Look at the digit to the right of that place.

Step 3 **a.** If that digit is 5 or more, that digit and all digits to the right become 0. The digit in the place you are rounding to is increased by 1.

b. If that digit is less than 5, that digit and all digits to the right become 0. The digit in the place you are rounding to remains the same.

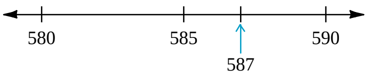
Example 3 Rounding to the Nearest Ten

NOTE

587 is between 580 and 590. It is closer to 590, so it makes sense to round up.

Round 587 to the nearest ten:

Tens
↓
5 8 7
↑
The digit to the right of the tens place
↓
5 8 7 is rounded to 590



We identify the tens digit. The digit to the right of the tens place, 7, is 5 or more. So round up.

Check Yourself 3

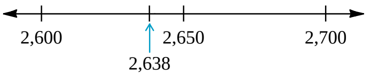
Round 847 to the nearest ten.

Example 4 Rounding to the Nearest Hundred

NOTE

2,638 is closer to 2,600 than to 2,700. So it makes sense to round down.

Round 2,638 to the nearest hundred:
↓
2, 6 38 is rounded to 2,600



We identify the hundreds digit. The digit to the right, 3, is less than 5. So round down.

Check Yourself 4

Round 3,482 to the nearest hundred.

Here are some further examples of using the rounding rule.

Example 5 Rounding Whole Numbers

(a) Round 2,378 to the nearest hundred:
↓
2, 3 78 is rounded to 2,400

We identified the hundreds digit. The digit to the right is 7. Because this is 5 or more, the hundreds digit is increased by 1. The 7 and all digits to the right of 7 become 0.

(b) Round 53,258 to the nearest thousand:

↓

5 3,258 is rounded to 53,000

We identified the thousands digit. Because the digit to the right is less than 5, the thousands digit remains the same. The 2 and all digits to its right become 0.

(c) Round 685 to the nearest ten:

↓

6 8 5 is rounded to 690

The digit to the right of the tens place is 5 or more. Round up by our rule.

(d) Round 52,813,212 to the nearest million:

↓

5 2,813,212 is rounded to 53,000,000

Check Yourself 5

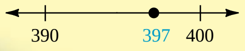
(a) Round 568 to the nearest ten.
(b) Round 5,446 to the nearest hundred.

Now, look at a case in which we round up a 9.

Example 6 Rounding to the Nearest Ten

NOTE

Which number is 397 closer to?



Suppose we want to round 397 to the nearest ten. We identify the tens digit and look at the next digit to the right.

↓

3 9 7

The digit to the right is 5 or more.
If this digit is 9, and it must be increased by 1, replace the 9 with 0 and increase the next digit to the *left* by 1.

So 397 is rounded to 400.

Check Yourself 6

Round 4,961 to the nearest hundred.

NOTE

An estimate is basically a good guess. If your answer is close to your estimate, then your answer is reasonable.

Whether you are doing an addition problem by hand or using a calculator, rounding numbers gives you a handy way of deciding whether an answer seems reasonable. The process is called **estimating,** which we illustrate with an example.

Example 7 Estimating a Sum

< Objective 2 >

NOTE

Placing an arrow above the column to be rounded can be helpful.

Begin by rounding to the nearest hundred.

$$\begin{array}{rr} 456 & 500 \\ 235 & 200 \\ 976 & 1{,}000 \\ +\ 344 & +\ 300 \\ \hline 2{,}011 & 2{,}000 \end{array}$$

← Estimate

By rounding to the nearest hundred and adding quickly, we get an estimate or guess of 2,000. Because this is close to the sum calculated, 2,011, the sum seems reasonable.

Check Yourself 7

Round each addend to the nearest hundred and estimate the sum. Then find the actual sum.

287 + 526 + 311 + 378

Estimation is a wonderful tool to use while you're shopping. Every time you go to the store, you should try to estimate the total bill by rounding the price of each item. If you do this regularly, both your addition skills and your rounding skills will improve. The same holds true when you eat in a restaurant. It is always a good idea to know approximately how much you are spending.

Example 8 Estimating a Sum in a Word Problem

Samantha has taken the family out to dinner, and she's now ready to pay the bill. The dinner check has no total, only the individual entries, as given below:

Soup	$2.95
Soup	2.95
Salad	1.95
Salad	1.95
Salad	1.95
Lasagna	7.25
Spaghetti	4.95
Ravioli	5.95

What is the approximate cost of the dinner?

Rounding each entry to the nearest whole dollar, we can estimate the total by finding the sum

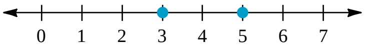
$3 + 3 + 2 + 2 + 2 + 7 + 5 + 6 = \$30$

Check Yourself 8

Jason is doing the weekly food shopping at FoodWay. So far his basket has items that cost $3.99, $7.98, $2.95, $1.15, $2.99, and $1.95. Approximate the total cost of these items.

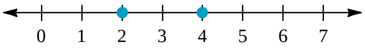
Earlier in this section, we used the number line to illustrate the idea of rounding numbers. The number line also gives us an excellent way to picture the concept of **order** for whole numbers, which means that numbers become larger as we move from left to right on the number line.

For instance, we know that 3 is less than 5. On the number line



we see that 3 lies *to the left* of 5.

We also know that 4 is greater than 2. On the number line



we see that 4 lies *to the right* of 2.

Two symbols, $<$ for "less than" and $>$ for "greater than," are used to indicate these relationships.

NOTE

The inequality symbol always "points at" the smaller number.

Definition

Inequalities

For whole numbers, we can write

1. $2 < 5$ (read "2 is less than 5") because 2 is *to the left* of 5 on the number line.
2. $8 > 3$ (read "8 is greater than 3") because 8 is *to the right* of 3 on the number line.

Example 9 illustrates the use of this notation.

Example 9 Indicating Order with $<$ or $>$

< Objective 3 >

Use the symbol $<$ or $>$ to complete each statement.

(a) 7 _____ 10 **(b)** 25 _____ 20

(c) 200 _____ 300 **(d)** 8 _____ 0

(a) $7 < 10$ 7 lies to the left of 10 on the number line.

(b) $25 > 20$ 25 lies to the right of 20 on the number line.

(c) $200 < 300$

(d) $8 > 0$

Check Yourself 9

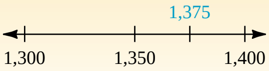
Use one of the symbols $<$ and $>$ to complete each of the following statements.

(a) 35 ___ 25 **(b)** 0 ___ 4

(c) 12 ___ 18 **(d)** 1,000 ___ 100

Check Yourself ANSWERS

1. 600 **2.**



Round 1,375 *up* to 1,400.

3. 850 **4.** 3,500 **5.** **(a)** 570; **(b)** 5,400 **6.** 5,000 **7.** 1,500; 1,502

8. $21 **9.** **(a)** $35 > 25$; **(b)** $0 < 4$; **(c)** $12 < 18$; **(d)** $1{,}000 > 100$

Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 1.4

(a) The practice of expressing numbers to the nearest hundred, thousand, and so on is called __________.

(b) The first step in rounding is to identify the ____________ ____________ of the digit to be rounded.

(c) The number line gives us an excellent way to picture the concept of ______________ for whole numbers.

(d) The symbol $<$ is read as "______________ than."

1.4 exercises

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Section __________ Date __________

Answers

1. __________ 2. __________

3. __________ 4. __________

5. __________ 6. __________

7. __________ 8. __________

9. __________ 10. __________

11. __________ 12. __________

13. __________ 14. __________

15. __________ 16. __________

17. __________ 18. __________

19. ____________________

20. ____________________

21. ____________________

22. ____________________

23. ____________________

24. ____________________

25. ____________________

26. ____________________

< Objective 1 >

Round each number to the indicated place.

1. 38, the nearest ten

> Videos

2. 72, the nearest ten

3. 253, the nearest ten

4. 578, the nearest ten

5. 696, the nearest ten

6. 683, the nearest hundred

7. 3,482, the nearest hundred

> Videos

8. 6,741, the nearest hundred

9. 5,962, the nearest hundred

10. 4,352, the nearest thousand

11. 4,927, the nearest thousand

12. 39,621, the nearest thousand

13. 23,429, the nearest thousand

> Videos

14. 38,589, the nearest thousand

15. 787,000, the nearest ten thousand

16. 582,000, the nearest hundred thousand

17. 21,800,000, the nearest million

18. 931,000, the nearest ten thousand

< Objective 2 >

In exercises 19 to 30, estimate each sum or difference by rounding to the indicated place. Then do the addition or subtraction and use your estimate to see if your actual sum or difference seems reasonable.

Round to the nearest ten.

19.
$$\begin{array}{r} 58 \\ 27 \\ +\ 33 \\ \hline \end{array}$$

20.
$$\begin{array}{r} 92 \\ 37 \\ 85 \\ +\ 64 \\ \hline \end{array}$$

21.
$$\begin{array}{r} 83 \\ -\ 27 \\ \hline \end{array}$$

22.
$$\begin{array}{r} 97 \\ -\ 31 \\ \hline \end{array}$$

Round to the nearest hundred.

23.
$$\begin{array}{r} 379 \\ 1{,}215 \\ +\ \ 528 \\ \hline \end{array}$$

24.
$$\begin{array}{r} 967 \\ 2{,}365 \\ 544 \\ +\ \ 738 \\ \hline \end{array}$$

25.
$$\begin{array}{r} 915 \\ -\ 411 \\ \hline \end{array}$$

26.
$$\begin{array}{r} 697 \\ -\ 539 \\ \hline \end{array}$$

Basic Skills | **Challenge Yourself** | Calculator/Computer | Career Applications | Above and Beyond

Round to the nearest thousand.

27.
$$\begin{array}{r} 2,238 \\ 3,925 \\ \underline{+\ 5,217} \end{array}$$

28.
$$\begin{array}{r} 3,678 \\ 4,215 \\ \underline{+\ 2,032} \end{array}$$

29.
$$\begin{array}{r} 4,822 \\ \underline{-\ 2,134} \end{array}$$

30.
$$\begin{array}{r} 6,120 \\ \underline{-\ 4,890} \end{array}$$

Solve each application.

31. Business and Finance Ed and Sharon go to lunch. The lunch check has no total but only lists individual items:

Soup $1.95	Soup $1.95
Salad $1.80	Salad $1.80
Salmon $8.95	Flounder $6.95
Pecan pie $3.25	Vanilla ice cream $2.25

Estimate the total amount of the lunch check.

32. Business and Finance Olivia will purchase several items at the stationery store. Thus far, the items she has collected cost $2.99, $6.97, $3.90, $2.15, $9.95, and $1.10. Approximate the total cost of these items.

33. Statistics Oscar scored 78, 91, 79, 67, and 100 on his arithmetic tests. Round each score to the nearest ten to estimate his total score.

> Videos

34. Business and Finance Luigi's pizza parlor makes 293 pizzas on an average day. Estimate (to the nearest hundred) how many pizzas were made on a three-day holiday weekend.

35. Business and Finance Mrs. Gonzalez went shopping for clothes. She bought a sweater for $32.95, a scarf for $9.99, boots for $68.29, a coat for $125.90, and socks for $18.15. Estimate the total amount of Mrs. Gonzalez's purchases.

36. Business and Finance Amir bought several items at the hardware store: hammer, $8.95; screwdriver, $3.15; pliers, $6.90; wire cutters, $4.25; and sandpaper; $1.89. Estimate the total cost of Amir's bill.

< Objective 3 >

Use the symbol $<$ or $>$ to complete each statement.

37. 500 _____ 400

> Videos

38. 20 _____ 15

39. 100 _____ 1,000

40. 3,000 _____ 2,000

Answers

27. ____________

28. ____________

29. ____________

30. ____________

31. ____________

32. ____________

33. ____________

34. ____________

35. ____________

36. ____________

37. ____________

38. ____________

39. ____________

40. ____________

Answers

41. ____________________

42. ____________________

43. ____________________

44. ____________________

45. ____________________

46. ____________________

47. ____________________

48. ____________________

Basic Skills | Challenge Yourself | Calculator/Computer | **Career Applications** | Above and Beyond

Use the following chart for exercises 41 and 42.

Appliance	Power Required (in Watts/hour [W/h])
Clock radio	10
Electric blanket	100
Clothes washer	500
Toaster oven	1,225
Laptop	50
Hair dryer	1,875
DVD player	25

41. Electronics Assuming all the appliances listed in the table are "on," estimate the total power required to the nearest hundred watts.

42. Electronics Which combination uses more power, the toaster oven and clothes washer or the hair dryer and DVD player?

43. Manufacturing Technology An inventory of machine screws shows that bin 1 contains 378 screws, bin 2 contains 192 screws, and bin 3 contains 267 screws. Estimate the total number of screws in the bins.

44. Manufacturing Technology A delivery truck must be loaded with the heaviest crates starting in the front to the lightest crates in the back. On Monday, crates weighing 378 pounds (lb), 221 lb, 413 lb, 231 lb, 208 lb, 911 lb, 97 lb, 188 lb, and 109 lb need to be shipped. In what order should the crates be loaded?

45. Number Problem A whole number rounded to the nearest ten is 60. **(a)** What is the smallest possible number? **(b)** What is the largest possible number?

46. Number Problem A whole number rounded to the nearest hundred is 7,700. **(a)** What is the smallest possible number? **(b)** What is the largest possible number?

Basic Skills | Challenge Yourself | Calculator/Computer | Career Applications | **Above and Beyond**

47. Statistics A bag contains 60 marbles. The number of blue marbles, rounded to the nearest 10, is 40, and the number of green marbles in the bag, rounded to the nearest 10, is 20. How many blue marbles are in the bag? (List all answers that satisfy the conditions of the problem.)

48. Social Science Describe some situations in which estimating and rounding would not produce a result that would be suitable or acceptable. Review the instructions for filing your federal income tax. What rounding rules are used in the preparation of your tax returns? Do the same rules apply to the filing of your state tax returns? If not, what are these rules?

49. The listed population of the United States on July 8, 2005, at 9:37 A.M. eastern standard time (EST) was 296,562,576 people. Round this number to the nearest ten million.

chapter 1 > Make the Connection

50. According to the U.S. Census Bureau, the population of the world was believed to be 6,457,380,056 on August 1, 2005. Round this number to the nearest million.

chapter 1 > Make the Connection

Answers

1. 40 **3.** 250 **5.** 700 **7.** 3,500 **9.** 6,000
11. 5,000 **13.** 23,000 **15.** 790,000 **17.** 22,000,000
19. Estimate: 120, actual sum: 118 **21.** Estimate: 50, actual difference: 56
23. Estimate: 2,100, actual sum: 2,122
25. Estimate: 500; actual difference: 504
27. Estimate: 11,000, actual sum: 11,380
29. Estimate: 3,000, actual difference: 2,688 **31.** $29 **33.** 420
35. $255 **37.** > **39.** < **41.** 3,800 W **43.** 900 screws
45. **(a)** 55; **(b)** 64 **47.** 36, 37, 38, 39, 40, 41, 42, 43, 44
49. 300,000,000 people

Answers

49. ____________________

50. ____________________

1.5

Multiplication

< **1.5 Objectives** >

1 > Multiply whole numbers
2 > Use the properties of multiplication
3 > Solve applications of multiplication
4 > Estimate products
5 > Find area and volume using multiplication

NOTE

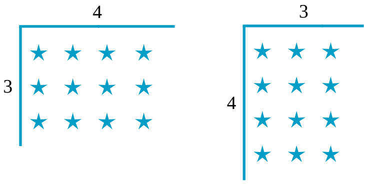
The use of the symbol × dates back to the 1600s.

NOTE

A centered dot is used the same as the times sign (×). We use the centered dot when we are using letters to represent numbers, as we have done with *a* and *b* here. We do that so the times sign will not be confused with the letter *x*.

Our work in this section deals with multiplication, another of the basic operations of arithmetic. Multiplication is closely related to addition. In fact, we can think of multiplication as a shorthand method for repeated addition. The symbol × is used to indicate multiplication.

3×4 can be interpreted as 3 rows of 4 objects. By counting we see that $3 \times 4 = 12$. Similarly, 4 rows of 3 means $4 \times 3 = 12$.



The fact that $3 \times 4 = 4 \times 3$ is an example of the **commutative property of multiplication,** which is given here.

Property

The Commutative Property of Multiplication

Given any two numbers, we can multiply them in either order and we get the same result.

In symbols, we say $a \cdot b = b \cdot a$.

Example 1 Multiplying Single-Digit Numbers

< **Objective 1** >

3×5 means 5 multiplied by 3. It is read 3 *times* 5. To find 3×5, we can add 5 three times.

$3 \times 5 = 5 + 5 + 5 = 15$

In a multiplication problem such as $3 \times 5 = 15$, we call 3 and 5 the **factors.** The answer, 15, is the **product** of the factors, 3 and 5.

$$\underset{\text{Factor}}{3} \times \underset{\text{Factor}}{5} = \underset{\text{Product}}{15}$$

Check Yourself 1

Name the factors and the product in the following statement.

2 × 9 = 18

Statements such as 3 × 4 = 12 and 3 × 5 = 15 are called the **basic multiplication facts.** If you have difficulty with multiplication, it may be that you do not know some of these facts. The following table will help you review before you go on. Notice that, because of the commutative property, you need memorize only half of these facts!

Basic Multiplication Facts Table

×	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

NOTE

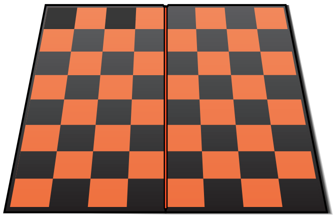
To use the table to find the product of 7 × 6: Find the row labeled 7, and then move to the right in this row until you are in the column labeled 6 at the top. We see that 7 × 6 is 42.

Armed with these facts, you can become a better, and faster, problem solver. Take a look at Example 2.

Example 2 Multiplying Instead of Counting

NOTE

This checkerboard is an example of a rectangular array, a series of rows or columns that form a rectangle. When you see such an arrangement, multiply to find the total number of units.

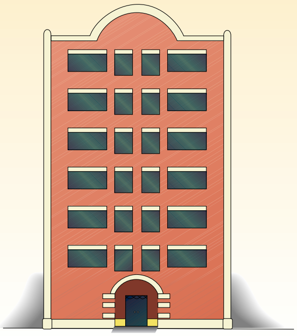


Find the total number of squares on the checkerboard.

You could find the number of squares by counting them. If you counted one per second, it would take you just over a minute. You could make the job a little easier by simply counting the squares in one row (8), and then adding 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8. Multiplication, which is simply repeated addition, allows you to find the total number of squares by multiplying 8 × 8. How long that takes depends on how well you know the basic multiplication facts! By now, you know that there are 64 squares on the checkerboard.

Check Yourself 2

Find the number of windows on the displayed side of the building.



The next property involves *both* multiplication and addition.

Example 3 Using the Distributive Property

< Objective 2 >

NOTE

Multiplication can also be indicated by using parentheses. A number followed by parentheses or back-to-back parentheses represent multiplication.

$2 \times (3 + 4)$ could be written as $2(3 + 4)$ or $(2)(3 + 4)$.

$$2 \times (3 + 4) = 2 \times 7 = 14$$

We have added 3 + 4 and then multiplied.

Also,

$$\begin{aligned} 2 \times (3 + 4) &= (2 \times 3) + (2 \times 4) \\ &= 6 + 8 \\ &= 14 \end{aligned}$$

We have multiplied 2 × 3 and 2 × 4 as the first step.

The result is the same.

We see that $2 \times (3 + 4) = (2 \times 3) + (2 \times 4)$. This is an example of the **distributive property of multiplication over addition** because we distributed the multiplication (in this case by 2) over the "plus" sign.

Property

The Distributive Property of Multiplication over Addition

To multiply a factor by a sum of numbers, multiply the factor by each number inside the parentheses. Then add the products. (The result will be the same if we find the sum and then multiply.)

In symbols, we say $a \cdot (b + c) = a \cdot b + a \cdot c$

Check Yourself 3

Show that

$3 \times (5 + 2) = (3 \times 5) + (3 \times 2)$

Regrouping must often be used to multiply larger numbers. We see how regrouping works in multiplication by looking at an example in the expanded form. When regrouping results in changing a digit to the left we sometimes say we "carry" the units.

Example 4 Multiplying by a Single-Digit Number

NOTE

3×25

$3 \cdot 25$

$(3)(25)$

all mean the same thing.

$$\begin{aligned} 3 \times 25 &= 3 \times (20 + 5) \\ &= 3 \times 20 + 3 \times 5 \\ &= 60 \quad \underbrace{+\ 15} \\ &= \underbrace{60 + 10} + 5 \\ &= 70 + 5 \\ &= 75 \end{aligned}$$

We use the distributive property again.

Write the 15 as 10 + 5.

Carry 10 ones or 1 ten to the tens place.

Here is the same multiplication problem using the short form.

Step 1

$$\begin{array}{r} {\scriptstyle 1} \\ 25 \\ \times\ 3 \\ \hline 5 \end{array}$$

1 ← Carry

Multiplying 3 × 5 gives us 15 ones. Write 5 ones and carry 1 ten.

Step 2

$$\begin{array}{r} {\scriptstyle 1} \\ 25 \\ \times\ 3 \\ \hline 75 \end{array}$$

Now multiply 3 × 2 tens and add the carry to get 7, the tens digit of the product.

Check Yourself 4

Multiply.

(a) $\begin{array}{r} 34 \\ \times\ 6 \\ \hline \end{array}$

(b) $\begin{array}{r} 43 \\ \times\ 7 \\ \hline \end{array}$

Units ANALYSIS

When you multiply a denominate number, such as 6 feet (ft), by an abstract number, such as 5, the result has the same units as the denominate number. Some examples are

5×6 ft $= 30$ ft

$3 \times \$7 = \$21$

9×4 A's $= 36$ A's

When you multiply two different denominate numbers, the units must also be multiplied. We will discuss this when we look at the area of geometric figures.

RECALL

It is best to write down the complete statement necessary for the solution of an application.

We briefly review our discussion of applications, or word problems.

As you will see, the process of solving applications is the same no matter which operation is required for the solution. In fact, the four-step procedure we suggested in Section 1.2 can be effectively applied here.

Step by Step

Solving Applications

Step 1	Read the problem carefully to determine the given information and what you are asked to find.
Step 2	Decide upon the operation or operations to be used.
Step 3	Write down the complete statement necessary to solve the problem and do the calculations.
Step 4	Check to make sure you have answered the question of the problem and that your answer seems reasonable.

Example 5 Solving an Application Involving Multiplication

< Objective 3 >

A car rental agency orders a fleet of 7 new subcompact cars at a cost of \$14,258 per automobile. What will the company pay for the entire order?

Step 1 We know the number of cars and the price per car. We want to find the total cost.

Step 2 Multiplication is the best approach to the solution.

Step 3 Write

$$7 \times \$14{,}258 = \$99{,}806$$

We could, of course, *add* \$14,258, the cost, 7 times, but multiplication is certainly more efficient.

Step 4 The total cost of the order is \$99,806.

Check Yourself 5

Tires sell for \$47 apiece. What is the total cost for 5 tires?

To multiply by numbers with more than one digit, we must multiply each digit of the first factor by each digit of the second. To do this, we form a series of partial products and then add them to arrive at the final product.

Example 6 Multiplying by a Two-Digit Number

Multiply 56 × 47.

Step 1

$$\begin{array}{r} {\scriptstyle 4}\\ 56\\ \times\ 47\\ \hline 392 \end{array}$$

The first partial product is 7 × 56, or 392. Note that we had to carry 4 to the tens column.

Step 2

$$\begin{array}{r} {\scriptstyle 2}\\ {\scriptstyle \cancel{4}}\\ 56\\ \times\ 47\\ \hline 392\\ 2240\\ \hline \end{array}$$

The second partial product is 40 × 56, or 2,240. We must carry 2 during the process.

Step 3

$$\begin{array}{r} {\scriptstyle 2}\\ {\scriptstyle \cancel{4}}\\ 56\\ \times\ 47\\ \hline 392\\ 2240\\ \hline 2{,}632 \end{array}$$

We add the partial products for our final result.

Check Yourself 6

Multiply.

$$\begin{array}{r} 38\\ \times\ 76\\ \hline \end{array}$$

If multiplication involves two three-digit numbers, another step is necessary. In this case we form three partial products. This will ensure that each digit of the first factor is multiplied by each digit of the second.

Example 7 Multiplying Two Three-Digit Numbers

Multiply.

$$\begin{array}{r} {\scriptstyle 22}\\ {\scriptstyle 33}\\ {\scriptstyle 22}\\ 278\\ \times\ 343\\ \hline 834\\ 11120\\ 83400\\ \hline 95{,}354 \end{array}$$

In forming the third partial product, we must multiply by 300.

NOTE

The three partial products are formed when we multiply by the ones, tens, and then the hundreds digits.

Check Yourself 7

Multiply.

$$\begin{array}{r} 352\\ \times\ 249\\ \hline \end{array}$$

Next, look at an example of multiplying by a number involving 0 as a digit. There are several ways to arrange the work, as our example shows.

Example 8 Multiplying Larger Numbers

Multiply 573 × 205.

Method 1

$$\begin{array}{r} \scriptstyle 1 \\ \scriptstyle 31 \\ 573 \\ \times\ 205 \\ \hline 2865 \\ 0000 \\ 114600 \\ \hline 117{,}465 \end{array}$$

We can write the second partial product as 0000 to indicate the multiplication by 0 in the tens place.

Here is a second approach to the problem.

Method 2

$$\begin{array}{r} \scriptstyle 1 \\ \scriptstyle 31 \\ 573 \\ \times\ 205 \\ \hline 2865 \\ 114600 \\ \hline 117{,}465 \end{array}$$

We can write a double 0 as our second step. If we place the third partial product on the same line, that product will be shifted *two* places left, indicating that we are multiplying by 200.

Because this second method is more compact, it is usually used.

Check Yourself 8

Multiply.

$$\begin{array}{r} 489 \\ \times\ 304 \\ \hline \end{array}$$

Example 9 will lead us to another property of multiplication.

Example 9 Using the Associative Property

$$\overbrace{(2 \times 3)} \times 4 = 6 \times 4 = 24$$

We do the multiplication in the parentheses first, 2 × 3 = 6. Then we multiply 6 × 4.

Also,

$$2 \times \overbrace{(3 \times 4)} = 2 \times 12 = 24$$

Here we multiply 3 × 4 as the first step. Then we multiply 2 × 12.

We see that

$$(2 \times 3) \times 4 = 2 \times (3 \times 4)$$

The product is the same no matter which way we *group* the factors. This is called the **associative property** of multiplication.

Property

The Associative Property of Multiplication

Multiplication is an *associative* operation. The way in which you group numbers in multiplication does not affect the final product.

Check Yourself 9

Find the products.

(a) (5 × 3) × 6 (b) 5 × (3 × 6)

There are some shortcuts that let you simplify your work when multiplying by a number that ends in 0. Let's see what we can discover by looking at some examples.

Example 10 Multiplying by 10

First we multiply by 10.

$$\begin{array}{r} 67 \\ \times\ 10 \\ \hline 670 \end{array}$$

$10 \times 67 = 670$

Next we multiply by 100.

$$\begin{array}{r} 537 \\ \times\ 100 \\ \hline 53{,}700 \end{array}$$

$100 \times 537 = 53{,}700$

Finally, we multiply by 1,000.

$$\begin{array}{r} 489 \\ \times\ 1{,}000 \\ \hline 489{,}000 \end{array}$$

$1{,}000 \times 489 = 489{,}000$

Check Yourself 10

Multiply.

(a) $\begin{array}{r} 257 \\ \times\ 100 \\ \hline \end{array}$ (b) $\begin{array}{r} 2{,}436 \\ \times\ 1{,}000 \\ \hline \end{array}$

NOTE

We talk about powers of 10 in greater detail in Section 1.7.

Do you see a pattern? Rather than writing out the multiplication, there is an easier way! We call the numbers 10, 100, 1,000, and so on **powers of 10.**

Property

Multiplying by Powers of 10

When a natural number is multiplied by a power of 10, the product is just that number followed by as many zeros as there are in the power of 10.

Example 11 Multiplying by Numbers That End in Zero

Multiply 400 × 678.

Write

$$\begin{array}{r} 678 \\ \times\quad 400 \\ \hline \end{array}$$

Shift 400 so that the two zeros are *to the right* of the digits above.

$$\begin{array}{r} {\scriptstyle 33} \\ 678 \\ \times\quad 400 \\ \hline 271{,}200 \end{array}$$

Bring down the two zeros, then multiply 4×678 to find the product.

There is no mystery about why this works. We know that 400 is 4 × 100. In this method, we are multiplying 678 by 4 and then by 100, adding two zeros to the product by our earlier rule.

Check Yourself 11

Multiply.

300 × 574

Your work in this section, together with our earlier rounding techniques, provides a convenient means of using estimation to check the reasonableness of our results in multiplication, as Example 12 illustrates.

Example 12 Estimating a Product by Rounding

< Objective 4 >

Estimate the following product by rounding each factor to the nearest hundred.

$$\begin{array}{rcr} & & \text{Rounded} \\ 512 & \longrightarrow & 500 \\ \times\ 289 & \longrightarrow & \times\ 300 \\ \hline & & 150{,}000 \end{array}$$

You might want to find the *actual* product and use our estimate to see if your result seems reasonable.

Check Yourself 12

Estimate the product by rounding each factor to the nearest hundred.

$$\begin{array}{r} 689 \\ \times\ 425 \\ \hline \end{array}$$

Rounding the factors can be a very useful way of estimating the solution to an application problem.

Example 13 Estimating the Solution to a Multiplication Application

Bart is thinking of running an ad in the local newspaper for an entire year. The ad costs $19.95 per week. Approximate the annual cost of the ad.

Rounding the charge to $20 and rounding the number of weeks in a year to 50, we get

$50 \times 20 = 1{,}000$

The ad would cost approximately $1,000.

Check Yourself 13

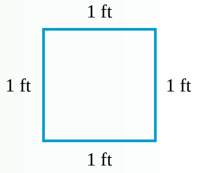
Phyllis is debating whether to join the health club for $400 per year or just pay $9 per visit. If she goes about once a week, approximately how much would she spend at $9 per visit?

Units ANALYSIS

What happens when we multiply two denominate numbers? The units of the result turn out to be the product of the units. This makes sense when we look at an example from geometry.

The area of a square is the square of one side. As a formula, we write that as

$A = s^2$



This tile is 1 ft by 1 ft.

$A = s^2 = (1\text{ ft})^2 = 1\text{ ft} \times 1\text{ ft} = 1\ (\text{ft}) \times (\text{ft}) = 1\text{ ft}^2$

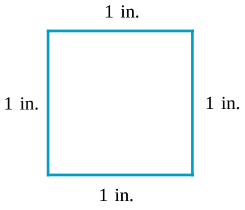
In other words, its area is one square foot (1 ft^2).

If we want to find the area of a room, we are actually finding how many of these square feet can be placed in the room.

NOTE

The small, raised 2 is called an *exponent* or *power.* This will be studied in Section 1.7. So in. × in. can be written in.^2 and is read "square inches."

Now look at the idea of **area.** Area is a measure that we give to a surface. It is measured in terms of **square units.** The area is the number of square units that are needed to cover the surface.



One square inch

One standard unit of area measure is the **square inch** (written in.^2). This is the measure of the surface contained in a square with sides of 1 in.

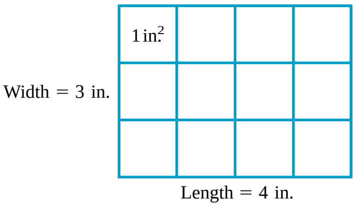
Other units of area measure are the square foot (ft^2), the square yard (yd^2), the square centimeter (cm^2), and the square meter (m^2).

Finding the area of a figure means finding the number of square units it contains. One simple case is a rectangle.

NOTE

The length and width must be in terms of the same unit.

The figure to the right shows a rectangle. The length of the rectangle is 4 in., and the width is 3 in. The area of the rectangle is measured in terms of square inches. We can simply count to find the area, 12 square inches (in.^2). However, because each of the four vertical strips contains 3 in.^2, we can multiply:



Area = 4 in. × 3 in. = 12 in.^2

Property

Formula for the Area of a Rectangle

In general, we can write the formula for the **area of a rectangle:** If the length of a rectangle is L units and the width is W units, then the formula for the area A of the rectangle can be written as

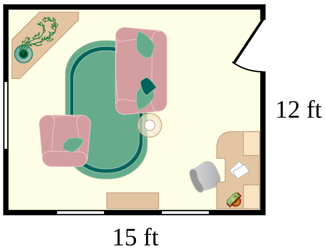
$A = L \cdot W$ (square units)

Example 14 Find the Area of a Rectangle

< Objective 5 >

A room has dimensions 12 ft by 15 ft. Find its area.

Use the area formula, with $L = 15$ ft and $W = 12$ ft.



$A = L \cdot W$

$= 15\ \text{ft} \times 12\ \text{ft} = 180\ \text{ft}^2$

The area of the room is 180 ft^2.

NOTE

s^2 is read "s squared."

Check Yourself 14

A desktop has dimensions 50 in. by 25 in. What is the area of its surface?

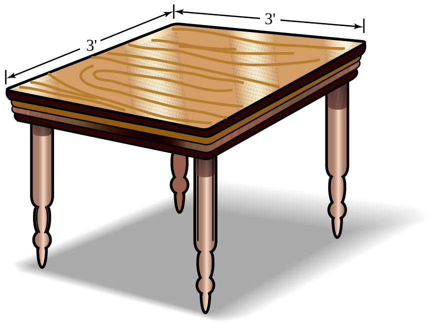
We can also write a convenient formula for the area of a square. If the sides of the square have length s, we can write

Property

Formula for the Area of a Square

$A = s \cdot s = s^2$

Example 15 Finding the Area



You wish to cover a square table with a plastic laminate that costs 60¢ a square foot. If each side of the table measures 3 ft, what will it cost to cover the table?

We first must find the area of the table. Use the area formula with $s = 3$ ft.

$$A = s^2$$
$$= (3 \text{ ft})^2 = 3 \text{ ft} \times 3 \text{ ft} = 9 \text{ ft}^2$$

Now, multiply by the cost per square foot.

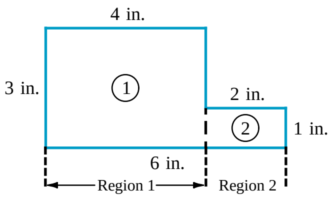
Cost = $9 \times 60¢ = 540¢ = \$5.40$

Check Yourself 15

You wish to carpet a room that is a square, 4 yd by 4 yd, with carpet that costs $12 per square yard. What will be the total cost of the carpeting?

Sometimes the total area of an oddly shaped figure is found by adding the smaller areas. Example 16 shows how this is done.

Example 16 Finding the Area of an Oddly Shaped Figure



Find the area of the figure.

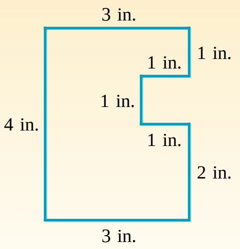
The area of the figure is found by adding the areas of regions 1 and 2. Region 1 is a 4 in. by 3 in. rectangle; the area of region 1 = 4 in. × 3 in. = 12 in.^{2}. Region 2 is a 2 in. by 1 in. rectangle; the area of region 2 = 2 in. × 1 in. = 2 in.^{2}.

The total area is the sum of the two areas:

Total area = 12 in.^{2} + 2 in.^{2} = 14 in.^{2}

Check Yourself 16

Find the area of the figure.

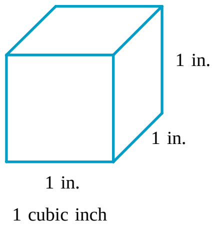


Hint: You can find the area by adding the areas of three rectangles, or by subtracting the area of the "missing" rectangle from the area of the "completed" larger rectangle.

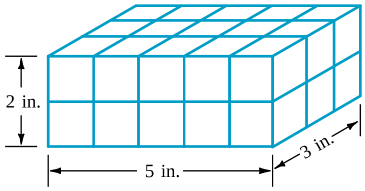
Our next measurement deals with finding the **volume of a solid.** The volume of a solid measures the amount of space contained by the solid.

Definition

Solid A **solid** is a three-dimensional figure. It has length, width, and height.



1 cubic inch



Volume is measured in **cubic units.** Examples include cubic inches (in.^{3}), cubic feet (ft^3), and cubic centimeters (cm^3). A cubic inch, for instance, is the measure of the space contained in a cube that is 1 in. on each edge. See the figure to the left.

In finding the volume of a figure, we want to know how many cubic units are contained in that figure. Let's start with a simple example, a **rectangular solid.** A rectangular solid is a very familiar figure. A box, a crate, and most rooms are rectangular solids. Say that the dimensions of the solid are 5 in. by 3 in. by 2 in. as pictured in the figure to the left. If we divide the solid into units of 1 in.^{3}, we have two layers, each containing 3 units by 5 units, or 15 in.^{3} Because there are two layers, the volume is 30 in.^{3}

In general, we can see that the volume of a rectangular solid is the product of its length, width, and height.

Property

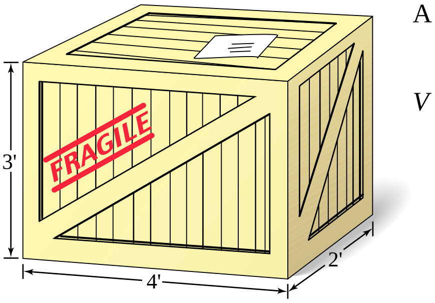
Formula for the Volume of a Rectangular Solid

The volume V of a rectangular solid with length L, width W, and height H is given by the product of the length, width, and height.

$$V = L \cdot W \cdot H$$

Volume is measured in cubic units.

Example 17 Finding the Volume



A crate has dimensions 4 ft by 2 ft by 3 ft. Find its volume.

We use the volume formula with $L = 4$ ft, $W = 2$ ft, and $H = 3$ ft.

$$V = L \cdot W \cdot H$$
$$= 4 \text{ ft} \times 2 \text{ ft} \times 3 \text{ ft}$$
$$= 24 \text{ ft}^3$$

The volume of the crate is 24 cubic feet.

Check Yourself 17

A room is 15 ft long, 10 ft wide, and 8 ft high. What is its volume?

Check Yourself ANSWERS

1. Factors 2, 9; product 18 **2.** 24 **3.** $3 \times 7 = 21$ and $15 + 6 = 21$
4. (a) 204; **(b)** 301 **5.** \$235 **6.** 2,888 **7.** 87,648 **8.** 148,656
9. **(a)** 90; **(b)** 90 **10.** **(a)** 25,700; **(b)** 2,436,000 **11.** 172,200
12. 280,000 **13.** \$500 **14.** 1,250 in.^{2} **15.** \$192 **16.** 11 in.^{2}
17. 1,200 ft^3

Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 1.5

(a) The final step in solving an application is to make certain that the answer is ________.

(b) The way in which you group numbers in multiplication does not affect the final ____________.

(c) The numbers 10, 100, and 1,000 are called ____________ of 10.

(d) We can write the equation for the area of a ____________ as $A = L \cdot W$.

1.5 exercises

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Name ______________

Section ________ Date ________

Answers

1. ________ 2. ________
3. ________ 4. ________
5. ________ 6. ________
7. ________ 8. ________
9. ________ 10. ________
11. ________ 12. ________
13. ________ 14. ________
15. ________ 16. ________
17. ________ 18. ________
19. ________ 20. ________
21. ________ 22. ________
23. ________ 24. ________
25. ________ 26. ________
27. ________ 28. ________
29. ________ 30. ________
31. ________ 32. ________
33. ________ 34. ________

35. ________________

36. ________________

37. ________________

38. ________________

< Objective 1 >

Multiply.

1. $\begin{array}{r} 5 \\ \times\ 3 \\ \hline \end{array}$

2. $\begin{array}{r} 7 \\ \times\ 4 \\ \hline \end{array}$

3. $6 \cdot 0$

4. $6 \cdot 6$

5. 4×48

6. 5×53

7. $\begin{array}{r} 508 \\ \times\ \ \ 6 \\ \hline \end{array}$

8. $\begin{array}{r} 903 \\ \times\ \ \ 9 \\ \hline \end{array}$ > Videos

9. $\begin{array}{r} 75 \\ \times\ 68 \\ \hline \end{array}$

10. 235×49

11. 327×59 > Videos

12. $2{,}364 \cdot 67$

13. $4{,}075 \cdot 84$

14. $\begin{array}{r} 315 \\ \times\ 243 \\ \hline \end{array}$

15. $\begin{array}{r} 124 \\ \times\ 225 \\ \hline \end{array}$

16. 345×267

17. 639×358

18. $547 \cdot 203$

19. $668 \cdot 305$

20. $\begin{array}{r} 2{,}458 \\ \times\ 135 \\ \hline \end{array}$

21. $\begin{array}{r} 3{,}219 \\ \times\ 207 \\ \hline \end{array}$

22. $\begin{array}{r} 2{,}534 \\ \times\ 3{,}106 \\ \hline \end{array}$

23. $\begin{array}{r} 3{,}158 \\ \times\ 2{,}034 \\ \hline \end{array}$

24. $\begin{array}{r} 43 \\ \times\ 70 \\ \hline \end{array}$

25. $\begin{array}{r} 58 \\ \times\ 40 \\ \hline \end{array}$

26. $\begin{array}{r} 562 \\ \times\ 400 \\ \hline \end{array}$

27. $\begin{array}{r} 907 \\ \times\ 900 \\ \hline \end{array}$

28. $\begin{array}{r} 345 \\ \times\ 230 \\ \hline \end{array}$

29. $\begin{array}{r} 362 \\ \times\ 310 \\ \hline \end{array}$

30. $\begin{array}{r} 157 \\ \times\ 3{,}200 \\ \hline \end{array}$

31. Find the product of 304 and 7. > Videos

32. Find the product of 8 and 5,679.

33. What is 21 multiplied by 551?

34. What is 135 multiplied by 507?

< Objective 2 >

Name the property of addition and/or multiplication that is illustrated.

35. $5 \times 8 = 8 \times 5$ > Videos

36. $3 \times (4 + 9) = (3 \times 4) + (3 \times 9)$ > Videos

37. $2 \times (3 \times 5) = (2 \times 3) \times 5$

38. $5 \times (6 + 2) = 5 \times (2 + 6)$

In exercises 39 and 40, complete the statement, using the given property.

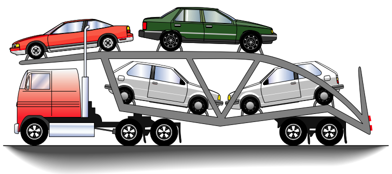
39. $7 + (3 \times 8) =$ Commutative property of multiplication

40. $3 \times (2 + 7) =$ Distributive property

< Objective 3 >

Solve each application.

41. Business and Finance A convoy company can transport 8 new cars on one of its trucks. If 34 truck shipments were made in one week, how many cars were shipped?



42. Business and Finance A computer printer can print 40 mailing labels per minute. How many labels can be printed in 1 hour (h)?

43. Social Science A rectangular parking lot has 14 rows of parking spaces, and each row contains 24 spaces. How many cars can be parked in the lot?

44. Statistics A petition to get Tom on the ballot for treasurer of student council has 28 signatures on each of 43 pages. How many signatures were collected?

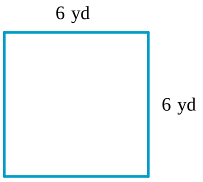
45. Business and Finance The manufacturer of wood-burning stoves can make 15 stoves in 1 day. How many stoves can be made in 28 days?

46. Business and Finance Each bundle of newspapers contains 25 papers. If 43 bundles are delivered to Jose's house, how many papers are delivered?

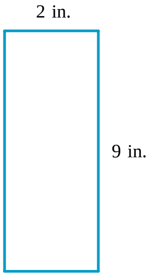
< Objective 5 >

Find the area of each figure.

47.



48.



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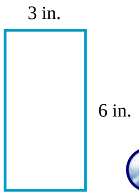
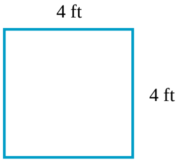
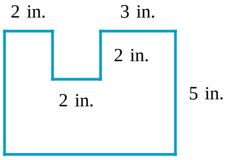


> Videos

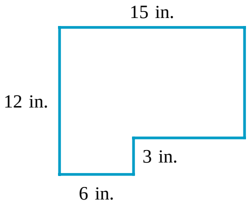
50.



51.



52.



Answers

39. ______

40. ______

41. ______

42. ______

43. ______

44. ______

45. ______

46. ______

47. ______

48. ______

49. ______

50. ______

51. ______

52. ______

Answers

53. ____________

54. ____________

55. ____________

56. ____________

57. ____________

58. ____________

59. ____________

60. ____________

61. ____________

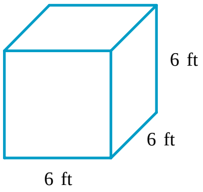
62. ____________

63. ____________

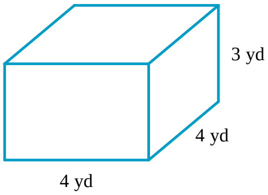
64. ____________

Find the volume of each figure.

53.



54.



< Objective 4 >

Estimate each product by rounding the factors to the nearest hundred.

55. $\begin{array}{r} 391 \\ \times\ 531 \\ \hline \end{array}$

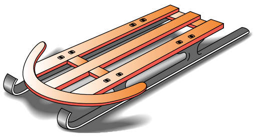
56. $\begin{array}{r} 729 \\ \times\ 481 \\ \hline \end{array}$

Solve each application.

57. **Social Science** A movie theater has its seats arranged so that there are 42 seats per row. The theater has 48 rows. Estimate the number of seats in the theater.

58. **Statistics** There are 52 mathematics classes with 28 students in each class. Estimate the total number of students in the mathematics classes.

59. **Business and Finance** A company can manufacture 45 sleds per day. Approximately how many can this company make in 128 days?



60. **Business and Finance** The attendance at a basketball game was 2,345. The cost of admission was $12 per person. Estimate the total gate receipts for the game.

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61. **Allied Health** A young male patient is to be administered an intravenous (IV) solution running on an infusion pump set for 125 milliliters (mL) per hour for 6 hours. What is the total volume of solution to be infused?

62. **Allied Health** To help assess breathing efficiency, respiratory therapists calculate the patient's alveolar minute ventilation, in milliliters per minute (mL/min), by taking the product of the patient's respiratory rate, in breaths per minute, and the difference between the patient's tidal volume and dead-space volume, both in milliliters. Calculate the alveolar minute ventilation for a male patient with lung disease given that his respiratory rate is 10 breaths per minute, his tidal volume is 575 mL, and his dead-space volume is 200 mL.

63. **Information Technology** Jack needs a thousand feet of twisted pair cable for a network installation project. He goes to a local electronics store. The store sells cable at 10¢ a foot. How much (in cents) will one thousand feet of cable cost? How much is that in dollars?

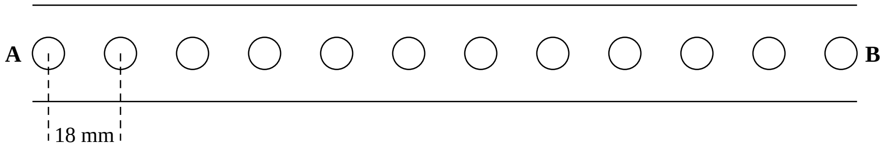
64. **Information Technology** Amber needs to buy 100 new computers for new employees that have been hired by ABC consulting. She finds the cost of a decent computer to be $655 from Dell Computers. How much will she spend to buy 100 new computers?

65. **Electronics** An electronics-component distributor sells resistors, small components that "resist" the flow of electric current, in presealed bags. Each bag contains 50 resistors. If you purchased 25 bags of resistors, how many resistors would you have?

66. **Electronics** Assume that is takes 4 hours to solder all the components on a given printed circuit board. If you are given 36 boards to solder, how many hours will the project take? If you worked nonstop, how many hours would the project take?

67. **Manufacturing Technology** A small shop has 6 machinists each earning \$14 per hour, 3 assembly workers earning \$8 per hour, and one supervisor-maintenance person earning \$18 per hour. What is the shop's payroll for a 40-hour week?

68. **Manufacturing Technology** What is the distance from the center of hole A to the center of hole B in the following diagram?



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69. **Social Science** We have seen that addition and multiplication are commutative operations. Decide which of the following activities are commutative.

(a) Taking a shower and eating breakfast
(b) Getting dressed and taking a shower
(c) Putting on your shoes and your socks
(d) Brushing your teeth and combing your hair
(e) Putting your key in the ignition and starting your car

70. **Social Science** The associative properties of addition and multiplication indicate that the result of the operation is the same regardless of where the grouping symbol is placed. This is not always the case in the use of the English language. Many phrases can have different meanings based on how the words are grouped. In each of the following, explain why the associative property would not hold.

(a) Cat fearing dog
(b) Hard test question
(c) Defective parts department
(d) Man eating animal

Write some phrases in which the associative property is satisfied.

71. **Construction** Suppose you wish to build a small, rectangular pen, and you have enough fencing for the pen's perimeter to be 36 ft. Assuming that the length and width are to be whole numbers, answer the following.

(a) List the possible dimensions that the pen could have. (*Note:* A square is a type of rectangle.)
(b) For each set of dimensions (length and width), find the area that the pen would enclose.
(c) Which dimensions give the greatest area?
(d) What is the greatest area?

Answers

65. ______
66. ______
67. ______
68. ______
69. ______
70. ______
71. ______

Answers

72. ____________________

73. ____________________

74. ____________________

72. Construction Suppose you wish to build a rectangular kennel that encloses 100 ft^2. Assuming that the length and width are to be whole numbers, answer the following.

(a) List the possible dimensions that the kennel could have. (*Note:* A square is a type of rectangle.)

(b) For each set of dimensions (length and width), find the perimeter that would surround the kennel.

(c) Which dimensions give the least perimeter?

(d) What is the least perimeter?

73. Social Science Most maps contain legends that allow you to convert the distance between two points on the map to actual miles. For instance, if a map uses a legend that equates 1 inch (in.) to 5 miles (mi) and the distance between two towns is 4 in. on the map, then the towns are actually 20 mi apart.

(a) Obtain a map of your state and determine the shortest distance between any two major cities.

(b) Could you actually travel the route you measured in part **(a)**?

(c) Plan a trip between the two cities you selected in part **(a)** over established roads. Determine the distance that you actually travel using this route.

74. Number Problem Complete the following number cross.

Across

1. 6×551
5. 7×8
6. 27×27
7. 19×50
10. 3×67
12. 6×25
13. 9×8
15. 16×303

Down

1. 5×7
2. 9×41
3. 67×100
4. $2 \times (49 + 100)$
8. $4 \times 1{,}301$
9. $100 + 10 + 1$
11. 2×87
14. $25 + 3$

1	2		3	■	4
5		■	6		
■	7	8		■	
9	■	10		11	■
12			■	13	14
	■	15			

Answers

1. 15 **3.** 0 **5.** 192 **7.** 3,048 **9.** 5,100 **11.** 19,293
13. 342,300 **15.** 27,900 **17.** 228,762 **19.** 203,740
21. 666,333 **23.** 6,423,372 **25.** 2,320 **27.** 816,300
29. 112,220 **31.** 2,128 **33.** 11,571
35. Commutative property of multiplication
37. Associative property of multiplication
39. $7 + (8 \times 3)$ **41.** 272 cars **43.** 336 cars **45.** 420 stoves
47. 36 yd^2 **49.** 18 in.^2 **51.** 31 in.^2 **53.** 216 ft^3
55. 200,000 **57.** 2,000 seats **59.** 6,500 sleds **61.** 750 mL
63. 10,000¢; $100 **65.** 1,250 resistors **67.** $5,040
69. Above and Beyond **71.** Above and Beyond **73.** Above and Beyond

1.6

Division

< 1.6 Objectives >

1 > Write a division problem as repeated subtraction
2 > Use the language of division
3 > Divide whole numbers
4 > Estimate quotients
5 > Solve applications of division

Now we examine a fourth arithmetic operation, division. Just as multiplication is repeated addition, division is repeated subtraction. Division asks *how many times* one number is contained in another.

Example 1 Dividing by Using Subtraction

< Objective 1 >

NOTE

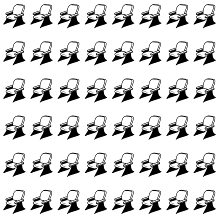
The 8 is subtracted six times.

Joel needs to set up 48 chairs in the student union for a concert. If there is room for 8 chairs per row, how many rows will it take to set up all 48 chairs?

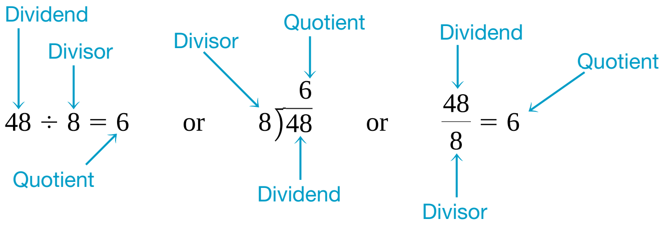
This problem can be solved by subtraction. Each row subtracts another 8 chairs.

$$\begin{array}{cccccc} 48 & 40 & 32 & 24 & 16 & 8 \\ -8 & -8 & -8 & -8 & -8 & -8 \\ \hline 40 & 32 & 24 & 16 & 8 & 0 \end{array}$$

Because 8 can be subtracted from 48 six times, there will be 6 rows.



This can also be seen as a division problem.



No matter which method we use, we call 48 the **dividend,** 8 the **divisor,** and 6 the **quotient.**

Check Yourself 1

Carlotta is creating a garden path made of bricks. She has 72 bricks. Each row will have 6 bricks in it. How many rows can she make?

Units ANALYSIS

When you divide a denominate number by an abstract number, the result will have the units of the denominate number. Here are a couple of examples:

76 trombones ÷ 4 = 19 trombones

\$55 ÷ 11 = \$5

When one denominate number is divided by another, the result will get the units of the dividend over the units of the divisor.

144 miles ÷ 6 gallons = 24 miles/gallon (which we read as "miles per gallon")

\$120 ÷ 8 hours = 15 dollars/hour ("dollars per hour")

To solve a problem requiring division, first set up the problem as a division statement. Example 2 illustrates this idea.

Example 2 Writing a Division Statement

< Objective 2 >

Write a division statement that corresponds to the following situation. You need not do the division.

The staff at the Wok Inn Restaurant splits all tips at the end of each shift. Yesterday's evening shift collected a total of \$224. How much should each of the 7 employees get in tips?

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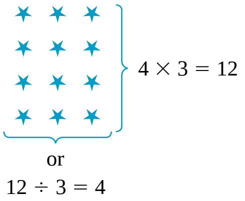
\$224 ÷ 7 employees (Note that the units for the answer will be "dollars per employee.")

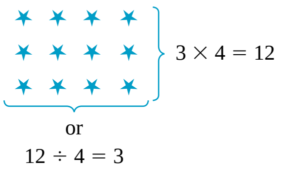
Check Yourself 2

Write a division statement that corresponds to the following situation. You need not do the division.

All nine sections of basic math skills at SCC (Sum Community College) are full. There are a total of 315 students in the classes. How many students are in each class? What are the units for the answer?

In Section 1.5, we used a rectangular array of stars to represent multiplication. These same arrays can represent division. Just as 3 × 4 = 12 and 4 × 3 = 12, so is it true that 12 ÷ 3 = 4 and 12 ÷ 4 = 3.





This relationship allows us to check our division results by doing multiplication.

Example 3 Checking Division by Using Multiplication

NOTE

For a division problem to check, the *product* of the divisor and the quotient *must equal the dividend.*

(a) $\begin{array}{r}3\\7\overline{)21}\end{array}$ Check: $7 \times 3 = 21$

(b) $48 \div 6 = 8$ Check: $6 \times 8 = 48$

Check Yourself 3

Complete the division statements and check your results.

(a) $9\overline{)45}$ **(b)** $28 \div 7$

NOTE

Because $36 \div 9 = 4$, we say that 36 is *exactly divisible* by 9.

In our examples so far, the product of the divisor and the quotient has been equal to the dividend. This means that the dividend is *exactly divisible* by the divisor. That is not always the case. Look at another example that uses repeated subtraction.

Example 4 Dividing by Using Subtraction, Leaving a Remainder

NOTE

The remainder must be smaller than the divisor or we could subtract again.

How many times is 5 contained in 23?

$$\begin{array}{r}23\\-\ 5\\\hline 18\end{array}\qquad\begin{array}{r}18\\-\ 5\\\hline 13\end{array}\qquad\begin{array}{r}13\\-\ 5\\\hline 8\end{array}\qquad\begin{array}{r}8\\-\ 5\\\hline 3\end{array}$$

We see that 5 is contained 4 times in 23, but 3 is "leftover."

The number 23 is not exactly divisible by 5. The "leftover" 3 is called the **remainder** in the division. To check the division operation when a remainder is involved, we have the following rule:

Definition

Remainder Dividend = divisor × quotient + remainder

Check Yourself 4

How many times is 7 contained in 38?

Example 5 Checking Division by a Single-Digit Number

NOTE

Another way to write the result is

$\begin{array}{r}4\text{ r3}\\5\overline{)23}\end{array}$ The "r" stands for remainder.

NOTE

The multiplication is done before the 3 is added.

Using the work of Example 4, we can write

$\begin{array}{r}4\\5\overline{)23}\end{array}$ with remainder 3

To apply our previous rule, we have

$$\text{Dividend} \longrightarrow 23 = \overset{\text{Divisor}}{5} \times \overset{\text{Quotient}}{4} + 3 \longleftarrow \text{Remainder}$$
$$23 = 20 + 3$$
$$23 = 23$$

The division checks.

Check Yourself 5

Evaluate $7\overline{)38}$. Check your answer.

We must be careful when 0 is involved in a division problem. There are two special cases.

Property

Division and Zero

1. Zero divided by any whole number (except 0) is 0.
2. Division by 0 is undefined.

The first case involving zero occurs when we are dividing into zero.

Example 6 Dividing into Zero

RECALL

There are three forms that are equivalent:

$0 \div 5 = 0$

$5\overline{)0} = 0$

$\frac{0}{5} = 0$

$0 \div 5 = 0$ because $0 = 5 \times 0$.

Check Yourself 6

(a) $0 \div 7$ (b) $9\overline{)0}$ (c) $\frac{0}{12}$

Our second case illustrates what happens when 0 is the *divisor.* Here we have a special problem.

Example 7 Dividing by Zero

$8 \div 0 = ?$ This means that $8 = 0 \times ?$

Can 0 times some number ever be 8? From our multiplication facts, the answer is *no!* There is no answer to this problem, so we say that $8 \div 0$ is undefined.

Check Yourself 7

Decide whether each problem results in 0 or is undefined.

(a) $\frac{9}{0}$ (b) $\frac{0}{9}$ (c) $15\overline{)0}$ (d) $0\overline{)15}$

It is easy to divide when small whole numbers are involved, because much of the work can be done mentally. In working with larger numbers, we turn to a process called **long division.** This is a shorthand method for performing the steps of repeated subtraction.

To start, look at an example in which we subtract multiples of the divisor.

Example 8 Dividing by a Single-Digit Number

< Objective 3 >

NOTE

With larger numbers, repeated subtraction is just too time-consuming to be practical.

Divide 176 by 8.

Because 20 eights are 160, we know that there are at least 20 eights in 176.

Step 1 Write

$$\begin{array}{rr} & 20 \\ & 8\overline{)176} \\ \text{20 eights} \longrightarrow & 160 \\ \cline{2-2} & 16 \end{array}$$

Subtracting 160 is just a shortcut for subtracting eight 20 times.

After subtracting the 20 eights, or 160, we are left with 16. There are 2 eights in 16, and so we continue.

Step 2

$$\begin{array}{r} \left.\begin{array}{r} 2 \\ \underline{20} \end{array}\right\} 22 \\ 8\overline{)176} \\ \underline{160} \\ 16 \\ \text{2 eights} \longrightarrow \underline{16} \\ 0 \end{array}$$

Adding 20 and 2 gives us the quotient, 22.

Subtracting the 2 eights, we have a 0 remainder. So 176 ÷ 8 = 22.

Check Yourself 8

Verify the result of Example 8, using multiplication.

The next step is to simplify this repeated-subtraction process one step further. The result will be the long-division method.

Example 9 Dividing by a Single-Digit Number

Divide 358 by 6.

The dividend is 358. We look at the first digit, 3. We cannot divide 6 into 3, and so we look at the *first two digits,* 35. There are 5 sixes in 35, and so we write 5 above the tens digit of the dividend.

$$\begin{array}{r} 5 \\ 6\overline{)358} \end{array}$$

When we place 5 as the tens digit, we really mean 5 tens, or 50.

Now multiply 5 × 6, place the product below 35, and subtract.

$$\begin{array}{r} 5 \\ 6\overline{)358} \\ \underline{30} \\ 5 \end{array}$$

We have actually subtracted 50 sixes (300) from 358.

Because the remainder, 5, is smaller than the divisor, 6, we bring down 8, the ones digit of the dividend.

$$\begin{array}{r} 5 \\ 6\overline{)358} \\ \underline{30{\downarrow}} \\ 58 \end{array}$$

Now divide 6 into 58. There are 9 sixes in 58, and so 9 is the ones digit of the quotient. Multiply 9 × 6 and subtract to complete the process.

$$\begin{array}{r} 59 \\ 6\overline{)358} \\ \underline{30{\downarrow}} \\ 58 \\ \underline{54} \\ 4 \end{array}$$

We now have:
358 ÷ 6 = 59 r4

To check: 358 = 6 × 59 + 4

NOTES

Because 4 is smaller than the divisor, we have a remainder of 4.

Verify that this is true and that the division checks.

Check Yourself 9

Divide $7\overline{)453}$.

Long division becomes a bit more complicated when we have a two-digit divisor. It is now a matter of trial and error. We round the divisor and dividend to form a *trial divisor and a trial dividend.* We then estimate the proper quotient and must determine whether our estimate was correct.

Example 10 Dividing by a Two-Digit Number

NOTE

Think: $4\overline{)29}$ with quotient 7

Divide.

$$38\overline{)293}$$

Round the divisor and dividend to the nearest ten. So 38 is rounded to 40, and 293 is rounded to 290. The trial divisor is then 40, and the trial dividend is 290.

Now look at the nonzero digits in the trial divisor and dividend. They are 4 and 29. We know that there are 7 fours in 29, and so 7 is our first estimate of the quotient. Now let's see if 7 works.

$$\begin{array}{r} 7 \longleftarrow \text{Your estimate} \\ 38\overline{)293} \\ \underline{266} \\ 27 \end{array}$$

Multiply 7 × 38. The product, 266, is less than 293, so we can subtract.

The remainder, 27, is less than the divisor, 38, and so the process is complete.

$293 \div 38 = 7 \text{ r}27$

Check: $293 = 38 \times 7 + 27$ You should verify that this statement is true.

Check Yourself 10

Divide.

$$57\overline{)482}$$

Because this process is based on estimation, our first guess is often incorrect.

Example 11 Dividing by a Two-Digit Number

NOTE

Think: $5\overline{)43}$ with quotient 8

Divide.

$$54\overline{)428}$$

Rounding to the nearest ten, we have a trial divisor of 50 and a trial dividend of 430.

If you look at the nonzero digits, how many fives are in 43? There are 8. This is our first estimate.

$$\begin{array}{r} 8 \\ 54\overline{)428} \\ \underline{432} \longleftarrow \text{Too large} \end{array}$$

We multiply 8 × 54. Do you see what's wrong? The product, 432, is too large. We can't subtract. Our estimate of the quotient must be adjusted *downward.*

We adjust the quotient downward to 7. We can now complete the division.

$$\begin{array}{r} 7 \\ 54\overline{)428} \\ \underline{378} \\ 50 \end{array}$$

We have

$428 \div 54 = 7 \text{ r}50$

Check: $428 = 54 \times 7 + 50$

Check Yourself 11

Divide.

$$63\overline{)557}$$

We must be careful when a 0 appears as a digit in the quotient. Here is an example in which this happens with a two-digit divisor.

Example 12 Dividing with Large Dividends

NOTE

Our divisor, 32, divides into 98, the first two digits of the dividend.

Divide.

$$32\overline{)9{,}871}$$

Rounding to the nearest ten, we have a trial divisor of 30 and a trial dividend of 100. Think, “How many threes are in 10?” There are 3, and this is our first estimate of the quotient.

$$\begin{array}{r} 3 \\ 32\overline{)9{,}871} \\ \underline{9\,6} \\ 2 \end{array}$$

Everything seems fine so far!

Bring down 7, the next digit of the dividend.

$$\begin{array}{r} 30 \\ 32\overline{)9{,}871} \\ \underline{9\,6{\downarrow}} \\ 27 \end{array}$$

Now do you see the difficulty? We cannot divide 32 into 27, and so we place 0 in the tens place of the quotient to indicate this fact.

We continue by multiplying by 0. After subtraction, we bring down 1, the last digit of the dividend.

$$\begin{array}{r} 30 \\ 32\overline{)9{,}871} \\ \underline{9\,6} \\ 27 \\ \underline{00}{\downarrow} \\ 271 \end{array}$$

Another problem develops here. We round 32 to 30 for our trial divisor, and we round 271 to 270, which is the trial dividend at this point. Our estimate of the last digit of the quotient must be 9.

$$\begin{array}{r} 309 \\ 32\overline{)9{,}871} \\ \underline{9\,6} \\ 27 \\ \underline{00} \\ 271 \\ \underline{288} \longleftarrow \text{Too large} \end{array}$$

We cannot subtract because 288 is larger than 271. The trial quotient must be adjusted downward to 8. We can now complete the division.

$$\begin{array}{r} 308 \\ 32\overline{)9{,}871} \\ \underline{9\,6} \\ 27 \\ \underline{00} \\ 271 \\ \underline{256} \\ 15 \end{array}$$

$9{,}871 \div 32 = 308 \text{ r}15$

Check: $9{,}871 = 32 \times 308 + 15$

Check Yourself 12

Divide.

$$43\overline{)8{,}857}$$

Because of the availability of calculators, it is rarely necessary that people find the exact answer when performing long division. On the other hand, it is frequently important that one be able to either estimate the result of long division or confirm that a given answer (particularly from a calculator) is reasonable. As a result, the emphasis in this section will be on improving your estimation skills in division.

Let's divide a four-digit number by a two-digit number. Generally, we will round the divisor to the nearest ten and the dividend to the nearest hundred.

Example 13 Estimating the Result of a Division Application

< Objective 4 >

The Ramirez family took a trip of 2,394 miles (mi) in their new car, using 77 gallons (gal) of gas. Estimate their gas mileage (mi/gal).

Our estimate will be based on dividing 2,400 by 80.

$$\begin{array}{r} 30 \\ 80\overline{)2{,}400} \end{array}$$

They got approximately 30 mi/gal.

Check Yourself 13

Troy flew a light plane on a trip of 2,844 mi that took 21 hours (h). What was his approximate speed in miles per hour (mi/h)?

As before, we may have to combine operations to solve an application of the mathematics you have learned.

Example 14 Estimating the Result of a Division Application

Charles purchases a used car for \$8,574. Interest charges will be \$978. He agrees to make payments for 4 years. Approximately what should his monthly payments be?

First, we find the amount that Charles owes:

$$\$8{,}574 + \$978 = \$9{,}552$$

Now, to find the monthly payment, we divide that amount by 48 (months). To estimate the payment, we'll divide \$9,600 by 50 months.

$$\begin{array}{r} 192 \\ 50\overline{)9{,}600} \end{array}$$

The payments will be approximately \$192 per month.

Check Yourself 14

One \$10 bag of fertilizer will cover 310 square feet (ft^2). Approximately what would it cost to cover 2,145 ft^2?

Check Yourself ANSWERS

1. 12 rows **2.** 315 students ÷ 9 classes; students per class
3. **(a)** 5; $9 \times 5 = 45$; **(b)** 4; $7 \times 4 = 28$ **4.** 5 **5.** 5 r3; $38 = 5 \times 7 + 3$
6. **(a)** 0; **(b)** 0; **(c)** 0 **7.** **(a)** Undefined; **(b)** 0; **(c)** 0; **(d)** undefined
8. $8 \times 22 = 176$ **9.** 64 r5 **10.** 8 r26 **11.** 8 r53 **12.** 205 r42
13. 140 mi/h **14.** $70

Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 1.6

(a) The result from division is called the ________________.

(b) The dividend is equal to the divisor times the quotient plus the ________________.

(c) Zero divided by any whole number (except ________________) is 0.

(d) Division is a shortened form for repeated ________________.

1.6 exercises

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Name ________________

Section ________ Date ________

Answers

1. ________________

2. ________________

3. ________ 4. ________

5. ________ 6. ________

7. ________ 8. ________

9. ________ 10. ________

11. ________ 12. ________

13. ________ 14. ________

15. ________ 16. ________

17. ________ 18. ________

19. ________ 20. ________

21. ________ 22. ________

23. ________ 24. ________

25. ________ 26. ________

27. ________ 28. ________

29. ________ 30. ________

31. ________ 32. ________

Basic Skills | Challenge Yourself | Calculator/Computer | Career Applications | Above and Beyond

< Objective 2 >

1. If 48 ÷ 8 = 6, 8 is the _______, 48 is the _______, and 6 is the _______.

2. In the statement $\begin{array}{r}9\\5\overline{)45}\end{array}$, 9 is the ________, 5 is the ________, and 45 is the _______.

3. Find 36 ÷ 9 by repeated subtraction.

4. Find 40 ÷ 8 by repeated subtraction.

5. **CRAFTS** Stefanie is planting rows of tomato plants. She wants to plant 63 plants with 9 plants per row. How many rows will she have?

> Videos

6. **CONSTRUCTION** Nick is designing a parking lot for a small office building. He must make room for 42 cars with 7 cars per row. How many rows should he plan for?

Divide. Identify the correct units for the quotient.

7. 36 pages ÷ 4

8. \$96 ÷ 8

9. 4,900 kilometers (km) ÷ 7

10. 360 gal ÷ 18

11. 160 miles ÷ 4 hours

12. 264 ft ÷ 3 s

13. 3,720 hours ÷ 5 months

14. 560 calories ÷ 7 grams

< Objective 3 >

Divide using long division, and check your work.

15. 54 ÷ 9

16. 21 ÷ 3

17. $6\overline{)42}$

18. $7\overline{)63}$

19. $4\overline{)32}$

20. 56 ÷ 8

21. $5\overline{)43}$

22. 40 ÷ 9

23. $9\overline{)65}$

24. $6\overline{)51}$

25. 57 ÷ 8

26. 74 ÷ 8

27. 0 ÷ 5

28. 5 ÷ 0

29. 4 ÷ 0

30. 0 ÷ 12

31. 0 ÷ 6

32. 18 ÷ 0

Divide.

33. $5\overline{)83}$ > Videos

34. $9\overline{)78}$

35. $162 \div 3$

36. $232 \div 4$

37. $\frac{293}{8}$

38. $\frac{346}{7}$

39. $8\overline{)3{,}136}$

40. $5\overline{)4{,}938}$

41. $5{,}438 \div 8$

42. $3{,}527 \div 9$

43. $\frac{22{,}153}{8}$

44. $\frac{43{,}287}{5}$

Basic Skills | **Challenge Yourself** | Calculator/Computer | Career Applications | Above and Beyond

45. $45\overline{)2{,}367}$

46. $53\overline{)3{,}480}$

47. $8{,}748 \div 34$ > Videos

48. $9{,}335 \div 27$

49. $\frac{7{,}902}{42}$ > Videos

50. $\frac{8{,}729}{53}$

< Objective 5 >

Solve each application.

51. **Business and Finance** There are 63 candy bars in 7 boxes. How many candy bars are in each box? > Videos

52. **Business and Finance** A total of 54 printers were shipped to 9 stores. How many printers were shipped to each store?

53. **Social Science** Joaquin is putting pictures in an album. He can fit 8 pictures on each page. If he has 77 pictures, how many will be left over after he has filled the last 8-picture page?

54. **Statistics** Kathy is separating a deck of 52 cards into 6 equal piles. How many cards will be left over?

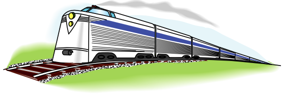
PLAYING CARDS

55. **Social Science** The records of an office show that 1,702 calls were made in 1 day. If there are 37 phones in the office, how many calls were placed per phone?

56. **Business and Finance** A television dealer purchased 23 sets, each the same model, for $5,267. What was the cost of each set?

57. **Business and Finance** A computer printer can print 340 lines per minute (min). How long will it take to complete a report of 10,880 lines?

58. **Statistics** A train traveled 1,364 mi in 22 h. What was the speed of the train? (*Hint:* Speed is the distance traveled divided by the time.)



< Objective 4 >

Estimate the result in each division problem. (Remember to round divisors to the nearest ten and dividends to the nearest hundred.)

59. 810 divided by 38

60. 458 divided by 18

61. 4,967 divided by 96

62. 3,971 divided by 39

Answers

33. ______ 34. ______
35. ______ 36. ______
37. ______ 38. ______
39. ______ 40. ______
41. ______ 42. ______
43. ______ 44. ______
45. ______ 46. ______
47. ______ 48. ______
49. ______ 50. ______
51. ______ 52. ______
53. ______ 54. ______
55. ______ 56. ______
57. ______ 58. ______
59. ______ 60. ______
61. ______ 62. ______

Answers

63. ______ 64. ______

65. ______ 66. ______

67. ______ 68. ______

69. ______ 70. ______

71. ______ 72. ______

73. ______ 74. ______

75. ______ 76. ______

77. ______ 78. ______

79. ______ 80. ______

81. ______

82. ______

83. ______

84. ______

85. ______

86. ______

63. 8,971 divided by 91

64. 3,981 divided by 78

65. 3,879 divided by 126

66. 8,986 divided by 178

67. 3,812 divided by 188

68. 5,245 divided by 255

Basic Skills | Challenge Yourself | **Calculator/Computer** | Career Applications | Above and Beyond

Use your calculator to perform the indicated operations.

69. $583{,}467 \div 129$

70. $464{,}184 \div 189$

71. $6 + 9 \div 3$

72. $18 - 6 \div 3$

73. $24 \div 6 \times 4$

74. $32 \div 8 \times 4$

75. $4{,}368 \div 56 + 726 \div 33$

76. $1{,}176 \div 42 - 1{,}572 \div 524$

77. $3 \times 8 \times 8 \times 8 \div 12$

78. $5 \times 6 \times 6 \div 18$

Basic Skills | Challenge Yourself | Calculator/Computer | **Career Applications** | Above and Beyond

79. Allied Health The doctor prescribes 525 milligrams (mg) of bexarotene to be given once daily to an adult female patient with lung cancer. How many pills should the nurse give her every day if each pill contains 75 mg of bexarotene?

80. Allied Health Determine the flow rate, in milliliters per hour (mL/h), needed for an electronic infusion pump to administer 180 mL of a saline solution via intravenous (IV) infusion over the course of 12 hours (h).

81. Information Technology Marcela is in the process of building a new testing computer lab for ABC software. This lab has five rows of computers and nine computers per row. The distance from each computer to a switch is about 4 feet (ft). She has a 200-ft roll of cable for the job. How many 4-ft cables can she make with a 200-ft roll? Does she have enough cable on the roll for the job?

82. Information Technology Your modem on your computer typically transmits at 56,000 bits per second. How long will it take to transmit five hundred sixty thousand bits?

83. Electronics An electronics component distributor sells resistors in two package sizes. A small package contains 500 resistors. A large package contains 1,250 resistors. If 10,000 resistors are needed to make a batch of parts, how many packages would you need to buy if you bought all small packages? All large packages?

84. Electronics A vendor that makes small-quantity batches of printed circuit boards will sell 25 boards for $400. What is the cost per board?

85. Manufacturing Technology An order of 24 parts weighs 1,752 lb. Assuming that the parts are identical, how much does each part weigh?

86. Manufacturing Technology Triplet Precision Machining has a 3,000-gallon (gal) liquid petroleum (LP) tank. If the cutting line consumes 125 gal of LP each day, how many days will the LP supply last?

87. **CONSTRUCTION** You are going to recarpet your living room. You have budgeted $1,500 for the carpet and installation.

(a) Determine how much carpet you will need to do the job. Draw a sketch to support your measurements.

(b) What is the highest price per square yard you can pay and still stay within budget?

(c) Go to a local store and determine the total cost of doing the job for three different grades of carpet. Be sure to include padding, labor costs, and any other expenses.

(d) What considerations (other than cost) would affect your decision about what type of carpet to install?

(e) Write a brief paragraph indicating your final decision and give supporting reasons.

88. **SOCIAL SCIENCE** Division is the inverse operation of multiplication. Many daily activities have inverses. For each of the following activities, state the inverse activity.

(a) Spending money
(b) Going to sleep
(c) Turning down the volume on your CD player
(d) Getting dressed

89. If you have no money in your pocket and want to divide it equally among your four friends, how much does each person get? Use this situation to explain division of zero by a nonzero number.

90. **NUMBER PROBLEM** Complete the following number cross.

Across
1. 48 ÷ 4
3. 1,296 ÷ 8
6. 2,025 ÷ 5
8. 4 × 5
9. 11 × 11
12. 15 ÷ 3 × 111
14. 144 ÷ (2 × 6)
16. 1,404 ÷ 6
18. 2,500 ÷ 5
19. 3 × 5

Down
1. (12 + 16) ÷ 2
2. 67 × 3
4. 744 ÷ 12
5. 2,600 ÷ 13
7. 6,300 ÷ 12
10. 304 ÷ 2
11. 5 × (161 ÷ 7)
13. 9,027 ÷ 17
15. 400 ÷ 20
17. 9 × 5

1	2	■	3	4	5
6		7	■	8	
■	9		10	■	
11	■	12		13	■
14	15	■	16		17
18			■	19	

Answers

1. Divisor, dividend, quotient **3.** 4 **5.** 7 **7.** 9 pages **9.** 700 km
11. 40 mi/h **13.** 744 h/month **15.** 6 **17.** 7 **19.** 8 **21.** 8 r3
23. 7 r2 **25.** 7 r1 **27.** 0 **29.** Undefined **31.** 0 **33.** 16 r3
35. 54 **37.** 36 r5 **39.** 392 **41.** 679 r6 **43.** 2,769 r1
45. 52 r27 **47.** 257 r10 **49.** 188 r6 **51.** 9 bars **53.** 5 pictures
55. 46 calls **57.** 32 min **59.** 20 **61.** 50 **63.** 100 **65.** 30
67. 20 **69.** 4,523 **71.** 9 **73.** 16 **75.** 100 **77.** 128
79. 7 pills **81.** Yes, 50 4-ft cables **83.** 20 packages; 8 packages
85. 73 lb **87.** Above and Beyond **89.** Above and Beyond

Answers

87. ______

88. ______

89. ______

90. ______

Activity 2 ::
Restaurant Management

In 2002 there were more than 30,000 McDonald's restaurants in the world. Of these, approximately 14,000 were in the United States. In Great Britain there were 1,116 McDonald's franchises. The following data are taken from those 1,116 McDonald's.

Total employees:	49,726
Office staff:	545
Management:	2,974
The rest are restaurant workers.	
Total employees 20 years old and under:	34,241
Total employees between 21 and 29 years old:	10,607
Total male employees:	27,546

Use these data to answer the questions that follow.

1. What is the total number of restaurant workers in McDonald's franchises in Great Britain?

2. How many of the employees mentioned are of age 30 or older?

3. How many of the employees were females?

4. Divide the total number of employees by the number of franchises to determine approximately how many employees are in each McDonald's in Great Britain.

5. Assume that each franchise decided to give each employee a \$125 bonus. If you use your result from question 4, how much would that cost each franchise?

6. What would the total cost of the \$125 bonus be for all the employees?

7. Assume that there are 45 employees in each of the U.S. McDonald's. Approximately how many people would be employed by McDonald's in the United States?

8. If health care costs \$400 per month for one employee, what would the annual cost for health care be?

9. Use the results of questions 7 and 8 to approximate the cost to McDonald's of providing health care for all its workers.

1.7

Exponential Notation and the Order of Operations

< **1.7 Objectives** >

1 > Use exponent notation

2 > Evaluate expressions containing powers of whole numbers

3 > Evaluate expressions that contain several operations

Tips for Student Success

Preparing for a Test

Preparation for a test really begins on the first day of class. Everything you do in class and at home is part of that preparation. However, there are a few things that you should focus on in the last few days before a scheduled test.

1. Plan your test preparation to end at least 24 hours before the test. The last 24 hours are too late, and besides, you need some rest before the test.
2. Go over your homework and class notes with pencil and paper in hand. Write down all the problem types, formulas, and definitions that you think might give you trouble on the test.
3. The day before the test, take the page(s) of notes from step 2, and transfer the most important ideas to a 3 × 5 card.
4. Just before the test, review the information on the card. You will be surprised at how much you remember about each concept.
5. Understand that, if you have been successful at completing your homework assignments, you can be successful on the test. This is an obstacle for many students, but it is an obstacle that can be overcome. Truly anxious students are often surprised that they scored as well as they did on a test. They tend to attribute this to blind luck. It is not. It is the first sign that you really do "get it." Enjoy the success.

Earlier we described multiplication as a shorthand for repeated addition. There is also a shorthand for repeated multiplication. It uses **powers of a whole number.**

Example 1 Writing Repeated Multiplication as a Power

< Objective 1.7 >

$3 \times 3 \times 3 \times 3$ can be written as 3^4.

This is read as "3 to the fourth power."

In this case, repeated multiplication is written as the power of a number.

In this example, 3 is the **base** of the expression, and the raised number, 4, is the **exponent,** or **power.**

Exponent or power

$$3^4 = \underbrace{3 \times 3 \times 3 \times 3}_{\text{4 factors}}$$

Base

We count the factors and make this the power (or exponent) of the base.

NOTE

Recall that

$3 + 3 + 3 + 3 = 4 \times 3$

Repeated addition was written as multiplication.

René Descartes, a French philosopher and mathematician, is generally credited with first introducing our modern exponent notation in about 1637.

Check Yourself 1

Write $2 \times 2 \times 2 \times 2 \times 2 \times 2$ as a power of 2.

Definition

Exponents The *exponent* tells us the number of times the base is to be used as a factor.

Example 2 Evaluating a Number Raised to a Power

< Objective 2 >

2^5 is read "2 to the fifth power."

$$2^5 = \underbrace{2 \times 2 \times 2 \times 2 \times 2}_{\text{5 times}} = 32$$

2^5 tells us to use 2 as a factor 5 times. The result is 32.

Here 2 is the base, and 5 is the exponent.

Check Yourself 2

Read and evaluate 3^4.

Example 3 Evaluating a Number Raised to a Power

> CAUTION

5^3 is *entirely different* from 5×3. $5^3 = 125$ whereas $5 \times 3 = 15$.

Evaluate 5^3 and 8^2.

$5^3 = 5 \times 5 \times 5 = 125$ Use 3 factors of 5.

5^3 is read "5 to the third power" or "5 cubed."

$8^2 = 8 \times 8 = 64$ Use 2 factors of 8.

And 8^2 is read "8 to the second power" or "8 squared."

Check Yourself 3

Evaluate.

(a) 6^2 **(b)** 2^4

We need two special definitions for powers of whole numbers.

Definition

Raising a Number to the First Power A number raised to the first power is just that number.

For example, $9^1 = 9$.

Definition

Raising a Number to the Zero Power A number, other than 0, raised to the zero power is 1.

For example, $7^0 = 1$.

Example 4 Evaluating Numbers Raised to the Power of 0 or 1

(a) $8^0 = 1$ **(b)** $4^0 = 1$ **(c)** $5^1 = 5$ **(d)** $3^1 = 3$

Check Yourself 4

Evaluate.

(a) 7^0 **(b)** 7^1

We talked about *powers of 10* earlier when we multiplied by numbers that end in 0. Because the powers of 10 have a special importance, we list some of them here.

NOTE

Notice that 10^3 is just a 1 followed by *three zeros*.

$10^0 = 1$

$10^1 = 10$

$10^2 = 10 \times 10 = 100$

$10^3 = 10 \times 10 \times 10 = 1{,}000$

$10^4 = 10 \times 10 \times 10 \times 10 = 10{,}000$

$10^5 = 10 \times 10 \times 10 \times 10 \times 10 = 100{,}000$

NOTE

10^5 is a 1 followed by *five zeros*.

Do you see why the powers of 10 are so important?

Definition

Powers of 10

The powers of 10 correspond to the place values of our number system—ones, tens, hundreds, thousands, and so on.

This is what we meant earlier when we said that our number system was based on the number 10.

NOTE

Archimedes (about 250 B.C.E.) reportedly estimated the number of grains of sand in the universe to be 10^{63}. This would be a 1 followed by 63 zeros!

If multiplication is combined with addition or subtraction, you must know which operation to do first in finding the expression's value. We can easily illustrate this problem. How should we simplify the following statement?

$3 + 4 \times 5 = ?$

Both multiplication and addition are involved in this expression, and we must decide which to do first to find the answer.

1. Multiplying first gives us

   $3 + 20 = 23$

2. Adding first gives us

   $7 \times 5 = 35$

> CAUTION

The answers differ depending on which operation is done first!

Only one of these results can be correct, which is why mathematicians developed some rules to tell us the order in which the operations should be performed. The rules are as follows.

Step by Step

The Order of Operations

If multiplication, division, addition, and subtraction are involved in the same expression, do the operations in the following order:

Step 1 Do all multiplication and division in order from left to right.

Step 2 Do all addition and subtraction in order from left to right.

Example 5 Using the Order of Operations

< Objective 3 >

NOTE

By this rule, we see that strategy 1 from before was correct.

(a) $3 \times 4 + 5 = 12 + 5 = 17$ Multiply *first*, then add or subtract.

(b) $5 + 3 \times 6 = 5 + 18 = 23$

(c) $16 - 2 \times 3 = 16 - 6 = 10$

(d) $7 \times 8 - 20 = 56 - 20 = 36$

(e) $5 \times 6 + 4 \times 3 = 30 + 12 = 42$

NOTE

(See the "Step by Step" below.)

When learning the order of operations, students sometimes remember this order by relating each step to part of the phrase

"Please Excuse My Dear Aunt Sally."
P
E
MD
AS

Check Yourself 5

Evaluate.

(a) $8 + 3 \times 5$

(b) $15 \times 5 - 3$

(c) $4 \times 3 + 2 \times 6$

We now want to extend our rule for the order of operations to see what happens when parentheses or exponents are involved in an expression.

Step by Step

The Order of Operations

Mixed operations in an expression should be done in the following order:

Step 1 Do any operations inside ***p***arentheses.

Step 2 Apply any ***e***xponents.

Step 3 Do all ***m***ultiplication and ***d***ivision in order from left to right.

Step 4 Do all ***a***ddition and ***s***ubtraction in order from left to right.

Example 6 Evaluating an Expression

Evaluate 4×2^3.

Step 1 There are no parentheses.

Step 2 Apply exponents.

$4 \times 2^3 = 4 \times 8$

Step 3 Multiply or divide.

$4 \times 8 = 32$

Check Yourself 6

Evaluate.

3×3^2

Example 7 Evaluating an Expression

Evaluate $(2 + 3)^2 + 4 \times 3$.

Step 1 Do operations inside parentheses.

$(2 + 3)^2 + 4 \times 3 = (5)^2 + 4 \times 3$

Step 2 Apply exponents.

$5^2 + 4 \times 3 = 25 + 4 \times 3$

Step 3 Multiply or divide.

$25 + 4 \times 3 = 25 + 12$

Step 4 Add or subtract.

$25 + 12 = 37$

Check Yourself 7

Evaluate.

(a) $4 + (8 - 5)^2$ **(b)** $(6 - 4)^3 + 3 \cdot 2$

Example 8 Using the Order of Operations

(a) Evaluate $20 \div 2 \times 5$.

$$\begin{aligned} &\underbrace{20 \div 2} \times 5 \\ ={}& \quad 10 \quad \times 5 \\ ={}& \quad 50 \end{aligned}$$

Because the multiplication and division appear next to each other, work in order from left to right. Try it the other way and see what happens!

So $20 \div 2 \times 5 = 50$.

(b) Evaluate $(5 + 13) \div 6$.

$$\begin{aligned} &\underbrace{(5 + 13)} \div 6 \\ ={}& \quad (18) \quad \div 6 \\ ={}& \quad 3 \end{aligned}$$

Do the addition in the parentheses as the first step.

So $(5 + 13) \div 6 = 3$.

(c) Evaluate $(3 + 4)^2 \div (2^3 - 1)$.

$$\begin{aligned} &\underbrace{(3 + 4)}^2 \div \underbrace{(2^3 - 1)} \\ ={}& \quad (7)^2 \quad \div (8 - 1) \\ ={}& \quad 7^2 \quad \div (7) \\ ={}& \quad 49 \quad \div 7 \\ ={}& \quad 7 \end{aligned}$$

Perform operations inside parentheses first.

So $(3 + 4)^2 \div (2^3 - 1) = 7$.

Check Yourself 8

Evaluate.

(a) $36 \div 4 \times 2$ **(b)** $(2 + 4)^2 \div (3^2 - 3)$ **(c)** $15 \div 3 + (3 - 2)^2 \times 4 - 2$

Tips for Student Success

Taking a Test

Earlier in this section, we discussed test preparation. Now that you are thoroughly prepared for the test, you must learn how to take it.

There is much to the psychology of anxiety that we can't readily address. There is, however, a physical aspect to anxiety that can be addressed rather easily. When people are in a stressful situation, they frequently start to panic. One symptom of the panic is shallow breathing. In a test situation, this starts a vicious cycle. If you breathe too shallowly, then not enough oxygen reaches your brain. When that happens, you are unable to think clearly. In a test situation, being unable to think clearly can cause you to panic. Hence, we have a vicious cycle.

How do you break that cycle? It's pretty simple. Take a few deep breaths. We have seen students whose performance on math tests improved markedly after they got in the habit of writing "remember to breathe!" at the bottom of every test page. Try breathing; it will almost certainly improve your math test scores!

Check Yourself ANSWERS

1. 2^6 **2.** "Three to the fourth power" is 81 **3.** **(a)** 36; **(b)** 16
4. **(a)** 1; **(b)** 7 **5.** **(a)** 23; **(b)** 72; **(c)** 24 **6.** 27 **7.** **(a)** 13; **(b)** 14
8. **(a)** 18; **(b)** 6; **(c)** 7

Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 1.7

(a) Preparation for a test really begins on the ______________ day of class.

(b) Another name for a power is an ______________.

(c) The first step in the order of operations involves doing operations ______________ parentheses.

(d) A whole number (other than zero) raised to the zero power is always equal to ______________.

1.7 exercises

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Name ______________________

Section __________ Date __________

Basic Skills | Challenge Yourself | Calculator/Computer | Career Applications | Above and Beyond

< Objectives 1–3 >

Evaluate.

1. 3^2

2. 2^3 > Videos

3. 5^1

4. 6^0

5. 10^3

6. 10^6

7. 2×4^3 > Videos

8. $(2 \times 4)^3$

9. $5 + 2^2$

10. $(5 + 2)^2$

11. $42 - 7 + 9$

12. $27 - 12 + 3$

13. $20 \div 5 \times 2$

14. $48 \div 3 \times 2$

15. $(3 + 2)^3 - 20$ > Videos

16. $5 + (9 - 5)^2$

17. $(7 - 4)^4 - 30$

18. $(5 + 2)^2 + 20$

19. $8^2 \div 4^2 + 2$

20. $3 \times 5^2 + 2^2$

21. $24 - 6 \div 3$ > Videos

22. $3 + 9 \div 3$

23. $(24 - 6) \div 3$

24. $(3 + 9) \div 3$

25. $12 + 3 \div (3^2 - 2 \cdot 3)$ > Videos

26. $(8^2 - 2^4) \div 2$

27. $8^2 - 2^4 \div 2$

28. $(5 - 3)^3 + (8 - 6)^2$

29. $30 \div 6 - 12 \div 3$

30. $5 + 8 \div 4 - 3$

Basic Skills | **Challenge Yourself** | Calculator/Computer | Career Applications | Above and Beyond

31. $16 - 12 \div 3 \cdot 2 + (16 - 12)^2 \cdot 3$

32. $3 \cdot 5 + 3 \cdot 4^2 \div (6 - 4)^2$

33. $6 + 3 \cdot 2^4 - (12 - 7)(10 - 7)$

34. $27 \div (2^2 + 5) - (35 - 33)(24 - 23)$

Answers

1. ____________ 2. ____________

3. ____________ 4. ____________

5. ____________ 6. ____________

7. ____________ 8. ____________

9. ____________ 10. ____________

11. ____________ 12. ____________

13. ____________ 14. ____________

15. ____________ 16. ____________

17. ____________ 18. ____________

19. ____________ 20. ____________

21. ____________ 22. ____________

23. ____________ 24. ____________

25. ____________ 26. ____________

27. ____________ 28. ____________

29. ____________ 30. ____________

31. ____________ 32. ____________

33. ____________ 34. ____________

35. $3 \times [(7 - 5)^3 - 8] + 5 \times 2$

36. $[(3 + 1) + (7 - 2)] \times 4 - 5 \times 7$

Basic Skills | Challenge Yourself | **Calculator/Computer** | Career Applications | Above and Beyond

Use your calculator to evaluate each expression.

37. $4 \times 5 - 7$

38. $3 \times 7 + 8$

39. $9 + 3 \times 7$

40. $6 \times 0 + 3$

41. $4 + 5 \times 0$

42. $23 - 4 \times 5$

43. $5 \times (4 + 7)$

44. $8 \times (6 + 5)$

45. $5 \times 4 + 5 \times 7$

46. $8 \times 6 + 8 \times 5$

Solve each application, using a calculator.

47. **BUSINESS AND FINANCE** A car dealer kept the following record of a month's sales. Complete the table.

Model	Number Sold	Profit per Sale	Monthly Profit
Subcompact	38	$528	________
Compact	33	647	________
Standard	19	912	________
		Monthly Total Profit	________

48. **BUSINESS AND FINANCE** You take a job paying $1 the first day. On each following day your pay doubles. That is, on day 2 your pay is $2, on day 3 the pay is $4, and so on. Complete the table.

Day	Daily Pay	Total Pay
1	$1	$1
2	2	3
3	4	7
4	________	________
5	________	________
6	________	________
7	________	________
8	________	________
9	________	________
10	________	________

Answers

35. ____________________

36. ____________________

37. ____________________

38. ____________________

39. ____________________

40. ____________________

41. ____________________

42. ____________________

43. ____________________

44. ____________________

45. ____________________

46. ____________________

47. ____________________

48. ____________________

Basic Skills | Challenge Yourself | Calculator/Computer | Career Applications | Above and Beyond

49. **ELECTRONICS** Resistors are commonly identified by colored bands to indicate their approximate resistance, measured in ohms. Each band's color and position corresponds to a specific component of the overall value. In resistors with four colored bands, the third band is typically considered to be the exponent for a base 10. If the first two bands are decoded as 43 and the third band is decoded as 5, what is the total resistance in ohms?

$43 \times 10^5 = ?$

Which of the three bands is most important to read correctly? Why?

50. **MANUFACTURING TECHNOLOGY** The kinetic energy (KE) of an object (in Joules) is given by the formula:

$$\text{KE} = \frac{mv^2}{2}$$

Find the kinetic energy of an object that has a mass (m) of 46 kg and is moving at a velocity (v) of 16 meters per second.

51. **MANUFACTURING TECHNOLOGY** The power (P) of a circuit (in Watts) can be given by any of the following formulas:

$$P = IV$$

$$P = \frac{V^2}{R}$$

$$P = I^2 R$$

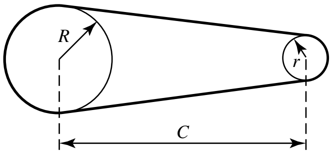
Find the power for each of the following circuits:

(a) Voltage (V) = 110 volts (V) and current (I) = 13 amperes (A).

(b) Voltage = 220 V and resistance (R) = 22 ohms.

(c) Current = 25 A and resistance = 9 ohms.

52. **MANUFACTURING TECHNOLOGY** A belt is used to connect two pulleys.



The length of the belt required is given by the formula:

$$\text{Belt length} = 2C + 3(R + r) + \frac{(2R + 2r)^2}{4C}$$

where C is the distance between the centers of the two pulleys.

Find the approximate belt length required to go around a 4-in. radius (r) pulley and a 6-in. radius (R) pulley that are 20 in. apart.

Answers

49. ______

50. ______

51. ______

52. ______

Answers

53. ______

54. ______

55. ______

56. ______

57. ______

58. ______

59. ______

60. ______

Numbers such as 3, 4, and 5 are called ***Pythagorean triples,*** *after the Greek mathematician Pythagoras (sixth century* B.C.*), because*

$3^2 + 4^2 = 5^2$

Which sets of numbers are Pythagorean triples?

53. 6, 8, 10 **54.** 6, 11, 12 **55.** 5, 12, 13

56. 7, 24, 25 **57.** 8, 16, 18 **58.** 8, 15, 17

59. Is $(a + b)^P$ equal to $a^P + b^P$?

Try a few numbers and decide if you think this is true for all whole numbers, for some whole numbers, or never true. Write an explanation of your findings and give examples.

60. Does $(a \cdot b)^P = a^P \cdot b^P$?

Try a few numbers and decide if you think this is true for all whole numbers, for some whole numbers, or never true. Write an explanation of your findings and give examples.

Answers

1. 9 **3.** 5 **5.** 1,000 **7.** 128 **9.** 9 **11.** 44
13. 8 **15.** 105 **17.** 51 **19.** 6 **21.** 22 **23.** 6
25. 13 **27.** 56 **29.** 1 **31.** 56 **33.** 39 **35.** 10
37. 13 **39.** 30 **41.** 4 **43.** 55 **45.** 55

47.
$$\begin{array}{r} \$20{,}064 \\ 21{,}351 \\ \underline{17{,}328} \\ \$58{,}743 \end{array}$$

49. 4,300,000 ohms; the third band; misreading it would lead to errors of powers of ten

51. **(a)** 1,430 W; **(b)** 2,200 W; **(c)** 5,625 W

53. Yes **55.** Yes **57.** No

59. Above and Beyond

Worldwide Services
Synchronizing the world of commerce

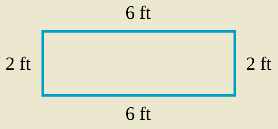
Activity 3 ::
Package Delivery

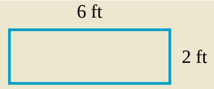
Read the following article and use the information you find there to answer the questions that follow.

> The 5-day workweek before Christmas is the busiest week of the year for package deliveries. In the year 2000, the U.S. Postal Service (USPS) averaged 750 million deliveries per day that week. In the same week, United Parcel Service (UPS) delivered about 18 million packages per day. Federal Express (Fed Ex) delivered nearly 5 million packages per day. Of the 5 million Federal Express deliveries, over 3 million of the orders were originated on the Internet.

1. Write out the number that represents the number of deliveries made by the USPS in the 5-day week before Christmas.
2. Write the word form for the answer to question 1.
3. Over the same 5 days, how many deliveries were made by UPS?
4. Over the same 5 days, how many deliveries were made by Fed Ex?
5. How many more deliveries were made by UPS than by Fed Ex that week?

summary :: chapter 1

Definition/Procedure	Example	Reference
The Decimal Place-Value System		**Section 1.1**
Digits Digits are the basic symbols of the system.	0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 are digits.	*p.* 2
Place Value The value of a digit in a number depends on its position or place.	52,589 Ones Tens Hundreds Thousands Ten thousands	*p.* 3
The value of a number is the sum of each digit multiplied by its place value.	$2{,}345 = (2 \times 1{,}000) + (3 \times 100) + (4 \times 10) + (5 \times 1)$	*p.* 3
Addition		**Section 1.2**
The Properties of Addition		
The Commutative Property The order in which you add two whole numbers does not affect the sum.	$5 + 4 = 4 + 5$	*p.* 12
The Associative Property The way in which you group whole numbers in addition does not affect the final sum.	$(2 + 7) + 8 = 2 + (7 + 8)$	*p.* 12
The Additive Identity The sum of 0 and any whole number is just that whole number.	$6 + 0 = 0 + 6 = 6$	*p.* 13
Measuring Perimeter		
The perimeter is the total distance around the outside edge of a shape. The perimeter of a rectangle is $P = 2 \cdot L + 2 \cdot W$.	 $P = 2 \times 6 \text{ ft} + 2 \times 2 \text{ ft}$ $= 12 \text{ ft} + 4 \text{ ft} = 16 \text{ ft}$	*p.* 18
Subtraction		**Section 1.3**
Minuend The number we are subtracting from. **Subtrahend** The number that is being subtracted. **Difference** The result of the subtraction.	15 ⟵Minuend − 9 ⟵Subtrahend 6 ⟵Difference	*p.* 26
Rounding, Estimation, and Order		**Section 1.4**
Step 1 To round a whole number to a certain decimal place, look at the digit to the right of that place.		*p.* 40
Step 2 **a.** If that digit is 5 or more, that digit and all digits to the right become 0. The digit in the place you are rounding to is increased by 1.	To the nearest hundred, 43,578 is rounded to 43,600.	
b. If that digit is less than 5, that digit and all digits to the right become 0. The digit in the place you are rounding to remains the same.	To the nearest thousand, 273,212 is rounded to 273,000.	
Multiplication		**Section 1.5**
Factors The numbers being multiplied. **Product** The result of the multiplication.	$\underbrace{7 \times 9}_{\text{Factors}} = 63 \longleftarrow$ Product	*p.* 48

Definition/Procedure	Example	Reference
The Properties of Multiplication		
The Commutative Property The order in which you multiply two whole numbers does not affect the product.	$7 \times 9 = 9 \times 7$	*p.* 48
The Distributive Property To multiply a factor by a sum of numbers, multiply the factor by each number inside the parentheses. Then add the products.	$2 \times (3 + 7) = (2 \times 3) + (2 \times 7)$	*p.* 50
The Associative Property The way in which you group numbers in multiplication does not affect the final product.	$(3 \times 5) \times 6 = 3 \times (5 \times 6)$	*p.* 53
Finding Area and Volume		
The area of a rectangle is found using the formula $A = L \cdot W$. The volume of a rectangular solid is found using the formula $V = L \cdot W \cdot H$.	 $A = L \times W = 6\text{ ft} \times 2\text{ ft} = 12\text{ ft}^2$	*p.* 56
Division		**Section 1.6**
Divisor The number we are dividing by. **Dividend** The number being divided. **Quotient** The result of the division. **Remainder** The number "left over" after the division.	Divisor Quotient $\begin{array}{rl} & 5 \\ 7 & \overline{)38} \longleftarrow \text{Dividend} \\ & \underline{35} \\ & 3 \longleftarrow \text{Remainder} \end{array}$	*pp.* 65–67
Dividend = divisor × quotient + remainder	$38 = 7 \times 5 + 3$	*p.* 67
Division by 0 is undefined.	$7 \div 0$ is undefined.	*p.* 68
Exponential Notation and the Order of Operations		**Section 1.7**
Using Exponents		
Base The number that is raised to a power. **Exponent** The exponent is written to the right and above the base. The exponent tells the number of times the base is to be used as a factor.	Exponent $5^3 = \underbrace{5 \times 5 \times 5}_{\text{Three factors}} = 125$ Base	*p.* 79
The Order of Operations Mixed operations in an expression should be done in the following order: **Step 1** Do any operations inside parentheses. **Step 2** Evaluate any exponents. **Step 3** Do all multiplication and division in order from left to right. **Step 4** Do all addition and subtraction in order from left to right. Remember *Please* Excuse *My* Dear *Aunt* Sally	$4 \times (2 + 3)^2 - 7$ $= 4 \times 5^2 - 7$ $= 4 \times 25 - 7$ $= 100 - 7$ $= 93$	*p.* 82

summary exercises :: chapter 1

This summary exercise set is provided to give you practice with each of the objectives of this chapter. Each exercise is keyed to the appropriate chapter section. When you are finished, you can check your answers to the odd-numbered exercises against those presented in the back of the text. If you have difficulty with any of these questions, go back and reread the examples from that section. The answers to the even-numbered exercises appear in the *Instructor's Solutions Manual*. Your instructor will give you guidelines on how to best use these exercises in your instructional setting.

1.1 *In exercises 1 and 2, give the place value of each of the indicated digits.*

1. 6 in the number 5,674

2. 5 in the number 543,400

In exercises 3 and 4, give word names of each number.

3. 27,428

4. 200,305

Write each number in standard form.

5. Thirty-seven thousand, five hundred eighty-three

6. Three hundred thousand, four hundred

1.2 *In exercises 7 and 8, name the property of addition illustrated.*

7. $4 + 9 = 9 + 4$

8. $(4 + 5) + 9 = 4 + (5 + 9)$

In exercises 9 to 13, perform the indicated operations.

9.
$$\begin{array}{r} 784 \\ 385 \\ +\ 247 \\ \hline \end{array}$$

10.
$$\begin{array}{r} 2{,}570 \\ 498 \\ 21{,}456 \\ +\quad 28 \\ \hline \end{array}$$

11.
$$\begin{array}{r} 367 \\ 289 \\ 1{,}463 \\ +\ 2{,}682 \\ \hline \end{array}$$

12.
$$\begin{array}{r} 6{,}389 \\ 1{,}567 \\ 315 \\ +\ 113{,}602 \\ \hline \end{array}$$

1.3 **13.** Find each value.

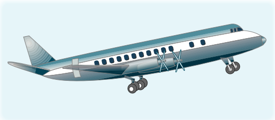
(a) 34 decreased by 7

(b) 7 more than 4

(c) The product of 9 and 5, divided by 3

Solve each application.

14. **STATISTICS** An airline had 173, 212, 185, 197, and 202 passengers on five morning flights between Washington, D.C. and New York. What was the total number of passengers?



15. **BUSINESS AND FINANCE** Future Stars summer camp employs five junior counselors. Their weekly salaries last week were \$108, \$135, \$81, \$135, and \$81. What was the total salary for the junior counselors?

1.3 *In exercises 16 to 20, perform the indicated operations.*

16.
$$\begin{array}{r} 5{,}325 \\ -\quad 847 \\ \hline \end{array}$$

17.
$$\begin{array}{r} 38{,}400 \\ -\ 19{,}600 \\ \hline \end{array}$$

18.
$$\begin{array}{r} 86{,}000 \\ -\quad 2{,}169 \\ \hline \end{array}$$

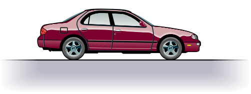
19.
$$\begin{array}{r} 2{,}682 \\ -\quad 108 \\ \hline \end{array}$$

20. Find the difference of 7,342 and 5,579.

Solve each application.

21. **Business and Finance** Chuck owes \$795 on a credit card after a trip. He makes payments of \$75, \$125, and \$90. Interest of \$31 is charged. How much remains to be paid on the account?

22. **Business and Finance** Juan bought a new car for \$16,785. The manufacturer offers a cash rebate of \$987. What was the cost after rebate?



1.4 *Round the numbers to the indicated place.*

23. 6,975 to the nearest hundred

24. 15,897 to the nearest thousand

25. 548,239 to the nearest ten thousand

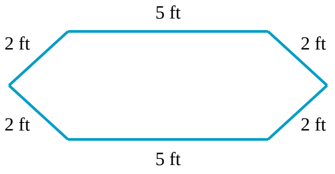
Complete the statements by using the symbol $<$ or $>$.

26. 60 ______ 70

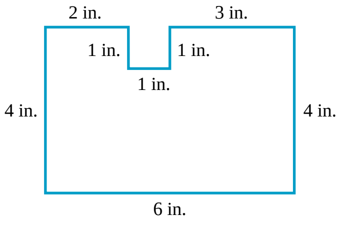
27. 38 ______ 35

Find the perimeter of each figure.

28.



29.



1.5 *In exercises 30 to 32, name the property of multiplication that is illustrated.*

30. $7 \times 8 = 8 \times 7$

31. $3 \times (4 + 7) = 3 \times 4 + 3 \times 7$

32. $(8 \times 9) \times 4 = 8 \times (9 \times 4)$

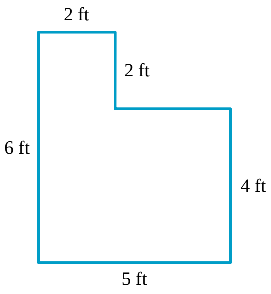
In exercises 33 to 35, perform the indicated operations.

33. $$\begin{array}{r} 58 \\ \times\ 32 \\ \hline \end{array}$$

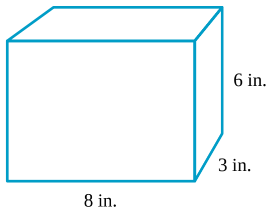
34. $$\begin{array}{r} 25 \\ \times\ 43 \\ \hline \end{array}$$

35. $$\begin{array}{r} 378 \\ \times\ 409 \\ \hline \end{array}$$

36. *Find the area of the figure.*

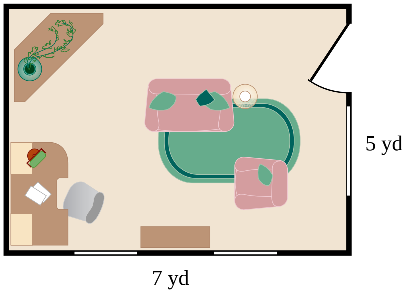


37. *Find the volume of the figure.*



Solve the application.

38. **CRAFTS** You wish to carpet a room that is 5 yd by 7 yd. The carpet costs \$18 per square yard. What will be the total cost of the materials?



Perform the indicated operation.

39.
$$\begin{array}{r} 129 \\ \times\ 240 \\ \hline \end{array}$$

Estimate the product by rounding each factor to the nearest hundred.

40.
$$\begin{array}{r} 1{,}217 \\ \times\ 494 \\ \hline \end{array}$$

1.6 *Divide if possible.*

41. $0 \div 8$

42. $5 \div 0$

In exercises 43 to 46, divide.

43. $8\overline{)2{,}469}$

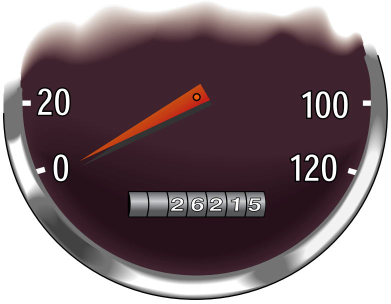
44. $39\overline{)2{,}157}$

45. $64\overline{)31{,}809}$

46. $362\overline{)86{,}915}$

Solve the application.

47. **STATISTICS** Hasina's odometer read 25,235 mi at the beginning of a trip and 26,215 mi at the end. If she used 35 gal of gas for the trip, what was her mileage (mi/gal)?



Estimate.

48. 356 divided by 37

49. 2,125 divided by 123

1.7 *In exercises 50 to 59, evaluate each expression.*

50. 5×2^3

51. $(5 \times 2)^3$

52. $4 + 8 \times 3$

53. $48 \div (2^3 + 4)$

54. $(4 + 8) \times 3$

55. $4 \times 3 + 8 \times 3$

56. $8 \div 4 \times 2 - 2 + 1$

57. $63 \times 2 \div 3 - 54 \div (12 \times 2 \div 4)$

58. $(3 \times 4)^2 - 100 \div 5 \times 6$

59. $(16 \times 2) \div 8 - (6 \div 3 \times 2)$

self-test 1

The purpose of this self-test is to help you assess your progress so that you can find concepts that you need to review before the next exam. Allow yourself about an hour to take this test. At the end of that hour, check your answers against those given in the back of this text. If you miss any, go back to the appropriate section to reread the examples until you have mastered that particular concept.

Perform the indicated operations.

1. $$\begin{array}{r} 489 \\ 562 \\ 613 \\ +254 \\ \hline \end{array}$$

2. $$\begin{array}{r} 13 \\ 2{,}543 \\ +10{,}547 \\ \hline \end{array}$$

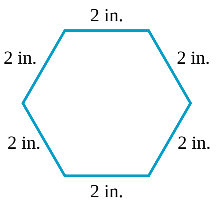
3. $$\begin{array}{r} 89 \\ \times 56 \\ \hline \end{array}$$

4. $$\begin{array}{r} 538 \\ \times 103 \\ \hline \end{array}$$

5. Give the word name for 302,525.

Find the perimeter of the figure shown.

6.



Complete the statements by using the symbol < or >.

7. 49 ______ 47

8. 80 ______ 90

9. Give the place value of 7 in 3,738,500.

Name the property that is illustrated.

10. $4 \times (3 + 6) = (4 \times 3) + (4 \times 6)$

11. $(7 + 3) + 8 = 7 + (3 + 8)$

12. $3 \times (2 \times 7) = (3 \times 2) \times 7$

13. $5 + 12 = 12 + 5$

Divide, using long division.

14. $28\overline{)2{,}135}$

15. $293\overline{)61{,}382}$

16. Write two million, four hundred thirty thousand as a number.

17. What is the total of 392, 95, 9,237, and 11,972?

Name ____________________

Section ________ Date ________

Answers

1. ____________________

2. ____________________

3. ____________________

4. ____________________

5. ____________________

6. ____________________

7. ____________________

8. ____________________

9. ____________________

10. ____________________

11. ____________________

12. ____________________

13. ____________________

14. ____________________

15. ____________________

16. ____________________

17. ____________________

Answers

18. ______________________

19. ______________________

20. ______________________

21. ______________________

22. ______________________

23. ______________________

24. ______________________

25. ______________________

26. ______________________

27. ______________________

28. ______________________

29. ______________________

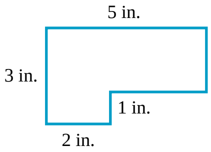
30. ______________________

18. **Business and Finance** A truck rental firm has ordered 25 new vans at a cost of $22,350 per van. What will be the total cost of the order?

19. **Business and Finance** Eight people estimate that the total expenses for a trip they are planning will be $1,784. If each person pays an equal amount, what will be each person's share?

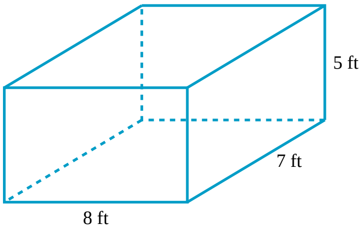
Find the area of the given figure.

20.



Find the volume of the given figure.

21.



22. **Statistics** The attendance for the games of a playoff series in basketball was 12,438, 14,325, 14,581, and 14,634. What was the total attendance for the series?

In exercises 23 to 26, subtract.

23. 289 − 54

24. 55,342 − 14,787

25. 32,345 − 1,575

26. 53,294 − 41,074

27. **Social Science** The maximum load for a light plane with full gas tanks is 500 pounds (lb). Mr. Whitney weighs 215 lb; his wife, 135 lb; and their daughter, 78 lb. How much luggage can they take on a trip without exceeding the load limit?

28. Evaluate the expression $(3 + 4)^2 - (2 + 3^2 - 1)$.

29. Evaluate the expression $15 - 12 \div 2^2 \times 3 + (12 \div 4 \times 3)$.

Estimate the sum by rounding each addend to the nearest hundred.

30.
$$\begin{array}{r} 943 \\ 3{,}281 \\ 778 \\ 2{,}112 \\ +570 \\ \hline \end{array}$$

chapter
2
> Make the Connection

CHAPTER 2

Multiplying and Dividing Fractions

INTRODUCTION

Rebecca got tired of working nine to five as a paralegal in a busy law office in Philadelphia. On many occasions, she had to stay at work until 6:00 or 6:30, and the commute back and forth was exhausting. She wanted a more flexible schedule and started looking for an opportunity to be her own boss.

Rebecca loved to cook and often made delicious meals when she had the time. After investigating certain job opportunities, Rebecca decided to become a personal chef. She read somewhere that personal chefs can make between $50,000 and $70,000 a year. She knew of many busy families in her area that did not have the time to cook meals at night, and she thought this might be a good opportunity. She quit her job and started making meals for some of the people in the law firm where she had previously worked. She planned menus, did the grocery shopping, and cooked the meals right in her clients' kitchens. Sometimes, she cooked enough meals for an entire week and left the meals in containers in the freezer with instructions on how to reheat. She had to adjust her recipes for larger or smaller amounts, depending on whom she was cooking for.

Before long, Rebecca's clients started referring her to friends and relatives and her business began to grow. She was very happy with her new schedule and the free time that she had.

CHAPTER 2 OUTLINE

Chapter 2 :: Prerequisite Test 98
2.1 Prime Numbers and Divisibility 99
2.2 Factoring Whole Numbers 108
2.3 Fraction Basics 117
2.4 Simplifying Fractions 128
2.5 Multiplying Fractions 139
2.6 Dividing Fractions 153
Chapter 2 :: Summary / Summary Exercises / Self-Test / Cumulative Review :: Chapters 1–2 167

2 prerequisite test

Name ______________________

Section __________ Date __________

This prerequisite test provides some exercises requiring skills that you will need to be successful in the coming chapter. The answers for these exercises can be found in the back of this text. This prerequisite test can help you identify topics that you will need to review before beginning the chapter.

Answers

1. ______________________

2. ______________________

3. ______________________

4. ______________________

5. ______________________

6. ______________________

7. ______________________

8. ______________________

9. ______________________

10. ______________________

1. Does 4 divide exactly into 30 (that is, with no remainder)?

2. Does 5 divide exactly into 29?

3. Does 6 divide exactly into 72?

4. Does 3 divide exactly into 412?

5. Does 2 divide exactly into 238?

6. List all the whole numbers that can divide exactly into 9.

7. List all the whole numbers that can divide exactly into 10.

8. List all the whole numbers that can divide exactly into 17.

9. List all the whole numbers that can divide exactly into 48.

10. List all the whole numbers that can divide exactly into 60.

2.1

Prime Numbers and Divisibility

< 2.1 Objectives >

1 > Find the factors of a number

2 > Determine whether a number is prime, composite, or neither

3 > Determine whether a number is divisible by 2, 3, 4, 5, 6, or 9

Tips for Student Success

Working Together

How many of your classmates do you know? Whether you are by nature gregarious or shy, you have much to gain by getting to know your classmates.

1. It is important to have someone to call when you miss class or if you are unclear on an assignment.
2. Working with another person is almost always beneficial to both people. If you don't understand something, it helps to have someone to ask about it. If you do understand something, nothing will cement that understanding more than explaining the idea to another person.
3. Sometimes we need to commiserate. If an assignment is particularly frustrating, it is reassuring to find that it is also frustrating for other students.
4. Have you ever thought you had the right answer, but it didn't match the answer in the text? Frequently the answers are equivalent, but that's not always easy to see. A different perspective can help you see that. Occasionally there is an error in a textbook (here, we are talking about *other* textbooks). In such cases it is wonderfully reassuring to find that someone else has the same answer as you do.

NOTE

Also, 2 and 5 can be called *divisors* of 10. They divide 10 exactly.

In Section 1.5 we said that because $2 \times 5 = 10$, we call 2 and 5 **factors** of 10.

Definition

Factor	A **factor** of a whole number is another whole number that *divides exactly* into the original number. This means that the division has a remainder of 0.

Example 1 Finding Factors

< Objective 1 >

List all the factors of 18.

When finding all the factors for any number, we always start with division by 1 because 1 is a factor of every number.

$1 \times 18 = 18$ Our list of factors starts with 1 and 18.

We continue with division by 2.

$2 \times 9 = 18$ Our list now contains 1, 2, 9, 18.

We check divisibility by each subsequent whole number until we get to a number that is already in the factor list.

$3 \times 6 = 18$ The list is now 1, 2, 3, 6, 9, 18.

NOTE

This is a complete list of the factors. There are no other whole numbers that divide 18 exactly. Note that the factors of 18, except for 18 itself, are *smaller* than 18.

Our number, 18, is not divisible by either 4 or 5. It is divisible by 6, but 6 is already in the list. The factor list is complete.

Check Yourself 1

List all the factors of 24.

Listing factors leads us to an important classification of whole numbers. Any whole number larger than 1 is either a *prime* or a *composite* number.

A whole number greater than 1 always has itself and 1 as factors. Sometimes these are the *only* factors. For instance, 1 and 3 are the only factors of 3.

Definition

Prime Number

A **prime number** is any whole number that has exactly two factors, 1 and itself.

NOTE

How large can a prime number be? There is no largest prime number. To date, the largest known prime is $2^{43,112,609} - 1$. This is a number with 12,978,189 digits. Of course, computers have to be used to verify that a number of this size is prime. If this number were printed out in 12-point type, it would be 30 miles long. By the time you read this, someone may very well have found an even larger prime number.

As examples, 2, 3, 5, and 7 are prime numbers. Their only factors are 1 and themselves.

To check whether a number is prime, one approach is simply to divide the smaller primes—2, 3, 5, 7, and so on—into the given number. If no factors other than 1 and the given number are found, the number is prime.

Here is the method known as the **sieve of Eratosthenes** for identifying prime numbers.

1. Write down a series of counting numbers, starting with the number 2. In this example, we stop at 50.
2. Start at the number 2. Delete every second number after 2.
3. Move to the number 3. Delete every third number after 3 (some numbers will be deleted twice).
4. Continue this process, deleting every fourth number after 4, every fifth number after 5, and so on.
5. When you have finished, the undeleted numbers are the prime numbers.

	2	3	~~4~~	5	~~6~~	7	~~8~~	~~9~~	~~10~~
11	~~12~~	13	~~14~~	~~15~~	~~16~~	17	~~18~~	19	~~20~~
~~21~~	~~22~~	23	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	29	~~30~~
31	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	37	~~38~~	~~39~~	~~40~~
41	~~42~~	43	~~44~~	~~45~~	~~46~~	47	~~48~~	~~49~~	~~50~~

The prime numbers less than 50 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47.

Example 2 Identifying Prime Numbers

< Objective 2 >

Which of the numbers 17, 29, and 33 are prime?

17 is a prime number. 1 and 17 are the only factors.

29 is a prime number. 1 and 29 are the only factors.

33 is *not* prime. 1, 3, 11, and 33 are all factors of 33.

Note: For two-digit numbers, if the number is *not* a prime, it has one or more of the numbers 2, 3, 5, or 7 as factors.

Check Yourself 2

Which of the following numbers are prime numbers?

2, 6, 9, 11, 15, 19, 23, 35, 41

We can now define a second class of whole numbers.

Definition

Composite Number

A **composite number** is any whole number greater than 1 that is not prime. Every composite number has more than two factors.

Example 3 Identifying Composite Numbers

Which of the numbers 18, 23, 25, and 38 are composite?

18 is a composite number.	1, 2, 3, 6, 9, and 18 are all factors of 18.
23 is *not* a composite number.	1 and 23 are the only factors. This means that 23 is a *prime number.*
25 is a composite number.	1, 5, and 25 are factors.
38 is a composite number.	1, 2, 19, and 38 are factors.

Check Yourself 3

Which of the following numbers are composite numbers?

2, 6, 10, 13, 16, 17, 22, 27, 31, 35

By the definitions of prime and composite numbers:

Definition

Zero and One

The whole numbers 0 and 1 are neither prime nor composite.

NOTE

Divisibility by 2 indicates that a number is *even.*

This is simply a matter of the way in which prime and composite numbers are defined in mathematics. The numbers 0 and 1 are the *only* two whole numbers that cannot be classified as one or the other.

For our work in this and the following sections, it is very useful to be able to tell whether a given number is divisible by 2, 3, or 5. The tests that follow will give you some tools to check divisibility without actually having to divide.

Tests for divisibility by other numbers are also available. However, we have limited this section to those tests involving 2, 3, 4, 5, 6, 9, and 10 because they are very easy to use and occur frequently in our work.

Property

Divisibility by 2

A whole number is divisible by 2 if its last digit is 0, 2, 4, 6, or 8.

Example 4 Determining If a Number Is Divisible by 2

< Objective 3 >

Which of the numbers 2,346, 13,254, 23,573, and 57,085 are divisible by 2?

2,346 is divisible by 2.	The final digit is 6.
13,254 is divisible by 2.	The final digit is 4.
23,573 is *not* divisible by 2.	The final digit is not 0, 2, 4, 6, or 8.
57,085 is *not* divisible by 2.	

Check Yourself 4

Which of the following are divisible by 2?

274 3,587 7,548 13,593

Property

Divisibility by 3 A whole number is divisible by 3 if the sum of its digits is divisible by 3.

Example 5 Determining If a Number Is Divisible by 3

Which of the numbers 345, 1,243, and 25,368 are divisible by 3?

345 is divisible by 3.	The sum of the digits, $3 + 4 + 5$, is 12, and 12 is divisible by 3.
1,243 is *not* divisible by 3.	The sum of the digits, $1 + 2 + 4 + 3$, is 10, and 10 is not divisible by 3.
25,368 is divisible by 3.	The sum of the digits, $2 + 5 + 3 + 6 + 8$, is 24, and 24 is divisible by 3. Note that 25,368 is also divisible by 2.

Check Yourself 5

(a) Is 372 divisible by 2? By 3? (b) Is 5,493 divisible by 2? By 3?

Property

Divisibility by 5 A whole number is divisible by 5 if its last digit is 0 or 5.

Example 6 Determining If a Number Is Divisible by 5

Determine which of the following are divisible by 5.

2,435 is divisible by 5.	Its last digit is 5.
23,123 is *not* divisible by 5.	Its last digit is 3.
123,240 is divisible by 5.	Its last digit is 0. Do you see that 123,240 is also divisible by 2 and 3?

Check Yourself 6

(a) Is 12,585 divisible by 5? By 2? By 3?

(b) Is 5,890 divisible by 5? By 2? By 3?

Combining some of the techniques we have developed in this section, we can come up with divisibility tests for composite numbers as well.

Property

Divisibility by 4 A whole number is divisible by 4 if its final two digits are divisible by 4.

Example 7 Determining If a Number Is Divisible by 4

Determine if the numbers 1,464 and 2,434 are divisible by 4.

1,464 is divisible by 4.	64 is divisible by 4.
2,434 is *not* divisible by 4.	34 is not divisible by 4.

Check Yourself 7

Determine if each number is divisible by 4.

(a) 6,456 **(b)** 242 **(c)** 22,100

Property

Divisibility by 6

A whole number is divisible by 6 if it is an even number that is divisible by 3.

Example 8 Determining If a Number Is Divisible by 6

Determine if the numbers 1,464 and 2,434 are divisible by 6.

1,464 is divisible by 6. It is an even number, and the sum of the digits, 15, indicates that it is divisible by 3.

2,434 is *not* divisible by 6. Although it is an even number, the sum of the digits, 13, indicates that it is not divisible by 3.

Check Yourself 8

Determine if each number is divisible by 6.

(a) 6,456 **(b)** 242 **(c)** 22,100

Property

Divisibility by 9

A whole number is divisible by 9 if the sum of its digits is divisible by 9.

Example 9 Determining If a Number Is Divisible by 9

Determine if the numbers 1,494 and 2,634 are divisible by 9.

1,494 is divisible by 9. The sum of the digits, 18, indicates that it is divisible by 9.

2,634 is *not* divisible by 9. The sum of the digits, 15, indicates that it is not divisible by 9.

Check Yourself 9

Determine if each number is divisible by 9.

(a) 3,456 **(b)** 243,000 **(c)** 22,200

Property

Divisibility by 10

A whole number is divisible by 10 if it ends with a zero.

Example 10 Determining If a Number Is Divisible by 10

Determine if the numbers 4,390,005 and 6,420 are divisible by 10.

4,390,005 is *not* divisible by 10. The number does not end with a zero.

6,420 is divisible by 10. The number does end with a zero.

Check Yourself 10

Determine whether each number is divisible by 10.

(a) 2,000,020 **(b)** 2,000,002 **(c)** 3,571,110

Check Yourself ANSWERS

1. 1, 2, 3, 4, 6, 8, 12, and 24 **2.** 2, 11, 19, 23, and 41 are prime numbers
3. 6, 10, 16, 22, 27, and 35 are composite numbers **4.** 274 and 7,548
5. **(a)** Yes in both cases; **(b)** only by 3 **6.** **(a)** By 5 and by 3; **(b)** by 5 and by 2
7. **(a)** Yes; **(b)** no; **(c)** yes **8.** **(a)** Yes; **(b)** no; **(c)** no
9. **(a)** Yes; **(b)** yes; **(c)** no **10.** **(a)** Yes; **(b)** no; **(c)** yes

Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 2.1

(a) ______________ is a factor of every number.

(b) A ______________ number is any whole number that has exactly two factors, 1 and itself.

(c) A __________ number is any whole number greater than 1 that is not prime.

(d) When a whole number is divisible by 2, we call it an ______________ number.

2.1 exercises

< Objective 1 >

List the factors of each number.

1. 4

2. 6

3. 10

4. 12

5. 15

6. 21

7. 24 > Videos

8. 32

9. 64

10. 66

11. 11 > Videos

12. 37

13. 135

14. 236

15. 256

16. 512

< Objective 2 >

Use the list of numbers for exercises 17 and 18.

0, 1, 15, 19, 23, 31, 49, 55, 59, 87, 91, 97, 103, 105

17. Which of the given numbers are prime?

18. Which of the given numbers are composite?

19. List all the prime numbers between 30 and 50.

20. List all the prime numbers between 55 and 75.

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Name ____________________

Section __________ Date __________

Answers

1. ________ 2. ________

3. ________

4. ________

5. ________ 6. ________

7. ________

8. ________

9. ________

10. ________

11. ________ 12. ________

13. ________

14. ________

15. ________

16. ________

17. ________

18. ________

19. ________

20. ________

Answers

21. ______________________

22. ______________________

23. ______________________

24. ______________________

25. ______________________

26. ______________________

27. ______________________

28. ______________________

29. ______________________

30. ______________________

31. ______________________

< Objective 3 >

Use the list of numbers for exercises 21 through 26.

45, 72, 158, 260, 378, 569, 570, 585, 3,541, 4,530, 8,300

21. Which of the given numbers are divisible by 2? > Videos

22. Which of the given numbers are divisible by 3?

23. Which of the given numbers are divisible by 6? > Videos

24. Which of the given numbers are divisible by 9?

25. Which of the given numbers are divisible by 4?

26. Which of the given numbers are divisible by 10?

27. **NUMBER PROBLEM** A school auditorium is to have 350 seats. The principal wants to arrange them in rows with the same number of seats in each row. Use divisibility tests to determine if it is possible to have rows of 10 seats each. Are 15 rows of seats possible?

28. **SOCIAL SCIENCE** Dr. Mento has a class of 80 students. For a group project, she wants to divide the students into groups of 6, 8, or 10. Is this possible? Explain your answer.

Basic Skills | Challenge Yourself | Calculator/Computer | Career Applications | **Above and Beyond**

29. **NUMBER PROBLEM** Use the *sieve of Eratosthenes* to determine all the prime numbers less than 100.

	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

30. Why is the statement not a valid divisibility test for 8?

"A number is divisible by 8 if it is divisible by 2 and 4."

Support your answer with an example. Give a valid divisibility test for 8.

31. Prime numbers that differ by 2 are called *twin primes*. Examples are 3 and 5, 5 and 7, and so on. Find one pair of twin primes between 85 and 105.

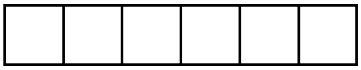
32. The following questions refer to *twin primes* (see exercise 31).

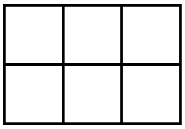
(a) Search for, and make a list of, several pairs of twin primes in which the primes are greater than 3.

(b) What do you notice about each number that lies *between* a pair of twin primes?

(c) Write an explanation for your observation in part **(b)**.

33. Obtain (or imagine that you have) a quantity of square tiles. Six tiles can be arranged in the shape of a rectangle in two different ways:





(a) Record the dimensions of the rectangles shown.

(b) If you use 7 tiles, how many different rectangles can you form?

(c) If you use 10 tiles, how many different rectangles can you form?

(d) What kind of number (of tiles) permits *only one* arrangement into a rectangle? *More than one* arrangement?

34. The number 10 has 4 factors: 1, 2, 5, and 10. We can say that 10 has an even number of factors. Investigate several numbers to determine which numbers have an *even number* of factors and which numbers have an *odd number* of factors.

35. **NUMBER PROBLEM** Suppose that a school has 1,000 lockers and that they are all closed. A person passes through, opening every other locker, beginning with locker 2. Then another person passes through, changing every third locker (closing it if it is open, opening it if it is closed), starting with locker 3. Yet another person passes through, changing every fourth locker, beginning with locker 4. This process continues until 1,000 people pass through.

(a) At the end of this process, which locker numbers are closed?

(b) Write an explanation for your answer to part **(a)**. (*Hint:* It may help to attempt exercise 34 first.)

Answers

1. 1, 2, 4 **3.** 1, 2, 5, 10 **5.** 1, 3, 5, 15 **7.** 1, 2, 3, 4, 6, 8, 12, 24
9. 1, 2, 4, 8, 16, 32, 64 **11.** 1, 11 **13.** 1, 3, 5, 9, 15, 27, 45, 135
15. 1, 2, 4, 8, 16, 32, 64, 128, 256 **17.** 19, 23, 31, 59, 97, 103
19. 31, 37, 41, 43, 47 **21.** 72, 158, 260, 378, 570, 4,530, 8,300
23. 72, 378, 570, 4,530 **25.** 72, 260, 8,300 **27.** Yes; no
29. Above and Beyond **31.** Above and Beyond **33.** Above and Beyond
35. Above and Beyond

Answers

32. ____________

33. ____________

34. ____________

35. ____________

2.2

Factoring Whole Numbers

< **2.2 Objectives** >

1 > Find the factors of a whole number
2 > Find the prime factorization for any number
3 > Find the greatest common factor (GCF) of two numbers
4 > Find the GCF for a group of numbers

To **factor a number** means to write the number as a product of its whole-number factors.

Example 1 Factoring a Composite Number

< Objective 1 >

Factor the number 10.

$10 = 2 \times 5$

The order in which you write the factors does not matter, so $10 = 5 \times 2$ would also be correct.

Of course, $10 = 10 \times 1$ is also a correct statement. However, in this section we are interested in factors other than 1 and the given number.

Factor the number 21.

$21 = 3 \times 7$

Check Yourself 1

Factor 35.

In writing composite numbers as a product of factors, there are a number of different possible factorizations.

Example 2 Factoring a Composite Number

NOTE

There have to be at least two different factorizations, because a composite number has factors other than 1 and itself.

Find three different factorizations of 72.

$$\begin{aligned} 72 &= 8 \times 9 \quad (1)\\ &= 6 \times 12 \quad (2)\\ &= 3 \times 24 \quad (3) \end{aligned}$$

Check Yourself 2

Find three different factorizations of 42.

We now want to write composite numbers as a product of their **prime factors.** Look again at the first factored line of Example 2. The process of factoring can be continued until all the factors are prime numbers.

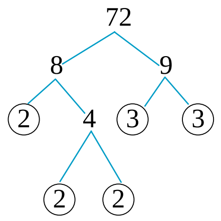
Example 3 Factoring a Composite Number

< Objective 3 >

NOTES

This is often called a **factor tree.**

Finding the prime factorization of a number will be important in our later work in adding fractions.

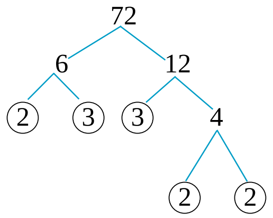


4 is still not prime, and so we continue by factoring 4.

72 is now written as a product of prime factors.

When we write 72 as 2 × 2 × 2 × 3 × 3, no further factorization is possible. This is called the *prime factorization* of 72.

Now, what if we start with a different factored line from the same example, 72 = 6 × 12?



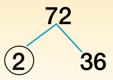
Continue to factor 6 and 12.

Continue again to factor 4. Other choices for the factors of 12 are possible. As we shall see, the end result will be the same.

No matter which pair of factors you start with, you will find the same prime factorization. In this case, there are three factors of 2 and two factors of 3. Because **multiplication is commutative,** the order in which we write the factors does not matter.

Check Yourself 3

We could also begin



Continue the factorization.

Property

The Fundamental Theorem of Arithmetic

There is exactly one prime factorization for any composite number.

The method of Example 3 always works. However, another method for factoring composite numbers exists. This method is particularly useful when numbers get large, in which case a factor tree becomes unwieldy.

Property

Factoring by Division

To find the prime factorization of a number, divide the number by a series of primes until the final quotient is a prime number.

The prime factorization is then the product of all the prime divisors and the final quotient, as we see in Example 4.

Example 4 Finding the Prime Factorization

< Objective 2 >

NOTE

Do you see how the divisibility tests are used here? 60 is divisible by 2, 30 is divisible by 2, and 15 is divisible by 3.

To write 60 as a product of prime factors, divide 2 into 60 for a quotient of 30. Continue to divide by 2 again for the quotient of 15. Because 2 won't divide evenly into 15, we try 3. Because the quotient 5 is prime, we are done.

$$\begin{array}{r}30\\2\overline{)60}\end{array} \searrow \begin{array}{r}15\\2\overline{)30}\end{array} \searrow \begin{array}{r}5\\3\overline{)15}\end{array} \quad \text{Prime}$$

Our factors are the prime divisors and the final quotient. We have

$60 = 2 \times 2 \times 3 \times 5$

Check Yourself 4

Complete the process to find the prime factorization of 90.

$$\begin{array}{r}45\\2\overline{)90}\end{array} \searrow \begin{array}{r}?\\?\overline{)45}\end{array}$$

Remember to continue until the final quotient is prime.

Writing composite numbers in their completely factored form can be simplified if we use a format called **continued division.**

Example 5 Finding Prime Factors Using Continued Division

NOTE

In each short division, we write the quotient *below* rather than above the dividend. This is just a convenience for the next division.

Use the continued-division method to divide 60 by a series of prime numbers.

$$\text{Primes}\begin{array}{l}\nearrow\\\rightarrow\\\searrow\end{array}\begin{array}{r}2\overline{)60}\\2\overline{)30}\\3\overline{)15}\\5\end{array}\quad \text{Stop when the final quotient is prime.}$$

To write the factorization of 60, we list each divisor used and the final prime quotient. In our example, we have

$60 = 2 \times 2 \times 3 \times 5$

Check Yourself 5

Find the prime factorization of 234.

NOTE

The factors of 20, other than 20 itself, are less than 20.

We know that a factor or a divisor of a whole number divides that number exactly. The factors or divisors of 20 are

1, 2, 4, 5, 10, 20

Each of these numbers divides 20 exactly; that is, with no remainder.

Our work in this section involves common factors or divisors. A **common factor** or **divisor** for two numbers is any factor that divides both numbers exactly.

Example 6 Finding Common Factors

< Objective 3 >

Look at the numbers 20 and 30. Is there a common factor for the two numbers?

First, we list the factors. Then we circle the ones that appear in both lists.

Factors

20: ①, ②, 4, ⑤, ⑩, 20

30: ①, ②, 3, ⑤, 6, ⑩, 15, 30

We see that 1, 2, 5, and 10 are common factors of 20 and 30. Each of these numbers divides both 20 and 30 exactly.

Our later work with fractions will require that we find the greatest common factor of a group of numbers.

Definition

Greatest Common Factor

The **greatest common factor** (GCF) of a group of numbers is the *largest* number that divides each of the given numbers exactly.

Example 6 (Continued) Finding the Greatest Common Factor

In the first part of Example 6, the common factors of the numbers 20 and 30 were listed as

1, 2, 5, 10 Common factors of 20 and 30

The greatest common factor of the two numbers is then 10, because 10 is the *largest* of the four common factors.

Check Yourself 6

List the factors of 30 and 36 and then find the greatest common factor.

The method of Example 6 also works in finding the greatest common factor of a group of more than two numbers.

Example 7 Finding the Greatest Common Factor by Listing Factors

< Objective 4 >

Find the GCF of 24, 30, and 36. We list the factors of each of the three numbers.

24: ①, ②, ③, 4, ⑥, 8, 12, 24

30: ①, ②, ③, 5, ⑥, 10, 15, 30

36: ①, ②, ③, 4, ⑥, 9, 12, 18, 36

So 6 is the greatest common factor of 24, 30, and 36.

NOTE

Looking at the three lists, we see that 1, 2, 3, and 6 are common factors.

Check Yourself 7

Find the greatest common factor of 16, 24, and 32.

NOTE

If there are no common prime factors, the GCF is 1.

The process shown in Example 7 is very time-consuming when larger numbers are involved. A better approach to the problem of finding the GCF of a group of numbers uses the prime factorization of each number. We outline the process here.

Step by Step

Finding the Greatest Common Factor

Step 1 Write the prime factorization for each of the numbers in the group.

Step 2 Locate the prime factors that are *common* to all the numbers.

Step 3 The greatest common factor (GCF) is the *product* of all the common prime factors.

Example 8 Finding the Greatest Common Factor

Find the GCF of 20 and 30.

Step 1 Write the prime factorization of 20 and 30.

$20 = 2 \times 2 \times 5$

$30 = 2 \times 3 \times 5$

Step 2 Find the prime factors common to each number.

$20 = \textcircled{2} \times 2 \times \textcircled{5}$

$30 = \textcircled{2} \times 3 \times \textcircled{5}$

2 and 5 are the common prime factors.

Step 3 Form the product of the common prime factors.

$2 \times 5 = 10$

So 10 is the greatest common factor.

Check Yourself 8

Find the GCF of 30 and 36.

To find the greatest common factor of a group of more than two numbers, we use the same process.

Example 9 Finding the Greatest Common Factor

Find the GCF of 24, 30, and 36.

$24 = \textcircled{2} \times 2 \times 2 \times \textcircled{3}$

$30 = \textcircled{2} \times \textcircled{3} \times 5$

$36 = \textcircled{2} \times 2 \times \textcircled{3} \times 3$

So 2 and 3 are the prime factors common to *all three numbers.*

And $2 \times 3 = 6$ is the GCF.

Check Yourself 9

Find the GCF of 15, 30, and 45.

Example 10 Finding the Greatest Common Factor

NOTE

If two numbers, such as 15 and 28, have no common factor other than 1, they are called **relatively prime.**

Find the greatest common factor of 15 and 28.

$15 = 3 \times 5$

$28 = 2 \times 2 \times 7$

There are no common prime factors listed. But remember that 1 is a factor of every whole number.

The greatest common factor of 15 and 28 is 1.

Check Yourself 10

Find the greatest common factor of 30 and 49.

Check Yourself ANSWERS

1. 5×7 **2.** $2 \times 21, 3 \times 14, 6 \times 7$ **3.** $2 \times 2 \times 2 \times 3 \times 3$

4. $\begin{array}{r} 45 \\ 2\overline{)90} \end{array} \rightarrow \begin{array}{r} 15 \\ 3\overline{)45} \end{array} \rightarrow \begin{array}{r} 5 \\ 3\overline{)15} \end{array}$ **5.** $2 \times 3 \times 3 \times 13$

$90 = 2 \times 3 \times 3 \times 5$

6. 30: ①, ②, ③, 5, ⑥, 10, 15, 30

36: ①, ②, ③, 4, ⑥, 9, 12, 18, 36

6 is the greatest common factor.

7. 16: ①, ②, ④, ⑧, 16

24: ①, ②, 3, ④, 6, ⑧, 12, 24

32: ①, ②, ④, ⑧, 16, 32

The GCF is 8.

8. $30 = \textcircled{2} \times \textcircled{3} \times 5$

$36 = \textcircled{2} \times 2 \times \textcircled{3} \times 3$

The GCF is $2 \times 3 = 6$.

9. $15 = \textcircled{3} \times \textcircled{5}$

$30 = 2 \times \textcircled{3} \times \textcircled{5}$

$45 = \textcircled{3} \times 3 \times \textcircled{5}$

The GCF is 15.

10. GCF is 1; 30 and 49 are relatively prime

Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 2.2

(a) Because multiplication is ________________, the order in which we write factors does not matter.

(b) There is exactly one ______________ factorization for any whole number.

(c) A ________________ factor for two numbers is any factor that divides both numbers exactly.

(d) GCF is an abbreviation for ________________ common factor.

2.2 exercises

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Name ____________________

Section __________ Date __________

Answers

1. ________ 2. ________

3. ________ 4. ________

5. ________ 6. ________

7. ____________________

8. ____________________

9. ____________________

10. ____________________

11. ____________________

12. ____________________

13. ____________________

14. ____________________

15. ____________________

16. ____________________

17. ________ 18. ________

19. ________ 20. ________

21. ________ 22. ________

23. ________ 24. ________

< Objectives 1 and 2 >

Find the prime factorization of each number.

1. 18

2. 22

3. 30

4. 35

5. 51

6. 42

7. 66 > Videos

8. 100

9. 130

10. 88

11. 315

12. 400

13. 225

14. 132

15. 189 > Videos

16. 330

In later mathematics courses, you sometimes want to find factors of a number with a given sum or difference. These exercises use this technique.

17. Find two factors of 48 with a sum of 14.

18. Find two factors of 48 with a sum of 26.

19. Find two factors of 48 with a difference of 8.

20. Find two factors of 48 with a difference of 2.

21. Find two factors of 24 with a sum of 10. > Videos

22. Find two factors of 15 with a difference of 2.

23. Find two factors of 30 with a difference of 1.

24. Find two factors of 28 with a sum of 11.

< Objective 3 >

Find the greatest common factor for each group of numbers.

25. 4 and 6 > Videos

26. 6 and 9

27. 10 and 15

28. 12 and 14

29. 21 and 24

30. 22 and 33

31. 20 and 21

32. 28 and 42

33. 18 and 24

34. 35 and 36

35. 18 and 54 > Videos

36. 12 and 48

Basic Skills | **Challenge Yourself** | Calculator/Computer | Career Applications | Above and Beyond

< Objective 4 >

Find the GCF for each group of numbers.

37. 12, 36, and 60

38. 15, 45, and 90

39. 105, 140, and 175

40. 17, 19, and 31

41. 25, 75, and 150 > Videos

42. 36, 72, and 144

For exercises 43 to 46 fill in each blank with either **always, sometimes,** *or* **never.**

43. Factors of a composite number ____________ include 1 and the number itself.

44. Factors of a prime number ____________ include 1 and the number itself.

45. A number with a repeated factor is ______________ a prime number.

46. Factors of an even number are ___________________ even numbers.

Answers

25. ______________________

26. ______________________

27. ______________________

28. ______________________

29. ______________________

30. ______________________

31. ______________________

32. ______________________

33. ______________________

34. ______________________

35. ______________________

36. ______________________

37. ______________________

38. ______________________

39. ______________________

40. ______________________

41. ______________________

42. ______________________

43. ______________________

44. ______________________

45. ______________________

46. ______________________

Basic Skills | Challenge Yourself | Calculator/Computer | Career Applications | Above and Beyond

Answers

47. ________

48. ________

49. ________

50. ________

47. A natural number is said to be *perfect* if it is equal to the sum of its factors, except itself.

(a) Show that 28 is a perfect number.

(b) Identify another perfect number less than 28.

48. Find the smallest natural number that is divisible by all the following: 2, 3, 4, 6, 8, 9.

49. **SOCIAL SCIENCE** Tom and Dick both work the night shift at the steel mill. Tom has every sixth night off, and Dick has every eighth night off. If they both have August 1 off, when will they both be off together again?

50. **SCIENCE AND MEDICINE** Mercury, Venus, and Earth revolve around the sun once every 3, 7, and 12 months, respectively. If the three planets are now in the same straight line, what is the smallest number of months that must pass before they line up again?

Answers

1. 2 × 3 × 3 **3.** 2 × 3 × 5 **5.** 3 × 17 **7.** 2 × 3 × 11
9. 2 × 5 × 13 **11.** 3 × 3 × 5 × 7 **13.** 3 × 3 × 5 × 5
15. 3 × 3 × 3 × 7 **17.** 6, 8 **19.** 4, 12 **21.** 4, 6 **23.** 5, 6
25. 2 **27.** 5 **29.** 3 **31.** 1 **33.** 6 **35.** 18 **37.** 12
39. 35 **41.** 25 **43.** always **45.** never
47. **(a)** 1 + 2 + 4 + 7 + 14 = 28; **(b)** 6 **49.** August 25

2.3 Fraction Basics

< **2.3 Objectives** >

1 > Identify the numerator and denominator of a fraction
2 > Use fractions to name parts of a whole
3 > Identify proper and improper fractions
4 > Write improper fractions as mixed numbers
5 > Write mixed numbers as improper fractions

Previous sections dealt with whole numbers and the operations that are performed on them. We are now ready to consider a new kind of number, a **fraction.**

Definition

Fraction	Whenever a unit or a whole quantity is divided into parts, we call those parts **fractions** of the unit.

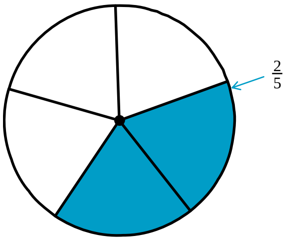
NOTE

Our word *fraction* comes from the Latin stem *fractio,* which means "breaking into pieces."

NOTE

Common fraction is technically the correct term. We just use *fraction* in these materials.

In this figure, the whole has been divided into five equal parts. We use the symbol $\frac{2}{5}$ to represent the shaded portion of the whole.



The symbol $\frac{2}{5}$ is called a **common fraction,** or more simply a fraction. A fraction is written in the form $\frac{a}{b}$, in which a and b represent whole numbers and b cannot be equal to 0.

We give the numbers a and b special names. The **denominator,** b, is the number on the bottom. This tells us into how many equal parts the unit or whole has been divided. The **numerator,** a, is the number on the top. This tells us how many parts of the unit are used.

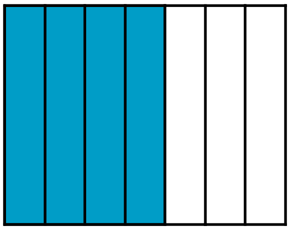
In the fraction $\frac{2}{5}$, the *denominator* is 5; the unit or whole (the circle) has been divided into five equal parts. The *numerator* is 2. We have shaded two parts of the unit.

$$\frac{2 \longleftarrow \text{Numerator}}{5 \longleftarrow \text{Denominator}}$$

Example 1 Labeling Fraction Components

< **Objectives 1, 2** >

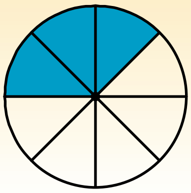
The fraction $\frac{4}{7}$ names the shaded part of the rectangle in the figure.



The unit or whole is divided into seven equal parts, so the denominator is 7. We have shaded four of those parts, and so we have a numerator of 4.

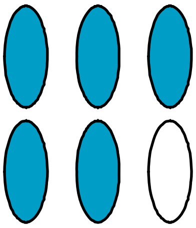
Check Yourself 1

What fraction names the shaded part of this diagram? Identify the numerator and denominator.



Fractions can also be used to name a part of a collection or a set of identical objects.

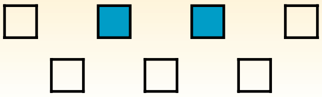
Example 2 Naming a Fractional Part



The fraction $\frac{5}{6}$ names the shaded part of the figure to the left. We have shaded five of the six identical objects.

Check Yourself 2

What fraction names the shaded part of this diagram?



Example 3 Naming a Fractional Part

NOTES

The fraction $\frac{8}{23}$ names the part of the class that is not women.

$\frac{a}{b}$ names the *quotient* when *a* is divided by *b*. Of course, *b* cannot be 0.

In a class of 23 students, 15 are women. We can name the part of the class that is women as $\frac{15}{23}$.

Check Yourself 3

Seven replacement parts out of a shipment of 50 were faulty. What fraction names the portion of the shipment that was faulty?

A fraction can also be thought of as indicating division. The symbol $\frac{a}{b}$ also means $a \div b$.

Example 4 Interpreting a Fraction as Division

The fraction $\frac{2}{3}$ names the quotient when 2 is divided by 3. So $\frac{2}{3} = 2 \div 3$.

Note: $\frac{2}{3}$ can be read as "two-thirds" or as "2 divided by 3."

Check Yourself 4

Write $\frac{5}{9}$ using division.

We can use the relative size of the numerator and denominator of a fraction to separate fractions into two different categories.

Definition

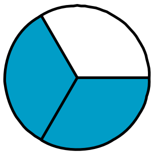
Proper Fraction If the numerator is *less than* the denominator, the fraction names a number less than 1 and is called a **proper fraction.**

Definition

Improper Fraction If the numerator is *greater than or equal to* the denominator, the fraction names a number greater than or equal to 1 and is called an **improper fraction.**

Example 5 Categorizing Fractions

< Objective 3 >



$\frac{2}{3}$ names less than 1 unit and $2 < 3$.

Numerator Denominator

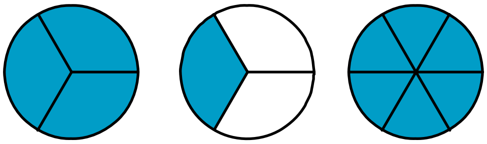
NOTE

In Figure 5, the circle on the left is divided into 3 parts, so it represents $\frac{3}{3}$.

(a) $\frac{2}{3}$ is a proper fraction because the numerator is less than the denominator.

(b) $\frac{4}{3}$ is an improper fraction because the numerator is larger than the denominator.

(c) Also, $\frac{6}{6}$ is an improper fraction because it names exactly 1 unit; the numerator is equal to the denominator.



$\frac{4}{3}$ names more than 1 unit and $4 > 3$.

Numerator Denominator

$\frac{6}{6} = 1$

Check Yourself 5

List the proper fractions and the improper fractions in the following list:

$$\frac{5}{4}, \frac{10}{11}, \frac{3}{4}, \frac{8}{5}, \frac{6}{6}, \frac{13}{10}, \frac{7}{8}, \frac{15}{8}$$

Another way to write a fraction that is larger than 1 is as a **mixed number.**

Definition

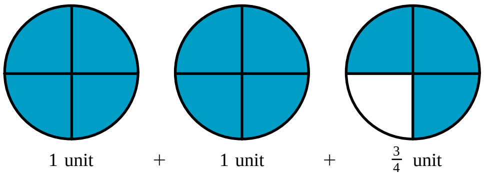
Mixed Number A **mixed number** is the sum of a whole number and a proper fraction.

Example 6 Identifying a Mixed Number

NOTE

$2\frac{3}{4}$ means $2 + \frac{3}{4}$. In fact, we read the mixed number as "two *and* three-fourths." The plus sign is usually not written.

The number $2\frac{3}{4}$ is a mixed number. It represents the sum of the whole number 2 and the fraction $\frac{3}{4}$. Look at the following diagram, which represents $2\frac{3}{4}$.

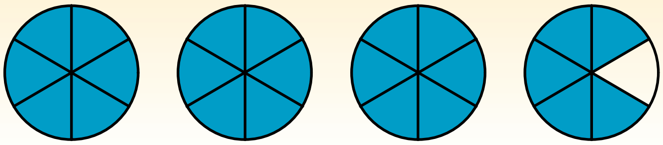


NOTE

In subsequent courses, you will find that improper fractions are preferred to mixed numbers.

Check Yourself 6

Give the mixed number that names the shaded portion of the given diagram.



For our later work it will be important to be able to change back and forth between improper fractions and mixed numbers. Because an improper fraction represents a number that is greater than or equal to 1, we have the following property:

Property

Improper Fractions to Mixed Numbers

An improper fraction can always be written as either a mixed number or a whole number.

To do this, remember that you can think of a fraction as indicating division. The numerator is divided by the denominator. This leads us to the following process:

Step by Step

To Change an Improper Fraction to a Mixed Number

Step 1 Divide the numerator by the denominator.

Step 2 If there is a remainder, write the remainder over the original denominator.

Example 7 Converting a Fraction to a Mixed Number

< Objective 4 >

NOTES

You can write the fraction $\frac{17}{5}$ as 17 ÷ 5. We divide the numerator by the denominator.

In step 1, the quotient gives the whole-number portion of the mixed number.
Step 2 gives the fractional portion of the mixed number.

Convert $\frac{17}{5}$ to a mixed number.

Divide 17 by 5.

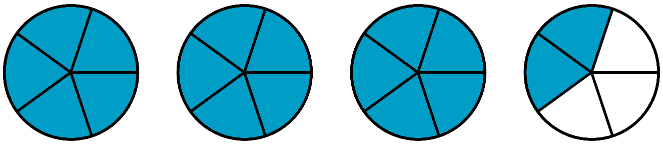
$$\begin{array}{r} 3 \\ 5\overline{)17} \\ \underline{15} \\ 2 \end{array} \qquad \frac{17}{5} = 3\frac{2}{5}$$

Remainder

Original denominator

Quotient

In diagram form:



$$\frac{17}{5} = 3\frac{2}{5}$$

Check Yourself 7

Convert $\frac{32}{5}$ to a mixed number.

Example 8 Converting a Fraction to a Mixed Number

NOTE

If there is *no* remainder, the improper fraction is equal to some whole number, in this case 3.

Convert $\frac{21}{7}$ to a mixed or a whole number.

Divide 21 by 7.

$$\begin{array}{r} 3 \\ 7\overline{)21} \\ \underline{21} \\ 0 \end{array} \qquad \frac{21}{7} = 3$$

Check Yourself 8

Convert $\frac{48}{6}$ to a mixed or a whole number.

You may also need to convert mixed numbers to improper fractions. Just use the following rule:

Step by Step

To Change a Mixed Number to an Improper Fraction

Step 1 Multiply the denominator of the fraction by the whole-number portion of the mixed number.

Step 2 Add the numerator of the fraction to that product.

Step 3 Write that sum over the original denominator to form the improper fraction.

Example 9 Converting Mixed Numbers to Improper Fractions

< Objective 5 >

(a) Convert $3\frac{2}{5}$ to an improper fraction.

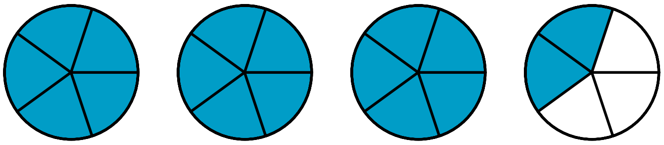
$$\overset{+}{\underset{\times}{3\frac{2}{5}}} = \frac{\overbrace{(5 \times 3)}^{\text{Denominator} \times \text{Whole number}} + \overset{\text{Numerator}}{2}}{\underset{\text{Denominator}}{5}}$$

Multiply the denominator by the whole number (5 × 3 = 15). Add the numerator. We now have 17.

$$= \frac{17}{5}$$

Write 17 over the original denominator.

In diagram form:



Each of the three units has 5 fifths, so the whole-number portion is 5 × 3, or 15, fifths. Then add the $\frac{2}{5}$ from the fractional portion for $\frac{17}{5}$.

NOTE

Multiply the denominator, 7, by the whole number, 4, and add the numerator, 5.

(b) Convert $4\frac{5}{7}$ to an improper fraction.

$$4\frac{5}{7} = \frac{(7 \times 4) + 5}{7} = \frac{33}{7}$$

Check Yourself 9

Convert $5\frac{3}{8}$ to an improper fraction.

One special kind of improper fraction should be mentioned at this point: a fraction with a denominator of 1.

Definition

Fractions with a Denominator of 1

Any fraction with a denominator of 1 is equal to the numerator alone. For example,

$$\frac{5}{1} = 5 \quad \text{and} \quad \frac{12}{1} = 12$$

You probably do many conversions between mixed and whole numbers without even thinking about the process that you follow, as Example 10 illustrates.

Example 10 Converting Quarter-Dollars to Dollars

Maritza has 53 quarters in her bank. How many dollars does she have?

Because there are 4 quarters in each dollar, 53 quarters can be written as

$$\frac{53}{4}$$

Converting the amount to dollars is the same as rewriting it as a mixed number.

$$\frac{53}{4} = 13\frac{1}{4}$$

She has $13\frac{1}{4}$ dollars, which you would probably write as \$13.25. (*Note:* We will discuss decimal point usage later in this text.)

Check Yourself 10

Kevin is doing the inventory in the convenience store in which he works. He finds there are 11 half-gallons (gal) of milk. Write the amount of milk as a mixed number of gallons.

Check Yourself ANSWERS

1. $\dfrac{3}{8}\begin{array}{l}\longleftarrow \text{Numerator}\\ \longleftarrow \text{Denominator}\end{array}$

2. $\frac{2}{7}$ **3.** $\frac{7}{50}$ **4.** $5 \div 9$

5. Proper fractions: $\frac{10}{11}, \frac{3}{4}, \frac{7}{8}$ Improper fractions: $\frac{5}{4}, \frac{8}{5}, \frac{6}{6}, \frac{13}{10}, \frac{15}{8}$ **6.** $3\frac{5}{6}$ **7.** $6\frac{2}{5}$ **8.** 8

9. $\frac{43}{8}$ **10.** $5\frac{1}{2}$ gal

Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 2.3

(a) Given a fraction like $\frac{3}{4}$, we call 4 the ______________ of the fraction.

(b) If the numerator is less than the denominator, the fraction names a number less than 1 and is called a ______________ fraction.

(c) An improper fraction can always be written as either a ______________ number or a whole number.

(d) Any fraction with a denominator of 1 is equal to the ______________ alone.

2.3 exercises

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Name ____________________

Section __________ Date __________

Answers

1. ____________________

2. ____________________

3. ____________________

4. ____________________

5. ____________________

6. ____________________

7. ____________________

8. ____________________

9. ____________________

10. ____________________

11. ____________________

12. ____________________

13. ____________________

14. ____________________

< Objective 1 >

Identify the numerator and denominator of each fraction.

1. $\frac{6}{11}$

2. $\frac{5}{12}$

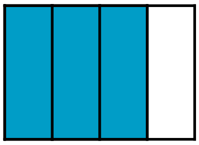
3. $\frac{3}{11}$

4. $\frac{9}{14}$

< Objective 2 >

What fraction names the shaded part of each figure?

5.



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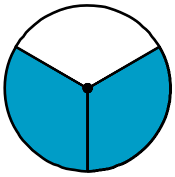

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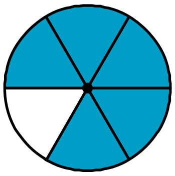

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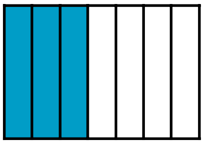

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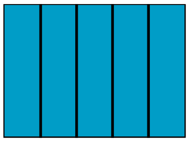

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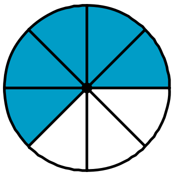

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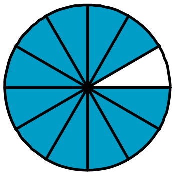

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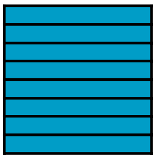

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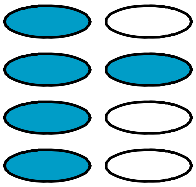

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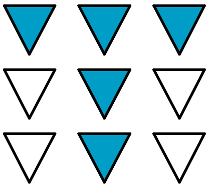


Solve each application.

15. Statistics You missed 7 questions on a 20-question test. What fraction names the part you got correct? The part you got wrong? > Videos

16. Statistics Of the 5 starters on a basketball team, 2 fouled out of a game. What fraction names the part of the starting team that fouled out?

17. Business and Finance A used-car dealer sold 11 of the 17 cars in stock. What fraction names the portion sold? What fraction names the portion *not* sold?

18. Business and Finance At lunch, 5 people out of a group of 9 had hamburgers. What fraction names the part of the group who had hamburgers? What fraction names the part who did *not* have hamburgers?

19. Use division to show another way of writing $\frac{2}{5}$.

20. Use division to show another way of writing $\frac{4}{5}$.

< **Objective 3** >

Identify each number as a proper fraction, an improper fraction, or a mixed number.

21. $\frac{3}{5}$

22. $\frac{9}{5}$

23. $2\frac{3}{5}$

24. $\frac{7}{9}$

25. $\frac{6}{6}$

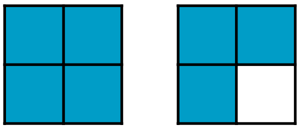
26. $1\frac{1}{5}$

27. $\frac{13}{17}$

28. $\frac{16}{15}$

Give the mixed number that names the shaded portion of each diagram. Also write each as an improper fraction.

29.



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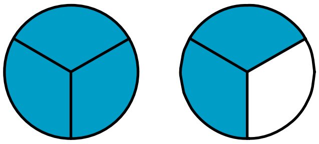

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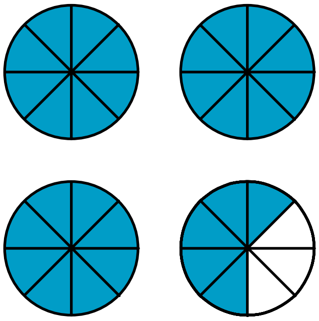

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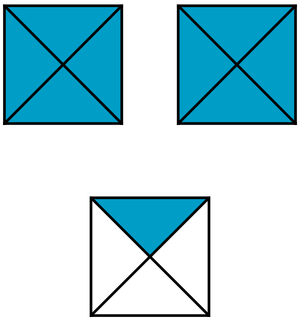

Answers

15. ____________________

16. ____________________

17. ____________________

18. ____________________

19. ____________________

20. ____________________

21. ____________________

22. ____________________

23. ____________________

24. ____________________

25. ____________________

26. ____________________

27. ____________________

28. ____________________

29. ____________________

30. ____________________

31. ____________________

32. ____________________

Answers

33. ________ 34. ________

35. ________ 36. ________

37. ________ 38. ________

39. ________ 40. ________

41. ________ 42. ________

43. ________ 44. ________

45. ________ 46. ________

47. ________ 48. ________

49. ________ 50. ________

51. ________ 52. ________

53. ________ 54. ________

55. ________ 56. ________

57. ________

Solve each application.

33. **Business and Finance** Clayton has 64 quarters in his bank. How many dollars does he have?

34. **Business and Finance** Amy has 19 quarters in her purse. How many dollars does she have?

35. **Business and Finance** Manuel counted 35 half-gallons of orange juice in his store. Write the amount of orange juice as a mixed number of gallons.

36. **Business and Finance** Sarah has 19 half-gallons of turpentine in her paint store. Write the amount of turpentine as a mixed number of gallons.

< Objective 4 >

Change to a mixed or whole number.

37. $\frac{22}{5}$ > Videos

38. $\frac{27}{8}$

39. $\frac{34}{5}$

40. $\frac{25}{6}$

41. $\frac{73}{8}$

42. $\frac{151}{12}$

43. $\frac{24}{6}$

44. $\frac{160}{8}$

45. $\frac{9}{1}$

46. $\frac{8}{1}$

< Objective 5 >

Change to an improper fraction.

47. $4\frac{2}{3}$

48. $2\frac{5}{6}$

49. 8

50. $4\frac{5}{8}$

51. $7\frac{6}{13}$

52. $7\frac{3}{10}$

53. $10\frac{2}{5}$

54. $13\frac{2}{5}$

55. $118\frac{3}{4}$

56. $250\frac{3}{4}$

Basic Skills | Challenge Yourself | Calculator/Computer | **Career Applications** | Above and Beyond

57. **Allied Health** A dilution contains 3 parts blood serum out of a total of 10 parts. Write this number as a fraction.

58. ELECTRONICS Write all the fractional values described in the following paragraph as fractions.

Betsy ordered electronic components from her favorite supplier. She bought two dozen, one-quarter watt resistors, 10 light-emitting diodes (LEDs) that require ten-thirds of a volt (forward voltage) to illuminate, and one, three-eighths henry inductor.

59. MANUFACTURING TECHNOLOGY In the packaging division of Early Enterprises, there are 36 packaging machines. At any given time, five of the machines are shut down for scheduled maintenance and service. What is the fraction of machines that are operating at one time?

60. INFORMATION TECHNOLOGY On a visit to a wiring closet, Joseph finds a rack of servers that has eight slots. He wants to know what fraction names the two already in the slots. Also, if he buys five more servers, what fraction names the total servers installed in the slots? If he has to remove two servers because of failure, what fraction names the total servers installed in the slots?

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61. SOCIAL SCIENCE The U.S. Census information can be found in your library, or on the Web, at www.census.gov. Use the 2000 census to determine the following:

(a) Fraction of the population of the United States contained in your state

(b) Fraction of the population of the United States 65 years of age or older

(c) Fraction of the United States that is female

62. SOCIAL SCIENCE Suppose the national debt had to be paid by individuals.

(a) How would the amount each individual owed be determined?

(b) Would this be a proper or an improper fraction?

Answers

58. ____________________

59. ____________________

60. ____________________

61. ____________________

62. ____________________

Answers

1. 6 is the numerator; 11 is the denominator

3. 3 is the numerator; 11 is the denominator **5.** $\frac{3}{4}$ **7.** $\frac{5}{6}$

9. $\frac{5}{5}$ **11.** $\frac{11}{12}$ **13.** $\frac{5}{8}$ **15.** Correct: $\frac{13}{20}$; wrong: $\frac{7}{20}$

17. Sold: $\frac{11}{17}$; not sold: $\frac{6}{17}$ **19.** $2 \div 5$ **21.** Proper

23. Mixed number **25.** Improper **27.** Proper

29. $1\frac{3}{4}$ or $\frac{7}{4}$ **31.** $3\frac{5}{8}$ or $\frac{29}{8}$ **33.** \$16 **35.** $17\frac{1}{2}$ gal **37.** $4\frac{2}{5}$

39. $6\frac{4}{5}$ **41.** $9\frac{1}{8}$ **43.** 4 **45.** 9 **47.** $\frac{14}{3}$ **49.** $\frac{8}{1}$ **51.** $\frac{97}{13}$

53. $10\frac{2}{5} = \frac{(5 \times 10) + 2}{5} = \frac{52}{5}$ **55.** $\frac{475}{4}$ **57.** $\frac{3}{10}$ **59.** $\frac{31}{36}$

61. Above and Beyond

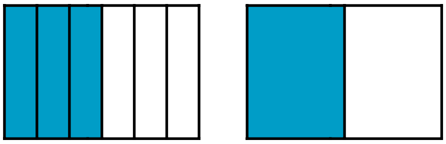
2.4

Simplifying Fractions

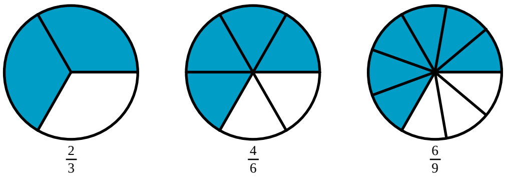
< 2.4 Objectives >

1 > Determine whether two fractions are equivalent

2 > Use the fundamental principle to simplify fractions

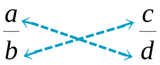


It is possible to represent the same portion of a whole by different fractions. Look at the figure, representing $\frac{3}{6}$ and $\frac{1}{2}$. The two fractions are simply different names for the same amount. They are called **equivalent fractions** for this reason.



Any fraction has many equivalent fractions. For instance, $\frac{2}{3}$, $\frac{4}{6}$, and $\frac{6}{9}$ are all equivalent fractions because they name the same part of a unit.

Many more fractions are equivalent to $\frac{2}{3}$. All these fractions can be used interchangeably. An easy way to find out if two fractions are equivalent is to use cross products.



We call $a \times d$ and $b \times c$ the **cross products.**

Property

Testing for Equivalence If the **cross products** for two fractions are equal, the two fractions are equivalent.

Example 1 Identifying Equivalent Fractions Using Cross Products

< Objective 1 >

(a) Are $\frac{3}{24}$ and $\frac{4}{32}$ equivalent fractions?

The cross products are 3×32, or 96, and 24×4, or 96. Because the cross products are equal, the fractions are equivalent.

(b) Are $\frac{2}{5}$ and $\frac{3}{7}$ equivalent fractions?

The cross products are 2×7 and 5×3.

$2 \times 7 = 14$ and $5 \times 3 = 15$

Because $14 \neq 15$, the fractions are *not* equivalent.

Check Yourself 1

(a) Are $\frac{3}{8}$ and $\frac{9}{24}$ equivalent fractions?

(b) Are $\frac{7}{8}$ and $\frac{8}{9}$ equivalent fractions?

In writing equivalent fractions, we use the following important principle.

Property

The Fundamental Principle of Fractions

For the fraction $\frac{a}{b}$ and any nonzero number c,

$$\frac{a}{b} = \frac{a \div c}{b \div c}$$

The Fundamental Principle of Fractions tells us that we can divide the numerator and denominator by the same nonzero number. The result is an equivalent fraction. For instance,

$$\frac{2}{4} = \frac{2 \div 2}{4 \div 2} = \frac{1}{2} \qquad \frac{3}{6} = \frac{3 \div 3}{6 \div 3} = \frac{1}{2} \qquad \frac{4}{8} = \frac{4 \div 4}{8 \div 4} = \frac{1}{2}$$

$$\frac{5}{10} = \frac{5 \div 5}{10 \div 5} = \frac{1}{2} \qquad \frac{6}{12} = \frac{6 \div 6}{12 \div 6} = \frac{1}{2} \qquad \frac{7}{14} = \frac{7 \div 7}{14 \div 7} = \frac{1}{2}$$

Simplifying a fraction or *reducing a fraction to lower terms* means finding an equivalent fraction with a *smaller* numerator and denominator than those of the original fraction. Dividing the numerator and denominator by the same nonzero number does exactly that.

Consider Example 2.

Example 2 Simplifying Fractions

< Objective 2 >

NOTES

We apply the fundamental principle to divide the numerator and denominator by 5.

We divide the numerator and denominator by 2.

Simplify each fraction.

(a) $\frac{5}{15} = \frac{5 \div 5}{15 \div 5} = \frac{1}{3}$

$\frac{5}{15}$ and $\frac{1}{3}$ are equivalent fractions.

Check this by finding the cross products.

(b) $\frac{4}{8} = \frac{4 \div 2}{8 \div 2} = \frac{2}{4}$

$\frac{4}{8}$ and $\frac{2}{4}$ are equivalent fractions.

Check Yourself 2

Write two fractions that are equivalent to $\frac{30}{45}$.

(a) Divide the numerator and denominator by 5.
(b) Divide the numerator and denominator by 15.

We say that a fraction is in **simplest form,** or in **lowest terms,** if the numerator and denominator have no common factors other than 1. This means that the fraction has the smallest possible numerator and denominator.

In Example 2, $\frac{1}{3}$ is in simplest form because the numerator and denominator have no common factors other than 1. The fraction is in lowest terms.

NOTE

In this case, the numerator and denominator are *not* as small as possible. The numerator and denominator have a common factor of 2.

$\frac{2}{4}$ is *not* in simplest form. Do you see that $\frac{2}{4}$ can also be written as $\frac{1}{2}$?

To write a fraction in simplest form or to *reduce a fraction to lowest terms,* divide the numerator and denominator by their greatest common factor (GCF).

Example 3 Simplifying Fractions

Write $\frac{10}{15}$ in simplest form.

From our work earlier in this chapter, we know that the greatest common factor of 10 and 15 is 5. To write $\frac{10}{15}$ in simplest form, divide the numerator and denominator by 5.

$$\frac{10}{15} = \frac{10 \div 5}{15 \div 5} = \frac{2}{3}$$

The resulting fraction, $\frac{2}{3}$, is in lowest terms.

Check Yourself 3

Write $\frac{12}{18}$ in simplest form by dividing the numerator and denominator by the GCF.

Many students prefer to simplify fractions by using the prime factorizations of the numerator and denominator. Example 4 uses this method.

Example 4 Factoring to Simplify a Fraction

(a) Simplify $\frac{24}{42}$.

To simplify $\frac{24}{42}$, factor.

NOTE

From the prime factorization of 24 and 42, we divide by the common factors of 2 and 3.

$$\frac{24}{42} = \frac{\overset{1}{\cancel{2}} \times 2 \times 2 \times \overset{1}{\cancel{3}}}{\underset{1}{\cancel{2}} \times \underset{1}{\cancel{3}} \times 7} = \frac{4}{7}$$

Note: The numerator of the simplified fraction is the *product* of the prime factors remaining in the numerator after the division by 2 and 3.

(b) Simplify $\frac{120}{180}$.

To reduce $\frac{120}{180}$ to lowest terms, write the prime factorizations of the numerator and denominator. Then divide by any common factors.

$$\frac{120}{180} = \frac{\overset{1}{\cancel{2}} \times \overset{1}{\cancel{2}} \times 2 \times \overset{1}{\cancel{3}} \times \overset{1}{\cancel{5}}}{\underset{1}{\cancel{2}} \times \underset{1}{\cancel{2}} \times \underset{1}{\cancel{3}} \times 3 \times \underset{1}{\cancel{5}}} = \frac{2}{3}$$

Check Yourself 4

Write each fraction in simplest form.

(a) $\frac{60}{75}$ **(b)** $\frac{210}{252}$

There is another way to organize your work in simplifying fractions. It again uses the fundamental principle to divide the numerator and denominator by any common factors. We illustrate this with the fractions considered in Example 4.

Example 5 Using Common Factors to Simplify Fractions

(a) Simplify $\frac{24}{42}$.

$$\frac{24}{42} = \frac{\overset{12}{\cancel{24}}}{\underset{21}{\cancel{42}}} = \frac{\overset{4}{\cancel{12}}}{\underset{7}{\cancel{21}}} = \frac{4}{7}$$

Divide by the common factor of 2.

Divide by the common factor of 3.

The original numerator and denominator are divisible by 2, and so we divide by that factor to arrive at $\frac{12}{21}$. Our divisibility tests tell us that a common factor of 3 still exists. (Do you remember why?) Divide again for the result $\frac{4}{7}$, which is in lowest terms.

Note: If we had seen the GCF of 6 at first, we could have divided by 6 and arrived at the same result in one step.

(b) Simplify $\frac{120}{180}$.

$$\frac{120}{180} = \frac{\overset{\overset{2}{\cancel{20}}}{\cancel{120}}}{\underset{\underset{3}{\cancel{30}}}{\cancel{180}}} = \frac{2}{3}$$

Our first step is to divide by the common factor of 6. We then have $\frac{20}{30}$. There is still a common factor of 10, so we again divide.

Again, we could have divided by the GCF of 60 in one step if we had recognized it.

Check Yourself 5

Using the method of Example 5, write each fraction in simplest form.

(a) $\frac{60}{75}$ **(b)** $\frac{84}{196}$

Check Yourself ANSWERS

1. **(a)** Yes; **(b)** no **2.** **(a)** $\frac{6}{9}$; **(b)** $\frac{2}{3}$

3. 6 is the GCF of 12 and 18, so $\frac{12}{18} = \frac{12 \div 6}{18 \div 6} = \frac{2}{3}$

4. **(a)** $\frac{60}{75} = \frac{2 \times 2 \times \overset{1}{\cancel{3}} \times \overset{1}{\cancel{5}}}{\underset{1}{\cancel{3}} \times \underset{1}{\cancel{5}} \times 5} = \frac{4}{5}$; **(b)** $\frac{210}{252} = \frac{\overset{1}{\cancel{2}} \times \overset{1}{\cancel{3}} \times 5 \times \overset{1}{\cancel{7}}}{\underset{1}{\cancel{2}} \times 2 \times 3 \times \underset{1}{\cancel{3}} \times \underset{1}{\cancel{7}}} = \frac{5}{6}$

5. **(a)** Divide by the common factors of 3 and 5, $\frac{60}{75} = \frac{4}{5}$

(b) Divide by the common factors of 4 and 7, $\frac{84}{196} = \frac{3}{7}$

Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 2.4

(a) If the ______________ products of two fractions are equal, the two fractions are equivalent.

(b) Two fractions that are simply different names for the same fraction are called ______________ fractions.

(c) In writing equivalent fractions, we use the ______________ Principle of Fractions.

(d) We say that a fraction is in simplest form if the numerator and denominator have no ______________ factors other than 1.

2.4 exercises

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< Objective 1 >

Are the pairs of fractions equivalent?

1. $\frac{1}{3}, \frac{3}{5}$

2. $\frac{3}{5}, \frac{9}{15}$

3. $\frac{1}{7}, \frac{4}{28}$

4. $\frac{2}{3}, \frac{3}{5}$

5. $\frac{5}{6}, \frac{15}{18}$ > Videos

6. $\frac{3}{4}, \frac{16}{20}$

7. $\frac{2}{21}, \frac{4}{25}$

8. $\frac{20}{24}, \frac{5}{6}$

9. $\frac{2}{7}, \frac{3}{11}$ > Videos

10. $\frac{12}{15}, \frac{36}{45}$

11. $\frac{16}{24}, \frac{40}{60}$

12. $\frac{15}{20}, \frac{20}{25}$

< Objective 2 >

Write each fraction in simplest form.

13. $\frac{15}{30}$

14. $\frac{100}{200}$

15. $\frac{8}{12}$

16. $\frac{12}{15}$

17. $\frac{10}{14}$

18. $\frac{15}{50}$

19. $\frac{12}{18}$ > Videos

20. $\frac{28}{35}$

21. $\frac{35}{40}$

22. $\frac{21}{24}$

23. $\frac{11}{44}$

24. $\frac{10}{25}$

25. $\frac{12}{36}$ > Videos

26. $\frac{18}{48}$

27. $\frac{24}{27}$

28. $\frac{30}{50}$

Answers

1. __________ 2. __________

3. __________ 4. __________

5. __________ 6. __________

7. __________ 8. __________

9. __________ 10. __________

11. __________ 12. __________

13. __________ 14. __________

15. __________ 16. __________

17. __________ 18. __________

19. __________ 20. __________

21. __________ 22. __________

23. __________ 24. __________

25. __________ 26. __________

27. __________ 28. __________

Answers

29. ______

30. ______

31. ______

32. ______

33. ______

34. ______

35. ______

36. ______

37. ______

38. ______

39. ______

40. ______

41. ______

42. ______

43. ______

44. ______

29. $\frac{32}{40}$

30. $\frac{17}{51}$

31. **Statistics** On a test of 72 questions, Sam answered 54 correctly. On another test, Sam answered 66 correctly out of 88. Did Sam get the same portion of each test correct?

32. **Statistics** Chipper Jones of the Atlanta Braves had 112 hits in 320 times at bat. Albert Pujols of the St. Louis Cardinals had 91 hits in 260 times at bat. Did they have the same batting average?

Solve each application.

33. **Number Problem** A quarter is what fractional part of a dollar? Simplify your result.

> Videos

34. **Number Problem** A dime is what fractional part of a dollar? Simplify your result.

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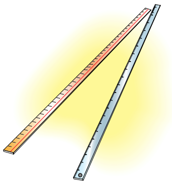
35. **Statistics** What fractional part of an hour is 15 minutes (min)? Simplify your result.

> Videos

36. **Statistics** What fractional part of a day is 6 hours (h)? Simplify your result.

37. **Science and Medicine** One meter is equal to 100 centimeters (cm). What fractional part of a meter is 70 cm? Simplify your result.



38. **Science and Medicine** One kilometer is equal to 1,000 meters (m). What fractional part of a kilometer is 300 m? Simplify your result.

39. **Technology** Susan did a tune-up on her automobile. She found that two of her eight spark plugs were fouled. What fraction represents the number of fouled plugs? Reduce to lowest terms.

40. **Statistics** Samantha answered 18 of 20 problems correctly on a test. What fractional part did she answer correctly? Reduce your answer to lowest terms.

Write each fraction in simplest form.

41. $\frac{75}{105}$

42. $\frac{62}{93}$

43. $\frac{48}{60}$

44. $\frac{48}{66}$

45. $\frac{105}{135}$

46. $\frac{54}{126}$

47. $\frac{66}{110}$

48. $\frac{280}{320}$

49. $\frac{16}{21}$

50. $\frac{21}{32}$

51. $\frac{31}{52}$

52. $\frac{42}{55}$

53. $\frac{96}{132}$

54. $\frac{33}{121}$

55. $\frac{85}{102}$

56. $\frac{133}{152}$

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Using Your Calculator to Simplify Fractions

If you have a calculator that supports fraction arithmetic, you should learn to use it to check your work. Here we look at two different types of these calculators.

Scientific Calculator

Before doing the following example, find the button on your scientific calculator that is labeled $\boxed{\textbf{a b/c}}$. This is the button that will be used to enter fractions.

Simplify the fraction $\frac{24}{68}$.

There are four steps in simplifying fractions using a scientific calculator.

(a) Enter the numerator, 24.
(b) Press the $\boxed{\textbf{a b/c}}$ key.
(c) Enter the denominator, 68.
(d) Press $\boxed{=}$.

The calculator will display the simplified fraction, $\frac{6}{17}$.

Graphing Calculator

We can simplify the same fraction, $\frac{24}{68}$, using a graphing calculator, such as the TI-84 Plus.

(a) Enter the fraction as a division problem: 24 ÷ 68. The calculator will display $\frac{24}{68}$ as 24/68.
(b) Press the $\boxed{\textbf{MATH}}$ key.
(c) Select $\boxed{\textbf{1: ▶ Frac}}$.
(d) Press $\boxed{\text{Enter}}$.

The calculator displays the simplified fraction, $\frac{6}{17}$.

The graphing calculator is particularly useful for simplifying fractions with large values in the numerator and denominator. Some scientific calculators cannot handle denominators larger than 999.

Answers

45. ____________________
46. ____________________
47. ____________________
48. ____________________
49. ____________________
50. ____________________
51. ____________________
52. ____________________
53. ____________________
54. ____________________
55. ____________________
56. ____________________

Answers

57. ____________

58. ____________

59. ____________

60. ____________

61. ____________

62. ____________

63. ____________

64. ____________

65. ____________

66. ____________

67. ____________

68. ____________

Use your calculator to simplify each fraction.

57. $\frac{28}{40}$

58. $\frac{121}{132}$

59. $\frac{96}{144}$

60. $\frac{445}{623}$

61. $\frac{299}{391}$

62. $\frac{289}{459}$

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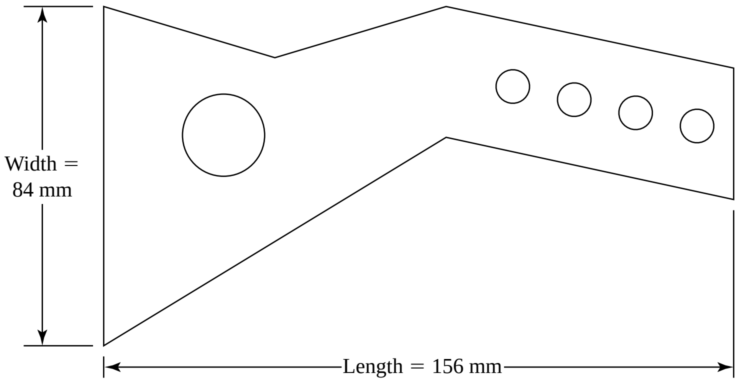
63. Allied Health Pepto-Bismol tablets contain 300 milligrams (mg) of medication; however, children 6 to 8 years of age should only take 200 mg at a time. What fractional part of a tablet should a 6-year-old child be given?

64. Allied Health The recommended adult dose of the laxative docusate is 500 milligrams (mg) per day, and the recommended dose for children 3 to 5 years old is 40 mg per day. The dose for a 4-year-old child is what fractional part of the adult dose?

65. Manufacturing Technology Express the width of the piece shown in the figure as a fraction of the length.



66. Manufacturing Technology In the packaging division of Early Enterprises, there are 36 packaging machines. At any given time, 4 of the machines are shut down for scheduled maintenance and service. What is the fraction of machines that are operating at one time?

67. Information Technology Jo, an executive vice president of information technology, has 10 people on staff, and she needs to hire two more people. What fraction names the new people of the total staff? Simplify your answer.

68. Information Technology Jason is responsible for the administration of the servers at his company. He measures that the average arrival rate of requests to the server is 50,000 requests per second, and he also finds out the servers can service 100,000 requests per second. The intensity of the traffic is measured by x, which is the quotient of the average arrival rate and the service rate. The intensity also shows how busy the servers are. What is the fraction that names this situation? Simplify your answer.

69. Can any of the following fractions be simplified?

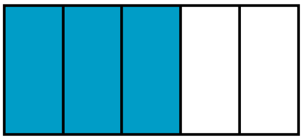
(a) $\frac{824}{73}$ **(b)** $\frac{59}{11}$ **(c)** $\frac{135}{17}$

What characteristic do you notice about the denominator of each fraction? What rule would you make up based on your observations?

70. Consider the given figures.

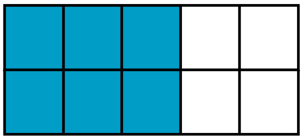
(a) Give the fraction that represents the shaded region.

(a)

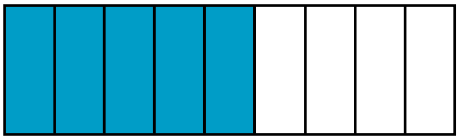


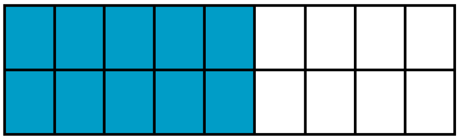
(b) Draw a horizontal line through the figure, as shown. Now give the fraction representing the shaded region.

(b)



71. Repeat exercise 70, using these figures.





72. A student is attempting to reduce the fraction $\frac{8}{12}$ to lowest terms. He produces the following argument:

$$\frac{8}{12} = \frac{4+4}{8+4} = \frac{4}{8} = \frac{1}{2}$$

What is the fallacy in this argument? What is the correct answer?

Answers

1. $1 \times 5 = 5$; $3 \times 3 = 9$. The fractions are not equivalent.
3. Yes **5.** Yes **7.** No **9.** No
11. $16 \times 60 = 960$, and $24 \times 40 = 960$. The fractions are equivalent.
13. $\frac{1}{2}$ **15.** $\frac{2}{3}$ **17.** $\frac{5}{7}$ **19.** $\frac{2}{3}$ **21.** $\frac{7}{8}$ **23.** $\frac{1}{4}$ **25.** $\frac{1}{3}$
27. $\frac{8}{9}$ **29.** $\frac{4}{5}$ **31.** Yes **33.** $\frac{1}{4}$ **35.** $\frac{1}{4}$ **37.** $\frac{7}{10}$ **39.** $\frac{1}{4}$
41. $\frac{5}{7}$ **43.** $\frac{4}{5}$ **45.** $\frac{7}{9}$ **47.** $\frac{3}{5}$ **49.** $\frac{16}{21}$ **51.** $\frac{31}{52}$ **53.** $\frac{8}{11}$
55. $\frac{5}{6}$ **57.** $\frac{7}{10}$ **59.** $\frac{2}{3}$ **61.** $\frac{13}{17}$ **63.** $\frac{2}{3}$ **65.** $\frac{7}{13}$ **67.** $\frac{1}{6}$
69. Above and Beyond **71.** (a) $\frac{5}{9}$; (b) $\frac{10}{18}$

Answers

69. ______________________

70. ______________________

71. ______________________

72. ______________________

Activity 4 ::
Daily Reference Values

chapter
2
> Make the Connection

According to the Food and Drug Administration (FDA) in 2003, the following table represents the daily reference values (DRV) for each dietary element, based on a 2,000-calorie diet.

	DRV
Total fat	65 grams
Saturated fat	20 grams
Cholesterol	300 milligrams
Sodium	2,400 milligrams
Carbohydrates	300 grams
Dietary fiber	25 grams (minimum)

A high-performance energy bar made by PowerBar has the following amounts of each food type.

	DRV
Total fat	3 grams
Saturated fat	1 gram
Cholesterol	0 milligrams
Sodium	1,000 milligrams
Carbohydrates	45 grams
Dietary fiber	3 grams

Use the tables to answer each of the following questions.

1. What fraction of the DRV for sodium is contained in the PowerBar?
2. What fraction of the DRV for carbohydrates is contained in the PowerBar?
3. What fraction of the DRV for dietary fiber is contained in the PowerBar?
4. What fraction of the DRV for total fat is contained in the PowerBar?
5. What fraction of the DRV for saturated fat is contained in the PowerBar?

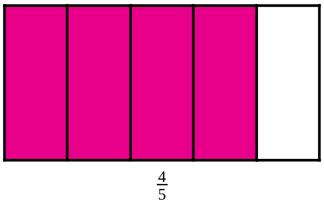
2.5

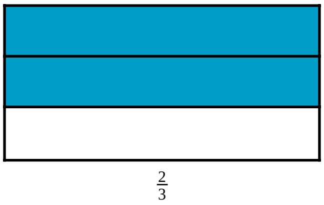
Multiplying Fractions

< 2.5 Objectives >

1 > Multiply two fractions
2 > Multiply mixed numbers and fractions
3 > Simplify before multiplying fractions
4 > Estimate products by rounding
5 > Solve applications involving multiplication of fractions

Multiplication is the easiest of the four operations with fractions. We can illustrate multiplication by picturing fractions as parts of a whole or unit. Using this idea, we show the fractions $\frac{4}{5}$ and $\frac{2}{3}$.

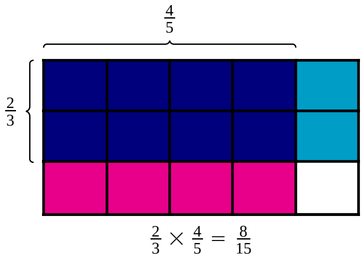




NOTE

A fraction followed by the word *of* means that we want to multiply by that fraction.

Suppose now that we wish to find $\frac{2}{3}$ of $\frac{4}{5}$. We can combine the diagrams as shown below. The part of the whole representing the product $\frac{2}{3} \times \frac{4}{5}$ is the purple region. The unit has been divided into 15 parts, and 8 of those parts are purple, so $\frac{2}{3} \times \frac{4}{5}$ must be $\frac{8}{15}$.



The purple parts represent $\frac{2}{3}$ of the red area, or $\frac{2}{3} \times \frac{4}{5}$.

The following rule is suggested by the diagrams.

Step by Step

To Multiply Fractions

Step 1 Multiply the numerators to find the numerator of the product.
Step 2 Multiply the denominators to find the denominator of the product.
Step 3 Simplify the resulting fraction if possible.

We need only use the first two steps in Example 1.

Example 1 Multiplying Two Fractions

< Objective 1 >

NOTE

We multiply fractions in this way *not* because it is easy, but because it works!

Multiply.

(a) $\frac{2}{3} \times \frac{4}{5} = \frac{2 \times 4}{3 \times 5} = \frac{8}{15}$

(b) $\frac{5}{8} \times \frac{7}{9} = \frac{5 \times 7}{8 \times 9} = \frac{35}{72}$

(c) $\left(\frac{2}{3}\right)^2 = \frac{2}{3} \times \frac{2}{3} = \frac{2 \times 2}{3 \times 3} = \frac{4}{9}$

Check Yourself 1

Multiply.

(a) $\frac{7}{8} \times \frac{3}{10}$ **(b)** $\frac{5}{7} \times \frac{3}{4}$ **(c)** $\left(\frac{3}{4}\right)^2$

Step 3 indicates that the product of fractions should always be simplified to lowest terms. Consider the following.

Example 2 Multiplying Two Fractions

Multiply and write the result in lowest terms.

$$\frac{3}{4} \times \frac{2}{9} = \frac{3 \times 2}{4 \times 9} = \frac{6}{36} = \frac{1}{6}$$

Noting that $\frac{6}{36}$ is not in simplest form, we divide numerator and denominator by 6 to write the product in lowest terms.

Check Yourself 2

Multiply and write the result in lowest terms.

$\frac{5}{7} \times \frac{3}{10}$

To find the product of a fraction and a whole number, write the whole number as a fraction (the whole number divided by 1) and apply the multiplication rule as before. Example 3 illustrates this approach.

Example 3 Multiplying a Whole Number and a Fraction

Do the indicated multiplication.

Remember that $5 = \frac{5}{1}$.

(a) $5 \times \frac{3}{4} = \frac{5}{1} \times \frac{3}{4} = \frac{5 \times 3}{1 \times 4}$

$= \frac{15}{4} = 3\frac{3}{4}$

NOTES

We have written the resulting improper fraction as a mixed number.

Write the product as a mixed number, and then write the fraction part in simplest form.

Alternatively, simplify $\frac{30}{12}$ to $\frac{5}{2}$, and then write this as $2\frac{1}{2}$.

(b) $\frac{5}{12} \times 6 = \frac{5}{12} \times \frac{6}{1}$

$$= \frac{5 \times 6}{12 \times 1}$$

$$= \frac{30}{12} = 2\frac{6}{12}$$

$$= 2\frac{1}{2}$$

Check Yourself 3

Multiply.

(a) $\frac{3}{16} \times 8$ **(b)** $4 \times \frac{5}{7}$

When mixed numbers are involved in multiplication, the problem requires an additional step. First change any mixed numbers to improper fractions. Then apply our multiplication rule for fractions.

Example 4 Multiplying a Mixed Number and a Fraction

< Objective 2 >

$$1\frac{1}{2} \times \frac{3}{4} = \frac{3}{2} \times \frac{3}{4}$$

Change the mixed number to an improper fraction.

Here $1\frac{1}{2} = \frac{3}{2}$.

$$= \frac{3 \times 3}{2 \times 4}$$

Multiply as before.

$$= \frac{9}{8} = 1\frac{1}{8}$$

The product is usually written in mixed-number form.

Check Yourself 4

Multiply.

$\frac{5}{8} \times 3\frac{1}{2}$

If two mixed numbers are involved, change both of the mixed numbers to improper fractions. Example 5 illustrates this.

Example 5 Multiplying Two Mixed Numbers

Multiply.

$$3\frac{2}{3} \times 2\frac{1}{2} = \frac{11}{3} \times \frac{5}{2}$$

Change the mixed numbers to improper fractions.

$$= \frac{11 \times 5}{3 \times 2} = \frac{55}{6} = 9\frac{1}{6}$$

> CAUTION

Be Careful! Students sometimes think of

$$3\frac{2}{3} \times 2\frac{1}{2} \quad \text{as} \quad (3 \times 2) + \left(\frac{2}{3} \times \frac{1}{2}\right)$$

This is *not* the correct multiplication pattern. You must first change the mixed numbers to improper fractions.

Check Yourself 5

Multiply.

$$2\frac{1}{3} \times 3\frac{1}{2}$$

When you multiply fractions, it is usually easier to simplify, that is, remove any common factors in the numerator and denominator, *before multiplying.* Remember that to simplify means to *divide* by the same common factor.

Example 6 Simplifying Before Multiplying Two Fractions

< Objective 3 >

NOTE

Once again we are applying the fundamental principle to divide the numerator and denominator by 3.

Simplify and then multiply.

$$\frac{3}{5} \times \frac{4}{9} = \frac{\overset{1}{\cancel{3}} \times 4}{5 \times \underset{3}{\cancel{9}}}$$

To simplify, we divide the *numerator* and *denominator* by the common factor 3. Remember that $\overset{1}{\cancel{3}}$ means 3 ÷ 3 = 1, and $\underset{3}{\cancel{9}}$ means 9 ÷ 3 = 3.

$$= \frac{1 \times 4}{5 \times 3}$$

$$= \frac{4}{15}$$

Because we divide by any common factors before we multiply, the resulting product is *in simplest form.*

Check Yourself 6

Simplify and then multiply.

$$\frac{7}{8} \times \frac{5}{21}$$

Our work in Example 6 leads to the following general rule about simplifying fractions in multiplication.

Property

Simplifying Fractions Before Multiplying

In multiplying two or more fractions, we can divide any factor of the numerator and any factor of the denominator by the same nonzero number to simplify the product.

When mixed numbers are involved, the process is similar. Consider Example 7.

Example 7 Simplifying Before Multiplying Two Mixed Numbers

Multiply.

$$2\frac{2}{3} \times 2\frac{1}{4} = \frac{8}{3} \times \frac{9}{4}$$

First, convert the mixed numbers to improper fractions.

$$= \frac{\overset{2}{\cancel{8}} \times \overset{3}{\cancel{9}}}{\underset{1}{\cancel{3}} \times \underset{1}{\cancel{4}}}$$

To simplify, divide by the common factors of 3 and 4.

$$= \frac{2 \times 3}{1 \times 1}$$

Multiply as before.

$$= \frac{6}{1} = 6$$

Check Yourself 7

Simplify and then multiply.

$$3\frac{1}{3} \times 2\frac{2}{5}$$

The ideas of our previous examples also allow us to find the product of more than two fractions.

Example 8 Simplifying Before Multiplying Three Numbers

RECALL

We can divide *any* factor of the numerator and *any* factor of the denominator by the same nonzero number.

Simplify and then multiply.

$$\frac{2}{3} \times 1\frac{4}{5} \times \frac{5}{8} = \frac{2}{3} \times \frac{9}{5} \times \frac{5}{8}$$

Write any mixed or whole numbers as improper fractions.

$$= \frac{\overset{1}{\cancel{2}} \times \overset{3}{\cancel{9}} \times \overset{1}{\cancel{5}}}{\underset{1}{\cancel{3}} \times \underset{1}{\cancel{5}} \times \underset{4}{\cancel{8}}}$$

To simplify, divide by the common factors in the numerator and denominator.

$$= \frac{3}{4}$$

Check Yourself 8

Simplify and then multiply.

$$\frac{5}{8} \times 4\frac{4}{5} \times \frac{1}{6}$$

We encountered estimation by rounding in our earlier work with whole numbers. Estimation can also be used to check the "reasonableness" of an answer when working with fractions or mixed numbers.

Example 9 Estimating the Product of Two Mixed Numbers

< Objective 4 >

Estimate the product of

$$3\frac{1}{8} \times 5\frac{5}{6}$$

Round each mixed number to the nearest whole number.

$$3\frac{1}{8} \rightarrow 3$$

$$5\frac{5}{6} \rightarrow 6$$

Our estimate of the product is then

$3 \times 6 = 18$

Note: The actual product in this case is $18\frac{11}{48}$, which certainly seems reasonable in view of our estimate.

Check Yourself 9

Estimate the product.

$$2\frac{7}{8} \times 8\frac{1}{3}$$

Units ANALYSIS

When you divide two denominate numbers, the units are also divided. This yields a unit in fraction form.

EXAMPLES:

$$250 \text{ mi} \div 10 \text{ gal} = \frac{250 \text{ mi}}{10 \text{ gal}} = \frac{25 \text{ mi}}{1 \text{ gal}} = 25 \text{ mi/gal (read “miles per gallon”)}$$

$$360 \text{ ft} \div 30 \text{ s} = \frac{360 \text{ ft}}{30 \text{ s}} = 12 \text{ ft/s (“feet per second”)}$$

When we multiply denominate numbers that have these units in fraction form, they behave just as fractions do.

EXAMPLES:

$$25 \text{ mi/gal} \times 12 \text{ gal} = \frac{25 \text{ mi}}{1 \cancel{\text{gal}}} \times \frac{12 \cancel{\text{gal}}}{1} = 300 \text{ mi}$$

(If we look at the units, we see that the gallons essentially “cancel” when one is in the numerator and the other in the denominator.)

$$12 \text{ ft/s} \times 60 \text{ s/min} = \frac{12 \text{ ft}}{1 \cancel{\text{s}}} \times \frac{60 \cancel{\text{s}}}{1 \text{ min}} = \frac{720 \text{ ft}}{1 \text{ min}} = 720 \text{ ft/min}$$

(Again, the seconds cancel, leaving feet in the numerator and minutes in the denominator.)

Now we can look at some applications of fractions that involve multiplication. In solving these word problems, we use the same approach we used earlier with whole numbers. Let’s review the four-step process introduced in Section 1.2.

Step by Step

Solving Applications Involving the Multiplication of Fractions

Step 1 Read the problem carefully to determine the given information and what you are asked to find.

Step 2 Decide upon the operation or operations to be used.

Step 3 Write down the complete statement necessary to solve the problem and do the calculations.

Step 4 Check to make sure that you have answered the question of the problem and that your answer seems reasonable.

We can work through some examples, using these steps.

Example 10 An Application Involving Multiplication

< Objective 5 >

Lisa worked $10\frac{1}{4}$ hours per day $\left(\frac{\text{h}}{\text{day}}\right)$ for 5 days. How many hours did she work?

Step 1 We are looking for the total hours Lisa worked.

Step 2 We will multiply the hours per day by the days.

Step 3 $10\frac{1}{4}\frac{\text{h}}{\text{day}} \times 5 \text{ days} = \frac{41}{4}\frac{\text{h}}{\text{day}} \times 5 \text{ days} = \frac{205}{4} \text{h} = 51\frac{1}{4} \text{h}$

Step 4 Note the days cancel, leaving only the unit of hours. The units should always be compared to the desired units from step 1. The answer also seems reasonable. An answer such as 5 h or 500 h would not seem reasonable.

Check Yourself 10

Carlos gets 30 mi/gal in his Miata. How far should he be able to drive with an 11-gal tank of gas?

In Example 11, we follow the four steps for solving applications, but do not label the steps. You should still think about these steps as we solve the problem.

Example 11 An Application Involving the Multiplication of Mixed Numbers

A sheet of notepaper is $6\frac{3}{4}$ inches (in.) wide and $8\frac{2}{3}$ in. long. Find the area of the paper.

Multiply the given length by the width. This gives the desired area. First, we will estimate the area.

$9 \text{ in.} \times 7 \text{ in.} = 63 \text{ in.}^2$

Now, we find the exact area.

RECALL

The area of a rectangle is the product of its length and width.

$$\begin{aligned} 8\frac{2}{3} \text{ in.} \times 6\frac{3}{4} \text{ in.} &= \frac{26}{3} \text{ in.} \times \frac{27}{4} \text{ in.} \\ &= \frac{117}{2} \text{ in.}^2 \\ &= 58\frac{1}{2} \text{ in.}^2 \end{aligned}$$

The units (square inches) are units of area. Note that from our estimate the result is reasonable.

Check Yourself 11

A window is $4\frac{1}{2}$ feet (ft) high by $2\frac{1}{3}$ ft wide. What is its area?

Example 12 reminds us that an abstract number multiplied by a denominate number yields the units of the denominate number.

Example 12 An Application Involving the Multiplication of a Mixed Number and a Fraction

A state park contains $38\frac{2}{3}$ acres. According to the plan for the park, $\frac{3}{4}$ of the park is to be left as a wildlife preserve. How many acres is this?

We want to find $\frac{3}{4}$ of $38\frac{2}{3}$ acres. We then multiply as shown:

RECALL

The word *of* usually indicates multiplication.

$$\frac{3}{4} \times 38\frac{2}{3} = \frac{3}{4} \times \frac{116}{3} = \frac{\overset{1}{\cancel{3}} \times \overset{29}{\cancel{116}}}{\underset{1}{\cancel{4}} \times \underset{1}{\cancel{3}}} \text{ acres} = 29 \text{ acres}$$

Check Yourself 12

A backyard has $25\frac{3}{4}$ square yards (yd^2) of open space. If Patrick wants to build a vegetable garden covering $\frac{2}{3}$ of the open space, how many square yards is this?

We have mentioned the word *of* usually indicates multiplication. You should also note that it indicates that the fraction preceding it is an abstract number (it has no units attached). There are even occasions, as in Example 13, when we are looking at the product of two abstract numbers.

Example 13 An Application Involving the Multiplication of Fractions

A grocery store survey shows that $\frac{2}{3}$ of the customers buy meat. Of these, $\frac{3}{4}$ will buy at least one package of beef. What portion of the store's customers buy beef?

Step 1 We know that $\frac{2}{3}$ of the customers buy meat and that $\frac{3}{4}$ of these customers buy beef.

Step 2 We wish to know $\frac{3}{4}$ of $\frac{2}{3}$. The operation here is multiplication.

Step 3 Multiplying, we have

$$\frac{3}{4} \times \frac{2}{3} = \frac{\overset{1}{\cancel{3}} \times \overset{1}{\cancel{2}}}{\underset{2}{\cancel{4}} \times \underset{1}{\cancel{3}}} = \frac{1}{2}$$

Step 4 From step 3 we have the result: $\frac{1}{2}$ of the store's customers buy beef.

NOTE

In this problem, *of* means to multiply.

Check Yourself 13

A supermarket survey shows that $\frac{2}{5}$ of the customers buy lunch meat. Of these, $\frac{3}{4}$ buy boiled ham. What portion of the store's customers buy boiled ham?

Example 14 An Application Involving the Multiplication of Mixed Numbers

Shirley drives at an average speed of 52 miles per hour (mi/h) for $3\frac{1}{4}$ h. How far has she traveled at the end of $3\frac{1}{4}$ h?

NOTE

Distance is the product of speed and time.

$$\underset{\text{Speed}}{52\,\frac{\text{mi}}{\text{h}}} \times \underset{\text{Time}}{3\frac{1}{4}\text{ h}} = \frac{52}{1}\,\frac{\text{mi}}{\cancel{\text{h}}} \times \frac{13}{4}\,\cancel{\text{h}}$$

$$= \frac{\overset{13}{\cancel{52}} \times 13}{1 \times \underset{1}{\cancel{4}}}\text{ mi}$$

$$= 169\text{ mi}$$

Check Yourself 14

(a) The scale on a map is 1 inch (in.) = 60 miles (mi). What is the distance in miles between two towns that are $3\frac{1}{2}$ in. apart on the map?

(b) Maria is ordering concrete for a new sidewalk that is to be $\frac{1}{9}$ yd thick, $22\frac{1}{2}$ yd long, and $1\frac{1}{3}$ yd wide. How much concrete should she order if she must order a whole number of cubic yards?

Check Yourself ANSWERS

1. **(a)** $\frac{7}{8} \times \frac{3}{10} = \frac{7 \times 3}{8 \times 10} = \frac{21}{80}$; **(b)** $\frac{5}{7} \times \frac{3}{4} = \frac{5 \times 3}{7 \times 4} = \frac{15}{28}$; **(c)** $\frac{9}{16}$

2. $\frac{5}{7} \times \frac{3}{10} = \frac{5 \times 3}{7 \times 10} = \frac{15}{70} = \frac{3}{14}$ **3.** **(a)** $1\frac{1}{2}$; **(b)** $2\frac{6}{7}$

4. $\frac{5}{8} \times 3\frac{1}{2} = \frac{5}{8} \times \frac{7}{2} = \frac{35}{16} = 2\frac{3}{16}$ **5.** $8\frac{1}{6}$

6. $\frac{7}{8} \times \frac{5}{21} = \frac{\overset{1}{\cancel{7}} \times 5}{8 \times \underset{3}{\cancel{21}}} = \frac{5}{24}$

7. $3\frac{1}{3} \times 2\frac{2}{5} = \frac{10}{3} \times \frac{12}{5} = \frac{\overset{2}{\cancel{10}} \times \overset{4}{\cancel{12}}}{\underset{1}{\cancel{3}} \times \underset{1}{\cancel{5}}} = \frac{8}{1} = 8$ **8.** $\frac{1}{2}$ **9.** 24

10. 330 mi **11.** $10\frac{1}{2}$ ft^2 **12.** $17\frac{1}{6}$ yd^2 **13.** $\frac{3}{10}$

14. **(a)** 210 mi; **(b)** the answer, $3\frac{1}{3}$ yd^3, is rounded up to 4 yd^3.

Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 2.5

(a) The product of fractions should always be expressed in ____________ terms.

(b) When multiplying fractions, it is usually easier to ____________ before multiplying.

(c) Estimation can be used to check the ________________ of an answer when working with fractions or mixed numbers.

(d) The final step in solving an application is to make sure that your answer seems ________________.

2.5 exercises

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Name ________________

Section __________ Date __________

Answers

1. __________ 2. __________

3. __________ 4. __________

5. __________ 6. __________

7. __________ 8. __________

9. __________ 10. __________

11. __________ 12. __________

13. __________ 14. __________

15. __________ 16. __________

17. __________ 18. __________

19. __________ 20. __________

21. __________ 22. __________

23. __________ 24. __________

25. __________ 26. __________

27. __________ 28. __________

29. __________ 30. __________

< Objective 1 >

Multiply. Be sure to write each answer in simplest form.

1. $\frac{3}{4} \times \frac{5}{11}$

2. $\frac{2}{7} \times \frac{5}{9}$

3. $\frac{3}{4} \times \frac{7}{11}$

> Videos

4. $\frac{2}{5} \times \frac{3}{7}$

5. $\frac{3}{5} \cdot \frac{5}{7}$

6. $\frac{6}{11} \cdot \frac{8}{6}$

7. $\left(\frac{4}{9}\right)^2$

8. $\left(\frac{5}{6}\right)^2$

9. $\frac{3}{11} \cdot \frac{7}{9}$

10. $\frac{7}{9} \cdot \frac{3}{5}$

11. $\frac{3}{10} \times \frac{5}{9}$

12. $\frac{5}{21} \times \frac{14}{25}$

13. $\frac{7}{9} \cdot \frac{6}{5}$

14. $\frac{48}{63} \cdot \frac{81}{60}$

15. $\frac{24}{33} \cdot \frac{55}{40}$

> Videos

< Objectives 2–3 >

16. $3\frac{1}{3} \times \frac{9}{11}$

17. $\left(\frac{2}{3}\right)\left(2\frac{2}{5}\right)$

> Videos

18. $\left(3\frac{1}{3}\right)\left(\frac{3}{7}\right)$

19. $\frac{2}{5} \times 3\frac{1}{4}$

20. $2\frac{1}{3} \times 2\frac{1}{6}$

21. $2\frac{1}{3} \cdot 2\frac{1}{2}$

22. $\frac{3}{7} \cdot 14$

23. $\left(\frac{5}{6}\right)^3$

> Videos

24. $\left(\frac{4}{5}\right)^3$

25. $\frac{12}{25} \times \frac{11}{18}$

26. $\frac{10}{12} \times \frac{16}{25}$

27. $\frac{14}{15} \cdot \frac{10}{21}$

28. $\frac{21}{25} \cdot \frac{30}{7}$

29. $\left(\frac{18}{28}\right)\left(\frac{35}{22}\right)$

30. $\left(3\frac{2}{3}\right)\left(\frac{9}{10}\right)$

< Objective 5 >

Evaluate. Be sure to use the proper units.

31. 36 mi/h × 4 h

32. 80 cal/g × 5 g

33. 55 joules/s × 11 s

34. 5 lb/ft × 3 ft

35. 88 ft/s × 1 mi/5,280 ft × 3,600 s/h

36. 24 h/day × 3,600 s/h × 365 days/yr

Solve each application.

37. Business and Finance Maria earns $11 per hour. Last week, she worked 9 hours/day for 6 days. What was her gross pay?

38. Statistics The gas tank in Luigi's Toyota Camry holds 17 gal when full. The car gets 21 mi/gal. How far can he travel on 3 full tanks?

39. Crafts A recipe calls for $\frac{2}{3}$ cup of sugar for each serving. How much sugar is needed for 6 servings?

chapter 2 > Make the Connection

40. Crafts Mom-Mom's French toast requires $\frac{3}{4}$ cup of batter for each serving. If 5 people are expected for breakfast, how much batter is needed?

chapter 2 > Make the Connection

Multiply and simplify.

41. $\frac{4}{9} \times 3\frac{3}{5}$

42. $5\frac{1}{3} \times \frac{7}{8}$

43. $\frac{10}{27} \times 3\frac{3}{5}$

44. $1\frac{1}{3} \times 1\frac{1}{5}$

45. $2\frac{2}{5} \cdot 3\frac{3}{4}$

46. $2\frac{2}{7} \cdot 2\frac{1}{3}$

47. $4\frac{1}{5} \times \frac{10}{21} \times \frac{9}{20}$

> Videos

48. $\frac{7}{8} \times 5\frac{1}{3} \times \frac{5}{14}$

49. $3\frac{1}{3} \cdot \frac{4}{5} \cdot 1\frac{1}{8}$

50. $4\frac{1}{2} \cdot 5\frac{5}{6} \cdot \frac{8}{15}$

51. Find $\frac{2}{3}$ of $\frac{3}{7}$.

52. What is $\frac{5}{6}$ of $\frac{9}{10}$?

< Objective 4 >

Estimate each product.

53. $3\frac{1}{5} \times 4\frac{2}{3}$

54. $5\frac{1}{7} \times 2\frac{2}{13}$

55. $11\frac{3}{4} \times 5\frac{1}{4}$

Answers

31. ________ 32. ________
33. ________ 34. ________
35. ________
36. ________
37. ________ 38. ________
39. ________ 40. ________
41. ________ 42. ________
43. ________ 44. ________
45. ________ 46. ________
47. ________ 48. ________
49. ________ 50. ________
51. ________ 52. ________
53. ________ 54. ________
55. ________

Answers

56. ____________________

57. ____________________

58. ____________________

59. ____________________

60. ____________________

61. ____________________

62. ____________________

63. ____________________

64. ____________________

65. ____________________

56. $3\frac{4}{5} \times 5\frac{6}{7}$

57. $8\frac{2}{9} \cdot 7\frac{11}{12}$

58. $\frac{9}{10} \cdot 2\frac{2}{7}$

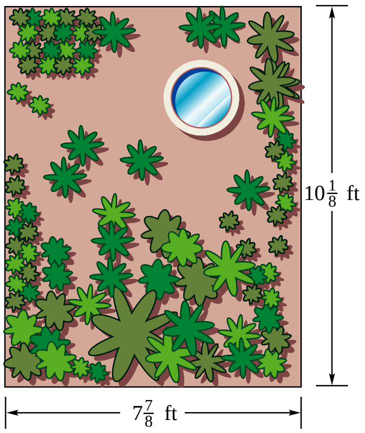
< Objective 5 >

59. **SCIENCE AND MEDICINE** A jet flew at an average speed of 540 mi/h on a $4\frac{2}{3}$-h flight. What was the distance flown?

60. **CONSTRUCTION** A piece of land that has $11\frac{2}{3}$ acres is being subdivided for home lots. It is estimated that $\frac{2}{7}$ of the area will be used for roads. What amount remains to be used for lots?

61. **GEOMETRY** Find the volume of a box that measures $2\frac{1}{4}$ in. by $3\frac{7}{8}$ in. by $4\frac{5}{6}$ in.

62. **CONSTRUCTION** Nico wishes to purchase mulch to cover his garden. The garden measures $7\frac{7}{8}$ ft by $10\frac{1}{8}$ ft. He wants the mulch to be $\frac{1}{3}$ ft deep. How much mulch should Nico order if he must order a whole number of cubic feet?



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Using Your Calculator to Multiply Fractions

Scientific Calculator

To multiply fractions on a scientific calculator, you enter the first fraction, using the $\boxed{\text{a b/c}}$ key, then press the multiplication sign, next enter the second fraction, and then press the equal sign. It is always a good idea to separate the fractions by using parentheses. Note that we do that in the example below.

Graphing Calculator

When using a graphing calculator, you must choose the fraction option $\boxed{1{:}\blacktriangleright\text{Frac}}$ from the $\boxed{\text{MATH}}$ menu before pressing $\boxed{\text{Enter}}$.

For the fraction problem $\frac{7}{15} \times \frac{5}{21}$, the keystroke sequence is

$\boxed{(}\ 7\ \boxed{\div}\ 15\ \boxed{)}\ \boxed{\times}\ \boxed{(}\ 5\ \boxed{\div}\ 21\ \boxed{)}\ \boxed{1{:}\blacktriangleright\text{Frac}}\ \boxed{\text{Enter}}$

The result is $\frac{1}{9}$.

Find each product, using your calculator.

63. $\frac{7}{12} \times \frac{36}{63}$

64. $\frac{8}{27} \times \frac{45}{64}$

65. $\frac{12}{45} \times \frac{27}{72}$

66. $\dfrac{18}{132} \times \dfrac{36}{63}$ **67.** $\dfrac{27}{72} \cdot \dfrac{24}{45}$ **68.** $\dfrac{81}{136} \cdot \dfrac{84}{135}$

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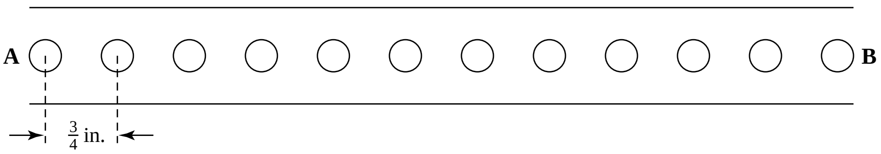
69. **Manufacturing Technology** Calculate the distance from the center of hole A to the center of hole B.



70. **Manufacturing Technology** A cut $3\frac{3}{8}$ in. long needs to be made in a piece of material. The cut rate is $\frac{3}{4}$ minute per in. How many minutes does it take to make the cut?

71. **Manufacturing Technology** A custom order requires $14\frac{3}{4}$ ounces of material that costs $7\frac{1}{2}$¢ per ounce. Find the total cost of the material.

72. **Manufacturing Technology** A customer order requires $8\frac{5}{8}$ ounces of material that costs $10\frac{1}{2}$¢ per ounce. Find the total cost of the material.

Answers

1. $\frac{15}{44}$ **3.** $\frac{3}{4} \times \frac{7}{11} = \frac{21}{44}$ **5.** $\frac{3}{7}$ **7.** $\frac{16}{81}$ **9.** $\frac{7}{33}$
11. $\frac{3}{10} \times \frac{5}{9} = \frac{15}{90} = \frac{1}{6}$ **13.** $\frac{14}{15}$ **15.** 1 **17.** $1\frac{3}{5}$ **19.** $1\frac{3}{10}$
21. $5\frac{5}{6}$ **23.** $\frac{125}{216}$ **25.** $\frac{22}{75}$ **27.** $\frac{4}{9}$ **29.** $1\frac{1}{44}$ **31.** 144 mi
33. 605 joules **35.** 60 mi/h **37.** $594 **39.** 4 cups
41. $1\frac{3}{5}$ **43.** $1\frac{1}{3}$ **45.** 9 **47.** $\frac{9}{10}$ **49.** 3 **51.** $\frac{2}{7}$ **53.** 15
55. 60 **57.** 64 **59.** 2,520 mi **61.** $42\frac{9}{64}$ in.^{3} **63.** $\frac{1}{3}$
65. $\frac{1}{10}$ **67.** $\frac{1}{5}$ **69.** $8\frac{1}{4}$ in. **71.** $110\frac{5}{8}$¢ or $1.11

Answers

66. ____________
67. ____________
68. ____________
69. ____________
70. ____________
71. ____________
72. ____________

Activity 5 ::
Overriding a Presidential Veto

Read the following article and use the information you find there to answer the questions that follow.

A bill is sent to the President of the United States when it has passed both houses of Congress. A majority vote in both the House of Representatives (218 of 435 members) and the Senate (51 of 100 members) is needed for the bill to be passed on to the President. The majority vote is a majority of the members present, as long as more than one-half of all the members are present. More than one-half of the members makes up what is called a quorum.

Once the bill comes to him, the President may either sign the bill, making it a law, or he can veto the bill. His veto sends the bill back to Congress.

Congress can still make the bill a law by overriding the Presidential veto. To override the veto, $\frac{2}{3}$ of the members of each legislative body must vote to override it. Again, this is $\frac{2}{3}$ of a quorum of members.

1. Assume that 420 members of the House are available to vote. How many votes are necessary to make up a majority, which is anything over $\frac{1}{2}$?
2. If 90 members of the Senate are available to vote, how many votes would constitute a majority?
3. How many votes from a group of 420 members of the House would be necessary to overturn a veto?
4. How many votes from a group of 90 members of the Senate would be necessary to overturn a veto?

2.6

Dividing Fractions

< **2.6 Objectives** >

1 > Find the reciprocal of a fraction

2 > Divide fractions

3 > Divide mixed numbers

4 > Solve applications involving division of fractions

We are now ready to look at the operation of division on fractions. First we need a new concept, the **reciprocal** of a fraction.

Property

The Reciprocal of a Fraction

We invert, or turn over, a fraction to write its **reciprocal.**

Example 1 Finding the Reciprocal of a Fraction

< **Objective 1** >

NOTE

In general, the reciprocal of the fraction $\frac{a}{b}$ is $\frac{b}{a}$. Neither a nor b can be 0.

Find the reciprocal of **(a)** $\frac{3}{4}$, **(b)** 5, and **(c)** $1\frac{2}{3}$.

(a) The reciprocal of $\frac{3}{4}$ is $\frac{4}{3}$.

Just invert, or turn over, the fraction.

(b) The reciprocal of 5, or $\frac{5}{1}$, is $\frac{1}{5}$.

Write 5 as $\frac{5}{1}$ and then turn over the fraction.

(c) The reciprocal of $1\frac{2}{3}$, or $\frac{5}{3}$, is $\frac{3}{5}$.

Write $1\frac{2}{3}$ as $\frac{5}{3}$ and then invert.

Check Yourself 1

Find the reciprocal of (a) $\frac{5}{8}$ and (b) $3\frac{1}{4}$.

An important property relating a number and its reciprocal follows.

Property

Reciprocal Products

The product of any number and its reciprocal is 1. (Every number except zero has a reciprocal.)

NOTE

$3 \div 5$ and $\frac{3}{5}$ both mean "3 divided by 5."

We are now ready to use the reciprocal to find a rule for dividing fractions. Recall that we can represent the operation of division in several ways. We used the symbol ÷ earlier. Remember that a fraction also indicates division. For instance,

$$3 \div 5 = \frac{3}{5}$$

In this statement, 5 is called the *divisor*. It follows the division sign ÷ and is written *below* the fraction bar.

Using this information, we can write a statement involving fractions and division as a *complex fraction*, which has a fraction as both its numerator and its denominator, as Example 2 illustrates.

Example 2 Writing a Quotient as a Complex Fraction

Write $\frac{2}{3} \div \frac{4}{5}$ as a complex fraction.

The numerator is $\frac{2}{3}$.

$$\frac{\frac{2}{3}}{\frac{4}{5}}$$

A *complex fraction* is written by placing the dividend in the numerator and the divisor in the denominator.

The denominator is $\frac{4}{5}$.

Check Yourself 2

Write $\frac{2}{5} \div \frac{3}{4}$ as a complex fraction.

Let's continue with the same division problem you looked at in Check Yourself 2.

Example 3 Rewriting a Division Problem

(I) $\frac{2}{5} \div \frac{3}{4} = \dfrac{\frac{2}{5}}{\frac{3}{4}}$

Write the original quotient as a complex fraction.

$$= \frac{\frac{2}{5} \times \frac{4}{3}}{\frac{3}{4} \times \frac{4}{3}}$$

Multiply the numerator and denominator by $\frac{4}{3}$, the reciprocal of the denominator. This does *not* change the value of the fraction.

$$= \frac{\frac{2}{5} \times \frac{4}{3}}{1}$$

The denominator becomes 1.

(II) $= \frac{2}{5} \times \frac{4}{3}$

Recall that a number divided by 1 is just that number.

We see from **(I)** and **(II)** that

$$\frac{2}{5} \div \frac{3}{4} = \frac{2}{5} \times \frac{4}{3}$$

NOTE

Do you see a rule suggested?

We would certainly like to be able to divide fractions easily without all the work of this example. Look carefully at the calculations. The following rule is suggested.

Property

To Divide Fractions

To divide one fraction by another, invert the divisor (the fraction after the division sign) and multiply.

Check Yourself 3

Write $\frac{3}{5} \div \frac{7}{8}$ as a multiplication problem.

Example 4 applies the rule for dividing fractions.

Example 4 Dividing Two Fractions

< Objective 2 >

RECALL

The number inverted is the divisor. It *follows* the division sign.

Divide.

(a) $\dfrac{\frac{1}{3}}{\frac{4}{7}}$

$$\frac{\frac{1}{3}}{\frac{4}{7}} = \frac{1}{3} \div \frac{4}{7} = \frac{1}{3} \times \frac{7}{4}$$

We invert the divisor, $\frac{4}{7}$, and then multiply.

$$= \frac{1 \times 7}{3 \times 4} = \frac{7}{12}$$

(b) $\dfrac{\frac{3}{4}}{6}$

$$\frac{\frac{3}{4}}{6} = \frac{3}{4} \div 6 = \frac{\overset{1}{\cancel{3}}}{4} \times \frac{1}{\underset{2}{\cancel{6}}}$$

$$= \frac{1}{8}$$

Check Yourself 4

Divide.

(a) $\dfrac{\frac{2}{5}}{\frac{3}{4}}$ **(b)** $\dfrac{\frac{2}{7}}{4}$

Here is a similar example.

Example 5 Dividing Two Fractions

Divide.

$$\frac{5}{8} \div \frac{3}{5} = \frac{5}{8} \times \frac{5}{3} = \frac{5 \times 5}{8 \times 3}$$

$$= \frac{25}{24} = 1\frac{1}{24}$$

Check Yourself 5

Divide.

$$\frac{5}{6} \div \frac{3}{7}$$

Simplifying will also be useful in dividing fractions. Consider Example 6.

Example 6 Dividing Two Fractions

> **CAUTION**

NOTE

Be careful! We must invert the divisor *before simplifying.*

Divide.

$$\frac{3}{5} \div \frac{6}{7} = \frac{3}{5} \times \frac{7}{6}$$

Invert the divisor *first!* Then you can divide by the common factor of 3.

$$= \frac{\overset{1}{\cancel{3}} \times 7}{5 \times \underset{2}{\cancel{6}}} = \frac{7}{10}$$

Check Yourself 6

Divide.

$$\frac{4}{9} \div \frac{8}{15}$$

When mixed or whole numbers are involved, the process is similar. Simply change the mixed or whole numbers to improper fractions as the first step. Then proceed with the division rule. Example 7 illustrates this approach.

Example 7 Dividing Two Mixed Numbers

< Objective 3 >

Divide.

$$2\frac{3}{8} \div 1\frac{3}{4} = \frac{19}{8} \div \frac{7}{4}$$

Write the mixed numbers as improper fractions.

$$\frac{19}{8} \times \frac{4}{7} = \frac{19 \times \overset{1}{\cancel{4}}}{\underset{2}{\cancel{8}} \times 7}$$

Invert the divisor and multiply as before.

$$= \frac{19}{14} = 1\frac{5}{14}$$

Check Yourself 7

Divide.

$$3\frac{1}{5} \div 2\frac{2}{5}$$

Example 8 illustrates the division process when a whole number is involved.

Example 8 Dividing a Mixed Number and a Whole Number

NOTE

Write the whole number 6 as $\frac{6}{1}$.

Divide and simplify.

$$1\frac{4}{5} \div 6 = \frac{9}{5} \div \frac{6}{1}$$

$$\frac{9}{5} \times \frac{1}{6} = \frac{\overset{3}{\cancel{9}} \times 1}{5 \times \underset{2}{\cancel{6}}}$$

Invert the divisor, and then divide by the common factor of 3.

$$= \frac{3}{10}$$

Check Yourself 8

Divide.

$$8 \div 4\frac{4}{5}$$

Units ANALYSIS

When dividing by denominate numbers that have fractional units, we multiply by the reciprocal of the number *and its units.*

EXAMPLES:

$$500\text{ mi} \div \frac{25\text{ mi}}{1\text{ gal}} = \frac{500\text{ mi}}{1} \times \frac{1\text{ gal}}{25\text{ mi}} = 20\text{ gal}$$

$$\$24{,}000 \div \frac{\$400}{1\text{ yr}} = \frac{24{,}000\text{ dol}}{1} \div \frac{400\text{ dol}}{1\text{ yr}} = \frac{24{,}000\ \cancel{\text{dol}}}{1} \times \frac{1\text{ yr}}{400\ \cancel{\text{dol}}} = 60\text{ yr}$$

(As always, note that in each case, the arithmetic of the units produces the final units.)

As was the case with multiplication, dividing fractions can be used in the solution of a variety of applications. The steps of the problem-solving process remain the same.

Example 9 An Application Involving the Division of Mixed Numbers

< Objective 4 >

NOTES

One kilometer, abbreviated km, is a metric unit of distance. It is about $\frac{6}{10}$ mi.

The important formula is

Speed = distance ÷ time.

Jack traveled 140 kilometers (km) in $2\frac{1}{3}$ hours (h). What was his average speed?

$$\text{Speed} = \overset{\substack{\text{Distance}\\ \downarrow}}{140\text{ km}} \div \overset{\substack{\text{Time}\\ \downarrow}}{2\frac{1}{3}\text{ h}}$$

$$= \frac{140}{1}\text{ km} \div \frac{7}{3}\text{ h}$$

We know the distance traveled and the time for that travel. To find the *average* speed, we must use division. Do you remember why?

$$\frac{140}{1} \times \frac{3}{7} = \frac{\overset{20}{\cancel{140}} \times 3}{1 \times \underset{1}{\cancel{7}}}\ \frac{\text{km}}{\text{h}}$$

$$= 60\text{ km/h}$$

$\frac{\text{km}}{\text{h}}$ is read "kilometers per hour."

This is a unit of speed.

Check Yourself 9

A light plane flew 280 mi in $1\frac{3}{4}$ h. What was its average speed?

Example 10 An Application Involving the Division of Mixed Numbers

NOTE

We must divide the length of the longer piece by the desired length of the shorter piece.

An electrician needs pieces of wire $2\frac{3}{5}$ in. long. If she has a $20\frac{4}{5}$-in. piece of wire, how many of the shorter pieces can she cut?

$$20\frac{4}{5}\text{ in.} \div 2\frac{3}{5}\ \frac{\text{in.}}{\text{piece}} = \frac{104}{5}\text{ in.} \div \frac{13}{5}\ \frac{\text{in.}}{\text{piece}}$$

$$\frac{104}{5}\text{ in.} \times \frac{5}{13}\ \frac{\text{pieces}}{\text{in.}} = \frac{\overset{8}{\cancel{104}} \times \overset{1}{\cancel{5}}}{\underset{1}{\cancel{5}} \times \underset{1}{\cancel{13}}}\text{ pieces}$$

$$= 8\text{ pieces}$$

Check Yourself 10

A piece of plastic water pipe 63 in. long is to be cut into lengths of $3\frac{1}{2}$ in. How many of the shorter pieces can be cut?

Units ANALYSIS

When you convert units, it is important, and helpful, to carry out all unit arithmetic.

EXAMPLES:

Convert $1\frac{1}{2}$ years into minutes.

To accomplish this, we must recall that there are $365 \frac{\text{days}}{\text{yr}}$, $24 \frac{\text{h}}{\text{day}}$, and $60 \frac{\text{min}}{\text{h}}$. This allows us to set up the following expression.

$$1\frac{1}{2}\ \text{yr} \times 365 \frac{\text{days}}{\text{yr}} \times 24 \frac{\text{h}}{\text{day}} \times 60 \frac{\text{min}}{\text{h}}$$

Before we work with the numbers, we should check the units to make certain that our result will be in "min(utes)."

Because the years, days, and hours all cancel, we are left with only minutes, so we can go ahead with the computation.

$$\frac{3}{2}\ \text{yr} \times 365 \frac{\text{days}}{\text{yr}} \times 24 \frac{\text{h}}{\text{day}} \times 60 \frac{\text{min}}{\text{h}} = 788{,}400\ \text{min}$$

There are 788,400 min in $1\frac{1}{2}$ yr.

Some applications require both multiplication and division. Example 11 is such an application.

Example 11 An Application Involving the Division of Mixed Numbers

A parcel of land that is $2\frac{1}{2}$ mi long and $1\frac{1}{3}$ mi wide is to be divided into tracts that are each $\frac{1}{3}$ square mile (mi^2). How many of these tracts will the parcel make?

The area of the parcel is its length times its width:

$$\begin{aligned}\text{Area} &= 2\frac{1}{2}\ \text{mi} \times 1\frac{1}{3}\ \text{mi}\\ &= \frac{5}{2}\ \text{mi} \times \frac{4}{3}\ \text{mi}\\ &= \frac{10}{3}\ \text{mi}^2\end{aligned}$$

We need to divide the total area of the parcel into $\frac{1}{3}$-mi^2 tracts.

$$\frac{10}{3}\text{ mi}^2 \div \frac{1}{3}\text{ mi}^2 = \frac{10}{3}\text{ mi}^2 \times \frac{3}{1\text{ mi}^2} = 10$$

The land will provide 10 tracts, each with an area of $\frac{1}{3}$ mi^2.

Check Yourself 11

A parcel of land that is $3\frac{1}{3}$ mi long and $2\frac{1}{2}$ mi wide is to be divided into $\frac{1}{3}$-mi^2 tracts. How many of these tracts will the parcel make?

In our final example, we look at a case in which the divisor has fractional units.

Example 12 An Application Involving Mixed Numbers

NOTE

We are using the gardener/contractor definition of a "yard" of mulch. It is actually 1 yd × 1 yd × 1 yd, *or* 1 yd^3.

Jackson has $6\frac{1}{2}$ yards of mulch. His garden needs $\frac{2}{3}$ yards per row. How many rows can he cover with the mulch?

We have $6\frac{1}{2}$ yards and $\frac{2}{3}\frac{\text{yards}}{\text{row}}$. Even if you don't immediately see how to solve the problem, units analysis can help. The units of the answer will be "rows." To get there, we need to have the yards units cancel. That will happen if we divide $6\frac{1}{2}$ by $\frac{2}{3}$!

$$6\frac{1}{2}\text{ yards} \div \frac{2}{3}\,\frac{\text{yards}}{\text{row}} = \frac{13}{2}\text{ yards} \div \frac{2}{3}\,\frac{\text{yards}}{\text{row}}$$

$$= \frac{13}{2}\text{ yards} \times \frac{3}{2}\,\frac{\text{rows}}{\text{yard}} = \frac{39}{4}\text{ rows} = 9\frac{3}{4}\text{ rows}$$

He can cover all of 9 rows and part $\left(\frac{3}{4}\right)$ of the tenth row.

Check Yourself 12

Tangela has $4,100 to invest in a certain stock. If the stock is selling at $\$25\frac{5}{8}$ per share, how many shares can she buy?

Check Yourself ANSWERS

1. **(a)** $\frac{8}{5}$; **(b)** $3\frac{1}{4}$ is $\frac{13}{4}$, so the reciprocal is $\frac{4}{13}$ **2.** $\frac{\frac{2}{5}}{\frac{3}{4}}$ **3.** $\frac{3}{5}\times\frac{8}{7}$

4. **(a)** $\frac{8}{15}$; **(b)** $\frac{1}{14}$ **5.** $1\frac{17}{18}$ **6.** $\frac{4}{9}\div\frac{8}{15}=\frac{4}{9}\times\frac{15}{8}=\frac{\overset{1}{\cancel{4}}\times\overset{5}{\cancel{15}}}{\underset{3}{\cancel{9}}\times\underset{2}{\cancel{8}}}=\frac{5}{6}$

7. $3\frac{1}{5}\div2\frac{2}{5}=\frac{16}{5}\div\frac{12}{5}=\frac{16}{5}\times\frac{5}{12}=\frac{\overset{4}{\cancel{16}}\times\overset{1}{\cancel{5}}}{\underset{1}{\cancel{5}}\times\underset{3}{\cancel{12}}}=\frac{4}{3}=1\frac{1}{3}$ **8.** $1\frac{2}{3}$

9. 160 mi/h **10.** 18 pieces **11.** 25 tracts **12.** 160 shares

Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 2.6

(a) The product of any number and its ________________ is 1.

(b) An expression that has a fraction as both its numerator and denominator is called a ________________ fraction.

(c) To divide one fraction by another, invert the ______________ and multiply.

(d) When dividing by denominate numbers that have fractional units, we multiply by the reciprocal of the number and its ______________.

2.6 exercises

< Objective 1 >

Find the reciprocal of each number.

1. $\frac{7}{8}$ > Videos

2. $\frac{9}{5}$

3. $\frac{1}{2}$

4. 6

5. $2\frac{1}{3}$ > Videos

6. $4\frac{3}{5}$

7. $9\frac{3}{4}$

8. $\frac{1}{8}$

< Objective 2 >

Divide. Write each result in simplest form.

9. $\frac{1}{5} \div \frac{3}{4}$ > Videos

10. $\frac{2}{5} \div \frac{1}{3}$

11. $\dfrac{\frac{2}{5}}{\frac{3}{4}}$

12. $\dfrac{\frac{5}{8}}{\frac{3}{4}}$

13. $\frac{8}{9} \div \frac{4}{3}$

14. $\frac{5}{9} \div \frac{8}{11}$

15. $\frac{7}{10} \div \frac{5}{9}$

16. $\frac{8}{9} \div \frac{11}{15}$

17. $\frac{8}{15} \div \frac{2}{5}$

18. $\frac{5}{27} \div \frac{15}{54}$

19. $\dfrac{\frac{5}{27}}{\frac{25}{36}}$

20. $\dfrac{\frac{9}{28}}{\frac{27}{35}}$

21. $\frac{4}{5} \div 4$

22. $27 \div \frac{3}{7}$

23. $12 \div \frac{2}{3}$

24. $\frac{5}{8} \div 5$

25. $\dfrac{\frac{12}{17}}{\frac{6}{7}}$

26. $\dfrac{\frac{3}{4}}{\frac{9}{10}}$

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Name ____________________

Section __________ Date __________

Answers

1. ______ 2. ______

3. ______ 4. ______

5. ______ 6. ______

7. ______ 8. ______

9. ______ 10. ______

11. ______ 12. ______

13. ______ 14. ______

15. ______ 16. ______

17. ______ 18. ______

19. ______ 20. ______

21. ______ 22. ______

23. ______ 24. ______

25. ______ 26. ______

Answers

27. ____________________

28. ____________________

29. ____________________

30. ____________________

31. ____________________

32. ____________________

33. ____________________

34. ____________________

35. ____________________

36. ____________________

37. ____________________

38. ____________________

39. ____________________

40. ____________________

41. ____________________

42. ____________________

43. ____________________

44. ____________________

45. ____________________

46. ____________________

< Objective 3 >

27. $15 \div 3\frac{1}{3}$

28. $2\frac{4}{7} \div 12$

29. $1\frac{3}{5} \div \frac{4}{15}$

30. $\frac{9}{14} \div 2\frac{4}{7}$

31. $\frac{7}{12} \div 2\frac{1}{3}$

32. $1\frac{3}{8} \div \frac{5}{12}$

33. $5\frac{3}{5} \div \frac{7}{15}$

34. $\frac{7}{18} \div 5\frac{5}{6}$

Basic Skills | **Challenge Yourself** | Calculator/Computer | Career Applications | Above and Beyond

35. $\left(\frac{2}{3}\right)^3 \div 4$

36. $\left(\frac{3}{5}\right)^3 \div 9$

37. $\frac{3}{5} \div \left(\frac{3}{10}\right)^2$ > Videos

38. $\left(\frac{2}{7}\right)^2 \div \frac{3}{14}$

< Objective 4 >

Divide. Be sure to attach the proper units.

39. $900 \text{ mi} \div 15 \frac{\text{mi}}{\text{gal}}$

40. $1{,}500 \text{ joules} \div 75 \frac{\text{joules}}{\text{s}}$

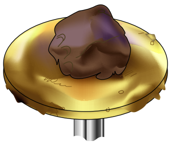
41. $8{,}750 \text{ watts} \div 350 \frac{\text{watts}}{\text{s}}$

42. $\$75{,}744 \div \frac{\$3{,}156}{\text{month}}$

Solve each application.

43. **CONSTRUCTION** A $5\frac{1}{4}$ ft long wire is to be cut into 7 pieces of the same length. How long is each piece? > Videos

44. **CRAFTS** A potter uses $\frac{2}{3}$ pound (lb) of clay in making a bowl. How many bowls can be made from 16 lb of clay?

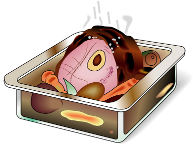


45. **STATISTICS** Virginia made a trip of 95 mi in $1\frac{1}{4}$ h. What was her average speed?

46. **BUSINESS AND FINANCE** A piece of land measures $3\frac{3}{4}$ acres and is for sale at \$60,000. What is the price per acre?

47. **CRAFTS** A roast weighs $3\frac{1}{4}$ lb. How many $\frac{1}{4}$-lb servings will the roast provide?

chapter 2 > Make the Connection



48. **CONSTRUCTION** A bookshelf is 55 in. long. If the books have an average thickness of $1\frac{1}{4}$ in., how many books can be put on the shelf?

Basic Skills | Challenge Yourself | **Calculator/Computer** | Career Applications | Above and Beyond

Using Your Calculator to Divide Fractions

Dividing fractions on your calculator is almost exactly the same as multiplying them. You simply press the $\boxed{\div}$ key instead of the $\boxed{\times}$ key. Again, parentheses are important when using a calculator to work with fractions.

Scientific Calculator

Dividing fractions on a scientific calculator requires only that you enter the problem followed by the equal sign. Recall that fractions are entered using the $\boxed{\textbf{a b/c}}$ key.

Graphing Calculator

When using a graphing calculator, you must choose the fraction option $\boxed{\textbf{1:}\blacktriangleright\textbf{Frac}}$ from the $\boxed{\textbf{MATH}}$ menu before pressing $\boxed{\text{Enter}}$.

Find each quotient, using your calculator.

49. $\frac{1}{5} \div \frac{2}{15}$

50. $\frac{13}{17} \div \frac{39}{34}$

51. $\frac{5}{7} \div \frac{15}{28}$

52. $\frac{3}{7} \div \frac{9}{28}$

53. $\frac{15}{18} \div \frac{45}{27}$

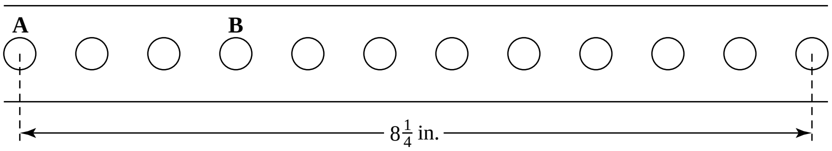
54. $\frac{19}{63} \div \frac{38}{9}$

55. $\frac{25}{45} \div \frac{100}{135}$

56. $\frac{86}{24} \div \frac{258}{96}$

Basic Skills | Challenge Yourself | Calculator/Computer | **Career Applications** | Above and Beyond

57. **MANUFACTURING TECHNOLOGY** Calculate the distance from the center of hole A to the center of hole B.



Answers

47. ______
48. ______
49. ______
50. ______
51. ______
52. ______
53. ______
54. ______
55. ______
56. ______
57. ______

Answers

58. ______

59. ______

60. ______

61. ______

62. ______

63. ______

64. ______

58. Manufacturing Technology A staircase is $95\frac{7}{8}$ in. tall and has 13 risers. What is the height of each riser? (*Hint:* Convert $95\frac{7}{8}$ into an improper fraction and divide by $\frac{13}{1}$.)

59. Manufacturing Technology A part that is $\frac{3}{4}$ in. wide is to be magnified for a detailed drawing. The scale is 1 in. $= \frac{3}{8}$ in. What is the width of the part in the drawing? (*Hint:* Divide the width by $\frac{3}{8}$.)

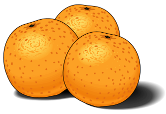
60. Manufacturing Technology A typical unified threaded bolt has one thread every $\frac{1}{20}$ in. How many threads are in $\frac{1}{4}$ in.?

61. Crafts Manuel has $7\frac{1}{2}$ yd of cloth. He wants to cut it into strips $1\frac{3}{4}$ yd long. How many strips will he have? How much cloth remains, if any?

62. Crafts Evette has $41\frac{1}{2}$ ft of string. She wants to cut it into pieces $3\frac{3}{4}$ ft long. How many pieces of string will she have? How much string remains, if any?

Basic Skills | Challenge Yourself | Calculator/Computer | Career Applications | **Above and Beyond** ▲

63. Crafts In squeezing oranges for fresh juice, 3 oranges yield about $\frac{1}{3}$ of a cup.



chapter 2 > Make the Connection

(a) How much juice could you expect to obtain from a bag containing 24 oranges?

(b) If you needed 8 cups of orange juice, how many bags of oranges should you buy?

64. Number Problem A farmer died and left 17 cows to be divided among three workers. The first worker was to receive $\frac{1}{2}$ of the cows, the second worker was to receive $\frac{1}{3}$ of the cows, and the third worker was to receive $\frac{1}{9}$ of the cows. The executor of the farmer's estate realized that 17 cows could not be divided into halves, thirds, or ninths and so added a neighbor's cow to the farmer's. With 18 cows, the executor gave 9 cows to the first worker, 6 cows to the second worker, and 2 cows to the third worker. This accounted for the 17 cows, so the executor returned the borrowed cow to the neighbor. Explain why this works.

65. In general division of fractions is not commutative.

For example, $\frac{3}{4} \div \frac{5}{6} \neq \frac{5}{6} \div \frac{3}{4}$.

There could be an exception. Can you think of a situation in which division of fractions would be commutative?

66. Josephine's boss tells her that her salary is to be divided by $\frac{1}{3}$. Should she quit?

67. Compare the English phrases "divide in half" and "divide by one-half." Do they say the same thing? Create examples to support your answer.

68. **(a)** Compute: $5 \div \frac{1}{10}$; $5 \div \frac{1}{100}$; $5 \div \frac{1}{1{,}000}$; $5 \div \frac{1}{10{,}000}$.

(b) As the divisor gets smaller (approaches 0), what happens to the quotient?

(c) What does this say about the answer to $5 \div 0$?

Answers

65. ____________________

66. ____________________

67. ____________________

68. ____________________

Answers

1. $\frac{8}{7}$ **3.** 2 **5.** $\frac{3}{7}$ **7.** $\frac{4}{39}$ **9.** $\frac{4}{15}$ **11.** $\frac{8}{15}$ **13.** $\frac{2}{3}$

15. $1\frac{13}{50}$ **17.** $1\frac{1}{3}$ **19.** $\frac{4}{15}$ **21.** $\frac{1}{5}$ **23.** 18 **25.** $\frac{14}{17}$

27. $4\frac{1}{2}$ **29.** 6 **31.** $\frac{1}{4}$ **33.** 12 **35.** $\frac{2}{27}$ **37.** $6\frac{2}{3}$

39. 60 gal **41.** 25 s **43.** $\frac{3}{4}$ ft **45.** 76 $\frac{\text{mi}}{\text{h}}$ **47.** 13 servings

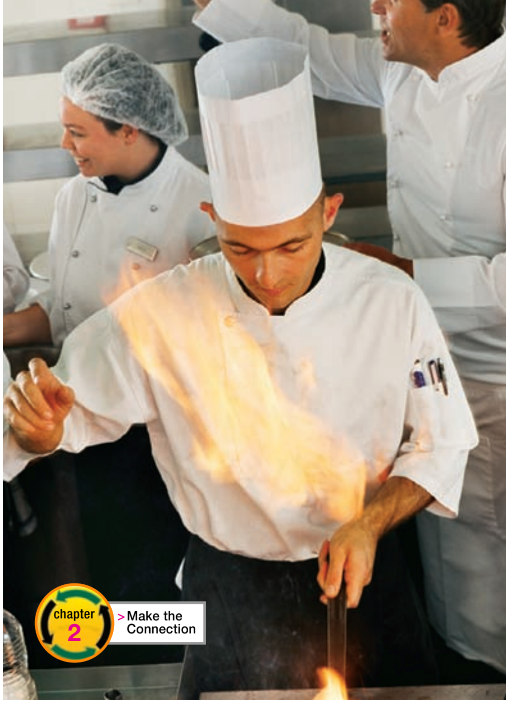
49. $\frac{3}{2}$ **51.** $\frac{4}{3}$ or $1\frac{1}{3}$ **53.** $\frac{1}{2}$ **55.** $\frac{3}{4}$ **57.** $2\frac{1}{4}$ in. **59.** 2 in.

61. 4 strips; $\frac{1}{2}$ yd **63.** $2\frac{2}{3}$ cups; 3 bags **65.** Above and Beyond

67. Above and Beyond

Activity 6 ::

Adapting a Recipe



Tom and Susan like eating in ethnic restaurants, so they were thrilled when Marco's Cafe, an Indian restaurant, opened in their neighborhood. The first time they ate there, Susan had a bowl of Mulligatawny soup and she loved it. She decided that it would be a great soup to serve her friends, so she asked Marco for the recipe. Marco said that was no problem. He had already had so many requests for the recipe that he had made up a handout. A copy of it is reproduced here (try it if you are adventurous):

Mulligatawny Soup

This recipe makes 10 gal; recommended serving size is a 12-ounce (12-oz) bowl. Sauté the following in a steam kettle until the onions are translucent:

10 lb	diced onion
10 lb	diced celery
$\frac{1}{2}$ cup	garlic puree
1 cup	madras curry
2 cups	mild curry

Add the following and bring to a boil:

4 cups	white wine
$\frac{1}{3}$ cup	sugar
1 #10 can	diced tomato
1 gal	fresh apple juice
$\frac{1}{3}$ cup	lemon juice
2 gal	water
1 #10 can	diced carrots
16 oz	chicken stock

Finish with:

Roux (1 lb butter and 1 lb flour) and 8 quarts cream (temper into hot liquid). Season to taste with salt, pepper, celery seed, basil, and garlic.

How many servings does this recipe make?

Visit local grocery stores to find out how much each item costs. Calculate the total cost for 10 gal of soup. What is the cost for each 12-oz serving? (This is called the *marginal cost*—it does not include the overhead for running the restaurant.)

What is roux? Call another restaurant to find out whether it would use the same definition.

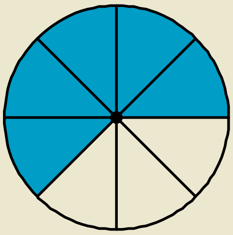
Susan does want to make this soup for a dinner party she is having. Rewrite the recipe so that it will serve six 12-oz bowls. Use reasonable measures, such as teaspoons and cups. Answering the following questions will help. For some items you may have to experiment.

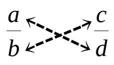
How many ounces in a #10 can?
How many cups in a gallon?
How many ounces in a pound?
How many teaspoons in a cup?
How many cups in a pound of diced onion?

summary :: chapter 2

Definition/Procedure	Example	Reference
Prime Numbers and Divisibility		**Section 2.1**
Prime Number Any whole number that has exactly two factors, 1 and itself.	7, 13, 29, and 73 are prime numbers.	*p.* 100
Composite Number Any whole number greater than 1 that is not prime.	8, 15, 42, and 65 are composite numbers.	*p.* 101
Zero and One Zero and one are neither classified as prime nor composite numbers.		*p.* 101
Divisibility Tests		
By 2 A whole number is divisible by 2 if its last digit is 0, 2, 4, 6, or 8.	932 is divisible by 2; 1,347 is not.	*p.* 101
By 3 A whole number is divisible by 3 if the sum of its digits is divisible by 3.	546 is divisible by 3; 2,357 is not.	*p.* 102
By 5 A whole number is divisible by 5 if its last digit is 0 or 5.	865 is divisible by 5; 23,456 is not.	*p.* 102
Factoring Whole Numbers		**Section 2.2**
Prime Factorization		
To find the prime factorization of a number, divide the number by a series of primes until the final quotient is a prime number. The prime factors include each prime divisor and the final quotient.	$2\overline{)630}$ $3\overline{)315}$ $3\overline{)105}$ $5\overline{)35}$ 7 So $630 = 2 \times 3 \times 3 \times 5 \times 7$.	*p.* 110
Greatest Common Factor (GCF) The GCF is the *largest* number that is a factor of each of a group of numbers.		*p.* 111

Continued

Definition/Procedure	Example	Reference
To Find the GCF		
Step 1 Write the prime factorization for each of the numbers in the group. **Step 2** Locate the prime factors that are *common* to all the numbers. **Step 3** The greatest common factor is the *product* of all of the common prime factors. If there are no common prime factors, the GCF is 1.	To find the GCF of 24, 30, and 36: $24 = \textcircled{2} \times 2 \times 2 \times \textcircled{3}$ $30 = \textcircled{2} \times \textcircled{3} \times 5$ $36 = \textcircled{2} \times 2 \times \textcircled{3} \times 3$ The GCF is $2 \times 3 = 6$.	*p.* 111
Fraction Basics		**Section 2.3**
Fraction Fractions name a number of equal parts of a unit or whole. A fraction is written in the form $\frac{a}{b}$, in which a and b are whole numbers and b cannot be zero.		*p.* 117
Denominator The number of equal parts into which the whole is divided. **Numerator** The number of parts of the whole that are used.	Numerator $\frac{5}{8}$ Denominator	*p.* 117
Proper Fraction A fraction whose numerator *is less than* its denominator. It names a number less than 1.	$\frac{2}{3}$ and $\frac{11}{15}$ are proper fractions.	*p.* 119
Improper Fraction A fraction whose numerator *is greater than* or equal to its denominator. It names a number greater than or equal to 1.	$\frac{7}{5}, \frac{21}{20},$ and $\frac{8}{8}$ are improper fractions.	*p.* 119
Mixed Number The sum of a whole number and a proper fraction.	$2\frac{1}{3}$ and $5\frac{7}{8}$ are mixed numbers. Note that $2\frac{1}{3}$ means $2 + \frac{1}{3}$.	*p.* 119

Definition/Procedure	Example	Reference
To Change an Improper Fraction into a Mixed Number		
Step 1 Divide the numerator by the denominator. The quotient is the whole-number portion of the mixed number. **Step 2** If there is a remainder, write the remainder over the original denominator. This gives the fractional portion of the mixed number.	To change $\frac{22}{5}$ to a mixed number: $\begin{array}{rl} 4 & \leftarrow \text{Quotient} \\ 5\overline{)22} & \\ \underline{20} & \\ 2 & \leftarrow \text{Remainder} \end{array}$ $\frac{22}{5} = 4\frac{2}{5}$	*p.* 120
To Change a Mixed Number to an Improper Fraction		
Step 1 Multiply the denominator of the fraction by the whole-number portion of the mixed number. **Step 2** Add the numerator of the fraction to that product. **Step 3** Write that sum over the original denominator to form the improper fraction.	$5\frac{3}{4} = \frac{(\overset{\text{Denominator}}{4} \times \overset{\text{Whole number}}{5}) + \overset{\text{Numerator}}{3}}{\underset{\text{Denominator}}{4}} = \frac{23}{4}$	*p.* 121
Simplifying Fractions		**Section 2.4**
Equivalent Fractions Two fractions that are equivalent (have equal value) are different names for the same number.		*p.* 128
Cross Products  $a \times d$ and $b \times c$ are called the *cross products*. If the cross products for two fractions are equal, the two fractions are equivalent.	$\frac{2}{3} = \frac{4}{6}$ because $2 \times 6 = 3 \times 4$	*p.* 128
The Fundamental Principle of Fractions For the fraction $\frac{a}{b}$, and any nonzero number c, $\frac{a}{b} = \frac{a \div c}{b \div c}$ **In words:** We can divide the numerator and denominator of a fraction by the same nonzero number. The result is an equivalent fraction.	$\frac{8}{12} = \frac{8 \div 4}{12 \div 4} = \frac{2}{3}$ $\frac{8}{12}$ and $\frac{2}{3}$ are equivalent fractions.	*p.* 129

Continued

Definition/Procedure	Example	Reference
Simplest Form A fraction is in simplest form, or in lowest terms, if the numerator and denominator have no common factors other than 1. This means that the fraction has the smallest possible numerator and denominator.	$\frac{2}{3}$ is in simplest form. $\frac{12}{18}$ is *not* in simplest form. The numerator and denominator have the common factor 6.	*p.* 130
To Write a Fraction in Simplest Form		
Divide the numerator and denominator by their greatest common factor.	$\frac{10}{15} = \frac{10 \div 5}{15 \div 5} = \frac{2}{3}$	*p.* 130
Multiplying Fractions		**Section 2.5**
To Multiply Two Fractions		
Step 1 Multiply numerator by numerator. This gives the numerator of the product. **Step 2** Multiply denominator by denominator. This gives the denominator of the product. **Step 3** Simplify the resulting fraction if possible. In multiplying fractions it is usually easiest to divide by any common factors in the numerator and denominator *before* multiplying.	$\frac{5}{8} \times \frac{3}{7} = \frac{5 \times 3}{8 \times 7} = \frac{15}{56}$ $\frac{5}{9} \times \frac{3}{10} = \frac{\overset{1}{\cancel{5}} \times \overset{1}{\cancel{3}}}{\underset{3}{\cancel{9}} \times \underset{2}{\cancel{10}}} = \frac{1}{6}$	*p.* 139
Dividing Fractions		**Section 2.6**
To Divide Two Fractions		
Invert the divisor and multiply.	$\frac{3}{7} \div \frac{4}{5} = \frac{3}{7} \times \frac{5}{4} = \frac{15}{28}$	*p.* 154
Multiplying or Dividing Mixed Numbers		
Convert any mixed or whole numbers to improper fractions. Then multiply or divide the fractions as before.	$6\frac{2}{3} \times 3\frac{1}{5} = \frac{\overset{4}{\cancel{20}} \times 16}{3 \times \underset{1}{\cancel{5}}}$ $= \frac{64}{3} = 21\frac{1}{3}$	*p.* 156

summary exercises :: chapter 2

This summary exercise set is provided to give you practice with each of the objectives of this chapter. Each exercise is keyed to the appropriate chapter section. When you are finished, you can check your answers to the odd-numbered exercises against those presented in the back of the text. If you have difficulty with any of these questions, go back and reread the examples from that section. The answers to the even-numbered exercises appear in the *Instructor's Solutions Manual.* Your instructor will give you guidelines on how to best use these exercises in your instructional setting.

2.1 *List all the factors of the given numbers.*

1. 52

2. 41

Use the group of numbers 2, 5, 7, 11, 14, 17, 21, 23, 27, 39, and 43.

3. List the prime numbers; then list the composite numbers.

Use the divisibility tests to determine which, if any, of the numbers 2, 3, and 5 are factors of each number.

4. 2,350

5. 33,451

2.2 *Find the prime factorization for the given numbers.*

6. 48

7. 420

8. 2,640

9. 2,250

Find the greatest common factor (GCF).

10. 15 and 20

11. 30 and 31

12. 24 and 40

13. 39 and 65

14. 49, 84, and 119

15. 77, 121, and 253

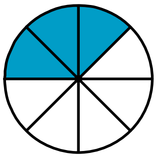
2.3 *Identify the numerator and denominator of each fraction.*

16. $\frac{5}{9}$

17. $\frac{17}{23}$

Give the fractions that name the shaded portions in each diagram. Identify the numerator and the denominator.

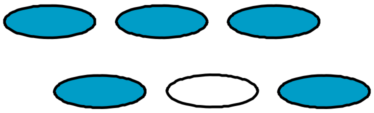
18.



Fraction ___________

Numerator ___________

Denominator ___________

19. 

Fraction: ____________

Numerator: ____________

Denominator: ____________

20. From the group of numbers:

$\frac{2}{3}, \frac{5}{4}, 2\frac{3}{7}, \frac{45}{8}, \frac{7}{7}, 3\frac{4}{5}, \frac{9}{1}, \frac{7}{10}, \frac{12}{5}, 5\frac{2}{9}$

List the proper fractions. ____________

List the improper fractions. ____________

List the mixed numbers. ____________

Convert to mixed or whole numbers.

21. $\frac{41}{6}$ **22.** $\frac{32}{8}$ **23.** $\frac{23}{3}$ **24.** $\frac{47}{4}$

Convert to improper fractions.

25. $7\frac{5}{8}$ **26.** $4\frac{3}{10}$ **27.** $5\frac{2}{7}$ **28.** $12\frac{8}{13}$

2.4 *Determine whether each pair of fractions is equivalent.*

29. $\frac{5}{8}, \frac{7}{12}$ **30.** $\frac{8}{15}, \frac{32}{60}$

Write each fraction in simplest form.

31. $\frac{24}{36}$ **32.** $\frac{45}{75}$ **33.** $\frac{140}{180}$ **34.** $\frac{16}{21}$

Decide whether each is a true statement.

35. $\frac{15}{25} = \frac{3}{5}$ **36.** $\frac{36}{40} = \frac{4}{5}$

2.5 *Multiply.*

37. $\frac{7}{15} \times \frac{5}{21}$ **38.** $\frac{10}{27} \times \frac{9}{20}$ **39.** $4 \cdot \frac{3}{8}$ **40.** $3\frac{2}{5} \cdot \frac{5}{8}$

41. $5\frac{1}{3} \times 1\frac{4}{5}$

42. $1\frac{5}{12} \times 8$

43. $3\frac{1}{5} \times \frac{7}{8} \times 2\frac{6}{7}$

Solve each application.

44. **Social Science** The scale on a map is 1 inch (in.) = 80 miles (mi). If two cities are $2\frac{3}{4}$ in. apart on the map, what is the actual distance between the cities?

45. **Construction** A kitchen measures $5\frac{1}{3}$ yards (yd) by $4\frac{1}{4}$ yd. If you purchase linoleum costing $9 per square yard ($\text{yd}^2$), what will it cost to cover the floor?

46. **Construction** Your living room measures $6\frac{2}{3}$ yd by $4\frac{1}{2}$ yd. If you purchase carpeting at $18 per square yard ($\text{yd}^2$), what will it cost to carpet the room?

47. **Business and Finance** Maria earns $72 per day. If she works $\frac{5}{8}$ of a day, how much will she earn?

48. **Science and Medicine** David drove at an average speed of 65 mi/h for $2\frac{2}{5}$ h. How many miles did he travel?

49. **Social Science** The scale on a map is 1 in. = 120 mi. What actual distance, in miles, does $3\frac{2}{5}$ in. on the map represent?

50. **Social Science** At a college, $\frac{2}{5}$ of the students take a science course. Of the students taking science, $\frac{1}{4}$ take biology. What fraction of the students take biology?

51. **Social Science** A student survey found that $\frac{3}{4}$ of the students have jobs while going to school. Of those who have jobs, $\frac{5}{6}$ work more than 20 h/week. What fraction of those surveyed work more than 20 h/week?

52. **Construction** A living room has dimensions $5\frac{2}{3}$ yd by $4\frac{1}{2}$ yd. How much carpeting must be purchased to cover the room?

2.6 *Divide.*

53. $\frac{5}{12} \div \frac{5}{8}$

54. $\dfrac{\frac{7}{15}}{\frac{14}{25}}$

55. $\dfrac{\frac{9}{20}}{\frac{12}{5}}$

56. $3\frac{3}{8} \div 2\frac{1}{4}$

57. $3\frac{3}{7} \div 8$

58. $6\frac{1}{7} \div \frac{3}{14}$

Solve each application.

59. Construction A piece of wire $3\frac{3}{4}$ ft long is to be cut into 5 pieces of the same length. How long will each piece be?

60. Crafts A blouse pattern requires $1\frac{3}{4}$ yd of fabric. How many blouses can be made from a piece of silk that is 28 yd long?

61. Science and Medicine If you drive 126 mi in $2\frac{1}{4}$ h, what is your average speed?

62. Science and Medicine If you drive 117 mi in $2\frac{1}{4}$ h, what is your average speed?

63. Construction An 18-acre piece of land is to be subdivided into home lots that are each $\frac{3}{8}$ acre. How many lots can be formed?

self-test 2

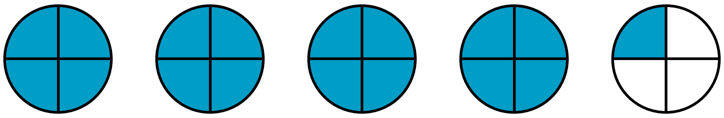
The purpose of this self-test is to help you assess your progress so that you can find concepts that you need to review before the next exam. Allow yourself about an hour to take this test. At the end of that hour, check your answers against those given in the back of this text. If you miss any, go back to the appropriate section to reread the examples until you have mastered that particular concept.

Find the greatest common factor (GCF) for the given numbers.

1. 36 and 84

2. 16, 24, and 72

3. Give the mixed number that names the shaded part of the following diagram.



Write the fractions in simplest form.

4. $\frac{21}{27}$

5. $\frac{36}{84}$

6. $\frac{8}{23}$

Perform the indicated operations.

7. $\frac{2}{3} \times \frac{5}{7}$

8. $5\frac{1}{3} \times \frac{3}{4}$

9. $\frac{7}{12} \div \frac{14}{15}$

10. $2\frac{2}{3} \times 1\frac{2}{7}$

11. $\frac{16}{35} \times \frac{14}{24}$

12. $5\frac{3}{5} \div 2\frac{1}{10}$

13. $\frac{9}{10} \times \frac{5}{8}$

14. $\frac{6}{7} \div \frac{3}{4}$

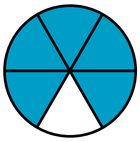
15. $3\frac{5}{6} \times 2\frac{2}{5}$

16. $1\frac{3}{4} \div 1\frac{3}{8}$

17. Which of the numbers 5, 9, 13, 17, 22, 27, 31, and 45 are prime numbers? Which are composite numbers?

What fraction names the shaded part of each diagram? Identify the numerator and denominator.

18.



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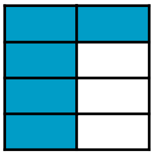

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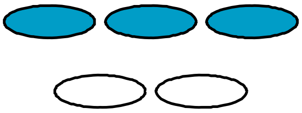


Name ____________________

Section ________ Date ________

Answers

1. ____________________
2. ____________________
3. ____________________
4. ____________________
5. ____________________
6. ____________________
7. ____________________
8. ____________________
9. ____________________
10. ____________________
11. ____________________
12. ____________________
13. ____________________
14. ____________________
15. ____________________
16. ____________________
17. ____________________
18. ____________________
19. ____________________
20. ____________________

Answers

21. ______

22. ______

23. ______

24. ______

25. ______

26. ______

27. ______

28. ______

29. ______

30. ______

31. ______

32. ______

33. ______

34. ______

35. ______

36. ______

37. ______

38. ______

Solve each application.

21. Business and Finance What is the cost of $2\frac{3}{4}$ pounds (lb) of apples if the price per pound is 48 cents?

22. Crafts A bookshelf is 66 in. long. If the thickness of each book on the shelf is $1\frac{3}{8}$ in., how many books can be placed on the shelf?

Convert the fractions to mixed or whole numbers.

23. $\frac{17}{4}$

24. $\frac{15}{1}$

25. $\frac{74}{8}$

26. $\frac{18}{6}$

27. Find the prime factorization for 264.

28. Identify the proper fractions, improper fractions, and mixed numbers in the following group.

$\frac{10}{11}, \frac{9}{5}, \frac{7}{7}, \frac{8}{1}, 2\frac{3}{5}, \frac{1}{8}$

Solve each application.

29. Construction A $31\frac{1}{3}$-acre piece of land is subdivided into home lots. Each home lot is to be $\frac{2}{3}$ acre. How many homes can be built?

30. Construction A room measures $5\frac{1}{3}$ yd by $3\frac{3}{4}$ yd. How many square yards (yd^2) of linoleum must be purchased to cover the floor?

31. Social Science The scale on a map is 1 inch (in.) = 80 miles (mi). If two towns are $2\frac{3}{8}$ in. apart on the map, what is the actual distance in miles between the towns?

32. Use the divisibility tests to determine which, if any, of the numbers 2, 3, and 5 are factors of 54,204.

Use the cross-product method to find out whether the pair of fractions is equivalent.

33. $\frac{2}{7}, \frac{8}{28}$

34. $\frac{8}{20}, \frac{12}{30}$

35. $\frac{3}{20}, \frac{2}{15}$

Convert the mixed numbers to improper fractions.

36. $5\frac{2}{7}$

37. $4\frac{3}{8}$

38. $8\frac{2}{9}$

cumulative review chapters 1-2

Name ____________________

Section ________ Date ________

The following exercises are presented to help you review concepts from earlier chapters. This is meant as review material and not as a comprehensive exam. The answers are presented in the back of the text. Beside each answer is a section reference for the concept. If you have difficulty with any of these exercises, be certain to at least read through the summary related to that section.

1. Give the place value of 7 in 3,738,500.

2. Give the word name for 302,525.

3. Write two million, four hundred thirty thousand as a numeral.

Name the property of addition that is illustrated.

4. $5 + 12 = 12 + 5$

5. $9 + 0 = 9$

6. $(7 + 3) + 8 = 7 + (3 + 8)$

Perform the indicated operations.

7.

$$\begin{array}{r} 593 \\ 275 \\ +\ \ 98 \\ \hline \end{array}$$

8. Find the sum of 58, 673, 5,325, and 17,295.

Round each number to the indicated place value.

9. 5,873 to the nearest hundred

10. 953,150 to the nearest ten thousand

Estimate the sum by rounding each addend to the nearest hundred.

11.

$$\begin{array}{r} 943 \\ 3{,}281 \\ 778 \\ 2{,}112 \\ +\ \ 570 \\ \hline \end{array}$$

Complete each statement by using the symbol $<$ or $>$.

12. 49 ______ 47

13. 80 ______ 90

Answers

1. ____________________

2. ____________________

3. ____________________

4. ____________________

5. ____________________

6. ____________________

7. ____________________

8. ____________________

9. ____________________

10. ____________________

11. ____________________

12. ____________________

13. ____________________

Answers

14. ______________

15. ______________

16. ______________

17. ______________

18. ______________

19. ______________

20. ______________

21. ______________

22. ______________

23. ______________

24. ______________

25. ______________

26. ______________

27. ______________

28. ______________

29. ______________

Perform the indicated operations.

14. $\begin{array}{r} 4{,}834 \\ -\ \ 973 \\ \hline \end{array}$

15. Find the difference of 25,000 and 7,535.

Solve each application.

16. **STATISTICS** Attendance for five performances of a play was 172, 153, 205, 193, and 182. How many people in total attended those performances?

17. **BUSINESS AND FINANCE** Alan bought a Volkswagen with a list price of $18,975. He added stereo equipment for $439 and an air conditioner for $615. If he made a down payment of $2,450, what balance remained on the car?

Name the property of addition and/or multiplication that is illustrated.

18. $3 \times (4 \times 7) = (3 \times 4) \times 7$

19. $3 \times 4 = 4 \times 3$

20. $5 \times (2 + 4) = 5 \times 2 + 5 \times 4$

Perform the indicated operations.

21. $\begin{array}{r} 538 \\ \times\ \ 703 \\ \hline \end{array}$

22. $\begin{array}{r} 1{,}372 \\ \times\ \ 500 \\ \hline \end{array}$

Solve the application.

23. **CONSTRUCTION** A classroom is 8 yards (yd) wide by 9 yd long. If the room is to be recarpeted with material costing $14 per square yard, find the cost of the carpeting.

Divide, using long division.

24. $48\overline{)3{,}259}$

25. $458\overline{)47{,}350}$

Evaluate each expression.

26. $3 + 5 \times 7$

27. $(3 + 5) \times 7$

28. 4×3^2

29. $2 + 8 \times 3 \div 4$

Solve each application.

30. **BUSINESS AND FINANCE** William bought a washer-dryer combination that, with interest charges, cost \$841. He paid \$145 down and agreed to pay the balance in 12 monthly payments. Find the amount of each payment.

31. **NUMBER PROBLEM** Which of the numbers 5, 9, 13, 17, 22, 27, 31, and 45 are prime numbers? Which are composite numbers?

32. Use the divisibility tests to determine which, if any, of the numbers 2, 3, and 5 are factors of 54,204.

33. Find the prime factorization for 264.

Find the greatest common factor (GCF) for the given numbers.

34. 36 and 96

35. 16, 40, and 72

Identify the proper fractions, improper fractions, and mixed numbers from the following group.

$$\frac{7}{12}, \frac{10}{8}, 3\frac{1}{5}, \frac{9}{9}, \frac{7}{1}, \frac{3}{7}, 2\frac{2}{3}$$

36. Proper: ___________ Improper: ____________

Mixed numbers: ___________

Convert to mixed or whole numbers.

37. $\frac{14}{5}$

38. $\frac{28}{7}$

Convert to improper fractions.

39. $4\frac{1}{3}$

40. $7\frac{7}{8}$

Determine whether each pair of fractions is equivalent.

41. $\frac{7}{21}, \frac{8}{24}$

42. $\frac{7}{12}, \frac{8}{15}$

Answers

30. ____________________

31. ____________________

32. ____________________

33. ____________________

34. ____________________

35. ____________________

36. ____________________

37. ____________________

38. ____________________

39. ____________________

40. ____________________

41. ____________________

42. ____________________

Answers

43. ____________

44. ____________

45. ____________

46. ____________

47. ____________

48. ____________

49. ____________

50. ____________

51. ____________

52. ____________

53. ____________

54. ____________

55. ____________

Write each fraction in simplest form.

43. $\frac{28}{42}$

44. $\frac{36}{96}$

Multiply.

45. $\frac{5}{9} \times \frac{8}{15}$

46. $\frac{20}{21} \cdot \frac{7}{25}$

47. $1\frac{1}{8} \cdot 4\frac{4}{5}$

48. $8 \times 2\frac{5}{6}$

49. $\frac{2}{3} \times 1\frac{4}{5} \times \frac{5}{8}$

Divide.

50. $\frac{5}{8} \div \frac{15}{32}$

51. $2\frac{5}{8} \div \frac{7}{12}$

52. $4\frac{1}{6} \div 5$

53. $2\frac{2}{7} \div 1\frac{11}{21}$

Solve each application.

54. **CONSTRUCTION** Your living room measures $6\frac{2}{3}$ yd by $4\frac{1}{2}$ yd. If you purchase carpeting at \$18 per square yard (yd^2), what will it cost to carpet the room?

55. **CONSTRUCTION** If a stack of $\frac{5}{8}$-in. plywood measures 55 in. high, how many sheets of plywood are in the stack?

chapter 3
> Make the Connection

CHAPTER 3

Adding and Subtracting Fractions

INTRODUCTION

Carpentry is one of the oldest and most important of all trades. It dates back to ancient times and the earliest use of primitive tools. It includes large-scale work such as architecture and smaller-scale work such as cabinetry and furniture making. Carpenters mostly work with wood, but they also use ceramic, metal, and plastic. Sometimes carpenters also do roofing, refinishing, remodeling, restoration, and flooring.

Carpenters need to have a vast knowledge of scale drawing and an understanding of blueprints. They have to be able to use a substantial amount of math for measuring and making scale drawings and drafts. They work with models that are hundreds of times smaller than the actual construction. Sometimes these are actual models, and sometimes they are drawings. Carpenters have to be extremely precise in their measuring, sometimes to tiny fractions of an inch. Errors in measurement can have dire consequences such as warping or cracking of materials later on.

Carpenters learn their trades from vocational schools or by serving as apprentices to a more experienced carpenter. Some carpenters are highly skilled and looked upon as artists, whereas others just do handy work.

CHAPTER 3 OUTLINE

Chapter 3 :: Prerequisite Test 182
3.1 **Adding and Subtracting Fractions with Like Denominators** 183
3.2 **Common Multiples** 191
3.3 **Adding and Subtracting Fractions with Unlike Denominators** 200
3.4 **Adding and Subtracting Mixed Numbers** 213
3.5 **Order of Operations with Fractions** 225
3.6 **Estimation Applications** 231
Chapter 3 :: Summary / Summary Exercises / Self-Test / Cumulative Review :: Chapters 1–3 237

3 prerequisite test

Name ____________

Section ________ Date ________

Answers

1. ____________
2. ____________
3. ____________
4. ____________
5. ____________
6. ____________
7. ____________
8. ____________
9. ____________
10. ____________

This prerequisite test provides some exercises requiring skills that you will need to be successful in the coming chapter. The answers for these exercises can be found in the back of this text. This prerequisite test can help you identify topics that you will need to review before beginning the chapter.

Write the prime factorization for each number.

1. 24

2. 36

3. 90

Change each mixed number to an improper fraction.

4. $4\frac{2}{3}$

5. $6\frac{2}{5}$

6. $9\frac{1}{10}$

Change each improper fraction to a mixed or whole number.

7. $\frac{29}{4}$

8. $\frac{42}{6}$

9. $\frac{8}{8}$

Evaluate.

10. $7 \times 11 - 2 \times (2 + 3)^2$

3.1 Adding and Subtracting Fractions with Like Denominators

< 3.1 Objectives >

1 > Add two like fractions

2 > Add a group of like fractions

3 > Subtract two like fractions

Recall from our work in Chapter 1 that adding can be thought of as combining groups of the *same kind* of objects. This is also true when you think about adding fractions.

Fractions can be added only if they name a number of the *same parts* of a whole. This means that we can add fractions only when they are **like fractions,** that is, when they have the *same* (*a common*) denominator. For instance, we can add two nickels and three nickels to get five nickels. We *cannot* directly add two nickels and three dimes!

As long as we are dealing with like fractions, addition is an easy matter. Just use the following rule.

Step by Step

To Add Like Fractions

Step 1 Add the numerators.
Step 2 Place the sum over the common denominator.
Step 3 Simplify the resulting fraction when necessary.

Example 1 illustrates the use of this rule.

Example 1 Adding Like Fractions

< Objective 1 >

Add.

$$\frac{1}{5} + \frac{3}{5}$$

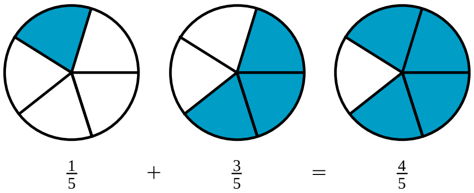
Step 1 Add the numerators.

$1 + 3 = 4$

Step 2 Write that sum over the common denominator, 5. We are done at this point because the answer, $\frac{4}{5}$, is in the simplest possible form.

$$\frac{1}{5} + \frac{3}{5} = \overset{\text{Step 1}}{\frac{1+3}{5}} = \overset{\text{Step 2}}{\frac{4}{5}}$$

We illustrate this addition with a diagram.



NOTE

Combining 1 of the 5 parts with 3 of the 5 parts gives a total of 4 of the 5 equal parts.

Check Yourself 1

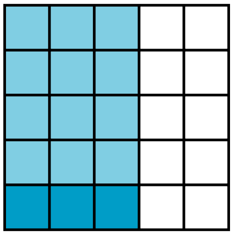
Add.

$$\frac{2}{9} + \frac{5}{9}$$

> CAUTION

Be Careful! In adding fractions, *do not* follow the rule for multiplying fractions. To multiply $\frac{1}{5} \times \frac{3}{5}$, you would multiply both the numerators and the denominators:

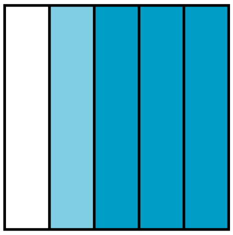
$$\frac{1}{5} \times \frac{3}{5} = \frac{3}{25}$$



$\frac{1}{5}$ of $\frac{3}{5} = \frac{3}{25}$

However, when you add two fractions, the sum will have the same like denominator. So, you **do not** add the denominators.

$$\frac{1}{5} + \frac{3}{5} \neq \frac{4}{10}$$



$\frac{1}{5} + \frac{3}{5} = \frac{4}{5}$

Step 3 of the addition rule for like fractions tells us to *simplify* the sum. Fractions should always be written in lowest terms. Consider Example 2.

Example 2 Adding Like Fractions That Require Simplifying

Add and simplify.

Step 3

$$\frac{3}{12} + \frac{5}{12} = \frac{8}{12} = \frac{2}{3}$$

The sum $\frac{8}{12}$ is *not* in lowest terms.

Divide the numerator and denominator by 4 to simplify the result.

Check Yourself 2

Add.

$$\frac{4}{15} + \frac{6}{15}$$

If the sum of two fractions is an improper fraction, we usually write that sum as a mixed number.

Example 3 Adding Like Fractions That Result in Mixed Numbers

NOTE

Add as before. Then convert the sum to a mixed number.

Add.

$$\frac{5}{9} + \frac{8}{9} = \frac{13}{9} = 1\frac{4}{9}$$

Write the sum $\frac{13}{9}$ as a mixed number.

Check Yourself 3

Add.

$\frac{7}{12} + \frac{10}{12}$

We can easily extend our addition rule to find the sum of more than two fractions as long as they all have the same denominator. This is shown in Example 4.

Example 4 Adding a Group of Like Fractions

< Objective 2 >

Add.

$$\frac{2}{7} + \frac{3}{7} + \frac{6}{7} = \frac{11}{7}$$

Add the numerators: 2 + 3 + 6 = 11.

$$= 1\frac{4}{7}$$

Check Yourself 4

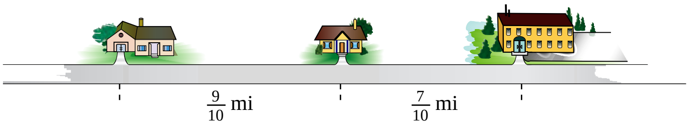
Add.

$\frac{1}{8} + \frac{3}{8} + \frac{5}{8}$

Many applications can be solved by adding fractions.

Example 5 An Application Involving the Adding of Like Fractions

Noel walked $\frac{9}{10}$ miles (mi) to Jensen's house and then walked $\frac{7}{10}$ mi to school. How far did Noel walk?



To find the total distance Noel walked, add the two distances.

$$\frac{9}{10} + \frac{7}{10} = \frac{16}{10} = 1\frac{6}{10} = 1\frac{3}{5}$$

Noel walked $1\frac{3}{5}$ mi.

NOTE

Like fractions have the same denominator.

Check Yourself 5

Emir bought $\frac{7}{16}$ pounds (lb) of candy at one store and $\frac{11}{16}$ lb at another store. How much candy did Emir buy?

If a problem involves like fractions, then subtraction, like addition, is not difficult.

Step by Step

To Subtract Like Fractions

Step 1	Subtract the numerators.
Step 2	Place the difference over the common denominator.
Step 3	Simplify the resulting fraction when necessary.

Example 6 Subtracting Like Fractions

< Objective 3 >

Subtract.

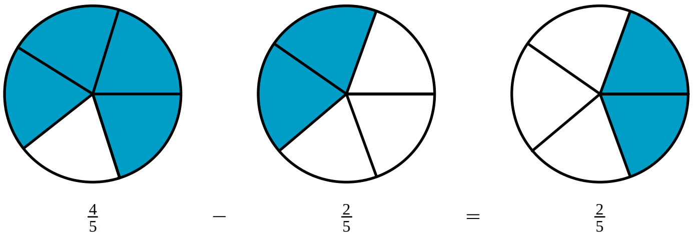
(a) $$\frac{4}{5} - \frac{2}{5} = \overset{\text{Step 1}}{\frac{4-2}{5}} = \overset{\text{Step 2}}{\frac{2}{5}}$$

Subtract the numerators: 4 − 2 = 2. Write the difference over the common denominator, 5. Step 3 is not necessary because the difference is in simplest form.

NOTE

Subtracting 2 of the 5 parts from 4 of the 5 parts leaves 2 of the 5 parts.

Illustrating with a diagram:



NOTE

Always write the result in simplest terms.

(b) $$\frac{5}{8} - \frac{3}{8} = \frac{5-3}{8} = \frac{2}{8} = \frac{1}{4}$$

Check Yourself 6

Subtract.

$$\frac{11}{12} - \frac{5}{12}$$

Check Yourself ANSWERS

1. $\frac{7}{9}$ **2.** $\frac{2}{3}$ **3.** $1\frac{5}{12}$ **4.** $1\frac{1}{8}$ **5.** $1\frac{1}{8}$ lb **6.** $\frac{1}{2}$

Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 3.1

(a) Fractions with the same (common) denominator are called ____________ fractions.

(b) When adding like fractions, add the ____________.

(c) After adding two fractions, ____________ the result when necessary.

(d) The result of subtraction is called the ____________.

3.1 exercises

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Name ____________________

Section ________ Date ________

< Objective 1 >

Add. Write all answers in simplest terms.

1. $\frac{3}{5} + \frac{1}{5}$

2. $\frac{4}{7} + \frac{1}{7}$

3. $\frac{4}{11} + \frac{6}{11}$

4. $\frac{5}{16} + \frac{4}{16}$

5. $\frac{2}{10} + \frac{3}{10}$ > Videos

6. $\frac{5}{12} + \frac{1}{12}$

7. $\frac{3}{7} + \frac{4}{7}$

8. $\frac{13}{20} + \frac{17}{20}$

9. $\frac{29}{30} + \frac{11}{30}$

10. $\frac{4}{9} + \frac{5}{9}$

11. $\frac{13}{48} + \frac{23}{48}$

12. $\frac{17}{60} + \frac{31}{60}$

13. $\frac{3}{7} + \frac{6}{7}$

14. $\frac{3}{5} + \frac{4}{5}$

15. $\frac{7}{10} + \frac{9}{10}$

16. $\frac{5}{8} + \frac{7}{8}$

17. $\frac{11}{12} + \frac{10}{12}$

18. $\frac{13}{18} + \frac{11}{18}$

< Objective 2 >

19. $\frac{1}{8} + \frac{1}{8} + \frac{3}{8}$

20. $\frac{1}{10} + \frac{3}{10} + \frac{3}{10}$

21. $\frac{1}{9} + \frac{4}{9} + \frac{5}{9}$ > Videos

22. $\frac{7}{12} + \frac{11}{12} + \frac{1}{12}$

< Objective 3 >

Subtract. Write all answers in simplest terms.

23. $\frac{3}{5} - \frac{1}{5}$

24. $\frac{5}{7} - \frac{2}{7}$

Answers

1. ________ 2. ________

3. ________ 4. ________

5. ________ 6. ________

7. ________ 8. ________

9. ________ 10. ________

11. ________ 12. ________

13. ________ 14. ________

15. ________ 16. ________

17. ________ 18. ________

19. ________ 20. ________

21. ________ 22. ________

23. ________ 24. ________

Answers

25. ______

26. ______

27. ______

28. ______

29. ______

30. ______

31. ______

32. ______

33. ______

34. ______

35. ______

36. ______

37. ______

38. ______

39. ______

40. ______

25. $\frac{7}{9} - \frac{4}{9}$

26. $\frac{7}{10} - \frac{3}{10}$

27. $\frac{13}{20} - \frac{3}{20}$ > Videos

28. $\frac{19}{30} - \frac{17}{30}$

29. $\frac{19}{24} - \frac{5}{24}$

30. $\frac{25}{36} - \frac{13}{36}$

31. $\frac{11}{12} - \frac{7}{12}$

32. $\frac{9}{10} - \frac{6}{10}$

33. $\frac{8}{9} - \frac{3}{9}$

34. $\frac{5}{8} - \frac{1}{8}$

Solve each application. Write each answer in simplest terms.

35. Number Problem You work 7 hours (h) one day, 5 h the second day, and 6 h the third day. How long did you work, as a fraction of a 24-h day?

> Videos

36. Number Problem One task took 7 minutes (min), a second task took 12 min, and a third task took 21 min. How long did the three tasks take, as a fraction of an hour?

37. Geometry What is the perimeter of a rectangle if the length is $\frac{7}{10}$ inches (in.) and the width is $\frac{2}{10}$ in.?

38. Geometry Find the perimeter of a rectangular picture if the width is $\frac{7}{9}$ yd and the length is $\frac{5}{9}$ yd.

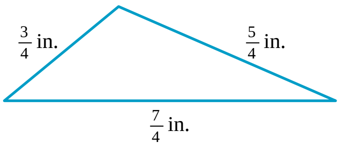
39. Statistics Patrick spent $\frac{4}{9}$ of an hour (h) in the batting cages on Friday and $\frac{7}{9}$ of an hour on Saturday. He wants to spend 2 h total on the weekend. How much time should he spend on Sunday to accomplish this goal?

40. Business and Finance Maria, a road inspector, must inspect $\frac{17}{30}$ of a mile of road. If she has already inspected $\frac{11}{30}$ of a mile, how much more does she need to inspect?

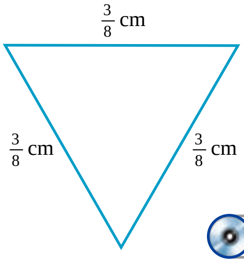
Basic Skills | **Challenge Yourself** | Calculator/Computer | Career Applications | Above and Beyond

GEOMETRY *Find the perimeter of each triangle.*

41.

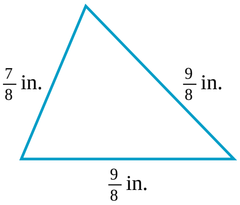

$\frac{3}{4}$ in.
$\frac{5}{4}$ in.
$\frac{7}{4}$ in.

42.

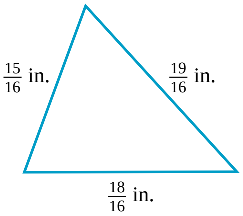

$\frac{3}{8}$ cm
$\frac{3}{8}$ cm
$\frac{3}{8}$ cm

> Videos

43.

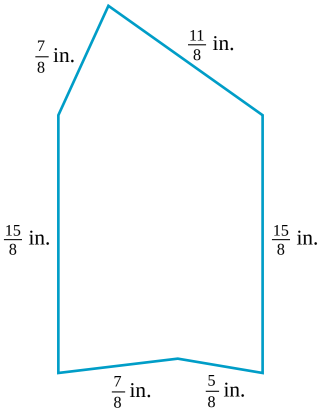

$\frac{7}{8}$ in.
$\frac{9}{8}$ in.
$\frac{9}{8}$ in.

44.

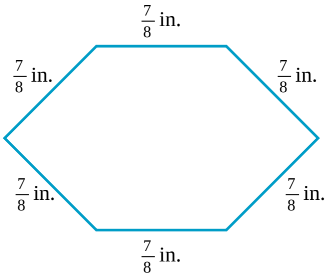

$\frac{15}{16}$ in.
$\frac{19}{16}$ in.
$\frac{18}{16}$ in.

GEOMETRY *Find the perimeter of each polygon.*

45.


$\frac{7}{8}$ in.
$\frac{11}{8}$ in.
$\frac{15}{8}$ in.
$\frac{15}{8}$ in.
$\frac{7}{8}$ in.
$\frac{5}{8}$ in.

46.


$\frac{7}{8}$ in.
$\frac{7}{8}$ in.
$\frac{7}{8}$ in.
$\frac{7}{8}$ in.
$\frac{7}{8}$ in.
$\frac{7}{8}$ in.

Evaluate. Write all answers in simplest terms.

47. $\frac{7}{12} - \frac{4}{12} + \frac{3}{12}$ > Videos

48. $\frac{8}{9} + \frac{3}{9} - \frac{5}{9}$

49. $\frac{6}{13} - \frac{3}{13} + \frac{11}{13}$

50. $\frac{9}{11} - \frac{3}{11} + \frac{7}{11}$

51. $\frac{18}{23} - \frac{13}{23} - \frac{3}{23}$

52. $\frac{17}{18} - \frac{11}{18} - \frac{5}{18}$

Answers

41. ____________

42. ____________

43. ____________

44. ____________

45. ____________

46. ____________

47. ____________

48. ____________

49. ____________

50. ____________

51. ____________

52. ____________

Answers

53. ____________________

54. ____________________

55. ____________________

56. ____________________

Basic Skills | Challenge Yourself | Calculator/Computer | **Career Applications** | Above and Beyond

53. **ALLIED HEALTH** Carla is to be given $\frac{1}{8}$ gram (g) of medication in the morning, $\frac{1}{8}$ g at 3 P.M. in the afternoon, and $\frac{3}{8}$ g before she goes to bed. How many grams of medication will she receive in one day?

54. **ALLIED HEALTH** Prior to chemotherapy, a patient's malignant tumor weighed $\frac{7}{8}$ pound (lb). After the initial round of chemotherapy, the tumor's weight had been reduced by $\frac{3}{8}$ lb. How much did the tumor weigh after the chemotherapy treatment?

55. **MANUFACTURING TECHNOLOGY** Triplet Precision Machine has ordered $\frac{5}{8}$ ton of steel. The first truck arrived with $\frac{1}{8}$ ton of the steel. How much is yet to arrive?

56. **MANUFACTURING TECHNOLOGY** A sidewalk will require $\frac{15}{16}$ yard (yd) of concrete. The concrete mixer is capable of mixing $\frac{3}{16}$ yd at a time. How much concrete is still needed after one mixer load is used?

chapter 3 > Make the Connection

Answers

1. $\frac{4}{5}$ **3.** $\frac{10}{11}$ **5.** $\frac{1}{2}$ **7.** 1 **9.** $\frac{4}{3} = 1\frac{1}{3}$ **11.** $\frac{3}{4}$ **13.** $1\frac{2}{7}$

15. $1\frac{3}{5}$ **17.** $1\frac{3}{4}$ **19.** $\frac{5}{8}$ **21.** $1\frac{1}{9}$ **23.** $\frac{2}{5}$ **25.** $\frac{1}{3}$ **27.** $\frac{1}{2}$

29. $\frac{7}{12}$ **31.** $\frac{1}{3}$ **33.** $\frac{5}{9}$ **35.** $\frac{3}{4}$ day **37.** $\frac{9}{5}$ in. $= 1\frac{4}{5}$ in.

39. $\frac{7}{9}$ h **41.** $\frac{15}{4}$ in. $= 3\frac{3}{4}$ in. **43.** $\frac{25}{8}$ in. $= 3\frac{1}{8}$ in.

45. $\frac{15}{2}$ in. $= 7\frac{1}{2}$ in. **47.** $\frac{1}{2}$ **49.** $\frac{14}{13} = 1\frac{1}{13}$ **51.** $\frac{2}{23}$

53. $\frac{5}{8}$ g **55.** $\frac{1}{2}$ ton

3.2

Common Multiples

< **3.2 Objectives** >

1 > Find the least common multiple (LCM) of two numbers

2 > Find the LCM of a group of numbers

3 > Compare the size of two fractions

In this chapter, we discuss the process used for adding or subtracting fractions. One of the most important concepts we use when we add or subtract fractions is that of **multiples.**

Definition

Multiples	The *multiples* of a number are the product of that number with the natural numbers 1, 2, 3, 4, 5, . . .

Example 1 Listing Multiples

NOTE

Notice that the multiples, except for 3 itself, are *larger* than 3.

List the multiples of 3.

The multiples of 3 are

$3 \times 1, 3 \times 2, 3 \times 3, 3 \times 4, \ldots$

or

3, 6, 9, 12, . . .   The three dots indicate that the list continues without stopping.

An easy way of listing the multiples of 3 is to think of *counting by threes.*

Check Yourself 1

List the first seven multiples of 4.

Sometimes we need to find common multiples of two or more numbers.

Definition

Common Multiples	If a number is a multiple of each of a group of numbers, it is called a *common multiple* of the numbers; that is, it is a number that is exactly divisible by each of the numbers in the group.

Example 2 Finding Common Multiples

NOTE

15, 30, 45, and 60 are multiples of *both* 3 and 5.

Find four common multiples of 3 and 5.

Some common multiples of 3 and 5 are

15, 30, 45, 60

Check Yourself 2

List the first six multiples of 6. Then look at your list from Check Yourself 1 and list some common multiples of 4 and 6.

For our later work, we will use the *least common multiple* of a group of numbers.

Definition

Least Common Multiple

The **least common multiple** (LCM) of a group of numbers is the *smallest* number that is divisible by each number in the group.

It is possible to simply list the multiples of each number and then find the LCM by inspection.

Example 3 Finding the Least Common Multiple (LCM)

< Objective 1 >

NOTE

48 is also a common multiple of 6 and 8, but we are looking for the *smallest* such number.

Find the least common multiple of 6 and 8.

Multiples

6: 6, 12, 18, ㉔, 30, 36, 42, 48, . . .

8: 8, 16, ㉔, 32, 40, 48, . . .

We see that 24 is the smallest number common to both lists. So 24 is the LCM of 6 and 8.

Check Yourself 3

Find the least common multiple of 20 and 30 by listing the multiples of each number.

The technique used in Example 3 will work for any group of numbers. However, it becomes tedious for larger numbers. Let's outline a different approach.

Step by Step

Finding the Least Common Multiple

Step 1 Write the prime factorization for each of the numbers in the group.
Step 2 Find all the prime factors that appear in any one of the prime factorizations.
Step 3 Form the product of those prime factors, using each factor the greatest number of times it occurs in any one factorization.

For instance, if a number appears three times in the factorization of a number, it must be included at least three times in forming the least common multiple.

This method is illustrated in Example 4.

Example 4 Finding the Least Common Multiple (LCM)

NOTE

Line up the *like* factors vertically.

To find the LCM of 10 and 18, factor:

$$\begin{array}{rl} 10 = & 2 \phantom{{}\times 3 \times 3} \times 5 \\ 18 = & 2 \times 3 \times 3 \\ \hline & 2 \times 3 \times 3 \times 5 \end{array}$$

Bring down the factors.

The numbers 2 and 5 appear, at most, one time in any one factorization. And 3 appears two times in one factorization.

$2 \times 3 \times 3 \times 5 = 90$

So 90 is the LCM of 10 and 18.

Check Yourself 4

Use the method of Example 4 to find the LCM of 24 and 36.

The procedure is the same for a group of more than two numbers.

Example 5 Finding the Least Common Multiple (LCM)

< Objective 2 >

NOTE

The different factors that appear are 2, 3, and 5.

To find the LCM of 12, 18, and 20, we factor:

$$\begin{array}{rcccccccccc} 12 &=& 2 &\times& 2 &\times& 3 &&&& \\ 18 &=& 2 &&&\times& 3 &\times& 3 && \\ 20 &=& 2 &\times& 2 &&&&&\times& 5 \\ \cline{3-11} &&2 &\times& 2 &\times& 3 &\times& 3 &\times& 5 \end{array}$$

The numbers 2 and 3 appear twice in one factorization, and 5 appears just once.

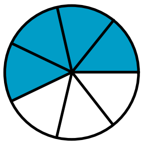
$$2 \times 2 \times 3 \times 3 \times 5 = 180$$

So 180 is the LCM of 12, 18, and 20.

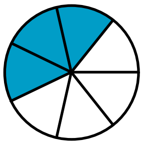
Check Yourself 5

Find the LCM of 3, 4, and 6.

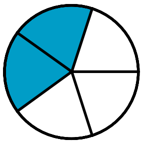
The process of finding the least common multiple is very useful when we are adding, subtracting, or comparing unlike fractions (fractions with different denominators).



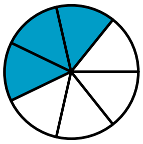
$\frac{4}{7}$



$\frac{3}{7}$



$\frac{2}{5}$



$\frac{3}{7}$

Suppose you are asked to compare the sizes of the fractions $\frac{3}{7}$ and $\frac{4}{7}$. Because each unit in the diagram is divided into seven parts, it is easy to see that $\frac{4}{7}$ is larger than $\frac{3}{7}$.

Four parts of seven are a greater portion than three parts. Now compare the size of the fractions $\frac{2}{5}$ and $\frac{3}{7}$.

We *cannot* compare fifths with sevenths! The fractions $\frac{2}{5}$ and $\frac{3}{7}$ are *not* like fractions. Because they name different ways of dividing the whole, deciding which fraction is larger is not nearly so easy.

To compare the sizes of fractions, we change them to equivalent fractions having a *common denominator*. This common denominator must be a common multiple of the original denominators.

We will use the following property to form such fractions.

Property

The Fundamental Principle of Fractions

$$\frac{a}{b} = \frac{a \times c}{b \times c}$$

Example 6 Comparing the Size of Fractions

< Objective 3 >

RECALL

$\frac{2}{5}$ and $\frac{14}{35}$ are **equivalent fractions.** They name the same part of a whole.

NOTE

15 of 35 parts represents a greater portion of the whole than 14 parts.

Compare the sizes of $\frac{2}{5}$ and $\frac{3}{7}$.

The original denominators are 5 and 7. Because 35 is a common multiple of 5 and 7, let's use 35 as our common denominator.

$$\begin{tikzcd}\frac{2}{5} \arrow[rr, bend left, "\times 7"] \arrow[rr, bend right, "\times 7"'] & = & \frac{14}{35}\end{tikzcd}$$

Think, "What must we multiply 5 by to get 35?" The answer is 7. Multiply the numerator and denominator by that number.

$$\begin{tikzcd}\frac{3}{7} \arrow[rr, bend left, "\times 5"] \arrow[rr, bend right, "\times 5"'] & = & \frac{15}{35}\end{tikzcd}$$

Multiply the numerator and denominator by 5.

Because $\frac{2}{5} = \frac{14}{35}$ and $\frac{3}{7} = \frac{15}{35}$, we see that $\frac{3}{7}$ is larger than $\frac{2}{5}$.

Check Yourself 6

Which is larger, $\frac{5}{9}$ or $\frac{4}{7}$?

Now, consider an example that uses inequality notation.

Example 7 Using an Inequality Symbol with Two Fractions

RECALL

The inequality symbols "point" to the smaller quantity.

NOTE

We use the Fundamental Principle of Fractions to convert these fractions.

Use the inequality symbols $<$ or $>$ to complete the statement.

$$\frac{5}{8} \ \square \ \frac{3}{5}$$

Once again we must compare the sizes of the two fractions, and we do this by converting the fractions to equivalent fractions with a common denominator. Here we use 40 as that denominator.

$$\begin{tikzcd}\frac{5}{8} \arrow[r, bend left, "\times 5"] \arrow[r, bend right, "\times 5"'] & \frac{25}{40} & \frac{3}{5} \arrow[r, bend left, "\times 8"] \arrow[r, bend right, "\times 8"'] & \frac{24}{40}\end{tikzcd}$$

Because $\frac{5}{8}\left(\text{or } \frac{25}{40}\right)$ is larger than $\frac{3}{5}\left(\text{or } \frac{24}{40}\right)$, we write

$$\frac{5}{8} > \frac{3}{5}$$

Check Yourself 7

Use the symbols $<$ or $>$ to complete the statement.

$$\frac{5}{9} \square \frac{6}{11}$$

Check Yourself ANSWERS

1. The first seven multiples of 4 are 4, 8, 12, 16, 20, 24, and 28.

2. 6, 12, 18, 24, 30, 36; some common multiples of 4 and 6 are 12 and 24.

3. The multiples of 20 are 20, 40, 60, 80, 100, 120, . . . ; the multiples of 30 are 30, 60, 90, 120, 150, . . . ; the least common multiple of 20 and 30 is 60, the smallest number common to both lists.

4. $2 \times 2 \times 2 \times 3 \times 3 = 72$ **5.** 12 **6.** $\frac{4}{7}$ is larger **7.** $\frac{5}{9} > \frac{6}{11}$

Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 3.2

(a) The ______________ of a number are the products of that number with the natural numbers.

(b) The LCM of a group of numbers is the ______________ number that is divisible by each number in that group.

(c) To compare the sizes of fractions, we change them to equivalent fractions having a common ______________.

(d) The statement $\frac{5}{8} > \frac{3}{5}$ is read "$\frac{5}{8}$ is ______________ than $\frac{3}{5}$."

3.2 exercises

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Name ____________

Section ________ Date ________

Answers

1. ________ 2. ________

3. ________ 4. ________

5. ________ 6. ________

7. ________ 8. ________

9. ________ 10. ________

11. ________ 12. ________

13. ________ 14. ________

15. ________ 16. ________

17. ________ 18. ________

19. ________ 20. ________

21. ________ 22. ________

23. ________________

24. ________________

25. ________________

26. ________________

< Objective 1 >

Find the least common multiple (LCM) for each group of numbers. Use whichever methods you wish.

1. 2 and 3

2. 3 and 5

3. 4 and 6

4. 6 and 9

5. 10 and 20

6. 12 and 36

7. 9 and 12

8. 20 and 30

9. 12 and 16

10. 10 and 15

11. 12 and 15

> Videos

12. 12 and 21

13. 18 and 36

> Videos

14. 25 and 50

15. 25 and 40

16. 10 and 14

< Objective 2 >

17. 3, 5, and 6

18. 2, 8, and 10

19. 18, 21, and 28

20. 8, 15, and 20

21. 20, 30, and 45

22. 12, 20, and 35

< Objective 3 >

Arrange the fractions from smallest to largest.

23. $\frac{12}{17}, \frac{9}{10}$

24. $\frac{4}{9}, \frac{5}{11}$

25. $\frac{5}{8}, \frac{3}{5}$

26. $\frac{9}{10}, \frac{8}{9}$

27. $\frac{3}{8}, \frac{1}{3}, \frac{1}{4}$ > Videos

28. $\frac{7}{12}, \frac{5}{18}, \frac{1}{3}$

29. $\frac{11}{12}, \frac{4}{5}, \frac{5}{6}$

30. $\frac{5}{8}, \frac{9}{16}, \frac{13}{32}$

Complete the statements, using the symbols < or >.

31. $\frac{5}{6} \square \frac{2}{5}$ > Videos

32. $\frac{3}{4} \square \frac{10}{11}$

33. $\frac{4}{9} \square \frac{3}{7}$

34. $\frac{7}{10} \square \frac{11}{15}$

35. $\frac{7}{20} \square \frac{9}{25}$

36. $\frac{5}{12} \square \frac{7}{18}$

37. $\frac{5}{16} \square \frac{7}{20}$

38. $\frac{7}{12} \square \frac{9}{15}$

Complete each equivalent fraction.

39. $\frac{4}{5} = \frac{\quad}{25}$ > Videos

40. $\frac{6}{13} = \frac{\quad}{26}$

41. $\frac{5}{6} = \frac{25}{\quad}$

42. $\frac{2}{5} = \frac{14}{\quad}$

43. $\frac{11}{37} = \frac{\quad}{111}$

44. $\frac{9}{31} = \frac{\quad}{248}$

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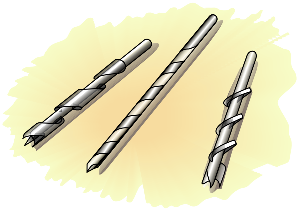
Solve each application.

45. **Crafts** Three drill bits are marked $\frac{3}{8}, \frac{5}{16}$, and $\frac{11}{32}$. Which drill bit is largest?

chapter 3 > Make the Connection



46. **Crafts** Bolts can be purchased with diameters of $\frac{3}{8}, \frac{1}{4}$, or $\frac{3}{16}$ inches (in.). Which is smallest?

chapter 3 > Make the Connection

47. **Construction** Plywood comes in thicknesses of $\frac{5}{8}, \frac{3}{4}, \frac{1}{2}$, and $\frac{3}{8}$ in. Which size is thickest?

chapter 3 > Make the Connection

48. **Construction** Dowels are sold with diameters of $\frac{1}{2}, \frac{9}{16}, \frac{5}{8}$, and $\frac{3}{8}$ in. Which size is smallest?

chapter 3 > Make the Connection

Answers

27.

28.

29.

30.

31.

32.

33.

34.

35.

36.

37.

38.

39.

40.

41.

42.

43.

44.

45.

46.

47.

48.

Answers

49. ____

50. ____

51. ____

52. ____

49. Elian is asked to create a fraction equivalent to $\frac{1}{4}$. His answer is $\frac{4}{7}$. What did he do wrong? What would be a correct answer?

50. A sign on a busy highway says Exit 5A is $\frac{3}{4}$ mile away and Exit 5B is $\frac{5}{8}$ mile away. Which exit is first?

Basic Skills | Challenge Yourself | Calculator/Computer | Career Applications | Above and Beyond

51. **ELECTRONICS** Imagine a guitar string vibrating back and forth. Each back-and-forth vibration is called a *cycle.* The number of times that a string vibrates back and forth per second is called the *frequency.* Accordingly, frequency is measured in cycles per second [CPS or hertz (Hz)]. If you had a camera that could zoom in on a vibrating guitar string, you'd see that it doesn't simply vibrate back and forth but actually vibrates differently in different sections of the string. The loudest vibration is called the fundamental frequency; quieter vibrations are called harmonics, which are multiples of the fundamental frequency.

I. If a note's fundamental frequency is 440 Hz, 1,760 Hz is a harmonic.

II. If a note's fundamental frequency is 440 Hz, 771.76 Hz is a harmonic.

Which of the following conclusions is valid?

(a) I is true and II is true.
(b) I is true and II is false.
(c) I is false and II is true.
(d) I is false and II is false.
(e) None of the above.

52. **ELECTRONICS** Music producers know that, when mixing tracks containing instruments and vocals, it is usually necessary to add some reverb and/or delay (echo) from each track into the mix. For a good mix, every instrument or vocal track usually has different degrees of reverb and delay. Delay, measured in milliseconds (ms), is the time it takes to hear the "echo," which can be quiet or sometimes quite loud. It is usually desirable to have the delay times be "in beat." If we let BPM denote the number of beats per minute the drum machine is set at, then the delay time is a multiple of $\frac{60{,}000}{\text{BPM}}$.

I. If the drum machine is set at 120 BPM, then 1,200 ms will produce an in-beat delay.

II. If the drum machine is set at 120 BPM, then 1,000 ms will produce an in-beat delay.

Which of the following conclusions is valid?

(a) I is true and II is true.
(b) I is true and II is false.
(c) I is false and II is true.
(d) I is false and II is false.
(e) None of the above.

53. **Business and Finance** A company uses two types of boxes, 8 cm and 10 cm long. They are packed in larger cartons to be shipped. What is the shortest length of a container that will accommodate boxes of either size without any room left over? (Each container can contain only boxes of one size—no mixing allowed.)

54. There is an alternate approach to finding the least common multiple of two numbers. The LCM of two numbers can be found by dividing the product of the two numbers by the greatest common factor (GCF) of those two numbers. For example, the GCF of 24 and 36 is 12. If we use the given formula, we obtain

$$\text{LCM of 24 and 36} = \frac{24 \cdot 36}{12} = 72$$

(a) Use the given formula to find the LCM of 150 and 480.

(b) Verify the result by finding the LCM using the method of prime factorization.

55. Complete the crossword puzzle.

Across

2. The LCM of 11 and 13
4. The GCF of 120 and 300
7. The GCF of 13 and 52
8. The GCF of 360 and 540

Down

1. The LCM of 8, 14, and 21
3. The LCM of 16 and 12
5. The LCM of 2, 5, and 13
6. The GCF of 54 and 90

1	■	2	3	
4		■		■
	■	■	■	5
■	6	■	7	
8			■	

Answers

53. ______

54. ______

55. ______

Answers

1. 6 **3.** 12 **5.** 20 **7.** 36 **9.** 48

11. $12 = 2 \times 2 \times 3$; $15 = 3 \times 5$; the LCM is $2 \times 2 \times 3 \times 5 = 60$ **13.** 36

15. 200 **17.** 30 **19.** 252 **21.** 180 **23.** $\frac{12}{17}, \frac{9}{10}$ **25.** $\frac{3}{5}, \frac{5}{8}$

27. $\frac{1}{4}, \frac{1}{3}, \frac{3}{8}$ **29.** $\frac{4}{5}, \frac{5}{6}, \frac{11}{12}$ **31.** $>$ **33.** $>$ **35.** $<$

37. $<$ **39.** 20 **41.** 30 **43.** 33 **45.** $\frac{3}{8}$ **47.** $\frac{3}{4}$ in.

49. He added 3 to both the numerator and denominator; $\frac{1 \times 3}{4 \times 3} = \frac{3}{12}$

51. b **53.** Above and Beyond **55.** Above and Beyond

3.3

Adding and Subtracting Fractions with Unlike Denominators

< **3.3 Objectives** >

1 > Add any two fractions

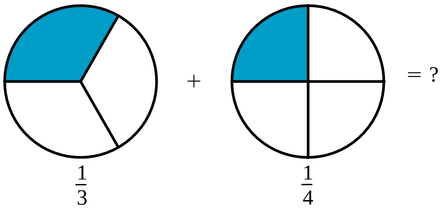
2 > Add any group of fractions

3 > Subtract any two fractions

In Section 3.1, you dealt with like fractions (fractions with a common denominator). What about a sum that deals with **unlike fractions,** such as $\frac{1}{3} + \frac{1}{4}$?

NOTE

Only *like* fractions can be added.

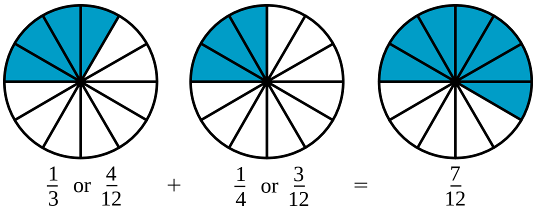


We cannot add unlike fractions because they have different denominators.

To add unlike fractions, write them as equivalent fractions with a common denominator. In this case, let's use 12 as the denominator.

NOTE

We can now add because we have like fractions.



$\frac{1}{3}$ or $\frac{4}{12}$
+
$\frac{1}{4}$ or $\frac{3}{12}$
=
$\frac{7}{12}$

We have chosen 12 because it is a *multiple* of 3 and 4.

$\frac{1}{3}$ is equivalent to $\frac{4}{12}$.

$\frac{1}{4}$ is equivalent to $\frac{3}{12}$.

Any common multiple of the denominators will work in forming equivalent fractions. For instance, we can write $\frac{1}{3}$ as $\frac{8}{24}$ and $\frac{1}{4}$ as $\frac{6}{24}$. Our work is simplest, however, if we use the smallest possible number for the common denominator. This is called the **least common denominator (LCD).**

The LCD is the least common multiple of the denominators of the fractions. This is the *smallest* number that is a multiple of all the denominators. For example, the LCD for $\frac{1}{3}$ and $\frac{1}{4}$ is 12, *not* 24.

Step by Step

To Find the Least Common Denominator

Step 1 Write the prime factorization for each of the denominators.

Step 2 Find all the prime factors that appear in any one of the prime factorizations.

Step 3 Form the product of those prime factors, using each factor the greatest number of times it occurs in any one factorization.

We are now ready to add unlike fractions. In this case, the fractions must be renamed as equivalent fractions that have the same denominator.

Step by Step

To Add Unlike Fractions

Step 1 Find the LCD of the fractions.

Step 2 Change each unlike fraction to an equivalent fraction with the LCD as a common denominator.

Step 3 Add the resulting like fractions as before.

Example 1 shows this process.

Example 1 Adding Unlike Fractions

< Objective 1 >

NOTE

See Section 3.2 to review how we arrived at 24.

Add the fractions $\frac{1}{6}$ and $\frac{3}{8}$.

Step 1 We find that the LCD for fractions with denominators of 6 and 8 is 24.

Step 2 Convert the fractions so that they have the denominator 24.

$$\begin{tikzcd} \frac{1}{6} \arrow[r, bend left, "\times 4"] \arrow[r, bend right, "\times 4"'] \arrow[r, phantom, "="] & \frac{4}{24} \end{tikzcd}$$

How many sixes are in 24? There are 4. So multiply the numerator and denominator by 4.

$$\begin{tikzcd} \frac{3}{8} \arrow[r, bend left, "\times 3"] \arrow[r, bend right, "\times 3"'] \arrow[r, phantom, "="] & \frac{9}{24} \end{tikzcd}$$

How many eights are in 24? There are 3. So multiply the numerator and denominator by 3.

Step 3 We can now add the equivalent like fractions.

$$\frac{1}{6} + \frac{3}{8} = \frac{4}{24} + \frac{9}{24} = \frac{13}{24}$$

Add the numerators and place that sum over the common denominator.

Check Yourself 1

Add.

$$\frac{3}{5} + \frac{1}{3}$$

Here is a similar example. Remember that the sum should always be written in simplest form.

Example 2 Adding Unlike Fractions That Require Simplifying

Add the fractions $\frac{7}{10}$ and $\frac{2}{15}$.

Step 1 The LCD for fractions with denominators of 10 and 15 is 30.

Step 2 $\frac{7}{10} = \frac{21}{30}$

$\frac{2}{15} = \frac{4}{30}$

Do you see how the equivalent fractions are formed?

Step 3 $\frac{7}{10} + \frac{2}{15} = \frac{21}{30} + \frac{4}{30}$

$= \frac{25}{30} = \frac{5}{6}$

Add the resulting like fractions. Be sure the sum is in simplest form.

Check Yourself 2

Add.

$$\frac{1}{6} + \frac{7}{12}$$

We can use the same procedure to add more than two fractions. Example 3 illustrates this approach.

Example 3 Adding a Group of Unlike Fractions

< Objective 2 >

Add $\frac{5}{6} + \frac{2}{9} + \frac{4}{15}$.

Step 1 The LCD is 90.

Step 2

$$\frac{5}{6} = \frac{75}{90}$$

Multiply the numerator and denominator by 15.

$$\frac{2}{9} = \frac{20}{90}$$

Multiply the numerator and denominator by 10.

$$\frac{4}{15} = \frac{24}{90}$$

Multiply the numerator and denominator by 6.

Step 3

$$\frac{75}{90} + \frac{20}{90} + \frac{24}{90} = \frac{119}{90}$$

Now add.

$$= 1\frac{29}{90}$$

Remember, if the sum is an improper fraction, it should be changed to a mixed number.

Check Yourself 3

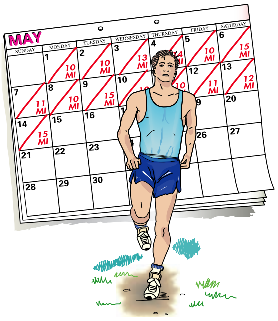
Add.

$$\frac{2}{5} + \frac{3}{8} + \frac{7}{20}$$

Many of the measurements you deal with in everyday life involve fractions. Following are some typical situations.

Example 4 An Application Involving the Addition of Unlike Fractions

Jack ran $\frac{1}{2}$ mi on Monday, $\frac{2}{3}$ mi on Wednesday, and $\frac{3}{4}$ mi on Friday. How far did he run during the week?



The three distances that Jack ran are the given information in the problem. We want to find a total distance, so we must add to find the solution.

$$\frac{1}{2} + \frac{2}{3} + \frac{3}{4} = \frac{6}{12} + \frac{8}{12} + \frac{9}{12}$$

$$= \frac{23}{12} = 1\frac{11}{12} \text{ mi}$$

Because we have no common denominator, we must convert to equivalent fractions before we can add.

Jack ran $1\frac{11}{12}$ mi during the week.

Check Yourself 4

Susan is designing an office complex. She needs $\frac{2}{5}$ acre for buildings, $\frac{1}{3}$ acre for driveways and parking, and $\frac{1}{6}$ acre for walks and landscaping. How much land does she need?

Example 5 An Application Involving the Addition of Unlike Fractions

NOTE

The abbreviation for pounds is "lb" from the Latin *libra,* meaning "balance" or "scales."

Sam bought three packages of spices weighing $\frac{1}{4}$, $\frac{5}{8}$, and $\frac{1}{2}$ pound (lb). What was the total weight?

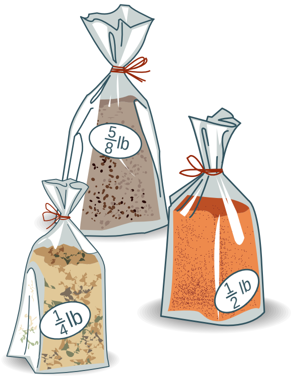
We need to find the total weight, so we must add.

$$\frac{1}{4} + \frac{5}{8} + \frac{1}{2} = \frac{2}{8} + \frac{5}{8} + \frac{4}{8}$$

Write each fraction with the denominator 8.

$$= \frac{11}{8} = 1\frac{3}{8} \text{ lb}$$

The total weight was $1\frac{3}{8}$ lb.



Check Yourself 5

For three different recipes, Max needs $\frac{3}{8}$, $\frac{1}{2}$, and $\frac{3}{4}$ gallon (gal) of tomato sauce. How many gallons should he buy altogether?

To subtract unlike fractions, which are fractions that do not have the same denominator, we use the following process.

Step by Step

To Subtract Unlike Fractions

Step 1 Find the LCD of the fractions.

Step 2 Change each unlike fraction to an equivalent fraction with the LCD as a common denominator.

Step 3 Subtract the resulting like fractions as before.

Example 6 Subtracting Unlike Fractions

< Objective 3 >

NOTE

You can use your calculator to check your result.

Subtract $\frac{5}{8} - \frac{1}{6}$.

Step 1 The LCD is 24.

Step 2 Convert the fractions so that they have the common denominator 24.

$$\frac{5}{8} = \frac{15}{24}$$

$$\frac{1}{6} = \frac{4}{24}$$

The first two steps are exactly the same as if we were adding.

Step 3 Subtract the equivalent like fractions.

$$\frac{5}{8} - \frac{1}{6} = \frac{15}{24} - \frac{4}{24} = \frac{11}{24}$$

> CAUTION

Be Careful! You *cannot* subtract the numerators and subtract the denominators.

$$\frac{5}{8} - \frac{1}{6} \quad \text{is } \textit{not} \quad \frac{4}{2}$$

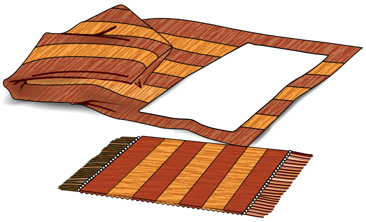
Check Yourself 6

Subtract.

$$\frac{7}{10} - \frac{1}{4}$$

Following is an application that involves subtracting unlike fractions.

Example 7 An Application Involving the Subtraction of Unlike Fractions



You have $\frac{7}{8}$ yards (yd) of a handwoven linen. A pattern for a placemat calls for $\frac{1}{2}$ yd. Will you have enough left for two napkins that will use $\frac{1}{3}$ yd?

First, find out how much fabric is left over after the placemat is made.

$$\frac{7}{8}\text{ yd} - \frac{1}{2}\text{ yd} = \frac{7}{8}\text{ yd} - \frac{4}{8}\text{ yd} = \frac{3}{8}\text{ yd}$$

Now compare the sizes of $\frac{1}{3}$ and $\frac{3}{8}$.

$$\frac{3}{8}\text{ yd} = \frac{9}{24}\text{ yd} \qquad \text{and} \qquad \frac{1}{3}\text{ yd} = \frac{8}{24}\text{ yd}$$

Because $\frac{3}{8}$ yd is *more than* the $\frac{1}{3}$ yd that is needed, there is enough material for the placemat *and* two napkins.

NOTE

Remember that $\frac{3}{8}$ yd is left over and that $\frac{1}{3}$ yd is needed.

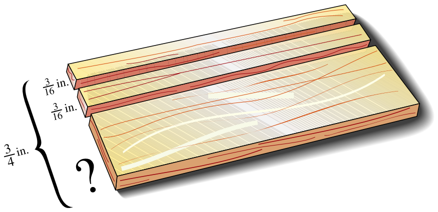
Check Yourself 7

A concrete walk will require $\frac{3}{4}$ cubic yard (yd^3) of concrete. If you have mixed $\frac{8}{9}$ yd^3, will enough concrete remain to do a project that will use $\frac{1}{6}$ yd^3?

Our next application involves measurements. Note that on a ruler or yardstick, the marks divide each inch into $\frac{1}{2}$-in., $\frac{1}{4}$-in., and $\frac{1}{8}$-in. sections, and on some rulers, $\frac{1}{16}$-in. sections. We will use denominators of 2, 4, 8, and 16 in our measurement applications.

Example 8 An Application Involving the Subtraction of Unlike Fractions

Alexei cut two $\frac{3}{16}$-in. slats from a piece of wood that is $\frac{3}{4}$ in. across. How much is left?


$\frac{3}{16}$ in.
$\frac{3}{16}$ in.
$\frac{3}{4}$ in.
?

chapter 3 > Make the Connection

The two $\frac{3}{16}$-in. pieces total

$$2 \times \frac{3}{16} = \frac{6}{16} = \frac{3}{8} \text{ in.}$$

$$\frac{3}{4} = \frac{6}{8}$$

$$\frac{6}{8} - \frac{3}{8} = \frac{3}{8}$$

The remaining strip is $\frac{3}{8}$ in. wide.

Check Yourself 8

Ricardo cut three strips from a 1-in. piece of metal. Each strip has a width of $\frac{3}{16}$ in. How much metal remains after the cuts?

Check Yourself ANSWERS

1. $\frac{14}{15}$ **2.** $\frac{1}{6} + \frac{7}{12} = \frac{2}{12} + \frac{7}{12} = \frac{9}{12} = \frac{3}{4}$ **3.** $1\frac{1}{8}$ **4.** $\frac{9}{10}$ acre

5. $1\frac{5}{8}$ gal **6.** $\frac{9}{20}$

7. $\frac{5}{36}$ yd^3 remains. You do *not* have enough concrete for both projects.

8. $\frac{7}{16}$ in.

Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 3.3

(a) The process for finding the LCD is nearly identical to the Step by Step for finding the ____________.

(b) To add ____________ fractions, we first find the LCD of the fractions.

(c) Two fractions with the same value but different denominators are called ____________ fractions.

(d) When adding fractions with a common denominator, we add the ____________ and put that sum over the common denominator.

3.3 exercises

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Name ______________________

Section __________ Date __________

Answers

1. __________ 2. __________
3. __________ 4. __________
5. __________ 6. __________
7. __________ 8. __________
9. __________ 10. __________
11. __________ 12. __________
13. __________ 14. __________
15. __________ 16. __________
17. __________ 18. __________
19. __________ 20. __________
21. __________ 22. __________
23. __________ 24. __________
25. __________ 26. __________
27. __________ 28. __________
29. __________ 30. __________
31. __________ 32. __________

< **Objective 1** >

Find the least common denominator (LCD) for fractions with the given denominators.

1. 3 and 4
2. 3 and 5
3. 4 and 8
4. 6 and 12
5. 9 and 27
6. 10 and 30
7. 8 and 12
8. 15 and 40
9. 14 and 21 > Videos
10. 15 and 20
11. 48 and 80
12. 60 and 84
13. 3, 4, and 5
14. 3, 4, and 6
15. 8, 10, and 15 > Videos
16. 6, 22, and 33
17. 5, 10, and 25
18. 8, 24, and 48

Add.

19. $\frac{2}{3} + \frac{1}{4}$
20. $\frac{3}{5} + \frac{1}{3}$
21. $\frac{1}{5} + \frac{3}{10}$
22. $\frac{1}{3} + \frac{1}{18}$
23. $\frac{3}{4} + \frac{1}{8}$
24. $\frac{4}{5} + \frac{1}{10}$
25. $\frac{1}{7} + \frac{3}{5}$ > Videos
26. $\frac{1}{6} + \frac{2}{15}$
27. $\frac{3}{7} + \frac{3}{14}$
28. $\frac{7}{20} + \frac{9}{40}$
29. $\frac{7}{15} + \frac{2}{35}$
30. $\frac{3}{10} + \frac{3}{8}$
31. $\frac{5}{8} + \frac{1}{12}$
32. $\frac{5}{12} + \frac{3}{10}$

Complete each statement with either **never, always,** *or* **sometimes.**

33. The sum of two like fractions is ____________ the sum of the numerators over the common denominator.

34. The LCD for two unlike proper fractions is ____________ the same as the GCF of their denominators.

35. The sum of two fractions can ____________ be simplified.

36. The difference of two proper fractions is ____________ less than either of the two fractions.

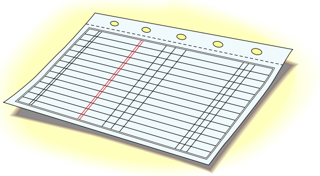
Solve each application.

37. NUMBER PROBLEM Paul bought $\frac{1}{2}$ pound (lb) of peanuts and $\frac{3}{8}$ lb of cashews. How many pounds of nuts did he buy?

38. CONSTRUCTION A countertop consists of a board $\frac{3}{4}$ inch (in.) thick and tile $\frac{3}{8}$ in. thick. What is the overall thickness?

chapter 3 > Make the Connection

39. BUSINESS AND FINANCE Amy budgets $\frac{2}{5}$ of her income for housing and $\frac{1}{6}$ of her income for food. What fraction of her income is budgeted for these two purposes? What fraction of her income remains?

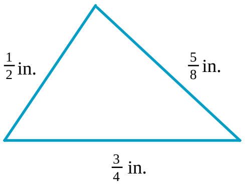


40. SOCIAL SCIENCE A person spends $\frac{3}{8}$ of a day at work and $\frac{1}{3}$ of a day sleeping. What fraction of a day do these two activities use? What fraction of the day remains?

< Objective 2 >

41. NUMBER PROBLEM Jose walked $\frac{3}{4}$ mile (mi) to the store, $\frac{1}{2}$ mi to a friend's house, and then $\frac{2}{3}$ mi home. How far did he walk?

42. GEOMETRY Find the perimeter of, or the distance around, the accompanying figure.



Answers

33. ____________

34. ____________

35. ____________

36. ____________

37. ____________

38. ____________

39. ____________

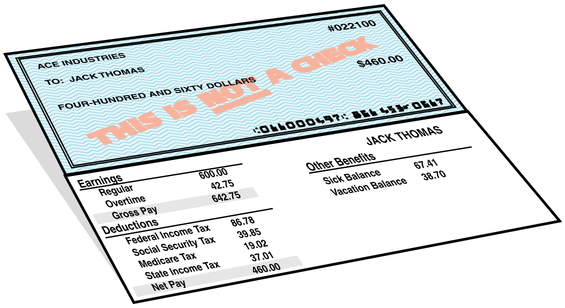
40. ____________

41. ____________

42. ____________

43. **BUSINESS AND FINANCE** A budget guide states that you should spend $\frac{1}{4}$ of your salary for housing, $\frac{3}{16}$ for food, $\frac{1}{16}$ for clothing, and $\frac{1}{8}$ for transportation. What total portion of your salary do these four expenses account for?

44. **BUSINESS AND FINANCE** Deductions from your paycheck are made roughly as follows: $\frac{1}{8}$ for federal tax, $\frac{1}{20}$ for state tax, $\frac{1}{20}$ for social security, and $\frac{1}{40}$ for a savings withholding plan. What portion of your pay is deducted?



ACE INDUSTRIES
TO: JACK THOMAS
FOUR-HUNDRED AND SIXTY DOLLARS
#022100
$460.00
THIS IS NOT A CHECK
JACK THOMAS

Earnings	
Regular	600.00
Overtime	42.75
Gross Pay	642.75
Deductions	
Federal Income Tax	86.78
Social Security Tax	39.85
Medicare Tax	19.02
State Income Tax	37.01
Net Pay	460.00

Other Benefits	
Sick Balance	67.41
Vacation Balance	38.70

45. $\frac{1}{5} + \frac{7}{10} + \frac{4}{15}$

46. $\frac{2}{3} + \frac{1}{4} + \frac{3}{8}$

47. $\frac{1}{9} + \frac{7}{12} + \frac{5}{8}$

48. $\frac{1}{3} + \frac{5}{12} + \frac{4}{5}$

< Objective 3 >

Subtract.

49. $\frac{4}{5} - \frac{1}{3}$

50. $\frac{7}{9} - \frac{1}{6}$

51. $\frac{11}{15} - \frac{3}{5}$ > Videos

52. $\frac{5}{6} - \frac{2}{7}$

53. $\frac{3}{8} - \frac{1}{4}$

54. $\frac{9}{10} - \frac{4}{5}$

55. $\frac{5}{12} - \frac{3}{8}$

56. $\frac{13}{15} - \frac{11}{20}$

Evaluate.

57. $\frac{33}{40} - \frac{7}{24} + \frac{11}{30}$ > Videos

58. $\frac{13}{24} - \frac{5}{16} + \frac{3}{8}$

59. $\frac{15}{16} + \frac{5}{8} - \frac{1}{4}$

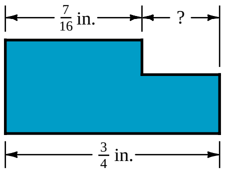
60. $\frac{9}{10} - \frac{1}{5} + \frac{1}{2}$

Answers

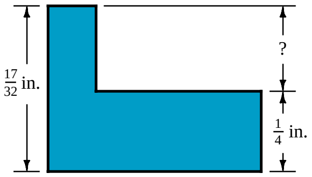
43. ____________
44. ____________
45. ____________
46. ____________
47. ____________
48. ____________
49. ____________
50. ____________
51. ____________
52. ____________
53. ____________
54. ____________
55. ____________
56. ____________
57. ____________
58. ____________
59. ____________
60. ____________

GEOMETRY *For exercises 61 and 62, find the missing dimension (?) in each figure.*

61.



62.



Basic Skills | Challenge Yourself | **Calculator/Computer** | Career Applications | Above and Beyond

Using Your Calculator to Add and Subtract Fractions

Adding or subtracting fractions on the calculator is very much like the multiplication and division you did in Chapter 2. The only thing that changes is the operation. Here's where the fraction calculator is a great tool for checking your work. No muss, no fuss, no searching for a common denominator. Just enter the fractions and get the right answer!

Find each sum or difference using your calculator.

63. $\frac{1}{10} + \frac{7}{12}$

64. $\frac{7}{15} + \frac{17}{24}$

65. $\frac{8}{9} + \frac{6}{7}$

66. $\frac{7}{15} + \frac{2}{5}$

67. $\frac{11}{18} + \frac{5}{12}$

68. $\frac{5}{8} + \frac{4}{9}$

69. $\frac{15}{17} - \frac{9}{11}$

70. $\frac{31}{43} - \frac{18}{53}$

71. $\frac{4}{9} - \frac{2}{5}$

72. $\frac{11}{13} - \frac{2}{3}$

Basic Skills | Challenge Yourself | Calculator/Computer | **Career Applications** | Above and Beyond

73. ALLIED HEALTH Prior to chemotherapy, a patient's malignant tumor weighed $\frac{5}{6}$ pound (lb). After a week of chemotherapy, the tumor's weight had been reduced by $\frac{1}{4}$ lb. How much did the tumor weigh at the end of the week?

74. ALLIED HEALTH Mrs. Lewis has three children who all need to take medication for the same illness. Charlie needs $\frac{3}{2}$ milliliters (mL), Sharon needs $\frac{2}{3}$ mL, and little Kevin needs $\frac{1}{4}$ mL of medication. How many milliliters of medication will Mrs. Lewis need to give each child a single dose of medicine?

Answers

61. ____________________

62. ____________________

63. ____________________

64. ____________________

65. ____________________

66. ____________________

67. ____________________

68. ____________________

69. ____________________

70. ____________________

71. ____________________

72. ____________________

73. ____________________

74. ____________________

Answers

75. ____________________

76. ____________________

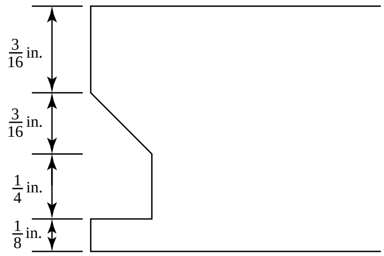
77. ____________________

78. ____________________

79. ____________________

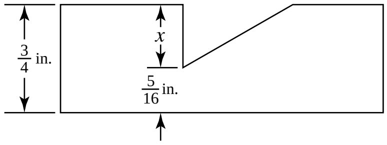
75. **MANUFACTURING TECHNOLOGY** Find the total thickness of this part.

chapter 3 > Make the Connection

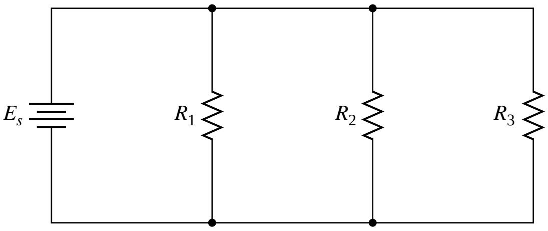


76. **MANUFACTURING TECHNOLOGY** Find the missing dimension (x).

chapter 3 > Make the Connection



77. **ELECTRONICS**



The circuit depicts three resistors (R_1, R_2, R_3) wired in parallel to a source, E_s. The circuit can be simplified by replacing the three separate resistors with a single equivalent resistor, R_{eq}, according to the following equation:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

If $R_1 = 10$ ohms (Ω), $R_2 = 20\ \Omega$, and $R_3 = 40\ \Omega$, what is $\frac{1}{R_{eq}}$?

What is R_{eq}?

78. **INFORMATION TECHNOLOGY** Amin wants to figure out how long it takes to transmit 1,000 bits over two networks that have transmission rates of 50,000 and 10,000 bits per second, respectively. What is the total time after the transmission? Use the fraction of the number of bits to the transmission time to figure out your answer. Represent your answer as a simplified fraction.

Basic Skills | Challenge Yourself | Calculator/Computer | Career Applications | **Above and Beyond**

79. **CONSTRUCTION** A door is $4\frac{1}{4}$ ft wide. Two hooks are to be attached to the door so that they are $1\frac{1}{2}$ in. apart and the same distance from each edge. How far from the edge of the door should each hook be located? Give your answer in feet.

chapter 3 > Make the Connection

80. Complete the following:

$\frac{1}{2} + \frac{1}{4} = $ ________.

$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} = $ ________.

$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} = $ ________.

Based on these results, predict the answer to

$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} = $ ________

Now, do the addition, and check your prediction.

Answers

80. ________________

Answers

1. 12 **3.** 8 **5.** 27

7. $8 = 2 \times 2 \times 2$; $12 = 2 \times 2 \times 3$; the LCD is $2 \times 2 \times 2 \times 3 = 24$ **9.** 42

11. 240 **13.** 60 **15.** 120 **17.** 50 **19.** $\frac{11}{12}$ **21.** $\frac{1}{2}$

23. $\frac{7}{8}$ **25.** $\frac{26}{35}$ **27.** $\frac{9}{14}$ **29.** $\frac{11}{21}$ **31.** $\frac{17}{24}$ **33.** always

35. sometimes **37.** $\frac{7}{8}$ lb **39.** $\frac{17}{30}, \frac{13}{30}$ **41.** $1\frac{11}{12}$ mi **43.** $\frac{5}{8}$

45. $\frac{1}{5} + \frac{7}{10} + \frac{4}{15} = \frac{6}{30} + \frac{21}{30} + \frac{8}{30} = \frac{35}{30} = 1\frac{5}{30} = 1\frac{1}{6}$ **47.** $1\frac{23}{72}$

49. $\frac{4}{5} - \frac{1}{3} = \frac{12}{15} - \frac{5}{15} = \frac{7}{15}$ **51.** $\frac{2}{15}$ **53.** $\frac{1}{8}$ **55.** $\frac{1}{24}$

57. $\frac{33}{40} - \frac{7}{24} + \frac{11}{30} = \frac{99}{120} - \frac{35}{120} + \frac{44}{120} = \frac{108}{120} = \frac{9}{10}$ **59.** $1\frac{5}{16}$

61. $\frac{5}{16}$ in. **63.** $\frac{41}{60}$ **65.** $1\frac{47}{63}$ **67.** $\frac{37}{36} = 1\frac{1}{36}$ **69.** $\frac{12}{187}$

71. $\frac{2}{45}$ **73.** $\frac{7}{12}$ lb **75.** $\frac{3}{4}$ in. **77.** $\frac{7}{40}; \frac{40}{7} = 5\frac{5}{7}$

79. 2 ft $\frac{3}{4}$ in. $= 2\frac{1}{16}$ ft

Activity 7 ::
Kitchen Subflooring

chapter 3
> Make the Connection

Benjamin and Olivia are putting a new floor in their kitchen. To get the floor up to the desired height, they need to add $1\frac{1}{8}$ in. of subfloor. They can do this in one of two ways. They can put $\frac{1}{2}$-in. sheet on top of $\frac{5}{8}$-in. board (note that the total would be $\frac{9}{8}$ in. or $1\frac{1}{8}$ in.). They could also put $\frac{3}{8}$-in. board on top of $\frac{3}{4}$-in. sheet.

The table gives the price for each sheet of plywood from a construction materials store.

Thickness	Cost for a 4 ft × 8 ft Sheet
$\frac{1}{8}$ in.	\$9.15
$\frac{1}{4}$ in.	13.05
$\frac{3}{8}$ in.	14.99
$\frac{1}{2}$ in.	17.88
$\frac{5}{8}$ in.	19.13
$\frac{3}{4}$ in.	21.36
$\frac{7}{8}$ in.	25.23
1 in.	28.49

1. What is the combined price for a $\frac{1}{2}$-in. sheet and a $\frac{5}{8}$-in. sheet?
2. What is the combined price for a $\frac{3}{8}$-in. sheet and a $\frac{3}{4}$-in. sheet?
3. What other combination of sheets of plywood, using two sheets, yields the needed $1\frac{1}{8}$-in. thickness?
4. Of the four combinations, which is most economical?
5. The kitchen is to be 12 ft × 12 ft. Find the total cost of the plywood you have suggested in question 4.

3.4 Adding and Subtracting Mixed Numbers

< **3.4 Objectives** >

1 > Add any two mixed numbers
2 > Add any group of mixed numbers
3 > Subtract any two mixed numbers
4 > Solve an application that involves addition or subtraction of mixed numbers

Once you know how to add fractions, adding mixed numbers is straightforward. Keep in mind that addition involves combining groups of the *same kind* of objects. Because mixed numbers consist of two parts—a whole number and a fraction—we can work with the whole numbers and the fractions separately.

Step by Step

To Add Mixed Numbers

Step 1 Find the LCD.
Step 2 Rewrite the fraction parts to match the LCD.
Step 3 Add the fraction parts.
Step 4 Add the whole number parts.
Step 5 Simplify if necessary.

Example 1 illustrates the use of this rule.

Example 1 Adding Mixed Numbers

< **Objective 1** >

Add $3\frac{1}{5} + 4\frac{2}{5}$.

$$\begin{array}{r} 3\frac{1}{5} \\ +\ 4\frac{2}{5} \\ \hline 7\frac{3}{5} \end{array}$$

$$3\frac{1}{5} + 4\frac{2}{5} = 7\frac{3}{5}$$

Check Yourself 1

Add $2\frac{3}{10} + 3\frac{4}{10}$.

When the fractional portions of the mixed numbers have different denominators, we must rename these fractions as equivalent fractions with the least common denominator to perform the addition in step 2. Consider Example 2.

Example 2 Adding Mixed Numbers with Different Denominators

Add.

$$\begin{array}{rcl} 3\frac{1}{6} & = & 3\frac{4}{24} \\ +\ 2\frac{3}{8} & = & 2\frac{9}{24} \\ \hline & & 5\frac{13}{24} \end{array}$$

The LCD of the fractions is 24. Rename them with that denominator.

Check Yourself 2

Add $5\frac{1}{10} + 3\frac{1}{6}$.

You follow the same procedure if more than two mixed numbers are involved in the problem.

Example 3 Adding Mixed Numbers with Different Denominators

< Objective 2 >

NOTE

The LCD of the three fractions is 40. Convert to equivalent fractions.

Add.

$$\begin{array}{rcl} 2\frac{1}{5} & = & 2\frac{8}{40} \\ 3\frac{3}{4} & = & 3\frac{30}{40} \\ +\ 4\frac{1}{8} & = & 4\frac{5}{40} \\ \hline & & 9\frac{43}{40} \end{array}$$

$$9\frac{43}{40} = 9 + \frac{43}{40} = 9 + 1 + \frac{3}{40} = 10\frac{3}{40}$$

Check Yourself 3

Add $5\frac{1}{2} + 4\frac{2}{3} + 3\frac{3}{4}$.

We can use a similar technique for *subtracting* mixed numbers. The process is like adding mixed numbers.

Step by Step

To Subtract Mixed Numbers

Step 1 Find the LCD.
Step 2 Rewrite the fraction parts to match the LCD.
Step 3 Subtract the fraction parts.
Step 4 Subtract the whole number parts.
Step 5 Simplify if necessary.

Example 4 illustrates the use of this process.

Example 4 Subtracting Mixed Numbers with Like Denominators

< Objective 3 >

Subtract.

$$\begin{array}{r} 5\frac{7}{12} \\ -\ 3\frac{5}{12} \\ \hline 2\frac{2}{12} \end{array}$$

$$2\frac{2}{12} = 2\frac{1}{6}$$

Check Yourself 4

Subtract $8\frac{7}{8} - 5\frac{3}{8}$.

Again, we must rename the fractions if different denominators are involved. This approach is shown in Example 5.

Example 5 Subtracting Mixed Numbers with Different Denominators

Subtract. $8\frac{7}{10} - 3\frac{3}{8}$.

$$\begin{array}{rcr} 8\frac{7}{10} & = & 8\frac{28}{40} \\ -\ 3\frac{3}{8} & = & 3\frac{15}{40} \\ \hline & & 5\frac{13}{40} \end{array}$$

Write the fractions with denominator 40.

Subtract as before.

Check Yourself 5

Subtract $7\frac{11}{12} - 3\frac{5}{8}$.

To subtract a mixed number from a whole number, we must use a form of regrouping, or borrowing.

Example 6 Subtracting Mixed Numbers by Borrowing

NOTE

$6 = 5 + 1$

$= 5 + \frac{4}{4}$

Subtract $6 - 2\frac{3}{4}$.

$$\begin{array}{rcr} 6 & = & 5\frac{4}{4} \\ -\ 2\frac{3}{4} & = & 2\frac{3}{4} \\ \hline & & 3\frac{1}{4} \end{array}$$

There is no fraction from which we can subtract the $\frac{3}{4}$. We regroup so that we can borrow 1 from the 6 and write it as $\frac{4}{4}$.

Check Yourself 6

Subtract.

(a) $7 - 3\frac{2}{5}$ **(b)** $9 - 2\frac{5}{7}$

A similar technique is used whenever the fraction of the minuend is smaller than the fraction of the subtrahend.

Example 7 Subtracting Mixed Numbers by Borrowing

Subtract.

$$\begin{array}{rcl} 5\frac{3}{8} & = & 5\frac{3}{8} \\ -\,3\frac{3}{4} & = & 3\frac{6}{8} \\ \hline \end{array}$$

Because you cannot subtract $\frac{6}{8}$ from $\frac{3}{8}$, borrow 1 from the 5 in the minuend.

$$\begin{array}{rcccl} 5\frac{3}{8} & = & 5\frac{3}{8} & = & 4\frac{11}{8} \\ -\,3\frac{3}{4} & = & 3\frac{6}{8} & = & 3\frac{6}{8} \\ \hline & & & & 1\frac{5}{8} \end{array}$$

Because $5 = 4 + 1$, rewrite the 1 as $\frac{8}{8}$ and add it to $\frac{3}{8}$ to get $\frac{11}{8}$.

Check Yourself 7

Subtract.

(a) $12\frac{5}{12} - 8\frac{2}{3}$ **(b)** $27\frac{7}{8} - 2\frac{9}{10}$

There is an alternative method that can be used when subtracting fractions. We can convert each mixed number into an improper fraction and then perform the subtraction. Although this technique requires an extra step, it eliminates the need to regroup.

Example 8 Subtracting by Converting to Improper Fractions

Subtract.

$$4\frac{5}{9} - 2\frac{11}{12}$$

First, convert to improper fractions.

$$\frac{41}{9} - \frac{35}{12}$$

The LCD is 36. Find the equivalent fractions and subtract.

$$\frac{164}{36} - \frac{105}{36} = \frac{59}{36}$$

Rewrite as a mixed number.

$$\frac{59}{36} = 1\frac{23}{36}$$

Check Yourself 8

Subtract $5\frac{1}{4} - 2\frac{5}{18}$.

Example 9 An Application of the Subtraction of Mixed Numbers

< Objective 4 >

Linda was $48\frac{1}{4}$ inches (in.) tall on her sixth birthday. By her seventh year she was $51\frac{5}{8}$ in. tall. How much did she grow during the year?

Because we want the difference in height, we must subtract $48\frac{1}{4}$ from $51\frac{5}{8}$.

$$\begin{array}{rcr} 51\frac{5}{8} & = & 51\frac{5}{8} \\ -\ 48\frac{1}{4} & = & 48\frac{2}{8} \\ \hline & & 3\frac{3}{8} \end{array}$$

Linda grew $3\frac{3}{8}$ in. during the year.

Check Yourself 9

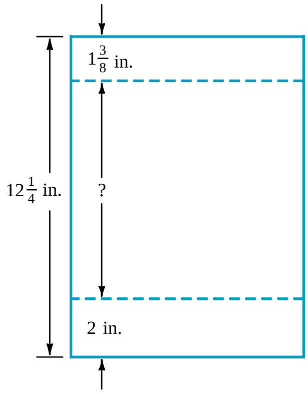
You use $4\frac{3}{4}$ yards (yd) of fabric from a 50-yd bolt. How much fabric remains on the bolt?

Often we will have to use more than one operation to find the solution to a problem. Consider Example 10.

Example 10 An Application Involving Mixed Numbers

A rectangular poster is to have a total length of $12\frac{1}{4}$ in. We want a $1\frac{3}{8}$-in. border on the top and a 2-in. border on the bottom. What is the length of the printed part of the poster?

First, we draw a sketch of the poster:



Now, we use this sketch to find the total width of the top and bottom borders.

$$\begin{array}{r} 1\frac{3}{8} \\ +\ 2 \\ \hline 3\frac{3}{8} \end{array}$$

Now *subtract* that sum (the top and bottom borders) from the total length of the poster.

$$\begin{array}{rcrcr} 12\frac{1}{4} & = & 12\frac{2}{8} & = & 11\frac{10}{8} \\ -\ \ 3\frac{3}{8} & = & 3\frac{3}{8} & = & 3\frac{3}{8} \\ \hline & & & & 8\frac{7}{8} \end{array}$$

The length of the printed part is $8\frac{7}{8}$ in.

Check Yourself 10

You cut one shelf $3\frac{3}{4}$ feet (ft) long and one $4\frac{1}{2}$ ft long from a 12-ft piece of lumber. Can you cut another shelf 4 ft long?

Check Yourself ANSWERS

1. $5\frac{7}{10}$ **2.** $8\frac{4}{15}$ **3.** $13\frac{11}{12}$ **4.** $3\frac{1}{2}$

5. $7\frac{11}{12} - 3\frac{5}{8} = 7\frac{22}{24} - 3\frac{15}{24} = 4\frac{7}{24}$ **6.** **(a)** $3\frac{3}{5}$; **(b)** $6\frac{2}{7}$

7. **(a)** $3\frac{3}{4}$; **(b)** $24\frac{39}{40}$ **8.** $2\frac{35}{36}$ **9.** $45\frac{1}{4}$ yd **10.** No, only $3\frac{3}{4}$ ft remains.

Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 3.4

(a) To add mixed numbers, we first find the ______________ of the fractions.

(b) To subtract a mixed number from a whole number, we must use a form of ______________, or borrowing.

(c) When the fractional portions of mixed numbers have different denominators, we rename the fractions as ______________ fractions.

(d) We will always need to borrow when subtracting mixed numbers if the minuend is ______________ than the subtrahend.

3.4 exercises

< Objective 1 >

Perform the indicated operations.

1. $2\frac{2}{9} + 3\frac{5}{9}$

2. $5\frac{2}{9} + 6\frac{4}{9}$

3. $2\frac{1}{9} + 5\frac{5}{9}$

4. $1\frac{1}{6} + 5\frac{5}{6}$

5. $6\frac{5}{9} + 4\frac{7}{9}$ > Videos

6. $5\frac{8}{9} + 4\frac{4}{9}$

7. $1\frac{1}{3} + 2\frac{1}{5}$

8. $2\frac{1}{4} + 1\frac{1}{6}$

< Objective 2 >

9. $2\frac{1}{4} + 3\frac{5}{8} + 1\frac{1}{6}$

10. $3\frac{1}{5} + 2\frac{1}{2} + 5\frac{1}{4}$

11. $3\frac{3}{5} + 4\frac{1}{4} + 5\frac{3}{10}$ > Videos

12. $4\frac{5}{6} + 3\frac{2}{3} + 7\frac{5}{9}$

< Objective 3 >

13. $7\frac{7}{8} - 3\frac{3}{8}$

14. $3\frac{5}{6} - 1\frac{1}{6}$

15. $3\frac{2}{5} - 1\frac{4}{5}$

16. $5\frac{3}{7} - 2\frac{5}{7}$

17. $3\frac{2}{3} - 2\frac{1}{4}$ > Videos

18. $5\frac{4}{5} - 2\frac{5}{6}$

19. $7\frac{5}{12} - 3\frac{11}{18}$ > Videos

20. $9\frac{3}{7} - 2\frac{13}{21}$

21. $5 - 2\frac{1}{4}$

22. $4 - 1\frac{2}{3}$

23. $17 - 8\frac{3}{4}$

24. $23 - 11\frac{5}{8}$

25. $3\frac{3}{4} + 5\frac{1}{2} - 2\frac{3}{8}$

26. $1\frac{5}{6} + 3\frac{5}{12} - 2\frac{1}{4}$

27. $2\frac{3}{8} + 2\frac{1}{4} - 1\frac{5}{6}$

28. $1\frac{1}{15} + 3\frac{3}{10} - 2\frac{4}{5}$

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Name ____________________

Section ________ Date ________

Answers

1. ________ **2.** ________

3. ________ **4.** ________

5. ________ **6.** ________

7. ________ **8.** ________

9. ________ **10.** ________

11. ________ **12.** ________

13. ________ **14.** ________

15. ________ **16.** ________

17. ________ **18.** ________

19. ________ **20.** ________

21. ________ **22.** ________

23. ________ **24.** ________

25. ________ **26.** ________

27. ________ **28.** ________

Answers

29. ____________

30. ____________

31. ____________

32. ____________

33. ____________

34. ____________

35. ____________

36. ____________

37. ____________

38. ____________

39. ____________

For exercises 29 to 32, label each statement as **true** *or* **false.**

29. The LCM of 3, 6, and 12 is 24.

30. The GCF of 15, 21, and 300 is 3.

31. The LCD for $\frac{3}{4}$, $\frac{9}{10}$, and $\frac{3}{20}$ is 40.

32. The LCD for $\frac{5}{6}$, $\frac{13}{15}$, and $\frac{1}{21}$ is 3.

< Objective 4 >

Solve each application.

33. Crafts Senta is working on a project that uses three pieces of fabric with lengths of $\frac{3}{4}$, $1\frac{1}{4}$, and $\frac{5}{8}$ yd. She needs to allow for $\frac{1}{8}$ yd of waste. How much fabric should she buy?

34. Construction The framework of a wall is $3\frac{1}{2}$ in. thick. We apply $\frac{5}{8}$-in. wallboard and $\frac{1}{4}$-in. paneling to the inside. Siding that is $\frac{3}{4}$ in. thick is applied to the outside. What is the finished thickness of the wall?

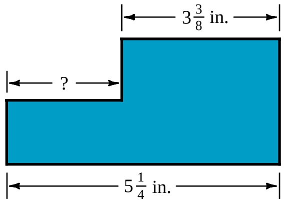
chapter 3 > Make the Connection

35. Business and Finance A stock was listed at $34\frac{3}{8}$ points on Monday. By closing time Friday, it was at $28\frac{3}{4}$. How much did it drop during the week?

36. Crafts A roast weighed $4\frac{1}{4}$ lb before cooking and $3\frac{3}{8}$ lb after cooking. How many pounds were lost during the cooking?

37. Crafts A roll of paper contains $30\frac{1}{4}$ yd. If $16\frac{7}{8}$ yd is cut from the roll, how much paper remains?

38. Geometry Find the missing dimension in the given figure.

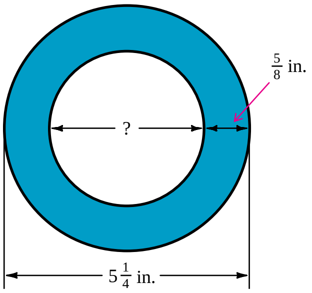


39. Crafts A $4\frac{1}{4}$-in. bolt is placed through a board that is $3\frac{1}{2}$ in. thick. How far does the bolt extend beyond the board?

chapter 3 > Make the Connection

40. Business and Finance Ben can work 20 h per week on a part-time job. He works $5\frac{1}{2}$ h on Monday and $3\frac{3}{4}$ h on Tuesday. How many more hours can he work during the week?

41. Geometry Find the missing dimension in the given figure.



42. Construction The Whites used $20\frac{3}{4}$ square yards (yd^2) of carpet for their living room, $15\frac{1}{2}$ yd^2 for the dining room, and $6\frac{1}{4}$ yd^2 for a hallway. How much remains if a 50-yd^2 roll of carpet is used?

Evaluate.

43. $4\frac{1}{8} + \frac{3}{7} - 2\frac{23}{28}$

44. $5\frac{1}{3} + 1\frac{3}{7} - 5\frac{23}{42}$

45. $6\frac{1}{11} + \frac{2}{3} - 2\frac{1}{6}$

46. $3\frac{1}{5} + 1\frac{7}{8} - 5\frac{1}{20}$

47. $6\frac{1}{11} - \frac{2}{3} + 2\frac{1}{6}$

48. $3\frac{1}{5} - 1\frac{7}{8} + 5\frac{1}{20}$

Basic Skills | Challenge Yourself | Calculator/Computer | Career Applications | Above and Beyond

Using Your Calculator to Add and Subtract Mixed Numbers

We have already seen how to add, subtract, multiply, and divide fractions using our calculators. Now we will use our calculators to add and subtract mixed numbers.

Scientific Calculator

To enter a mixed number on a scientific calculator, press the fraction key between both the whole number and the numerator and denominator. For example, to enter $3\frac{7}{12}$, press

3 $\boxed{\text{a b/c}}$ 7 $\boxed{\text{a b/c}}$ 12

Graphing Calculator

As with multiplying and dividing fractions, when using a graphing calculator, you must choose the fraction option from the math menu before pressing $\boxed{\text{Enter}}$.

Answers

40. ______
41. ______
42. ______
43. ______
44. ______
45. ______
46. ______
47. ______
48. ______

Answers

49. ______

50. ______

51. ______

52. ______

53. ______

54. ______

55. ______

56. ______

57. ______

58. ______

59. ______

For the problem $3\frac{7}{12} + 2\frac{11}{16}$, the keystroke sequence is

$\boxed{(}\ 3\ \boxed{+}\ 7\ \boxed{\div}\ 12\ \boxed{)}\ \boxed{+}\ \boxed{(}\ 2\ \boxed{+}\ 11\ \boxed{\div}\ 16\ \boxed{)}$ **▸Frac** Enter

Note that the parentheses are very important when doing subtraction. The display will read $\frac{301}{48}$. This is equivalent to $6\frac{13}{48}$.

Add or subtract.

49. $4\frac{7}{9} - 2\frac{11}{18}$

50. $7\frac{8}{11} - 4\frac{13}{22}$

51. $5\frac{11}{16} - 2\frac{5}{12}$

52. $18\frac{5}{24} - 11\frac{3}{40}$

53. $6\frac{2}{3} - 1\frac{5}{6}$

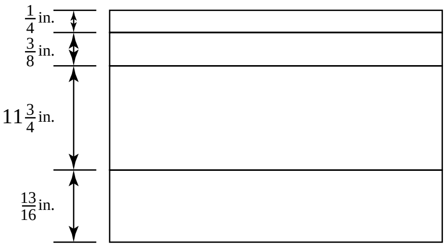
54. $131\frac{43}{45} - 99\frac{27}{60}$

55. $10\frac{2}{3} + 4\frac{1}{5} + 7\frac{2}{15}$

56. $7\frac{1}{5} + 3\frac{2}{3} + 1\frac{1}{5}$

Basic Skills | Challenge Yourself | Calculator/Computer | **Career Applications** | Above and Beyond

57. Manufacturing Technology A factory floor is made up of several layers shown in the drawing.



What is the total thickness of the floor?

chapter 3 > Make the Connection

58. Manufacturing Technology Pieces that are $2\frac{1}{4}$, $5\frac{1}{16}$, $4\frac{3}{4}$, and $1\frac{7}{8}$ in. long need to be cut from round stock. How long of a piece of round stock is required? (Allow $\frac{3}{32}$ in. for each saw kerf.)

chapter 3 > Make the Connection

59. Information Technology Joy is running cable in a new office building in downtown Washington, D.C. She has $60\frac{1}{4}$ ft of cable but she needs 100 ft. How much more cable is needed?

60. **INFORMATION TECHNOLOGY** Abraham has a part-time technician job while he is going to college. He works the following hours in one week: $3\frac{1}{4}$, $5\frac{3}{4}$, $4\frac{1}{2}$, 4, and $2\frac{1}{4}$. For the week, how many hours did Abraham work?

Answers

60. ____________________

Answers

1. $5\frac{7}{9}$ **3.** $7\frac{2}{3}$ **5.** $11\frac{1}{3}$ **7.** $3\frac{8}{15}$ **9.** $7\frac{1}{24}$ **11.** $13\frac{3}{20}$

13. $4\frac{1}{2}$ **15.** $1\frac{3}{5}$ **17.** $1\frac{5}{12}$ **19.** $3\frac{29}{36}$ **21.** $2\frac{3}{4}$ **23.** $8\frac{1}{4}$

25. $6\frac{7}{8}$ **27.** $2\frac{19}{24}$ **29.** False **31.** False **33.** $2\frac{3}{4}$ yd

35. $5\frac{5}{8}$ points **37.** $13\frac{3}{8}$ yd **39.** $\frac{3}{4}$ in. **41.** 4 in. **43.** $1\frac{41}{56}$

45. $4\frac{13}{22}$ **47.** $7\frac{13}{22}$ **49.** $2\frac{1}{6}$ **51.** $3\frac{13}{48}$ **53.** $4\frac{5}{6}$ **55.** 22

57. $13\frac{3}{16}$ in. **59.** $39\frac{3}{4}$ ft

Activity 8 ::
Sharing Costs

The Associated Student Government at CCC is sending six students to the national conference in Washington, D.C. Two of the students, Mikaila and Courtney, are in charge of making the room reservations. Looking at the hotels that are either hosting or adjacent to the conference site, they come up with the following information. Accommodations were not available for places on the table that are blank.

Hotel	Price for a Single	Price for a Double	Price for a Triple	Suite (Sleeps 6)
Wyndham	$180	$180	$210	
St. Gregory	168	198	240	$450
Hyatt	190	190	222	
Marriott	159	174		

1. If they get three double rooms at the Marriott and each pays $\frac{1}{6}$ of the bill, what is the cost per person each night?
2. If they stay at the Wyndham in a triple room, they need only two rooms. If they each pay $\frac{1}{6}$ of that total bill, what is the cost per night?
3. If they get the suite at the St. Gregory, what is the per-person cost per night?
4. The Hyatt and the St. Gregory each offer a free breakfast for each person staying there. Is it now cheaper to stay at the Hyatt in a triple room than at the Wyndham? What information would you need to make the decision?
5. What about the St. Gregory suite with a free breakfast? How do you now make the decision?

3.5 Order of Operations with Fractions

< **3.5 Objectives** >

1 > Evaluate an expression with grouping symbols

2 > Solve an application that involves evaluating an expression

In Chapter 1, we introduced the order of operations. We repeat them here with a small addition to step 1.

Step by Step

Evaluating an Expression

Step 1 Do any operations within parentheses or other grouping symbols.

Step 2 Apply any exponents.

Step 3 Do all multiplication and division in order from left to right.

Step 4 Do all addition and subtraction in order from left to right.

The following examples demonstrate the use of the order of operations in evaluating expressions involving fractions. Refer to the steps as we evaluate each expression.

Example 1 Evaluating an Expression

Evaluate.

$$\frac{14}{15} - \left(\frac{1}{2}\right)^2 \cdot \left(\frac{2}{3} + \frac{4}{5}\right)$$

Perform operations in parentheses.

$$= \frac{14}{15} - \left(\frac{1}{2}\right)^2 \cdot \left(\frac{22}{15}\right)$$

The next step is to apply the exponents.

$$= \frac{14}{15} - \left(\frac{1}{4}\right) \cdot \left(\frac{22}{15}\right)$$

The next step is to perform the multiplication.

$$= \frac{14}{15} - \frac{11}{30}$$

The final step is to perform addition and subtraction.

$$= \frac{17}{30}$$

Check Yourself 1

Evaluate.

$$\left(\frac{2}{3}\right)^3 + \left(\frac{1}{3}\right)^2 \cdot \left(\frac{1}{2} + \frac{2}{3}\right)$$

Recall that, once parentheses are removed, multiplication and division are always performed left to right. Example 2 illustrates.

Example 2 Evaluating an Expression

Evaluate.

$$\left(\frac{1}{13}\right)^2 \cdot \left(\frac{1}{4} + \frac{1}{6}\right) \div \frac{5}{13}$$ Add the expression inside the parentheses.

$$= \left(\frac{1}{13}\right)^2 \cdot \left(\frac{5}{12}\right) \div \frac{5}{13}$$ Apply the exponent.

$$= \frac{1}{169} \cdot \frac{5}{12} \div \frac{5}{13}$$ Invert and multiply.

$$= \frac{1}{\underset{13}{\cancel{169}}} \cdot \frac{\overset{1}{\cancel{5}}}{12} \cdot \frac{\overset{1}{\cancel{13}}}{\underset{1}{\cancel{5}}} = \frac{1}{156}$$

Check Yourself 2

Evaluate.

$$\left(\frac{1}{5}\right)^2 \cdot \left(\frac{3}{10} + \frac{1}{2}\right) \div \frac{2}{5}$$

When mixed numbers are involved in a complex expression, it is almost always best to convert them to improper fractions before continuing.

Example 3 Evaluating an Expression with Mixed Numbers

Evaluate.

$$3\frac{3}{4} + 5 \cdot \left(2\frac{1}{2}\right)$$ Rewrite the mixed numbers as improper fractions.

$$= \frac{15}{4} + 5 \cdot \left(\frac{5}{2}\right)$$ Multiplication precedes addition in the order of operations.

$$= \frac{15}{4} + \frac{25}{2}$$ To add, use the common denominator of 4.

$$= \frac{15}{4} + \frac{50}{4} = \frac{65}{4} = 16\frac{1}{4}$$

Check Yourself 3

Evaluate.

$$4\frac{1}{3} - \frac{1}{2} \cdot \left(5\frac{1}{8}\right)$$

Example 4 illustrates an application of the material in this section.

Example 4 Solving an Application

One formula for calculating children's dosages based on the recommended adult dosage and the child's age in years is Young's rule:

$$\text{Child's dose} = \left(\frac{\text{age}}{\text{age} + 12}\right) \times \text{adult dose}$$

According to this rule, the dose prescribed to a 3-year-old child if the recommended adult dose is 24 milligrams (mg) can be found by evaluating the expression

$$\left(\frac{3}{3 + 12}\right) \times 24 \text{ mg}$$

To evaluate this expression, first we do the operations inside the parentheses.

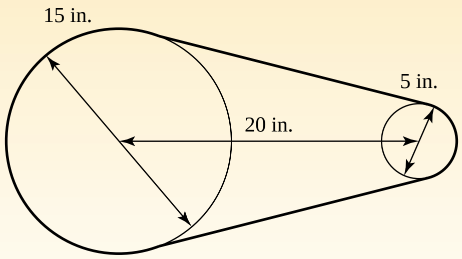
$$\left(\frac{3}{3 + 12}\right) \times 24 \text{ mg}$$

$$= \left(\frac{3}{15}\right) \times 24 \text{ mg}$$

$$= \frac{1}{5} \times \frac{24}{1} \text{ mg} = \frac{24}{5} \text{ mg} = 4\frac{4}{5} \text{ mg}$$

Check Yourself 4

The approximate length of the belt pictured is given by



$$\frac{22}{7}\left(\frac{1}{2} \cdot 15 + \frac{1}{2} \cdot 5\right) + 2 \cdot 21$$

Find the length of the belt.

Check Yourself ANSWERS

1. $\frac{23}{54}$ **2.** $\frac{2}{25}$ **3.** $1\frac{37}{48}$ **4.** $73\frac{3}{7}$ in.

Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 3.5

(a) In evaluating an expression, first do operations inside parentheses or other ______________ symbols.

(b) The second step in evaluating an expression is to evaluate all ______________.

(c) When dividing by a fraction, ______________ and multiply by that fraction.

(d) When mixed numbers are involved in a complex expression, it is almost always best to convert them to __________ fractions before continuing.

3.5 exercises

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Name ____________

Section ________ Date ________

Answers

1. ________ 2. ________

3. ________ 4. ________

5. ________ 6. ________

7. ________ 8. ________

9. ________ 10. ________

11. ________ 12. ________

13. ________ 14. ________

15. ________ 16. ________

17. ________

18. ________

19. ________

20. ________

21. ________

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Evaluate.

1. $\frac{1}{3} - \left(\frac{1}{2} - \frac{1}{4}\right)$ > Videos

2. $\frac{2}{3} - \left(\frac{3}{4} - \frac{1}{2}\right)$

3. $\frac{3}{4} - \left(\frac{1}{2}\right)\left(\frac{1}{3}\right)$ > Videos

4. $\frac{5}{6} - \left(\frac{3}{4}\right)\left(\frac{2}{3}\right)$

5. $\left(\frac{1}{2}\right)^2 - \left(\frac{1}{4} - \frac{1}{5}\right)$

6. $\left(\frac{3}{4}\right)^2 - \left(\frac{1}{2} - \frac{1}{3}\right)$

7. $\frac{1}{2} + \frac{2}{3} \cdot \frac{9}{16}$

8. $\frac{2}{3} + \frac{1}{2} \div \frac{3}{4}$

9. $\left(\frac{1}{3}\right)\left(\frac{1}{2} + \frac{1}{4}\right)$

10. $\left(\frac{3}{4}\right)\left(\frac{1}{2} - \frac{1}{3}\right)$

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11. $\left(\frac{1}{2}\right)^3 + \left(\frac{1}{3}\right) \cdot \left(\frac{1}{2} + \frac{1}{4}\right)$ > Videos

12. $\left(\frac{1}{3}\right) + \left(\frac{1}{2}\right)^2 \cdot \left(\frac{1}{3} + \frac{1}{6}\right)$

13. $\left(\frac{3}{4}\right)^2 + \left(\frac{1}{2}\right)^3 \cdot \left(\frac{1}{2} + \frac{3}{4}\right)$

14. $\left(\frac{1}{10}\right) + \left(\frac{1}{2}\right)^3 \cdot \left(\frac{1}{5} + \frac{1}{15}\right)$

15. $\left(\frac{2}{3}\right)^2 \cdot \left(\frac{2}{15} + \frac{1}{2}\right) \div \frac{3}{5}$

16. $\left(\frac{1}{3}\right)^3 \cdot \left(\frac{1}{4} + \frac{1}{2}\right) \div \frac{2}{9}$

17. $2\frac{1}{5} - \frac{1}{2} \cdot \frac{1}{4}$

18. $4\frac{3}{8} - \frac{1}{2} \cdot \frac{1}{5}$

19. $11\frac{1}{10} - \frac{1}{2} \cdot \left(6\frac{2}{3}\right)$

20. $12\frac{2}{7} - \frac{1}{3} \cdot \left(3\frac{3}{7}\right)$

21. **CONSTRUCTION** A construction company has bids for paving roads of $1\frac{1}{2}$, $\frac{3}{4}$, and $3\frac{1}{3}$ miles (mi) for the month of July. With their present equipment, they can pave 8 mi in 1 month. How much more work can they take on in July?

22. **Statistics** On an 8-h trip, Jack drives $2\frac{3}{4}$ h and Pat drives $2\frac{1}{2}$ h. How many hours are left to drive?

23. **Statistics** A runner has told herself that she will run 20 mi each week. She runs $5\frac{1}{2}$ mi on Sunday, $4\frac{1}{4}$ mi on Tuesday, $4\frac{3}{4}$ mi on Wednesday, and $2\frac{1}{8}$ mi on Friday. How far must she run on Saturday to meet her goal?

24. **Science and Medicine** If paper takes up $\frac{1}{2}$ of the space in a landfill and plastic takes up $\frac{1}{10}$ of the space, how much of the landfill is used for other materials?

25. **Science and Medicine** If paper takes up $\frac{1}{2}$ of the space in a landfill and organic waste takes up $\frac{1}{8}$ of the space, how much of the landfill is used for other materials?

26. **Business and Finance** The interest rate on an auto loan in May was $12\frac{3}{8}$%. By September the rate was up to $14\frac{1}{4}$%. By how many percentage points did the interest rate increase over the period?

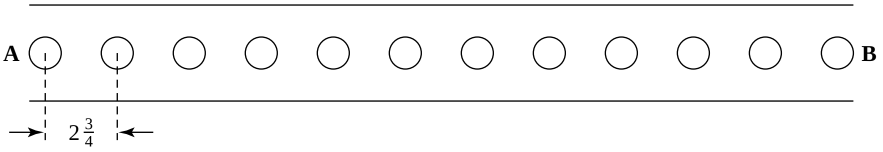
Basic Skills | Challenge Yourself | Calculator/Computer | **Career Applications** | Above and Beyond

27. **Allied Health** Simone suffers from Gaucher's disease. The recommended dosage of Cerezyme is $2\frac{1}{2}$ units per kilogram (kg) of the patient's weight. How much Cerezyme should the doctor prescribe if Simone weighs $15\frac{1}{3}$ kg?

28. **Information Technology** Kendra is running cable in a new office building in downtown Kansas City. She has $110\frac{1}{4}$ ft of cable, but she needs 150 ft. How much more cable is needed?

29. **Manufacturing Technology** A cut $3\frac{3}{8}$ in. long needs to be made in a piece of material. The cut rate is $\frac{3}{4}$ in. per minute. How many minutes does it take to make the cut?

30. **Manufacturing Technology** Calculate the distance from the center of hole A to the center of hole B.



Answers

22. ______

23. ______

24. ______

25. ______

26. ______

27. ______

28. ______

29. ______

30. ______

Answers

31. ____________________

32. ____________________

31. **INFORMATION TECHNOLOGY** On average, $18\frac{1}{4}$ printed circuit boards can be completely populated with components (with all parts soldered to the board) by Amara in 1 h. Burt can complete $34\frac{2}{3}$ printed circuit boards in 2 h. If both workers continue at their respective average paces, how many total printed circuit boards can be populated in 8 h?

32. **INFORMATION TECHNOLOGY** If Carlos joins Amara and Burt in soldering components on circuit boards, the three workers can average 410 complete boards in 8 h. Assuming the other two workers perform at their respective averages stated in exercise 31, how many boards can Carlos average in 8 h? How many boards does Carlos average in 1 h?

Answers

1. $\frac{1}{12}$
3. $\frac{7}{12}$
5. $\frac{1}{5}$
7. $\frac{7}{8}$
9. $\frac{1}{4}$
11. $\frac{3}{8}$
13. $\frac{23}{32}$
15. $\frac{38}{81}$
17. $2\frac{3}{40}$
19. $7\frac{23}{30}$
21. $2\frac{5}{12}$ mi
23. $3\frac{3}{8}$ mi
25. $\frac{3}{8}$
27. $38\frac{1}{3}$ units
29. $4\frac{1}{2}$ min
31. $284\frac{2}{3}$ or 284 completed boards

3.6 Estimation Applications

< **3.6 Objective** >

1 > Use estimation to solve application problems

Units ANALYSIS

Every denominate number with a fractional unit, such as

$$25\ \frac{\text{mi}}{\text{gal}}$$

has a dual denominate number, which is the reciprocal number and the reciprocal units. In this case we have

$$\frac{1}{25}\ \frac{\text{gal}}{\text{mi}}$$

The $25\ \frac{\text{mi}}{\text{gal}}$ indicates that we can drive 25 mi on 1 gal. The $\frac{1}{25}\ \frac{\text{gal}}{\text{mi}}$ indicates that we use $\frac{1}{25}$ of a gallon for each mile.

EXAMPLES:

Denominate Number	Dual
$55\ \frac{\text{mi}}{\text{h}}$	$\frac{1}{55}\ \frac{\text{h}}{\text{mi}}$
$\frac{2}{3}\ \frac{\text{page}}{\text{min}}$	$\frac{3}{2}\ \frac{\text{min}}{\text{page}}$

Of all the skills you develop in the study of arithmetic, perhaps the most useful is that of estimation. Every day, you have occasion to estimate. Here are a few estimation exercises that you may have gone through this morning.

How much flour should I put in the pancakes?
How much cash am I likely to need today?
How long will it take me to walk to the bus stop?
How long is the ride to school?
How many miles can I get on just over $\frac{1}{4}$ tank of gas?

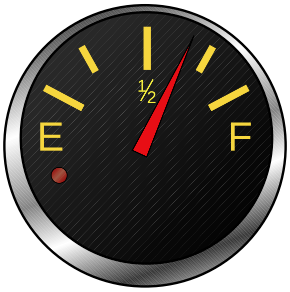
Although you may not think of these as math problems, they are. As your estimation skills improve, so will your ability to come up with good answers to everyday problems.

Example 1 Estimating Measurements

< Objective 1 >

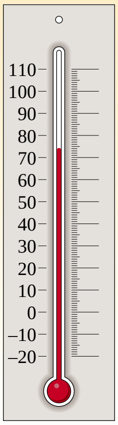
Based on the gauge, is the gas tank closer to $\frac{3}{4}$, $\frac{2}{3}$, $\frac{1}{3}$, or $\frac{1}{4}$ full?

At slightly more than $\frac{1}{2}$, the gauge indicates close to $\frac{2}{3}$ of a tank remains.



Check Yourself 1

Is the reading of the thermometer closer to 72°, 74°, or 76°?

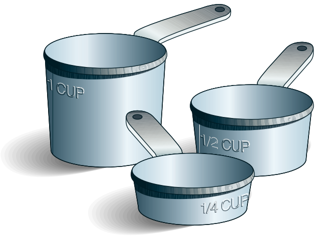


Cooking provides many opportunities for estimation. Example 2 illustrates one.

Example 2 Estimating Measurement

A recipe calls for $\frac{1}{3}$ cup of sugar. The only measuring cups you have are 1 cup, $\frac{1}{2}$ cup, and $\frac{1}{4}$ cup. What should you do?

To solve this problem you must believe that recipes are only approximations anyway. Once you accept that idea, you can estimate $\frac{1}{3}$ cup. It is between $\frac{1}{4}$ cup and $\frac{1}{2}$ cup, so fill the $\frac{1}{4}$ cup, dump it into the $\frac{1}{2}$ cup, and add a little more sugar.



Check Yourself 2

A recipe calls for $\frac{3}{4}$ cup of flour, but you have only a 1-cup, a $\frac{2}{3}$-cup, and a $\frac{1}{3}$-cup measure. How should you proceed?

Almost every shopping trip presents many estimation opportunities. Whether you are estimating the impact of car payments on your monthly budget or estimating the number of bananas you can buy for $3, you are practicing your arithmetic skills. Example 3 provides an opportunity to practice these skills.

Example 3 Estimating Total Cost

DOG FOOD
DOG FOOD
DOG FOOD
3 FOR $1.00

Dog food is on special at three cans for a dollar. Your puppy eats about $\frac{1}{2}$ can per day. How much should you budget for dog food over the next semester (almost 5 months)?

First, estimate the number of days in the semester. At 30 days per month, we'll use

$$30\ \frac{\text{days}}{\text{month}} \times 5 \text{ months} = 150 \text{ days}$$

Next, estimate the amount of food to be consumed. We could try using $\frac{1}{2}$ can per day, which yields

$$\frac{1}{2}\ \frac{\text{can}}{\text{day}} \times 150 \text{ days} = 75 \text{ cans}$$

but remember that this is a puppy, so we will assume his food intake will increase over the next 5 months. To be safe, we will add 25 cans and estimate the total to be 100 cans.

Finally, we can estimate the total cost. If we buy all the food today, we can buy it at 3 cans per dollar. At $\frac{1}{3}$ dollar per can, we get

$$\frac{1}{3}\ \frac{\text{dollar}}{\text{can}} \times 100 \text{ cans} \approx \$33$$

The cost will be about \$33.

Check Yourself 3

Cat food is also on special at seven cans for \$2 $\left(\frac{7}{2}\ \frac{\text{cans}}{\text{dollars}}\right)$. If the cat eats about $\frac{1}{2}$ can per day, what is the approximate cost of the cat food for a 5-month semester?

Check Yourself ANSWERS

1. It is closer to 74°.
2. Fill the $\frac{2}{3}$-cup measure, dump it into the 1-cup measure, and add a little flour.
3. The cost is a little over \$20.

Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 3.6

(a) Every denominate number with a fractional unit has a ________________ denominate number.

(b) Of all the skills you develop in the study of arithmetic, perhaps the most useful is ________________.

(c) Measurements in a recipe are ________________.

(d) Almost every shopping trip represents many ____________ opportunities.

3.6 exercises

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Name ____________________

Section __________ Date __________

Answers

1. ____________________

2. ____________________

3. ____________________

4. ____________________

5. ____________________

6. ____________________

7. ____________________

8. ____________________

9. ____________________

10. ____________________

11. ____________________

12. ____________________

13. ____________________

14. ____________________

< Objective 1 >

Find the dual of each denominate number.

1. $36\ \frac{\text{ft}}{\text{s}}$ > Videos

2. $42\ \frac{\text{kilowatts}}{\text{h}}$

3. $5\ \frac{\text{pages}}{\text{min}}$ > Videos

4. $125\ \frac{\text{joules}}{\text{s}}$

5. $18\ \frac{\text{mi}}{\text{gal}}$

6. $84\ \frac{\text{coulombs}}{\text{s}}$

7. $68\ \frac{\text{lumens}}{\text{cm}^2}$

8. $55\ \frac{\text{ergs}}{\text{min}}$

Solve each application.

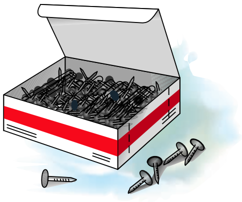
9. CRAFTS Mark wants to make $300 by selling cupcakes at a bake sale, and his mother has offered to pay for the ingredients. He plans to sell them for $5 per dozen. A box of cake mix makes about 50 cupcakes. Approximately how many boxes should Mark buy?

10. STATISTICS Amy is traveling to a city 418 miles away at a speed of roughly 55 miles per hour. About how long should her trip take?

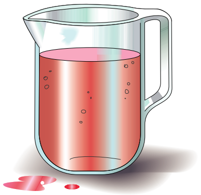
11. SOCIAL SCIENCE The map tells Manuel that it is 423 miles from Eastwick to West Goshen. He knows that, with mixed city and freeway driving, he can average about 50 miles per hour. He is currently in Eastwick and needs to be in West Goshen by noon. Make a rough estimate of the time that Manuel should leave Eastwick.

12. BUSINESS AND FINANCE Sam works in a shipping department for a manufacturer. He is filling a customer's order for several small items. The items ordered weigh 21, 23, 18, and 7 ounces (oz). The company policy is to use a stronger box to ship products totaling more than 3 pounds (lb). Knowing 16 oz is 1 lb, estimate whether Sam should use the stronger box.

13. CONSTRUCTION Lauren is a contractor for a roofing job. She estimates that she will need about 4,800 roofing nails. According to a handbook, the roofing nails that are needed count out at about 189 nails per pound. Estimate how many pounds Lauren will need for the job.



14. BUSINESS AND FINANCE Julio works as a quality control expert in a beverage factory. The assembly line that he monitors produces about 20,000 bottles in a 24-hour period. Julio samples about 120 bottles an hour and rejects the line if he finds more than $\frac{1}{50}$ of the sample to be defective. About how many defective bottles should Julio allow before rejecting the entire line?



15. CRAFTS Based on the amount of liquid in the pitcher, is the pitcher closer to $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{4}$, or $\frac{3}{4}$ full?

16. BUSINESS AND FINANCE Lunch meat sells for about \$3 per pound. You use about $\frac{1}{2}$ pound per day for sandwiches. How much should you budget for lunch meat over the next month (about 20 workdays)?

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Estimate each sum or difference.

17. $2\frac{1}{8} + 7\frac{9}{11} + 4\frac{6}{7} + 5\frac{1}{12}$ > Videos

18. $4\frac{5}{6} + 3\frac{1}{5} + 8\frac{1}{7} + 11\frac{8}{9}$

19. $15\frac{4}{5} - 3\frac{9}{10} - 6\frac{1}{7}$

20. $18\frac{1}{5} + 11\frac{7}{8} - 14\frac{10}{11} - 10\frac{6}{7}$

Estimate each product or quotient.

21. $2\frac{5}{6} \times 4\frac{1}{3}$

22. $3\frac{1}{7} \times 6\frac{8}{9}$

23. $9\frac{2}{9} \div 2\frac{6}{7}$

24. $17\frac{11}{12} \div 6\frac{1}{10}$

25. $15\frac{6}{7} \div 4\frac{1}{8} \times 1\frac{3}{4}$

Answers

1. $\frac{1\text{ s}}{36\text{ ft}}$ **3.** $\frac{1\text{ min}}{5\text{ page}}$ **5.** $\frac{1\text{ gal}}{18\text{ mi}}$ **7.** $\frac{1\text{ cm}^2}{68\text{ lumen}}$ **9.** 15 boxes

11. 3 A.M. **13.** 24 lb **15.** $\frac{3}{4}$ **17.** 20 **19.** 6 **21.** 12

23. 3 **25.** 8

Answers

15. ______

16. ______

17. ______

18. ______

19. ______

20. ______

21. ______

22. ______

23. ______

24. ______

25. ______

Activity 9 ::
Aerobic Exercise

According to the website www.Fitnesszone.com, **aerobic training zone** refers to the training intensity range that will produce improvement in your level of aerobic fitness without overtaxing your cardiorespiratory system. Your aerobic training zone is based on a percentage of your maximal heart rate. As a general rule, your maximal heart rate is measured directly or estimated by subtracting your age from 220. Depending upon how physically fit you are, the lower and upper limits of your aerobic training zone are then based on a fraction of the maximal heart rate, approximately $\frac{3}{5}$ to $\frac{9}{10}$, respectively.

The chart below uses this information to determine maximal heart rate (MHR) by subtracting the age from 220. Complete the table.

Age	Maximal Heart Rate
20	200
25	
30	
35	
40	
45	
50	
55	

Using the information provided in the first paragraph, complete the following table to find the lower and upper limits for the training zone for each age. Note that when we compute the upper limit of the training zone, we round up to the next integer. For example, we will compute the upper limit for age 25.

$$195 \times \frac{9}{10} = \frac{195}{1} \times \frac{9}{10} = \frac{39}{1} \times \frac{9}{2} = \frac{351}{2} = 175\frac{1}{2}$$

Rounding up, we get an upper limit of 176.

Age	Maximal Heart Rate	Lower Limit of Zone $\left(\frac{3}{5}\text{ MHR}\right)$	Upper Limit of Zone $\left(\frac{9}{10}\text{ MHR}\right)$
20	200	120	180
25	195	117	176
30	190		
35	185		
40	180		
45	175		
50	170		
55	165		

Definition/Procedure	Example	Reference
Adding and Subtracting Fractions with Like Denominators *To Add Like Fractions*		Section 3.1
Step 1 Add the numerators. **Step 2** Place the sum over the common denominator. **Step 3** Simplify the resulting fraction if necessary.	$\frac{5}{18} + \frac{7}{18} = \frac{12}{18} = \frac{2}{3}$	*p.* 183
To Subtract Like Fractions **Step 1** Subtract the numerators. **Step 2** Place the difference over the common denominator. **Step 3** Simplify the resulting fraction when necessary.	$\frac{17}{20} - \frac{7}{20} = \frac{10}{20} = \frac{1}{2}$	*p.* 185
Common Multiples		Section 3.2
Least Common Multiple (LCM) The LCM is the *smallest* number that is a multiple of each of a group of numbers.		*p.* 192
To Find the LCM **Step 1** Write the prime factorization for each of the numbers in the group. **Step 2** Find all the prime factors that appear in any one of the prime factorizations. **Step 3** Form the product of those prime factors, using each factor the greatest number of times it occurs in any one factorization.	To find the LCM of 12, 15, and 18: $\begin{array}{rccccccccc} 12 = & 2 & \times & 2 & \times & 3 & & & & \\ 15 = & & & & & 3 & \times & & & 5 \\ 18 = & 2 & & & \times & 3 & \times & 3 & & \\ \hline & 2 & \times & 2 & \times & 3 & \times & 3 & \times & 5 \end{array}$ The LCM is 2 × 2 × 3 × 3 × 5, or 180.	*p.* 192
Adding and Subtracting Fractions with Unlike Denominators *To Find the LCD of a Group of Fractions*		Section 3.3
Step 1 Write the prime factorization for each of the denominators. **Step 2** Find all the prime factors that appear in any one of the prime factorizations. **Step 3** Form the product of those prime factors, using each factor the greatest number of times it occurs in any one factorization.	To find the LCD of fractions with denominators 4, 6, and 15: $\begin{array}{rccccccc} 4 = & 2 & \times & 2 & & & & \\ 6 = & 2 & & & \times & 3 & & \\ 15 = & & & & & 3 & \times & 5 \\ \hline & 2 & \times & 2 & \times & 3 & \times & 5 \end{array}$ The LCD = 2 × 2 × 3 × 5, or 60.	*p.* 200

Continued

Definition/Procedure	Example	Reference
To Add Unlike Fractions		
Step 1 Find the LCD of the fractions. **Step 2** Change each unlike fraction to an equivalent fraction with the LCD as a common denominator. **Step 3** Add the resulting like fractions as before.	$\frac{3}{4}+\frac{7}{10}=\frac{15}{20}+\frac{14}{20}$ $=\frac{29}{20}=1\frac{9}{20}$	*p.* 201
To Subtract Unlike Fractions **Step 1** Find the LCD of the fractions. **Step 2** Change each unlike fraction to an equivalent fraction with the LCD as a common denominator. **Step 3** Subtract the resulting like fractions as before.	$\frac{8}{9}-\frac{5}{6}=\frac{16}{18}-\frac{15}{18}=\frac{1}{18}$	*p.* 203
Adding and Subtracting Mixed Numbers		**Section 3.4**
To Add or Subtract Mixed Numbers		
Step 1 Find the LCD. **Step 2** Rewrite the fraction parts to match the LCD. **Step 3** Add or subtract the fractions. **Step 4** Add or subtract the whole number part. **Step 5** Simplify if necessary.	$2\frac{1}{4}+3\frac{4}{5}=2\frac{5}{20}+3\frac{16}{20}$ $=2+3+\frac{5}{20}+\frac{16}{20}=5+\frac{21}{20}=6\frac{1}{20}$	*p.* 213
Order of Operations with Fractions		**Section 3.5**
Order of Operations **Step 1** Do any operations within parentheses or other grouping symbols. **Step 2** Evaluate all powers. **Step 3** Do all multiplication and division in order from left to right. **Step 4** Do all addition and subtraction in order from left to right.	$\frac{2}{3}+\left(\frac{1}{2}\right)^2\left(\frac{1}{3}+\frac{1}{2}\right)=\frac{2}{3}+\left(\frac{1}{2}\right)^2\left(\frac{5}{6}\right)$ $=\frac{2}{3}+\left(\frac{1}{4}\right)\left(\frac{5}{6}\right)$ $=\frac{2}{3}+\frac{5}{24}$ $=\frac{16}{24}+\frac{5}{24}$ $=\frac{21}{24}=\frac{7}{8}$	*p.* 225
Estimation Applications		**Section 3.6**
Every denominate number with a fractional unit has a dual denominate number that is the reciprocal number and the reciprocal units.	Denominate Number — Dual $50\,\frac{\text{mi}}{\text{h}}$ — $\frac{1}{50}\,\frac{\text{h}}{\text{mi}}$ $\frac{10\text{ dollars}}{\text{ticket}}$ — $\frac{1}{10}\,\frac{\text{ticket}}{\text{dollars}}$	*p.* 231

summary exercises :: chapter 3

This summary exercise set is provided to give you practice with each of the objectives of this chapter. Each exercise is keyed to the appropriate chapter section. When you are finished, you can check your answers to the odd-numbered exercises against those presented in the back of the text. If you have difficulty with any of these questions, go back and reread the examples from that section. The answers to the even-numbered exercises appear in the *Instructor's Solutions Manual.* Your instructor will give you guidelines on how to best use these exercises in your instructional setting.

3.1 *Add. Simplify when possible.*

1. $\frac{8}{15} + \frac{2}{15}$

2. $\frac{4}{7} + \frac{3}{7}$

3. $\frac{8}{13} + \frac{7}{13}$

4. $\frac{17}{18} + \frac{5}{18}$

5. $\frac{19}{24} + \frac{13}{24}$

6. $\frac{1}{9} + \frac{2}{9} + \frac{4}{9}$

7. $\frac{2}{9} + \frac{5}{9} + \frac{4}{9}$

8. $\frac{4}{15} + \frac{7}{15} + \frac{7}{15}$

3.2 *Find the least common multiple (LCM) for each group of numbers.*

9. 4 and 12

10. 8 and 16

11. 18 and 24

12. 12 and 18

13. 15 and 20

14. 14 and 21

15. 9, 12, and 24

16. 14, 21, and 28

Arrange the fractions in order from smallest to largest.

17. $\frac{5}{8}, \frac{7}{12}$

18. $\frac{5}{6}, \frac{4}{5}, \frac{7}{10}$

Complete each statement using the symbols $<$, $=$, *or* $>$.

19. $\frac{5}{12} \square \frac{3}{8}$

20. $\frac{3}{7} \square \frac{9}{21}$

21. $\frac{9}{16} \square \frac{7}{12}$

3.3 *Write as equivalent fractions with the LCD as a common denominator.*

22. $\frac{1}{6}, \frac{7}{8}$

23. $\frac{3}{10}, \frac{5}{8}, \frac{7}{12}$

Find the least common denominator (LCD) for fractions with the given denominators.

24. 6 and 24

25. 12 and 18

26. 20 and 24

27. 25 and 40

28. 4, 5, and 9

29. 3, 4, and 11

30. 2, 5, and 8

31. 3, 6, and 8

Add.

32. $\frac{3}{10} + \frac{7}{12}$

33. $\frac{3}{8} + \frac{5}{12}$

34. $\frac{5}{36} + \frac{7}{24}$

35. $\frac{2}{15} + \frac{9}{20}$

36. $\frac{9}{14} + \frac{10}{21}$

37. $\frac{7}{15} + \frac{13}{18}$

38. $\frac{12}{25} + \frac{19}{30}$

39. $\frac{1}{2} + \frac{1}{4} + \frac{1}{8}$

40. $\frac{1}{3} + \frac{1}{5} + \frac{1}{10}$

41. $\frac{3}{8} + \frac{5}{12} + \frac{7}{18}$

42. $\frac{5}{6} + \frac{8}{15} + \frac{9}{20}$

Subtract.

43. $\frac{8}{9} - \frac{3}{9}$

44. $\frac{9}{10} - \frac{6}{10}$

45. $\frac{5}{8} - \frac{1}{8}$

46. $\frac{11}{12} - \frac{7}{12}$

47. $\frac{7}{8} - \frac{2}{3}$

48. $\frac{5}{6} - \frac{3}{5}$

49. $\frac{11}{18} - \frac{2}{9}$

50. $\frac{5}{6} - \frac{1}{4}$

51. $\frac{5}{8} - \frac{1}{6}$

52. $\frac{13}{18} - \frac{5}{12}$

53. $\frac{8}{21} - \frac{1}{14}$

54. $\frac{13}{18} - \frac{7}{15}$

55. $\frac{11}{12} - \frac{1}{4} - \frac{1}{3}$

56. $\frac{13}{15} + \frac{2}{3} - \frac{3}{5}$

3.4 *Perform the indicated operations.*

57. $4\frac{5}{8} - 4\frac{3}{8}$

58. $2\frac{11}{18} - 2\frac{2}{9}$

59. $3\frac{7}{10} - 3\frac{7}{12}$

60. $9\frac{11}{27} - 9\frac{5}{18}$

61. $5\frac{4}{9} + 2\frac{5}{12} - 7\frac{3}{8}$

62. $6\frac{5}{7} + 3\frac{4}{7}$

63. $5\frac{7}{10} + 3\frac{11}{12}$

64. $2\frac{1}{2} + 3\frac{5}{6} + 3\frac{3}{8}$

65. $7\frac{7}{9} - 3\frac{4}{9}$

66. $9\frac{1}{6} - 3\frac{1}{8}$

67. $6\frac{5}{12} - 3\frac{5}{8}$

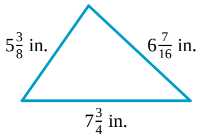
68. $2\frac{1}{3} + 5\frac{1}{6} - 2\frac{4}{5}$

Solve each application.

69. CRAFTS A recipe calls for $\frac{1}{3}$ cup of milk. You have $\frac{3}{4}$ cup. How much milk will be left over?

70. CONSTRUCTION Bradley needs two shelves, one $32\frac{3}{8}$ in. long and the other $36\frac{11}{16}$ in. long. What is the total length of shelving that is needed?

71. GEOMETRY Find the perimeter of the triangle.



72. STATISTICS At the beginning of a year Miguel was $51\frac{3}{4}$ in. tall. In June, he measured $53\frac{1}{8}$ in. How much did he grow during that period?

73. CONSTRUCTION A bookshelf that is $42\frac{5}{16}$ in. long is cut from a board with a length of 8 ft. If $\frac{1}{8}$ in. is wasted in the cut, what length board remains?

74. CRAFTS Amelia buys an 8-yd roll of wallpaper on sale. After measuring, she finds that she needs the following amounts of the paper: $2\frac{1}{3}$, $1\frac{1}{2}$, and $3\frac{3}{4}$ yd. Does she have enough for the job? If so, how much will be left over?

75. **Crafts** Roberto used $1\frac{3}{4}$ gallon (gal) of paint in his living room, $1\frac{1}{3}$ gal in the dining room, and $\frac{1}{2}$ gal in a hallway. How much paint did he use?

76. **Construction** A sheet of plywood consists of two outer sections that are $\frac{3}{16}$ in. thick and a center section that is $\frac{3}{8}$ in. thick. How thick is the plywood overall?

77. **Crafts** A pattern calls for four pieces of fabric with lengths $\frac{1}{2}$, $\frac{3}{8}$, $\frac{1}{4}$, and $\frac{5}{8}$ yd. How much fabric must be purchased to use the pattern?

3.5 *Evaluate.*

78. $\frac{3}{4} + \left(\frac{1}{2}\right)\left(\frac{1}{3}\right)$

79. $\frac{2}{3} - \left(\frac{3}{4}\right)^2\left(\frac{1}{2}\right)$

80. $\left(4\frac{3}{8}\right)\left(1\frac{1}{2}\right) \div \left(\frac{3}{4} + \frac{1}{2}\right)$

81. $\frac{1}{3} \cdot \left(\frac{3}{4} - \frac{1}{2}\right)^2$

82. $\left(\frac{2}{3}\right)^3 - \frac{1}{2} \cdot \frac{1}{9}$

83. $2\frac{1}{3} - \left[\frac{1}{2} \cdot \left(2 - \frac{1}{3}\right)\right]$

3.6

84. **Business and Finance** Jared had about $2,800 in his checking account. He wrote five checks for equal amounts, and he knows his balance is about $200. Estimate the amount of each check that he wrote.

85. **Business and Finance** An appliance store has the following items on sale:

19″ color television for $399
Entertainment center for $509.95
A mini sound system for $369.95

Estimate the cost of one entertainment center, two color televisions, and three mini sound systems.

Estimate.

86. $3\frac{5}{6} + 4\frac{1}{7} - 2\frac{6}{7}$

87. $4\frac{1}{8} \times 5\frac{11}{12}$

88. $18\frac{1}{9} \div 1\frac{8}{9} \times 4\frac{5}{6}$

self-test 3

CHAPTER 3

Name ______________________

Section __________ Date __________

The purpose of this self-test is to help you assess your progress so that you can find concepts that you need to review before the next exam. Allow yourself about an hour to take this test. At the end of that hour, check your answers against those given in the back of this text. If you miss any, go back to the appropriate section to reread the examples until you have mastered that particular concept.

Answers

1. ______________________
2. ______________________
3. ______________________
4. ______________________
5. ______________________
6. ______________________
7. ______________________
8. ______________________
9. ______________________
10. ______________________
11. ______________________
12. ______________________
13. ______________________
14. ______________________
15. ______________________

Find the least common denominator for fractions with the given denominators.

1. 12 and 15

2. 3, 4, and 18

Perform the indicated operations.

3. $7\frac{3}{8} + 2\frac{7}{8}$

4. $3\frac{5}{6} - 2\frac{2}{9}$

5. $\frac{3}{10} + \frac{6}{10}$

6. $\frac{1}{6} + \frac{3}{7}$

7. $\frac{11}{12} - \frac{3}{20}$

8. $4\frac{2}{7} + 3\frac{3}{7} + 1\frac{3}{7}$

9. $4\frac{1}{6} + 3\frac{3}{4}$

10. $\frac{5}{12} + \frac{3}{12}$

11. $3\frac{1}{2} + 4\frac{3}{4} + 5\frac{3}{10}$

12. $\frac{11}{15} + \frac{9}{20}$

Solve each application.

13. **STATISTICS** You have $\frac{5}{6}$ hour (h) to take a three-part test. You use $\frac{1}{3}$ h for the first section and $\frac{1}{4}$ h for the second. How much time do you have left to finish the last section of the test?

14. **CRAFTS** A recipe calls for $\frac{1}{2}$ cup of raisins, $\frac{1}{4}$ cup of walnuts, and $\frac{2}{3}$ cup of rolled oats. What is the total amount of these ingredients?

15. Find the least common multiple of 18, 24, and 36.

Evaluate.

16. $\frac{1}{4} + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{3} + \frac{1}{12}\right)$

17. $\left(1\frac{1}{3} + 2\frac{1}{2}\right) \cdot 3\frac{1}{4}$

Perform the indicated operations.

18. $\frac{2}{5} + \frac{4}{10}$

19. $7\frac{3}{8} - 5\frac{5}{8}$

20. $7 - 5\frac{7}{15}$

21. $\frac{7}{18} - \frac{5}{18}$

22. $\frac{3}{8} + \frac{5}{12}$

23. $7\frac{1}{8} - 3\frac{1}{6}$

24. $\frac{7}{9} - \frac{4}{9}$

25. $\frac{1}{4} + \frac{5}{8} + \frac{7}{10}$

26. $5\frac{3}{10} + 2\frac{4}{10}$

27. $\frac{5}{24} + \frac{3}{8}$

28. $6\frac{3}{8} + 5\frac{7}{10}$

Solve each application.

29. **STATISTICS** The average person drinks about $3\frac{1}{5}$ cups of coffee per day. If a person works 5 days a week for 50 weeks every year, estimate how many cups of coffee that person will drink in a working lifetime of $51\frac{3}{4}$ years.

30. **BUSINESS AND FINANCE** A worker has $2\frac{1}{6}$ h of overtime on Tuesday, $1\frac{3}{4}$ h on Wednesday, and $1\frac{5}{6}$ h on Friday. What is the total overtime for the week?

Answers

16. ______

17. ______

18. ______

19. ______

20. ______

21. ______

22. ______

23. ______

24. ______

25. ______

26. ______

27. ______

28. ______

29. ______

30. ______

cumulative review chapters 1-3

Name ______________________

Section ________ Date ________

Answers

1. ______________________

2. ______________________

3. ______________________

4. ______________________

5. ______________________

6. ______________________

7. ______________________

8. ______________________

9. ______________________

10. ______________________

11. ______________________

12. ______________________

13. ______________________

14. ______________________

15. ______________________

16. ______________________

17. ______________________

18. ______________________

19. ______________________

The following exercises are presented to help you review concepts from earlier chapters. This is meant as review material and not as a comprehensive exam. The answers are presented in the back of the text. Beside each answer is a section reference for the concept. If you have difficulty with any of these exercises, be certain to at least read through the summary related to that section.

Perform the indicated operations.

1. $$\begin{array}{r} 1,369 \\ +\ 5,804 \\ \hline \end{array}$$

2. $$\begin{array}{r} 489 \\ 562 \\ 613 \\ +\ 254 \\ \hline \end{array}$$

3. $$\begin{array}{r} 357 \\ 28 \\ +\ 2,346 \\ \hline \end{array}$$

4. $$\begin{array}{r} 13 \\ 2,543 \\ +\ 10,547 \\ \hline \end{array}$$

5. $289 - 54$

6. $53,294 - 41,074$

7. $503 - 74$

8. $5,731 - 2,492$

9. $$\begin{array}{r} 58 \\ \times\ \ 3 \\ \hline \end{array}$$

10. Find the product of 273 and 7.

11. $$\begin{array}{r} 89 \\ \times\ 56 \\ \hline \end{array}$$

12. $$\begin{array}{r} 538 \\ \times\ 103 \\ \hline \end{array}$$

13. $281\overline{)6,935}$

14. $571\overline{)12,583}$

15. $293\overline{)61,382}$

Evaluate each expression.

16. $12 \div 6 + 3$

17. $4 + 12 \div 4$

18. $3^3 \div 9$

19. $28 \div 7 \times 4$

20. $26 - 2 \times 3$

21. $36 \div (3^2 + 3)$

Identify the proper fractions, improper fractions, and mixed numbers from the following group.

$\frac{5}{7}, \frac{15}{9}, 4\frac{5}{6}, \frac{8}{8}, \frac{11}{1}, \frac{2}{5}, 3\frac{5}{6}$

22. Proper: ___ Improper: _______ Mixed numbers: _____

Convert to mixed or whole numbers.

23. $\frac{16}{9}$

24. $\frac{36}{5}$

Convert to improper fractions.

25. $5\frac{3}{4}$

26. $6\frac{1}{9}$

Determine whether each pair of fractions is equivalent.

27. $\frac{8}{32}, \frac{9}{36}$

28. $\frac{6}{11}, \frac{7}{9}$

Multiply.

29. $\frac{7}{15} \times \frac{5}{21}$

30. $\frac{10}{27} \times \frac{9}{20}$

31. $4 \times \frac{3}{8}$

32. $3\frac{2}{5} \times \frac{5}{8}$

33. $5\frac{1}{3} \times 1\frac{4}{5}$

34. $1\frac{5}{12} \times 8$

35. $3\frac{1}{5} \times \frac{7}{8} \times 2\frac{6}{7}$

Answers

20. ____________

21. ____________

22. ____________

23. ____________

24. ____________

25. ____________

26. ____________

27. ____________

28. ____________

29. ____________

30. ____________

31. ____________

32. ____________

33. ____________

34. ____________

35. ____________

Answers

36.

37.

38.

39.

40.

41.

42.

43.

44.

45.

46.

47.

48.

49.

50.

51.

52.

53.

Divide.

36. $\frac{5}{12} \div \frac{5}{8}$

37. $\frac{7}{15} \div \frac{14}{25}$

38. $\frac{9}{20} \div 2\frac{2}{5}$

Add.

39. $\frac{4}{15} + \frac{8}{15}$

40. $\frac{7}{25} + \frac{8}{15}$

41. $\frac{2}{5} + \frac{3}{4} + \frac{5}{8}$

Perform the indicated operations.

42. $\frac{17}{20} - \frac{7}{20}$

43. $\frac{5}{9} - \frac{5}{12}$

44. $\frac{5}{18} + \frac{4}{9} - \frac{1}{6}$

Perform the indicated operations.

45. $3\frac{5}{7} + 2\frac{4}{7}$

46. $4\frac{7}{8} + 3\frac{1}{6}$

47. $8\frac{1}{9} - 3\frac{5}{9}$

48. $7\frac{7}{8} - 3\frac{5}{6}$

49. $9 - 5\frac{3}{8}$

50. $3\frac{1}{6} + 3\frac{1}{4} - 2\frac{7}{8}$

Solve each application.

51. **Business and Finance** In his part-time job, Manuel worked $3\frac{5}{6}$ hours (h) on Monday, $4\frac{3}{10}$ h on Wednesday, and $6\frac{1}{2}$ h on Friday. Find the number of hours that he worked during the week.

52. **Crafts** A $6\frac{1}{2}$-in. bolt is placed through a wall that is $5\frac{7}{8}$ in. thick. How far does the bolt extend beyond the wall?

53. **Statistics** On a 6-hour (h) trip, Carlos drove $1\frac{3}{4}$ h and then Maria drove for another $2\frac{1}{3}$ h. How many hours remained on the trip?

chapter
4
> Make the Connection

CHAPTER

4

Decimals

INTRODUCTION

When Barry is not at his job, he loves to be outside partaking in many outdoor activities. Some of these activities include bicycling, hiking, and tennis. Lately, Barry has been spending a lot of time on his bike. He bought a new road bike because of the long distances he was riding. Two years ago, Barry noticed a sign in his neighborhood advertising a 150-mile bike ride to raise money for muscular dystrophy. He thought it was a good cause and started training for the ride. Barry completed the ride and experienced a great feeling of accomplishment in doing so.

The MD ride inspired Barry to look for more rides that raised money for important causes. He has since ridden for cerebral palsy and cancer research. Barry likes the idea that he can raise money for a good cause while getting exercise doing something he enjoys.

Lance Armstrong is one of Barry's inspirational heroes. Armstrong is a cancer survivor who battled the disease and came back to win the Tour de France seven times. He founded the Lance Armstrong Foundation for Cancer Research and the yellow Livestrong wristbands.

You can learn more about the Tour de France by doing Activity 12 on page 303. For more information on the Tour de France or Lance Armstrong you can also go to www.letour.fr or lancewins.com.

CHAPTER 4 OUTLINE

Chapter 4 :: Prerequisite Test 248
4.1 Place Value and Rounding 249
4.2 Converting Between Fractions and Decimals 258
4.3 Adding and Subtracting Decimals 268
4.4 Multiplying Decimals 281
4.5 Dividing Decimals 291
Chapter 4 :: Summary / Summary Exercises / Self-Test / Cumulative Review :: Chapters 1–4 304

4 prerequisite test

Name ____________

Section ______ Date ______

Answers

1. ____________

2. ____________

3. ____________

4. ____________

5. ____________

6. ____________

7. ____________

8. ____________

9. ____________

10. ____________

This prerequisite test provides some exercises requiring skills that you will need to be successful in the coming chapter. The answers for these exercises can be found in the back of this text. This prerequisite test can help you identify topics that you will need to review before beginning the chapter.

Write the name of each number in words.

1. $3\frac{7}{10}$

2. $6\frac{29}{100}$

3. $17\frac{89}{1,000}$

Perform the indicated operations.

4. $183 + 5 + 69$

5. $213 - 49$

6. 426×15

7. $792 \div 10$

8. $792 \div 100$

Find the area of each figure described.

9. A rectangle has length 17 feet and width 8 feet.

10. A square has sides of length 15 inches.

4.1

Place Value and Rounding

< **4.1 Objectives** >

1 > Write a number in decimal form
2 > Identify place value in a decimal fraction
3 > Write a decimal in words
4 > Write a decimal as a fraction or mixed number
5 > Compare the size of several decimals
6 > Round a decimal to the nearest tenth
7 > Round a decimal to any specified decimal place

In Chapters 2 and 3, we looked at common fractions. Now we turn to a special kind of fraction called **decimal fractions.** A decimal fraction is a fraction whose denominator is a *power of* 10. Some examples of decimal fractions are $\frac{3}{10}$, $\frac{45}{100}$, and $\frac{123}{1{,}000}$.

Earlier we talked about the idea of place value. Recall that in our decimal place-value system, each place has *one-tenth* of the value of the place to its left.

Example 1 Identifying Place Values

RECALL

The powers of 10 are 1, 10, 100, 1,000, and so on.
You might want to review Section 1.7 before going on.

Label the place values for the number 538.

5	3	8
↑	↑	↑
Hundreds	Tens	Ones

The ones place value is $\frac{1}{10}$ of the tens place value; the tens place value is $\frac{1}{10}$ of the hundreds place value; and so on.

Check Yourself 1

Label the place values for the number 2,793.

NOTE

The decimal point separates the whole-number part and the fractional part of a decimal fraction.

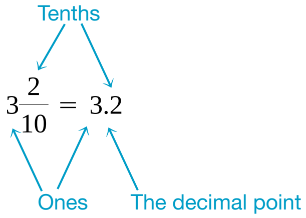
We want to extend this idea *to the right* of the ones place. Write a period to the *right* of the ones place. This is called the **decimal point.** Each digit to the right of that decimal point will represent a fraction whose denominator is a power of 10. The first place to the right of the decimal point is the tenths place:

$$0.1 = \frac{1}{10}$$

Example 2 Writing a Number in Decimal Form

< **Objective 1** >

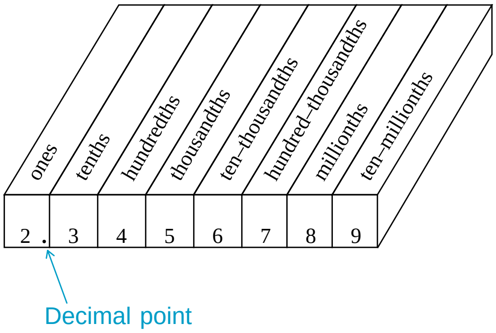
Write the mixed number $3\frac{2}{10}$ in decimal form.



Check Yourself 2

Write $5\frac{3}{10}$ in decimal form.

As you move farther to the *right*, each place value must be $\frac{1}{10}$ of the value before it. The second place value is hundredths $\left(0.01 = \frac{1}{100}\right)$. The next place is thousandths, the fourth position is the ten-thousandths place, and so on. The figure illustrates the value of each position as we move to the right of the decimal point.



Example 3 Identifying Place Values in a Decimal Fraction

< Objective 2 >

NOTE

For convenience we shorten the term *decimal fraction* to *decimal* from this point on.

What are the place values of the 4 and 6 in the decimal 2.34567?

The place value of 4 is hundredths, and the place value of 6 is ten-thousandths.

Check Yourself 3

What is the place value of 5 in the decimal of Example 3?

Understanding place values allows you to read and write decimals. You can use the following steps.

Step by Step

Reading or Writing Decimals in Words

Step 1 Read the digits *to the left* of the decimal point as a whole number.

Step 2 Read the decimal point as the word *and*.

Step 3 Read the digits *to the right* of the decimal point as a whole number followed by the place value of the rightmost digit.

Example 4 Writing a Decimal Number in Words

< Objective 3 >

NOTE

If there are *no* nonzero digits to the left of the decimal point, start directly with step 3.

Write each decimal number in words.

5.03 is read "five and three hundredths."

Hundredths

The rightmost digit, 3, is in the hundredths position.

12.057 is read "twelve and fifty-seven thousandths."

Thousandths

The rightmost digit, 7, is in the thousandths position.

0.5321 is read "five thousand three hundred twenty-one ten–thousandths."

NOTES

An informal way of reading decimals is to simply read the digits in order and use the word *point* to indicate the decimal point. 2.58 can be read “two point five eight.” 0.689 can be read “zero point six eight nine.”

The number of digits to the right of the decimal point is called the number of **decimal places** in a decimal number. So, 0.35 has two decimal places.

When the decimal has no whole-number part, we have chosen to write a 0 to the left of the decimal point. This simply makes sure that you don’t miss the decimal point. However, both 0.5321 and .5321 are correct.

Check Yourself 4

Write 2.58 in words.

One quick way to write a decimal as a common fraction is to remember that the number of decimal places must be the same as the number of zeros in the denominator of the common fraction.

Example 5 Writing a Decimal Number as a Mixed Number

< Objective 4 >

Write each decimal as a common fraction or mixed number.

$$\underset{\substack{\uparrow\\ \text{Two}\\ \text{places}}}{0.35} = \underset{\substack{\uparrow\\ \text{Two}\\ \text{zeros}}}{\frac{35}{100}}$$

NOTE

The 0 to the right of the decimal point is a “placeholder” that is not needed in the common-fraction form.

The same method can be used with decimals that are greater than 1. Here the result will be a mixed number.

$$2.\underbrace{058}_{\substack{\uparrow\\ \text{Three}\\ \text{places}}} = 2\underset{\substack{\uparrow\\ \text{Three}\\ \text{zeros}}}{\frac{58}{1{,}000}}$$

Check Yourself 5

Write as common fractions or mixed numbers.

(a) 0.528 **(b)** 5.08

RECALL

By the Fundamental Principle of Fractions, multiplying the numerator and denominator of a fraction by the same nonzero number does not change the value of the fraction.

It is often useful to compare the sizes of two decimal fractions. One approach to comparing decimals uses the following fact.

Adding zeros to the right *does not change* the value of a decimal. The number 0.53 is the same as 0.530. Look at the fractional form:

$$\frac{53}{100} = \frac{530}{1{,}000}$$

The fractions are equivalent because we multiplied both the numerator and denominator by 10.

This allows us to compare decimals as shown in Example 6.

Example 6 Comparing the Sizes of Two Decimal Numbers

< Objective 5 >

Which is larger?

0.84 or 0.842

Write 0.84 as 0.840. Then we see that 0.842 (or 842 thousandths) is greater than 0.840 (or 840 thousandths), and we can write

$0.842 > 0.84$

Check Yourself 6

Complete the statement, using the symbols $<$ or $>$.

0.588 ______ 0.59

Whenever a decimal represents a measurement made by some instrument (a rule or a scale), the measurement is not exact. It is accurate only to a certain number of places and is called an **approximate number.** Usually, we want to make all decimals in a particular problem accurate to a specified decimal place or tolerance. This requires **rounding** the decimals. We can picture the process on a number line.

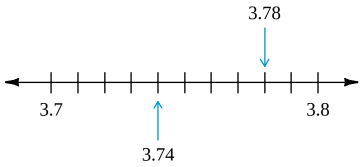
Example 7 Rounding to the Nearest Tenth

< Objective 6 >

NOTE

3.74 is closer to 3.7 than it is to 3.8, while 3.78 is closer to 3.8.

3.74 is rounded down to the nearest tenth, 3.7, and 3.78 is rounded up to 3.8.



Check Yourself 7

Use the number line in Example 7 to round 3.77 to the nearest tenth.

Rather than using the number line, the following rule can be applied.

Step by Step

To Round a Decimal

Step 1 Find the place to which the decimal is to be rounded.

Step 2 If the next digit to the right is 5 or more, increase the digit in the place you are rounding to by 1. Discard remaining digits to the right.

Step 3 If the next digit to the right is less than 5, just discard that digit and any remaining digits to the right.

Example 8 Rounding to the Nearest Tenth

NOTE

Many students find it easiest to mark the digit they are rounding to with an arrow.

Round 34.58 to the nearest tenth.

34.58 Locate the digit you are rounding to. The 5 is in the tenths place.

Because the next digit to the right, 8, is 5 or more, increase the tenths digit by 1. Then discard the remaining digits.

34.58 is rounded to 34.6.

Check Yourself 8

Round 48.82 to the nearest tenth.

Example 9 Rounding to the Nearest Hundredth

< Objective 7 >

Round 5.673 to the nearest hundredth.

5.673 The 7 is in the hundredths place.

The next digit to the right, 3, is less than 5. Leave the hundredths digit as it is and discard the remaining digits to the right.

5.673 is rounded to 5.67.

Check Yourself 9

Round 29.247 to the nearest hundredth.

Example 10 Rounding to a Specified Decimal Place

NOTE

The fourth place to the *right* of the decimal point is the ten-thousandths place.

Round 3.14159 to four decimal places.

3.14159 The 5 is in the ten–thousandths place.

The next digit to the right, 9, is 5 or more, so increase the digit you are rounding to by 1. Discard the remaining digits to the right.

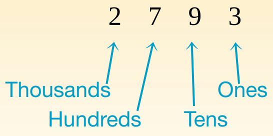
3.14159 is rounded to 3.1416.

Check Yourself 10

Round 0.8235 to three decimal places.

Check Yourself ANSWERS

1.



2. 5.3 **3.** Thousandths

4. Two and fifty-eight hundredths **5.** **(a)** $\frac{528}{1,000}$; **(b)** $5\frac{8}{100}$

6. $0.588 < 0.59$ **7.** 3.8 **8.** 48.8 **9.** 29.25 **10.** 0.824

Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 4.1

(a) A ______________ fraction is a fraction whose denominator is a power of 10.

(b) The number of digits to the right of the decimal point is called the number of decimal ______________.

(c) Whenever a decimal represents a measure made by some instrument, the decimals are not ______________.

(d) The fourth place to the right of the decimal is called the ______________ place.

4.1 exercises

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Name ____________________

Section __________ Date __________

Answers

1. ____________________

2. ____________________

3. ____________________

4. ____________________

5. __________ 6. __________

7. __________ 8. __________

9. __________ 10. __________

11. ____________________

12. ____________________

13. ____________________

14. ____________________

15. ____________________

16. ____________________

17. __________ 18. __________

19. __________ 20. __________

< Objective 2 >

For the decimal 8.57932:

1. What is the place value of 7?

2. What is the place value of 5?

3. What is the place value of 3? > Videos

4. What is the place value of 2?

< Objective 1 >

Write in decimal form.

5. $\frac{23}{100}$

6. $\frac{371}{1{,}000}$

7. $\frac{209}{10{,}000}$ > Videos

8. $3\frac{5}{10}$

9. $23\frac{56}{1{,}000}$ > Videos

10. $7\frac{431}{10{,}000}$

< Objective 3 >

Write in words.

11. 0.23

12. 0.371

13. 0.071 > Videos

14. 0.0251

15. 12.07

16. 23.056

< Objective 1 >

Write in decimal form.

17. Fifty-one thousandths > Videos

18. Two hundred fifty-three ten-thousandths

19. Seven and three tenths

20. Twelve and two hundred forty-five thousandths

< **Objective 4** >

Write each number as a common fraction or mixed number.

21. 0.65

22. 0.00765

23. 5.231 > Videos

24. 4.0171

< **Objective 5** >

Complete each statement, using the symbols $<$, $=$, *or* $>$.

25. 0.69 __________ 0.689

26. 0.75 __________ 0.752

27. 1.23 __________ 1.230

28. 2.451 __________ 2.45

29. 10 __________ 9.9

30. 4.98 __________ 5

31. 1.459 __________ 1.46 > Videos

32. 0.235 __________ 0.2350

Arrange in order from smallest to largest.

33. 4.0339, 4.034, $4\frac{3}{10}$, $\frac{432}{100}$, 4.33

34. $\frac{38}{1{,}000}$, 0.0382, 0.04, 0.37, $\frac{39}{100}$

35. 0.71, 0.072, $\frac{7}{10}$, 0.007, 0.0069, $\frac{7}{100}$, 0.0701, 0.0619, 0.0712

36. 2.05, $\frac{25}{10}$, 2.0513, 2.059, $\frac{251}{100}$, 2.0515, 2.052, 2.051

< **Objectives 6–7** >

Round to the indicated place.

37. 21.534 hundredths > Videos

38. 5.842 tenths

39. 0.342 hundredths

40. 2.3576 thousandths

41. 2.71828 thousandths

42. 1.543 tenths

43. 0.0475 tenths

44. 0.85356 ten-thousandths

Answers

21. ______________________

22. ______________________

23. ______________________

24. ______________________

25. __________ 26. __________

27. __________ 28. __________

29. __________ 30. __________

31. __________ 32. __________

33. ______________________

34. ______________________

35. ______________________

36. ______________________

37. __________ 38. __________

39. __________ 40. __________

41. __________ 42. __________

43. __________ 44. __________

Answers

45. __________

46. __________

47. __________

48. __________

49. __________

50. __________

51. __________

52. __________

53. __________

54. __________

55. __________

56. __________

57. __________

58. __________

59. __________

60. __________

61. __________

62. __________

63. __________

64. __________

45. 4.85344 ten-thousandths

46. 52.8728 thousandths

47. 6.734 two decimal places

48. 12.5467 three decimal places

49. 6.58739 four decimal places

50. 503.824 two decimal places

Round to the nearest cent or dollar, as indicated.

51. \$235.1457 cent

52. \$1,847.9895 cent

53. \$752.512 dollar

54. \$5,642.4958 dollar

Label each statement as **true** *or* **false.**

55. The only number between 8.6 and 8.8 is 8.7.

56. The smallest number that is greater than 98.6 is 98.7.

57. The place value to the right of the decimal point is tenths.

58. The number 4.586 would always be rounded as 4.6.

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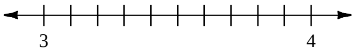
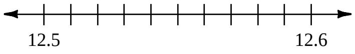
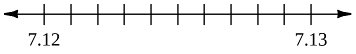
59. Plot (draw a dot on the number line) the following: 3.2 and 3.7. Then estimate the location for 3.62.



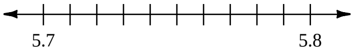
60. Plot the following on a number line: 12.51 and 12.58. Then estimate the location for 12.537.



61. Plot the following on a number line: 7.124 and 7.127. Then estimate the location of 7.1253.



62. Plot the following on a number line: 5.73 and 5.74. Then estimate the location for 5.782.



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63. **ALLIED HEALTH** A nurse calculates a child's dose of Reglan to be 1.53 milligrams (mg). Round this dose to the nearest tenth of a milligram.

64. **ALLIED HEALTH** A nurse calculates a young boy's dose of Dilantin to be 23.375 mg every 5 minutes. Round this dose to the nearest hundredth of a milligram.

65. **ELECTRONICS** Write in decimal form:

- **(a)** Ten and thirty-five hundredths volts (V)
- **(b)** Forty-seven hundred-thousandths of a farad (F)
- **(c)** One hundred fifty-eight ten-thousandths of a henry (H)

66. **INFORMATION TECHNOLOGY** Josie needed to check connectivity of a PC on the network. She uses a tool called ping to see if the PC is configured properly. She receives three readings from ping: 2.1, 2.2, and 2.3 seconds. Convert the decimals to fractions and simplify if needed.

67. **MANUFACTURING** Put the following mill bits in order from smallest to largest:

0.308, 0.297, 0.31, 0.3, 0.311, 0.32

68. **MANUFACTURING** A drill size is listed as $\frac{372}{1{,}000}$. Express this as a decimal.

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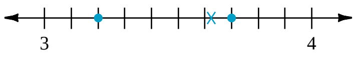
69. **(a)** What is the difference in the values of the following: 0.120, 0.1200, and 0.12000?

(b) Explain in your own words why placing zeros to the right of a decimal point does not change the value of the number.

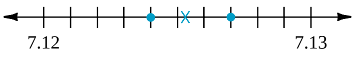
70. Lula wants to round 76.24491 to the nearest hundredth. She first rounds 76.24491 to 76.245 and then rounds 76.245 to 76.25 and claims that this is the final answer. What is wrong with this approach?

Answers

1. Hundredths **3.** Ten-thousandths **5.** 0.23 **7.** 0.0209
9. 23.056 **11.** Twenty-three hundredths **13.** Seventy-one thousandths
15. Twelve and seven hundredths **17.** 0.051 **19.** 7.3
21. $\frac{65}{100}\left(\text{or } \frac{13}{20}\right)$ **23.** $5\frac{231}{1{,}000}$ **25.** $0.69 > 0.689$ **27.** $1.23 = 1.230$
29. $10 > 9.9$ **31.** $1.459 < 1.46$ **33.** 4.0339, 4.034, $4\frac{3}{10}$, $\frac{432}{100}$, 4.33
35. 0.0069, 0.007, 0.0619, $\frac{7}{100}$, 0.0701, 0.0712, 0.072, $\frac{7}{10}$, 0.71 **37.** 21.53
39. 0.34 **41.** 2.718 **43.** 0.0 **45.** 4.8534 **47.** 6.73
49. 6.5874 **51.** $235.15 **53.** $753 **55.** False **57.** True
59.



61.



63. 1.5 mg **65.** **(a)** 10.35 V; **(b)** 0.00047 F; **(c)** 0.0158 H
67. 0.297, 0.3, 0.308, 0.31, 0.311, 0.32 **69.** Above and Beyond

Answers

65. ____________________

66. ____________________

67. ____________________

68. ____________________

69. ____________________

70. ____________________

4.2 Converting Between Fractions and Decimals

< **4.2 Objectives** >

1 > Convert a common fraction to a decimal
2 > Convert a common fraction to a repeating decimal
3 > Convert a mixed number to a decimal
4 > Convert a decimal to a common fraction

Because a common fraction can be interpreted as division, you can divide the numerator of the common fraction by its denominator to convert a common fraction to a decimal. The result is called a **decimal equivalent.**

Example 1 Converting a Fraction to a Decimal Equivalent

< Objective 1 >

RECALL

5 can be written as 5.0, 5.00, 5.000, and so on.

Write $\frac{5}{8}$ as a decimal.

To begin, write a decimal point to the right of 5. The decimal point in the quotient is placed directly above this decimal point. We continue the division process by adding zeros to the right of the decimal point in the dividend until a zero remainder is reached.

$$\begin{array}{r} 0.625 \\ 8\overline{)5.000} \\ \underline{4\,8} \\ 20 \\ \underline{16} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

Because $\frac{5}{8}$ means $5 \div 8$, divide 8 into 5.

We see that $\frac{5}{8} = 0.625$; 0.625 is the decimal equivalent of $\frac{5}{8}$.

Check Yourself 1

Find the decimal equivalent of $\frac{7}{8}$.

Some fractions are used so often that we have listed their decimal equivalents here as a reference.

NOTE

The division used to find these decimal equivalents stops when a zero remainder is reached. The equivalents are called **terminating decimals.**

Some Common Decimal Equivalents

$\frac{1}{2} = 0.5$	$\frac{1}{4} = 0.25$	$\frac{1}{5} = 0.2$	$\frac{1}{8} = 0.125$
	$\frac{3}{4} = 0.75$	$\frac{2}{5} = 0.4$	$\frac{3}{8} = 0.375$
		$\frac{3}{5} = 0.6$	$\frac{5}{8} = 0.625$
		$\frac{4}{5} = 0.8$	$\frac{7}{8} = 0.875$

If a decimal equivalent does not terminate, you can round the result to approximate the fraction to some specified number of decimal places. Consider Example 2.

Example 2 Converting a Fraction to a Decimal Equivalent

Write $\frac{3}{7}$ as a decimal. Round the answer to the nearest thousandth.

$$\begin{array}{r} 0.4285 \\ 7\overline{)3.0000} \\ \underline{2\,8} \\ 20 \\ \underline{14} \\ 60 \\ \underline{56} \\ 40 \\ \underline{35} \\ 5 \end{array}$$

In this example, we are choosing to round to three decimal places, so we must add enough zeros to carry the division to four decimal places.

So $\frac{3}{7} = 0.429$ (to the nearest thousandth)

Check Yourself 2

Find the decimal equivalent of $\frac{5}{11}$ to the nearest thousandth.

If the decimal equivalent of a fraction does *not* terminate, it will *repeat* a sequence of digits. These decimals are called **repeating decimals.**

Example 3 Converting a Fraction to a Repeating Decimal

< Objective 2 >

(a) Write $\frac{1}{3}$ as a decimal.

$$\begin{array}{r} 0.333 \\ 3\overline{)1.000} \\ \underline{9} \\ 10 \\ \underline{9} \\ 10 \\ \underline{9} \end{array}$$

The digit 3 will just repeat indefinitely because each new remainder will be 1.

Adding more zeros and going on will simply lead to more 3s in the quotient.

We can say $\frac{1}{3} = 0.333\ldots$

The three dots mean "and so on" and tell us that 3 will repeat itself indefinitely.

(b) Write $\frac{5}{12}$ as a decimal.

$$\begin{array}{r} 0.4166\ldots \\ 12\overline{)5.0000} \\ \underline{4\,8} \\ 20 \\ \underline{12} \\ 80 \\ \underline{72} \\ 80 \\ \underline{72} \\ 8 \end{array}$$

In this example, the digit 6 will just repeat itself because the remainder, 8, will keep occurring if we add more zeros and continue the division.

so $\frac{5}{12} = 0.4166\ldots$

Check Yourself 3

Find the decimal equivalent of each fraction.

(a) $\frac{2}{3}$ **(b)** $\frac{7}{12}$

Some important decimal equivalents (rounded to the nearest thousandth) are shown here as a reference.

$\frac{1}{3} = 0.333$ $\frac{1}{6} = 0.167$ $\frac{2}{3} = 0.667$ $\frac{5}{6} = 0.833$

Another way to write a repeating decimal is with a bar placed over the digit or digits that repeat. For example, we can write

0.37373737 . . .

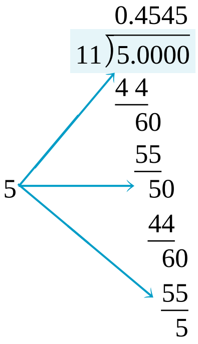
as

$0.\overline{37}$

The bar placed over the digits indicates that 37 repeats indefinitely.

Example 4 Converting a Fraction to a Repeating Decimal

Write $\frac{5}{11}$ as a decimal.



As soon as a remainder repeats itself, as 5 does here, the pattern of digits will repeat in the quotient.

$\frac{5}{11} = 0.\overline{45}$

$= 0.4545\ldots$

Check Yourself 4

Use the bar notation to write the decimal equivalent of $\frac{5}{7}$. (Be patient. You'll have to divide for a while to find the repeating pattern.)

You can find the decimal equivalents for mixed numbers in a similar way. Find the decimal equivalent of the fractional part of the mixed number and then combine that with the whole-number part. Example 5 illustrates this approach.

Example 5 Converting a Mixed Number to a Decimal Equivalent

< Objective 3 >

Find the decimal equivalent of $3\frac{5}{16}$.

$\frac{5}{16} = 0.3125$ First find the equivalent of $\frac{5}{16}$ by division.

$3\frac{5}{16} = 3.3125$ Add 3 to the result.

Check Yourself 5

Find the decimal equivalent of $2\frac{5}{8}$.

We have learned something important in this section. To find the decimal equivalent of a fraction, we use long division. Because the remainder must be less than the divisor, the remainder must *either repeat or become zero.* Thus, *every common fraction* has a *repeating* or a *terminating* decimal as its decimal equivalent.

Next, using what we learned about place values, you can write decimals as common fractions. The following rule is used.

Step by Step

To Convert a Terminating Decimal Less Than 1 to a Common Fraction

Step 1 Write the digits of the decimal without the decimal point. This will be the numerator of the common fraction.

Step 2 The denominator of the fraction is a 1 followed by as many zeros as there are places in the decimal.

Example 6 Converting a Decimal to a Common Fraction

< Objective 4 >

$$0.\underbrace{7}_{\text{One place}} = \frac{7}{\underbrace{10}_{\text{One zero}}} \qquad 0.\underbrace{09}_{\text{Two places}} = \frac{9}{\underbrace{100}_{\text{Two zeros}}} \qquad 0.\underbrace{257}_{\text{Three places}} = \frac{257}{\underbrace{1{,}000}_{\text{Three zeros}}}$$

Check Yourself 6

Write as common fractions.

(a) 0.3 (b) 0.311

When a decimal is converted to a common fraction, that resulting common fraction should be written in lowest terms.

Example 7 Converting a Decimal to a Common Fraction

NOTE

Divide the numerator and denominator by 5.

Convert 0.395 to a fraction and write the result in lowest terms.

$$0.395 = \frac{395}{1{,}000} = \frac{79}{200}$$

Check Yourself 7

Write 0.275 as a common fraction.

If the decimal has a whole-number portion, write the digits to the right of the decimal point as a proper fraction and then form a mixed number for your result.

Example 8 Converting a Decimal to a Mixed Number

Write 12.277 as a mixed number.

$$0.277 = \frac{277}{1{,}000}$$

NOTE

Repeating decimals can also be written as common fractions, although the process is more complicated. We will limit ourselves to the conversion of terminating decimals in this textbook.

so

$$12.277 = 12\frac{277}{1{,}000}$$

Check Yourself 8

Write 32.433 as a mixed number.

Comparing the sizes of common fractions and decimals requires finding the decimal equivalent of the common fraction and then comparing the resulting decimals.

Example 9 Comparing the Sizes of Common Fractions and Decimals

RECALL

0.38 can be written as 0.380. Comparing this to 0.375, we see that 0.380 > 0.375.

Which is larger, $\frac{3}{8}$ or 0.38?

Write the decimal equivalent of $\frac{3}{8}$. That decimal is 0.375. Now comparing 0.375 and 0.38, we see that 0.38 is the larger of the numbers:

$$0.38 > \frac{3}{8}$$

Check Yourself 9

Which is larger, $\frac{3}{4}$ or 0.8?

Check Yourself ANSWERS

1. 0.875 **2.** $\frac{5}{11} = 0.455$ (to the nearest thousandth)

3. **(a)** 0.666 . . . ; **(b)** 0.583 . . . The digit 3 will continue indefinitely.

4. $\frac{5}{7} = 0.\overline{714285}$ **5.** 2.625 **6.** **(a)** $\frac{3}{10}$; **(b)** $\frac{311}{1{,}000}$ **7.** $\frac{11}{40}$

8. $32\frac{433}{1{,}000}$ **9.** $0.8 > \frac{3}{4}$

Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 4.2

(a) You can ______________ the numerator of a fraction by its denominator to convert a common fraction to a decimal.

(b) We can write a repeating decimal with a ______________ placed over the digit or digits that repeat.

(c) Every common fraction has a repeating or a ______________ decimal as its equivalent.

(d) The denominator of a fraction that is equivalent to a given decimal is a 1 followed by as many zeros as there are ______________ in the decimal.

4.2 exercises

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< Objective 1 >

Find the decimal equivalent for each fraction.

1. $\frac{3}{4}$

2. $\frac{4}{5}$

3. $\frac{9}{20}$

4. $\frac{3}{10}$

5. $\frac{1}{5}$

6. $\frac{1}{8}$

7. $\frac{5}{16}$

8. $\frac{11}{20}$

9. $\frac{7}{10}$ > Videos

10. $\frac{7}{16}$

11. $\frac{27}{40}$

12. $\frac{17}{32}$

Find the decimal equivalents rounded to the indicated place.

13. $\frac{5}{6}$ thousandths > Videos

14. $\frac{7}{12}$ hundredths

15. $\frac{4}{15}$ thousandths

< Objective 2 >

Write the decimal equivalents, using the bar notation.

16. $\frac{1}{18}$

17. $\frac{4}{9}$

18. $\frac{3}{11}$

< Objective 3 >

Find the decimal equivalent for each mixed number.

19. $5\frac{3}{5}$ > Videos

20. $4\frac{7}{16}$

< Objective 4 >

Write each number as a common fraction or mixed number, in simplest terms.

21. 0.9

22. 0.3

23. 0.8

24. 0.6

25. 0.37

26. 0.97

27. 0.587

28. 0.379

29. 0.48 > Videos

30. 0.75

Answers

1. __________ **2.** __________

3. __________ **4.** __________

5. __________ **6.** __________

7. __________ **8.** __________

9. __________ **10.** __________

11. __________ **12.** __________

13. __________ **14.** __________

15. __________ **16.** __________

17. __________ **18.** __________

19. __________ **20.** __________

21. __________ **22.** __________

23. __________ **24.** __________

25. __________ **26.** __________

27. __________ **28.** __________

29. __________ **30.** __________

Answers

31. ______________________

32. ______________________

33. ______________________

34. ______________________

35. ______________________

36. ______________________

37. ______________________

38. ______________________

39. ______________________

40. ______________________

41. ______________________

42. ______________________

43. ______________________

44. ______________________

31. 0.58

32. 0.65

33. 0.425

34. 0.116

35. 0.375

36. 0.225

37. 0.136

38. 0.575

39. 0.059 > Videos

40. 0.067

Basic Skills | **Challenge Yourself** | Calculator/Computer | Career Applications | Above and Beyond

41. **STATISTICS** In a weekend baseball tournament, Joel had 4 hits in 13 times at bat. That is, he hit safely $\frac{4}{13}$ of the time. Write the decimal equivalent for Joel's hitting, rounding to three decimal places. (That number is Joel's batting average.)

42. **STATISTICS** The following table gives the wins and losses of some teams in the National League East as of mid-September in a recent season. The winning percentage of each team is calculated by writing the number of wins over the total games played and converting this fraction to a decimal. Convert this fraction to a decimal for every team, rounding to three decimal places.

Team	Wins	Losses
Atlanta	92	56
New York	90	58
Philadelphia	70	77
Florida	57	89

43. **STATISTICS** The following table gives the wins and losses of all the teams in the Western Division of the National Football Conference for a recent season. Determine the fraction of wins over total games played for every team, rounding to three decimal places for each of the teams.

Team	Wins	Losses
San Francisco	10	6
St. Louis	7	9
Seattle	7	9
Arizona	5	11

44. **STATISTICS** The following table gives the free throws attempted (FTA) and the free throws made (FTM) for the top five players in the NBA for a recent season. Calculate the free throw percentage for each player by writing the FTM over the FTA and converting this fraction to a decimal. Round to three decimal places.

Player	FTM	FTA
Allan Houston	363	395
Ray Allen	316	345
Steve Nash	308	339
Troy Hudson	208	231
Reggie Miller	207	230

Find the decimal equivalent for each fraction. Use the bar notation.

45. $\frac{1}{11}$ **46.** $\frac{1}{111}$ **47.** $\frac{1}{1,111}$

48. From the pattern of exercises 45 to 47, can you guess the decimal representation for $\frac{1}{11,111}$?

Insert $>$ or $<$ to form a true statement.

49. $\frac{31}{34}$ ☐ 0.9118 **50.** $\frac{21}{37}$ ☐ 0.5664 **51.** $\frac{13}{17}$ ☐ 0.7657

52. $\frac{7}{8}$ ☐ 0.87 > Videos **53.** $\frac{5}{16}$ ☐ 0.313 **54.** $\frac{9}{25}$ ☐ 0.4

Basic Skills | Challenge Yourself | **Calculator/Computer** | Career Applications | Above and Beyond

A calculator is very useful in converting common fractions to decimals. Just divide the numerator by the denominator, and the decimal equivalent will be in the display. Often, you will have to round the result in the display. For example, to find the decimal equivalent of $\frac{5}{24}$ to the nearest hundredth, enter

$5 \boxed{\div} 24 \boxed{=}$

The display may show 0.2083333, and rounding, we have $\frac{5}{24} = 0.21$.

When you are converting a mixed number to a decimal, addition must also be used. To change $7\frac{5}{8}$ to a decimal, for example, enter

$7 \boxed{+} 5 \boxed{\div} 8 \boxed{=}$

The result is 7.625.

Use your calculator to find the decimal equivalents.

55. $\frac{7}{8}$ **56.** $\frac{9}{16}$

57. $\frac{5}{32}$ to the thousandth **58.** $\frac{11}{75}$ to the thousandth

59. $\frac{3}{11}$ using bar notation **60.** $\frac{16}{33}$ using bar notation

61. $3\frac{7}{8}$ **62.** $8\frac{3}{16}$

Answers

45. ____________
46. ____________
47. ____________
48. ____________
49. ____________
50. ____________
51. ____________
52. ____________
53. ____________
54. ____________
55. ____________
56. ____________
57. ____________
58. ____________
59. ____________
60. ____________
61. ____________
62. ____________

Answers

63. ____________________

64. ____________________

65. ____________________

66. ____________________

67. ____________________

Basic Skills | Challenge Yourself | Calculator/Computer | **Career Applications** | Above and Beyond

63. ALLIED HEALTH The internal diameter, in millimeters (mm), of an endotracheal tube for a child is calculated using the formula $\frac{\text{Height}}{20}$, which is based on the child's height in centimeters (cm). Determine the size of endotracheal tube needed for a girl who is 110 cm tall.

64. ALLIED HEALTH The stroke volume, which measures the average cardiac output per heartbeat (liters/beat), is based on a patient's cardiac output (CO), in liters per minute (L/min), and heart rate (HR), in beats per minute (beats/min). It is calculated using the fraction $\frac{\text{CO}}{\text{HR}}$. Determine the stroke volume for a patient whose cardiac output is 4 L/min and whose heart rate is 80 beats/min. Write your answer as a decimal.

65. INFORMATION TECHNOLOGY The propagation delay for a satellite connection is 0.350 seconds (s). Convert to a fraction and simplify.

66. INFORMATION TECHNOLOGY From your computer, it takes 0.0021 s to transmit a ping packet to another computer. Convert to a fraction and simplify.

Basic Skills | Challenge Yourself | Calculator/Computer | Career Applications | **Above and Beyond**

67. Write the decimal equivalent of each fraction, using bar notation.

$$\frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}$$

Describe any patterns that you see. Predict the decimal equivalent of $\frac{6}{7}$.

Answers

1. 0.75 **3.** 0.45 **5.** 0.2 **7.** 0.3125 **9.** 0.7 **11.** 0.675

13. 0.833 **15.** 0.267 **17.** $0.\overline{4}$ **19.** 5.6 **21.** $\frac{9}{10}$ **23.** $\frac{4}{5}$

25. $\frac{37}{100}$ **27.** $\frac{587}{1{,}000}$ **29.** $\frac{12}{25}$ **31.** $\frac{29}{50}$ **33.** $\frac{17}{40}$ **35.** $\frac{3}{8}$

37. $\frac{17}{125}$ **39.** $\frac{59}{1{,}000}$ **41.** 0.308 **43.** 0.625, 0.438, 0.438, 0.313

45. $0.\overline{09}$ **47.** $0.\overline{0009}$ **49.** $<$ **51.** $<$ **53.** $<$ **55.** 0.875

57. 0.156 **59.** $0.\overline{27}$ **61.** 3.875 **63.** 5.5 mm **65.** $\frac{7}{20}$

67. Above and Beyond

Activity 10 ::
Terminate or Repeat?

Every fraction has a decimal equivalent that either terminates (for example, $\frac{1}{4} = 0.25$) or repeats (for example, $\frac{2}{9} = 0.\overline{2}$). Work with a group to discover which fractions have terminating decimals and which have repeating decimals. You may assume that the numerator of each fraction you consider is one and focus your attention on the denominator. As you complete the following table, you will find that the key to this question lies with the prime factorization of the denominator.

Fraction	Decimal Form	Terminate?	Prime Factorization of the Denominator
$\frac{1}{2}$			
$\frac{1}{3}$			
$\frac{1}{4}$			
$\frac{1}{5}$			
$\frac{1}{6}$			
$\frac{1}{7}$			
$\frac{1}{8}$			
$\frac{1}{9}$			
$\frac{1}{10}$			
$\frac{1}{11}$			
$\frac{1}{12}$			

State a general rule describing which fractions have decimal forms that terminate and which have decimal forms that repeat.

Now test your rule on at least three new fractions. That is, be able to predict whether a fraction such as $\frac{1}{25}$ or $\frac{1}{30}$ has a terminating decimal or a repeating decimal. Then confirm your prediction.

4.3 Adding and Subtracting Decimals

< **4.3 Objectives** >

1 > Add two or more decimals
2 > Use addition of decimals to solve application problems
3 > Subtract one decimal from another
4 > Use subtraction of decimals to solve application problems

Working with decimals rather than common fractions makes the basic operations much easier. First we will look at addition. One method for adding decimals is to write the decimals as common fractions, add, and then change the sum back to a decimal.

$$0.34 + 0.52 = \frac{34}{100} + \frac{52}{100} = \frac{86}{100} = 0.86$$

It is much more efficient to leave the numbers in decimal form and perform the addition in the same way as we did with whole numbers. You can use the following rule.

Step by Step

To Add Decimals

Step 1 Write the numbers being added in column form *with their decimal points in a vertical line.*

Step 2 Add just as you would with whole numbers.

Step 3 Place the decimal point of the sum in line with the decimal points of the addends.

Example 1 illustrates the use of this rule.

Example 1 Adding Decimals

< Objective 1 >

NOTE

Placing the decimal points in a vertical line ensures that we are adding digits of the same place value.

Add 0.13, 0.42, and 0.31.

$$\begin{array}{r} 0.13 \\ 0.42 \\ +\ 0.31 \\ \hline 0.86 \end{array}$$

Check Yourself 1

Add 0.23, 0.15, and 0.41.

When adding decimals, you can use the *carrying process* just as you did when adding whole numbers. Consider the following.

Example 2 Adding Decimals by Carrying

Add 0.35, 1.58, and 0.67.

$$\begin{array}{r} {\scriptstyle 1\ 2}\ \ \ \\ 0.35 \\ 1.58 \\ +\ 0.67 \\ \hline 2.60 \end{array} \quad \longleftarrow \text{Carries}$$

In the hundredths column:
5 + 8 + 7 = 20
Write 0 and carry 2 to the tenths column.
In the tenths column:
2 + 3 + 5 + 6 = 16
Write 6 and carry 1 to the ones column.

Note: The carrying process works with decimals, just as it did with whole numbers, because each place value is again $\frac{1}{10}$ the value of the place to its left.

Check Yourself 2

Add 23.546, 0.489, 2.312, and 6.135.

When adding decimals, the numbers may not have the same number of decimal places. Just fill in as many zeros as needed so that all the numbers added have the same number of decimal places.

Recall that adding zeros to the right *does not change* the value of a decimal. The number 0.53 is the same as 0.530. We see this in Example 3.

Example 3 Adding Decimals

NOTE

Be sure that the decimal points are in a vertical line.

Add 0.53, 4, 2.7, and 3.234.

$$\begin{array}{r@{}l} & 0.53 \\ & 4. \\ & 2.7 \\ + & 3.234 \\ \hline \end{array}$$

Note that for a whole number, the decimal is understood to be to its right. So 4 = 4.

Now fill in the missing zeros and add as before.

$$\begin{array}{r@{}l} & 0.530 \\ & 4.000 \\ & 2.700 \\ + & 3.234 \\ \hline & 10.464 \end{array}$$

Now all the numbers being added have *three* decimal places.

Check Yourself 3

Add 6, 2.583, 4.7, and 2.54.

Many applied problems require working with decimals. For instance, filling up at a gas station means reading decimal amounts.

Example 4 An Application of the Addition of Decimals

< Objective 2 >

NOTE

Because we want a total amount, we use addition to find the solution.

On a trip the Chang family kept track of the gas purchases. If they bought 12.3, 14.2, 10.7, and 13.8 gallons (gal), how much gas did they use on the trip?

$$\begin{array}{r@{}l} & 12.3 \\ & 14.2 \\ & 10.7 \\ + & 13.8 \\ \hline & 51.0 \text{ gal} \end{array}$$

Check Yourself 4

The Higueras kept track of the gasoline they purchased on a recent trip. If they bought 12.4, 13.6, 9.7, 11.8, and 8.3 gal, how much gas did they buy on the trip?

Every day you deal with amounts of money. Because our system of money is a decimal system, most problems involving money also involve operations with decimals.

Example 5 An Application of the Addition of Decimals

Andre makes deposits of \$3.24, \$15.73, \$50, \$28.79, and \$124.38 during May. What is the total of his deposits for the month?

$$\begin{array}{r}
\$\quad 3.24 \\
15.73 \\
50.00 \\
28.79 \\
+\ 124.38 \\
\hline
\$222.14
\end{array}$$

Simply add the amounts of money deposited as decimals. Note that we write \$50 as \$50.00.

← The total of deposits for May

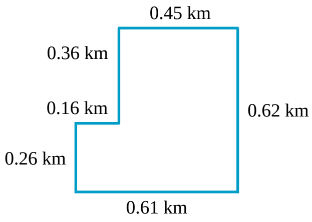
Check Yourself 5

Your textbooks for the fall term cost \$63.50, \$78.95, \$43.15, \$82, and \$85.85. What was the total cost of textbooks for the term?

In Chapter 1, we defined *perimeter* as the distance around the outside of a straight-edged shape. Finding the perimeter often requires that we add decimal numbers.

Example 6 An Application Involving the Addition of Decimals

Rachel is going to put a fence around the perimeter of her farm. The figure represents the land, measured in kilometers (km). How much fence does she need to buy?



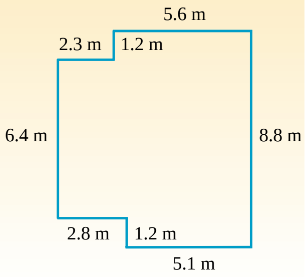
The perimeter is the sum of the lengths of the sides, so we add those lengths to find the total fencing needed.

$$0.26 + 0.16 + 0.36 + 0.45 + 0.62 + 0.61 = 2.46$$

Rachel needs 2.46 km of fence for the perimeter of her farm.

Check Yourself 6

Manuel intends to build a walkway around the perimeter of his garden. What will the total length of the walkway be?



Much of what we have said about adding decimals is also true of subtraction. To subtract decimals, we use the following rule:

Step by Step

To Subtract Decimals

Step 1 Write the numbers being subtracted in column form *with their decimal points in a vertical line.*

Step 2 Subtract just as you would with whole numbers.

Step 3 Place the decimal point of the difference in line with the decimal points of the numbers being subtracted.

Example 7 illustrates the use of this rule.

Example 7 Subtracting a Decimal

< Objective 3 >

RECALL

The number that follows the word *from,* here 3.58, is written first. The number we are subtracting, here 1.23, is then written beneath 3.58.

Subtract 1.23 from 3.58.

$$\begin{array}{r} 3.58 \\ -\;1.23 \\ \hline 2.35 \end{array}$$

Subtract in the hundredths, the tenths, and then the ones columns.

Check Yourself 7

Subtract 9.87 − 5.45.

Because each place value is $\frac{1}{10}$ the value of the place to its left, borrowing, when you are subtracting decimals, works just as it did in subtracting whole numbers.

Example 8 Decimal Subtraction That Involves Borrowing

Subtract 1.86 from 6.54.

$$\begin{array}{r} 5\,{}^{1}4_{1} \\ \cancel{6}.\cancel{5}4 \\ -\;1.86 \\ \hline 4.68 \end{array}$$

Here, borrow from the tenths and ones places to do the subtraction.

Check Yourself 8

Subtract 35.35 − 13.89.

In subtracting decimals, as in adding, we can add zeros to the right of the decimal point so that both decimals have the same number of decimal places.

Example 9 Subtracting a Decimal

(a) Subtract 2.36 from 7.5.

$$\begin{array}{r} 4\;1 \\ 7.\cancel{5}\,0 \\ -\;2.36 \\ \hline 5.14 \end{array}$$

We have added a 0 to 7.5. Next, borrow 1 tenth from the 5 tenths in the minuend.

NOTES

When you are subtracting, align the decimal points, and then add zeros to the right to align the digits.

9 has been rewritten as 9.000.

(b) Subtract 3.657 from 9.

$$\begin{array}{r} 8\;99 \\ 1\,1\,1 \\ \cancel{9}.\cancel{0}\cancel{0}0 \\ -\;3.657 \\ \hline 5.343 \end{array}$$

In this case, move left to the ones place to begin the borrowing process.

Check Yourself 9

Subtract 5 − 2.345.

We can apply the subtraction methods of the previous examples in solving applications.

Example 10 An Application of the Subtraction of a Decimal Number

< Objective 4 >

NOTE

We want to find the difference between the two measurements, so we subtract.

Jonathan was 98.3 centimeters (cm) tall on his sixth birthday. On his seventh birthday he was 104.2 cm. How much did he grow during the year?

$$\begin{array}{r} 104.2\text{ cm} \\ -\ \ 98.3\text{ cm} \\ \hline 5.9\text{ cm} \end{array}$$

Jonathan grew 5.9 cm during the year.

Check Yourself 10

A car's highway mileage before a tune-up was 28.8 miles per gallon (mi/gal). After the tune-up, it measured 30.1 mi/gal. What was the increase in mileage?

The same methods can be used in working with money.

Example 11 An Application of the Subtraction of a Decimal Number

NOTE

Sally's change is the *difference* between the price of the roast and the $20 paid. We must use subtraction for the solution.

At the grocery store, Sally buys a roast that is marked $12.37. She pays for her purchase with a $20 bill. How much change does she receive?

$$\begin{array}{r} \$20.00 \\ -\ 12.37 \\ \hline \$\ \ 7.63 \end{array}$$

Add zeros to write $20 as $20.00. Then subtract as before.

Sally receives $7.63 in change after her purchase.

Check Yourself 11

A stereo system that normally sells for $549.50 is discounted (or marked down) to $499.95 for a sale. What is the savings?

Balancing a checkbook requires addition and subtraction of decimal numbers.

Example 12 An Application Involving the Addition and Subtraction of Decimals

For the following check register, find the running balance.

Beginning balance	$234.15
Check # 301	23.88
Balance	______
Check # 302	38.98
Balance	______
Check # 303	114.66
Balance	______
Deposit	175.75
Balance	______
Check # 304	212.55
Ending balance	______

To keep a running balance, we add the deposits and subtract the checks.

Beginning balance	$234.15
Check # 301	23.88
Balance	210.27
Check # 302	38.98
Balance	171.29
Check # 303	114.66
Balance	56.63
Deposit	175.75
Balance	232.38
Check # 304	212.55
Ending balance	19.83

Check Yourself 12

For the following check register, add the deposits and subtract the checks to find the balance.

	Beginning balance	$398.00
	Check # 401	19.75
(a)	Balance	______
	Check # 402	56.88
(b)	Balance	______
	Check # 403	117.59
(c)	Balance	______
	Deposit	224.67
(d)	Balance	______
	Check # 404	411.48
(e)	Ending balance	______

Check Yourself ANSWERS

1. 0.79 **2.** 32.482 **3.**

$$\begin{array}{r} 6.000 \\ 2.583 \\ 4.700 \\ +\ 2.540 \\ \hline 15.823 \end{array}$$

4. 55.8 gal **5.** \$353.45 **6.** 33.4 m

7. 4.42 **8.** 21.46 **9.** 2.655 **10.** 1.3 mi/gal **11.** \$49.55

12. **(a)** \$378.25; **(b)** \$321.37; **(c)** \$203.78; **(d)** \$428.45; **(e)** \$16.97

Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 4.3

(a) To add decimals, write the numbers being added in column form with their ________ ________ in a vertical line.

(b) Adding zeros to the right does not change the ________ of a decimal.

(c) ________ is the distance around the outside of a straight-edged shape.

(d) When subtracting one number from another, the number ________ the word *from* is written first.

4.3 exercises

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Section __________ Date __________

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< Objective 1 >

Add.

1. $\begin{array}{r} 0.28 \\ +\ 0.79 \\ \hline \end{array}$ > Videos

2. $\begin{array}{r} 2.59 \\ +\ 0.63 \\ \hline \end{array}$

3. $\begin{array}{r} 13.58 \\ 7.239 \\ +\quad 1.5 \\ \hline \end{array}$

4. $\begin{array}{r} 8.625 \\ 2.45 \\ +\ 12.6 \\ \hline \end{array}$

5. $\begin{array}{r} 25.3582 \\ 6.5 \\ 1.898 \\ +\quad 0.69 \\ \hline \end{array}$

6. $\begin{array}{r} 1.336 \\ 15.6857 \\ 7.9 \\ +\ 0.85 \\ \hline \end{array}$

7. $0.43 + 0.8 + 0.561$ > Videos

8. $1.25 + 0.7 + 0.259$

9. $42.731 + 1.058 + 103.24$

10. $27.4 + 213.321 + 39.38$

< Objective 3 >

Subtract.

11. $\begin{array}{r} 0.85 \\ -\ 0.59 \\ \hline \end{array}$

12. $\begin{array}{r} 5.68 \\ -\ 2.65 \\ \hline \end{array}$

13. $\begin{array}{r} 3.82 \\ -\ 1.565 \\ \hline \end{array}$

14. $\begin{array}{r} 8.59 \\ -\ 5.6 \\ \hline \end{array}$

15. $\begin{array}{r} 7.02 \\ -\ 4.7 \\ \hline \end{array}$

16. $\begin{array}{r} 45.6 \\ -\quad 8.75 \\ \hline \end{array}$

17. $\begin{array}{r} 12 \\ -\quad 5.35 \\ \hline \end{array}$ > Videos

18. $\begin{array}{r} 15 \\ -\quad 8.85 \\ \hline \end{array}$

19. Subtract 2.87 from 6.84. > Videos

20. Subtract 3.69 from 10.57.

21. Subtract 7.75 from 9.4.

22. Subtract 5.82 from 12.

23. Subtract 0.24 from 5.

24. Subtract 8.7 from 16.32.

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< Objectives 2, 4 >

Solve each application.

25. **BUSINESS AND FINANCE** On a 3-day trip, Dien bought 12.7, 15.9, and 13.8 gallons (gal) of gas. How many gallons of gas did he buy? > Videos

Answers

1. ____________ 2. ____________
3. ____________ 4. ____________
5. ____________ 6. ____________
7. ____________ 8. ____________
9. ____________ 10. ____________
11. ____________ 12. ____________
13. ____________ 14. ____________
15. ____________ 16. ____________
17. ____________ 18. ____________
19. ____________ 20. ____________
21. ____________ 22. ____________
23. ____________ 24. ____________
25. ____________

Answers

26. ____________________

27. ____________________

28. ____________________

29. ____________________

30. ____________________

31. ____________________

32. ____________________

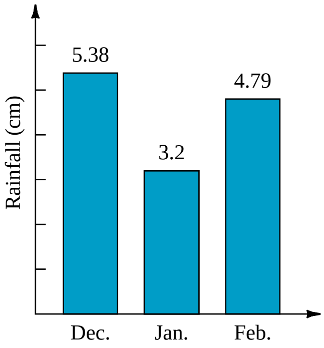
33. ____________________

26. **SCIENCE AND MEDICINE** Felix ran 2.7 miles (mi) on Monday, 1.9 mi on Wednesday, and 3.6 mi on Friday. How far did he run during the week?

27. **SOCIAL SCIENCE** Rainfall was recorded in centimeters (cm) during the winter months, as indicated on the bar graph.

(a) How much rain fell during those months?

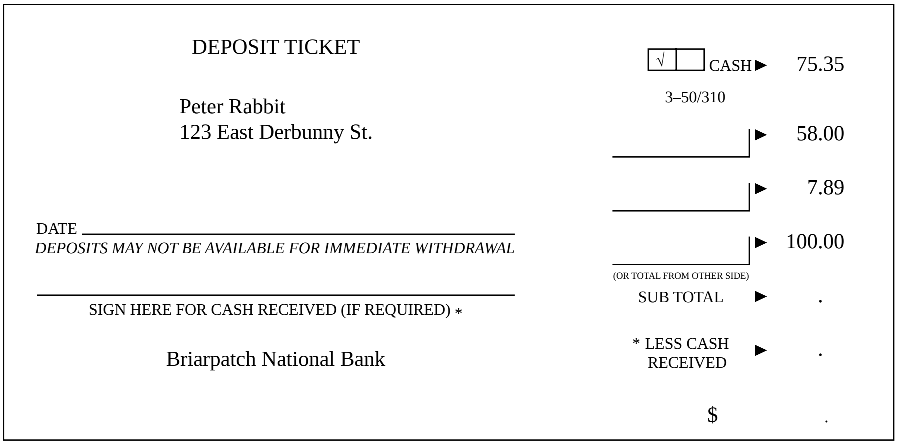
(b) How much more rain fell in December than in February?



28. **CONSTRUCTION** A metal fitting has three sections, with lengths 2.5, 1.775, and 1.45 inches (in.). What is the total length of the fitting?

29. **BUSINESS AND FINANCE** Nicole had the following expenses on a business trip: gas, $45.69; food, $123; lodging, $95.60; and parking and tolls, $8.65. What were her total expenses during the trip?

30. **BUSINESS AND FINANCE** The deposit slip shown indicates the amounts that made up a deposit Peter Rabbit made. What was the total amount of his deposit?



DEPOSIT TICKET

☑ CASH ► 75.35

3–50/310

Peter Rabbit
123 East Derbunny St.

► 58.00

► 7.89

► 100.00

DATE ____________________

DEPOSITS MAY NOT BE AVAILABLE FOR IMMEDIATE WITHDRAWAL

(OR TOTAL FROM OTHER SIDE)

SUB TOTAL ► .

SIGN HERE FOR CASH RECEIVED (IF REQUIRED) *

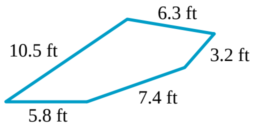
* LESS CASH RECEIVED ► .

Briarpatch National Bank

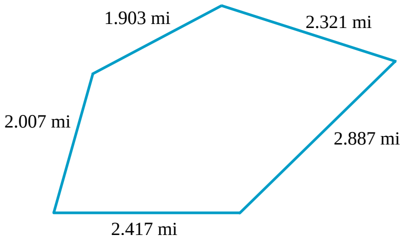
$.

31. **CONSTRUCTION** Lupe is putting a fence around her yard. Her yard is rectangular and measures 8.16 yards (yd) long and 12.68 yd wide. How much fence should Lupe purchase?

32. **GEOMETRY** Find the perimeter of the given figure.



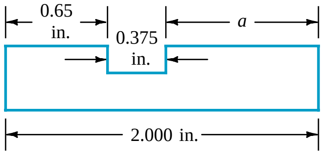
33. **CONSTRUCTION** The figure gives the distance in miles (mi) of the boundary sections around a ranch. How much fencing is needed for the property?



34. **Business and Finance** A television set selling for $399.50 is discounted (or marked down) to $365.75. What is the savings?

35. **Crafts** The outer radius of a piece of tubing is 2.8325 inches (in.). The inner radius is 2.775 in. What is the thickness of the wall of the tubing?

36. **Geometry** Given the following figure, find dimension *a*.



37. **Business and Finance** You make charges of $37.25, $8.78, and $53.45 on a credit card. If you make a payment of $73.50, how much do you still owe?

38. **Business and Finance** For the following check register, find the running balance.

Beginning balance	$896.74
Check # 501	$425.69
Balance	______
Check # 502	$ 56.34
Balance	______
Check # 503	$ 41.89
Balance	______
Deposit	$123.91
Balance	______
Check # 504	$356.98
Ending balance	______

39. **Business and Finance** For the following check register, find the running balance.

Beginning balance	$456.00
Check # 601	$199.29
Balance	______
Service charge	$ 18.00
Balance	______
Check # 602	$ 85.78
Balance	______
Deposit	$250.45
Balance	______
Check # 603	$201.24
Ending balance	______

Answers

34. ______

35. ______

36. ______

37. ______

38. ______

39. ______

Answers

40. ____________

41. ____________

42. ____________

40. Business and Finance For the following check register, find the running balance.

Beginning balance	$589.21
Check # 678	$175.63
Balance	
Check # 679	$ 56.92
Balance	
Deposit	$121.12
Balance	
Check # 680	$345.99
Ending balance	

41. Business and Finance For the following check register, find the running balance.

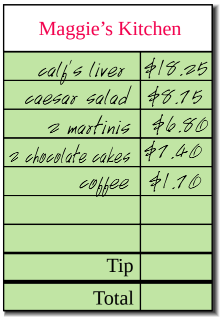
Beginning balance	$1,345.23
Check # 821	$ 234.99
Balance	
Check # 822	$ 555.77
Balance	
Deposit	$ 126.77
Balance	
Check # 823	$ 53.89
Ending balance	

Estimation can be a useful tool when working with decimal fractions. To estimate a sum, one approach is to round the addends to the nearest whole number and add for your estimate. For instance, to estimate the following sum:

$$\begin{array}{r} 19.8 \\ 3.5 \\ 24.2 \\ +\ 10.4 \\ \hline \end{array} \quad \xrightarrow{\text{Round}} \quad \begin{array}{r} 20 \\ 4 \\ 24 \\ +\ 10 \\ \hline 58 \end{array} \quad \text{Add to get the estimate}$$

Use estimation to solve this application.

42. Business and Finance Alem's restaurant bill is pictured here. Estimate his total by rounding to the nearest dollar.



Maggie's Kitchen

calf's liver	$18.25
caesar salad	$8.15
2 martinis	$6.80
2 chocolate cakes	$7.40
coffee	$1.10
Tip	
Total	

Basic Skills | Challenge Yourself | **Calculator/Computer** | Career Applications | Above and Beyond

Entering decimals in your calculator is similar to entering whole numbers. There is just one difference: The decimal point key $\boxed{\bullet}$ is used to place the decimal point as you enter the number.

Often both addition and subtraction are involved in a calculation. In this case, just enter the decimals and the operation signs, + or −, as they appear in the problem. To find 23.7 − 5.2 + 3.87 − 2.341, enter

$23.7\ \boxed{-}\ 5.2\ \boxed{+}\ 3.87\ \boxed{-}\ 2.341\ \boxed{=}$

The display should show 20.029.

Solve each exercise, using your calculator.

43. 10,345.2 + 2,308.35 + 153.58

44. 8.7675 + 2.8 − 3.375 − 6

Solve each application, using your calculator.

45. **Business and Finance** Your checking account has a balance of $532.89. You write checks of $50, $27.54, and $134.75 and make a deposit of $50. What is your ending balance?

46. **Business and Finance** Your checking account has a balance of $278.45. You make deposits of $200 and $135.46. You write checks for $389.34, $249, and $53.21. What is your ending balance? Be careful with this exercise. A negative balance means that your account is overdrawn.

47. **Business and Finance** A small store makes a profit of $934.20 in the first week of a given month, $1,238.34 in the second week, and $853 in the third week. If the goal is a profit of $4,000 for the month, what profit must the store make during the remainder of the month?

Basic Skills | Challenge Yourself | Calculator/Computer | **Career Applications** | Above and Beyond

48. **Allied Health** Respiratory therapists calculate the humidity deficit, in milligrams per liter (mg/L), for a patient by subtracting the actual humidity content of inspired air from 43.9 mg/L, which is the maximum humidity content at body temperature. Determine the humidity deficit if the inspired air has a humidity content of 32.7 mg/L.

49. **Allied Health** A patient is given three capsules of Tc99m sodium pertechnetate containing 79.4, 15.88, and 3.97 millicuries (mCi), respectively. What was the total activity, in millicuries, administered to the patient?

50. **Manufacturing** A dimension on a computer-aided design (CAD) drawing is given as 3.084 in. ± 0.125 in. What is the minimum and maximum length the feature may be?

51. **Electronics** Sandy purchased a lot of 12-volt (V) batteries from an online auction. There were 10 batteries in the lot. If the batteries were connected in series, the total open-voltage would be the sum of the battery voltages; therefore, she expected an open-voltage of 120 V. Unfortunately, her voltmeter (used to measure voltage) doesn't read above 100 V. So she measured each battery's voltage and recorded it in the table on the next page. Calculate the actual open-voltage of the batteries if they are connected in series.

Answers

43. ____________

44. ____________

45. ____________

46. ____________

47. ____________

48. ____________

49. ____________

50. ____________

51. ____________

Answers

52. ______________________

53. ______________________

54. ______________________

55. ______________________

56. ______________________

57. ______________________

Battery	Measured Voltage (in volts)	Battery	Measured Voltage (in volts)
1	12.20	6	12.82
2	13.84	7	11.93
3	11.42	8	11.01
4	13.00	9	12.77
5	12.45	10	12.03

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52. **BUSINESS AND FINANCE** Following are charges on a credit card account: $8.97, $32.75, $15.95, $67.32, $215.78, $74.95, $83.90, and $257.28

(a) Estimate the total bill for the charges by rounding each number to the nearest dollar and adding the results.

(b) Estimate the total bill by adding the charges and then rounding to the nearest dollar.

(c) What are the advantages and disadvantages of the methods in (a) and (b)?

53. **NUMBER PROBLEM** Find the next number in the following sequence: 3.125, 3.375, 3.625, . . .

Recall that a magic square is one in which the sum of every row, column, and diagonal is the same. Complete each magic square.

54.

2.4		7.2
10.8		
4.8		

55.

1.6		1.2
	1	
0.8		

56. **(a)** Determine the average amount of rainfall (to the nearest hundredth of an inch) in your town or city for each of the past 24 months.

(b) Determine the difference in rainfall amounts per month for each month from one year to the next.

57. Find the next two numbers in each of the following sequences:

(a) 0.75 0.62 0.5 0.39 **(b)** 1.0 1.5 0.9 3.5 0.8

Answers

1. 1.07 **3.** 22.319 **5.** 34.4462 **7.** 1.791 **9.** 147.029
11. 0.26 **13.** 2.255 **15.** 2.32 **17.** 6.65 **19.** 3.97
21. 1.65 **23.** 4.76 **25.** 42.4 gal **27.** **(a)** 13.37 cm; **(b)** 0.59 cm
29. $272.94 **31.** 41.68 yd **33.** 11.535 mi **35.** 0.0575 in.
37. $25.98 **39.** $256.71; $238.71; $152.93; $403.38; $202.14
41. $1,110.24; $554.47; $681.24; $627.35 **43.** 12,807.13
45. $370.60 **47.** $974.46 **49.** 99.25 mCi **51.** 123.47 V
53. 3.875 **55.**

1.6	0.2	1.2
0.6	1	1.4
0.8	1.8	0.4

57. **(a)** 0.29, 0.20; **(b)** 5.5, 0.7

4.4

Multiplying Decimals

< **4.4 Objectives** >

1 > Multiply two or more decimals

2 > Use multiplication of decimals to solve application problems

3 > Multiply a decimal by a power of 10

4 > Use multiplication by a power of 10 to solve an application problem

To start our discussion of the multiplication of decimals, write the decimals in common-fraction form and then multiply.

Example 1 Multiplying Two Decimals

< **Objective 1** >

$$0.32 \times 0.2 = \frac{32}{100} \times \frac{2}{10} = \frac{64}{1{,}000} = 0.064$$

Note:

$$\begin{array}{ccc} 2 & + \; 1 & = \; 3 \\ \uparrow & \nwarrow & \nwarrow \\ \text{Places in } 0.32 & \text{Place in } 0.2 & \text{Places in the product } 0.064 \end{array}$$

Here 0.32 has *two* decimal places, and 0.2 has *one* decimal place. The product 0.064 has *three* decimal places.

Check Yourself 1

Find the product and the number of decimal places.

0.14 × 0.054

You do not need to write decimals as common fractions to multiply. Our work suggests the following rule.

Step by Step

To Multiply Decimals

Step 1	Multiply the decimals as though they were whole numbers. Temporarily ignore the decimal points.
Step 2	Count the number of decimal places in the numbers being multiplied.
Step 3	Place the decimal point in the product so that the number of decimal places in the product is the sum of the number of decimal places in the factors.

Example 2 illustrates this rule.

Example 2 Multiplying Two Decimals

Multiply 0.23 by 0.7.

$$\begin{array}{rl} 0.23 & \longleftarrow \text{Two places} \\ \times \;\; 0.7 & \longleftarrow \text{One place} \\ \hline 0.161 & \longleftarrow \text{Three places} \end{array}$$

Check Yourself 2

Multiply 0.36 × 1.52.

You may have to affix zeros to the left in the product to place the decimal point. Consider Example 3.

Example 3 Multiplying Two Decimals

Multiply.

$$\begin{array}{r l l}
0.136 & \longleftarrow \text{Three places} & \\
\times\ \ 0.28 & \longleftarrow \text{Two places} & 3+2=5 \\
\hline
1088 & & \\
272 & & \\
\hline
0.03808 & \longleftarrow \text{Five places} & \text{Insert a 0 to mark off five decimal places.} \\
\uparrow & & \\
\text{Insert 0} & &
\end{array}$$

Check Yourself 3

Multiply 0.234 × 0.24.

Estimation is also helpful in multiplying decimals.

Example 4 Estimating the Product of Two Decimals

Estimate the product 24.3 × 5.8.

$$\begin{array}{r c r l}
24.3 & \xrightarrow{\text{Round}} & 24 & \\
\times\ \ 5.8 & & \times\ \ 6 & \\
\cline{1-1}\cline{3-3}
 & & 144 & \text{Multiply to get the estimate.}
\end{array}$$

Check Yourself 4

Estimate the product.

17.95 × 8.17

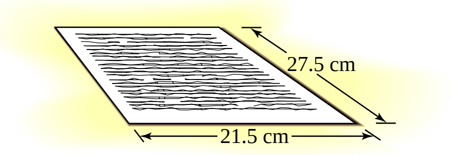
Many applications require decimal multiplication.

Example 5 An Application Involving the Multiplication of Two Decimals

< Objective 2 >

A sheet of paper has dimensions 27.5 by 21.5 centimeters (cm). What is its area?

We multiply to find the required area.



RECALL

The area of a rectangle is length times width, so multiplication is the necessary operation.

$$\begin{array}{r}
27.5\ \text{cm} \\
\times\ 21.5\ \text{cm} \\
\hline
1375\phantom{\ \text{cm}} \\
275\phantom{0\ \text{cm}} \\
550\phantom{00\ \text{cm}} \\
\hline
591.25\ \text{cm}^2
\end{array}$$

The area of the paper is 591.25 cm^2.

Check Yourself 5

If 1 kilogram (kg) is 2.2 pounds (lb), how many pounds equal 5.3 kg?

Example 6 An Application Involving the Multiplication of Two Decimals

NOTE

Usually in problems dealing with money we round the result to the nearest cent (hundredth of a dollar).

Jack buys 8.7 gallons (gal) of propane at 98.9 cents per gallon. Find the cost of the propane.

We multiply the cost per gallon by the number of gallons. Then we round the result to the nearest cent. Note that the units of the answer will be cents.

$$\begin{array}{r} 98.9 \\ \times\ \ 8.7 \\ \hline 69\,23 \\ 791\,2 \\ \hline 860.43 \end{array}$$

The product 860.43 (cents) is rounded to 860 (cents), or $8.60.

The cost of Jack's propane is $8.60.

Check Yourself 6

One liter (L) is approximately 0.265 gal. On a trip to Europe, the Bernards purchased 88.4 L of gas for their rental car. How many gallons of gas did they purchase, to the nearest tenth of a gallon?

Sometimes we will have to use more than one operation for a solution, as Example 7 shows.

Example 7 An Application Involving Two Operations

Steve purchased a television set for $299.50. He agreed to pay for the set by making payments of $27.70 for 12 months. How much extra does he pay on the installment plan?

First we multiply to find the amount actually paid.

$$\begin{array}{r} \$\ \ 27.70 \\ \times\qquad 12 \\ \hline 55\,40 \\ 277\,0 \\ \hline \$332.40 \end{array} \longleftarrow \text{Amount paid}$$

Now subtract the listed price. The difference will give the extra amount Steve paid.

$$\begin{array}{r} \$332.40 \\ -\ \ 299.50 \\ \hline \$\ \ 32.90 \end{array} \longleftarrow \text{Extra amount}$$

Steve will pay an additional $32.90 on the installment plan.

Check Yourself 7

Sandy's new car had a list price of $10,985. She paid $1,500 down and will pay $305.35 per month for 36 months on the balance. How much extra will she pay with this loan arrangement?

There are enough applications involving multiplication by the powers of 10 to make it worthwhile to develop a special rule so you can do such operations quickly and easily. Look at the patterns in some of these special multiplications.

$$\begin{array}{r} 0.679 \\ \times\qquad 10 \\ \hline 6.790, \text{ or } 6.79 \end{array} \qquad\qquad \begin{array}{r} 23.58 \\ \times\qquad 10 \\ \hline 235.80, \text{ or } 235.8 \end{array}$$

NOTE

The digits remain the same. Only the *position* of the decimal point is changed.

Do you see that multiplying by 10 has moved the decimal point *one place to the right?* What happens when we multiply by 100?

$$\begin{array}{r} 0.892 \\ \times \quad 100 \\ \hline 89.200 \end{array}\text{, or } 89.2 \qquad\qquad \begin{array}{r} 5.74 \\ \times \quad 100 \\ \hline 574.00 \end{array}\text{, or } 574$$

Multiplying by 100 shifts the decimal point *two places to the right*. The pattern of these examples gives us the following rule:

Property

To Multiply by a Power of 10

Move the decimal point to the right the same number of places as there are zeros in the power of 10.

Example 8 Multiplying by Powers of 10

< Objective 3 >

NOTE

Multiplying by 10, 100, or any other larger power of 10 makes the number *larger*. Move the decimal point to *the right*.

RECALL

10^5 is just a 1 followed by five zeros.

$$2.356 \times 10 = 23.56$$

One zero

The decimal point has moved one place to the right.

$$34.788 \times 100 = 3{,}478.8$$

Two zeros

The decimal point has moved two places to the right.

$$3.67 \times 1{,}000 = 3{,}670.$$

Three zeros

The decimal point has moved three places to the right. Note that we added a 0 to place the decimal point correctly.

$$0.005672 \times 10^5 = 567.2$$

Five zeros

The decimal point has moved five places to the right.

Check Yourself 8

Multiply.

(a) 43.875×100 **(b)** 0.0083×10^3

Example 9 is just one of many applications that require multiplying by a power of 10.

Example 9 An Application Involving Multiplication by a Power of 10

< Objective 4 >

NOTES

There are 1,000 meters in 1 kilometer.

If the result is a whole number, there is no need to write the decimal point.

To convert from kilometers to meters, multiply by 1,000. Find the number of meters (m) in 2.45 kilometers (km).

$$2.45 \text{ km} = 2{,}450. \text{ m}$$

Just move the decimal point three places right to make the conversion. Note that we added a zero to place the decimal point correctly.

Check Yourself 9

To convert from kilograms to grams, multiply by 1,000. Find the number of grams (g) in 5.23 kilograms (kg).

Check Yourself ANSWERS

1. 0.00756, five decimal places **2.** 0.5472 **3.** 0.05616 **4.** 144
5. 11.66 lb **6.** 23.4 gal **7.** \$1,507.60 **8.** **(a)** 4,387.5; **(b)** 8.3
9. 5,230 g

Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 4.4

(a) When multiplying decimals, ______________ the number of decimal places in the numbers being multiplied.

(b) The decimal point in the ______________ is placed so that the number of places is the sum of the number of decimal places in the factors.

(c) It is sometimes necessary to affix ______________ to the left of the product of two decimals to accurately place the decimal point.

(d) Multiplying by 100 shifts the decimal point two places to the ______________.

4.4 exercises

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Name ____________________

Section __________ Date __________

Answers

1. __________
2. __________
3. __________
4. __________
5. __________
6. __________
7. __________
8. __________
9. __________
10. __________
11. __________
12. __________
13. __________
14. __________
15. __________
16. __________
17. __________
18. __________
19. __________
20. __________
21. __________
22. __________
23. __________
24. __________
25. __________

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< Objective 1 >

Multiply.

1. $\begin{array}{r} 2.3 \\ \times\ 3.4 \\ \hline \end{array}$

2. $\begin{array}{r} 6.5 \\ \times\ 4.3 \\ \hline \end{array}$

3. $\begin{array}{r} 8.4 \\ \times\ 5.2 \\ \hline \end{array}$

> Videos

4. $\begin{array}{r} 9.2 \\ \times\ 4.6 \\ \hline \end{array}$

5. $\begin{array}{r} 2.56 \\ \times\ \ 72 \\ \hline \end{array}$

6. $\begin{array}{r} 56.7 \\ \times\ \ 35 \\ \hline \end{array}$

7. $\begin{array}{r} 0.78 \\ \times\ \ 2.3 \\ \hline \end{array}$

8. $\begin{array}{r} 9.5 \\ \times\ 0.45 \\ \hline \end{array}$

9. $\begin{array}{r} 15.7 \\ \times\ 2.35 \\ \hline \end{array}$

10. $\begin{array}{r} 28.3 \\ \times\ 0.59 \\ \hline \end{array}$

11. $\begin{array}{r} 0.354 \\ \times\ \ 0.8 \\ \hline \end{array}$

12. $\begin{array}{r} 0.624 \\ \times\ \ 0.85 \\ \hline \end{array}$

13. $\begin{array}{r} 3.28 \\ \times\ 5.07 \\ \hline \end{array}$

14. $\begin{array}{r} 0.582 \\ \times\ \ 6.3 \\ \hline \end{array}$

15. $\begin{array}{r} 5.238 \\ \times\ \ 0.48 \\ \hline \end{array}$

16. $\begin{array}{r} 0.372 \\ \times\ \ 58 \\ \hline \end{array}$

17. $\begin{array}{r} 1.053 \\ \times\ 0.552 \\ \hline \end{array}$

18. $\begin{array}{r} 2.375 \\ \times\ \ 0.28 \\ \hline \end{array}$

19. $\begin{array}{r} 0.0056 \\ \times\ \ 0.082 \\ \hline \end{array}$

20. $\begin{array}{r} 1.008 \\ \times\ 0.046 \\ \hline \end{array}$

21. 0.8 × 2.376

22. 3.52 × 58

23. 0.3085 × 4.5

> Videos

24. 0.028 × 0.685

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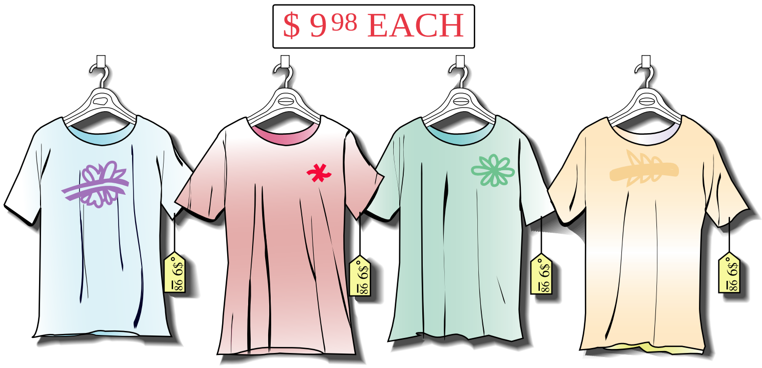
< Objective 2 >

Solve each application.

25. **Business and Finance** Kurt bought four shirts on sale as pictured. What was the total cost of the purchase?



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26. **BUSINESS AND FINANCE** Juan makes monthly payments of \$123.65 on his car. What will he pay in 1 year?

27. **SCIENCE AND MEDICINE** If 1 gallon (gal) of water weighs 8.34 pounds (lb), how much does 2.5 gal weigh?

28. **BUSINESS AND FINANCE** Malik worked 37.4 hours (h) in 1 week. If his hourly rate of pay is \$8.75, what was his pay for the week?

29. **BUSINESS AND FINANCE** To find the amount of simple interest on a loan at $9\frac{1}{2}$%, we have to multiply the amount of the loan by 0.095. Find the simple interest on a \$1,500 loan for 1 year.

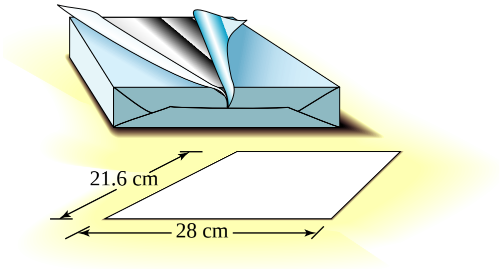
30. **BUSINESS AND FINANCE** A beef roast weighing 5.8 lb costs \$3.25 per pound. What is the cost of the roast?

31. **BUSINESS AND FINANCE** Tom's state income tax is found by multiplying his income by 0.054. If Tom's income is \$23,450, find the amount of his tax.

> Videos

32. **BUSINESS AND FINANCE** Claudia earns \$9.40 per hour (h). For overtime (each hour over 40 h) she earns \$14.10. If she works 48.5 h in a week, what pay should she receive?

33. **GEOMETRY** A sheet of typing paper has dimensions as shown. What is its area?



34. **BUSINESS AND FINANCE** A rental car costs \$24 per day plus 18 cents per mile (mi). If you rent a car for 5 days and drive 785 mi, what will the total car rental bill be?

Label each statement as **true** *or* **false.**

35. The decimal points must be aligned when finding the sum of two decimals.

36. The decimal points must be aligned when finding the difference of two decimals.

37. The decimal points must be aligned when finding the product of two decimals.

38. The decimal points must be aligned when finding the better looking of two decimals.

39. The number of decimal places in the product of two factors will be the product of the number of places in the two factors.

40. When multiplying a decimal by a power of 10, the decimal point is moved to the left the number of places as there are zeros in the power of 10.

Answers

26. ______
27. ______
28. ______
29. ______
30. ______
31. ______
32. ______
33. ______
34. ______
35. ______
36. ______
37. ______
38. ______
39. ______
40. ______

Answers

41. ____________
42. ____________
43. ____________
44. ____________
45. ____________
46. ____________
47. ____________
48. ____________
49. ____________
50. ____________
51. ____________
52. ____________
53. ____________
54. ____________
55. ____________
56. ____________
57. ____________
58. ____________
59. ____________
60. ____________
61. ____________
62. ____________
63. ____________
64. ____________

< Objective 3 >

Multiply.

41. 5.89×10 > Videos

42. 0.895×100

43. 23.79×100

44. 2.41×10

45. 0.045×10

46. 5.8×100

47. 0.431×100

48. 0.025×10

49. 0.471×100

50. $0.95 \times 10{,}000$

51. $0.7125 \times 1{,}000$

52. $23.42 \times 1{,}000$

53. 4.25×10^2 > Videos

54. 0.36×10^3

55. 3.45×10^4

56. 0.058×10^5

< Objective 4 >

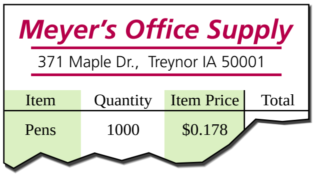
Solve each application.

57. **Business and Finance** A store purchases 100 items at a cost of \$1.38 each. Find the total cost of the order.

58. **Science and Medicine** To convert from meters (m) to centimeters (cm), multiply by 100. How many centimeters are there in 5.3 m?

59. **Science and Medicine** How many grams (g) are there in 2.2 kilograms (kg)? Multiply by 1,000 to make the conversion.

60. **Business and Finance** An office purchases 1,000 pens at a cost of 17.8 cents each. What is the cost of the purchase in dollars?



Meyer's Office Supply

371 Maple Dr., Treynor IA 50001

Item	Quantity	Item Price	Total
Pens	1000	$0.178	

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The steps for finding the product of decimals on a calculator are similar to the ones we used for multiplying whole numbers. To multiply $2.8 \times 3.45 \times 3.725$, enter

$2.8 \boxed{\times} 3.45 \boxed{\times} 3.725 \boxed{=}$

The display should read 35.9835.

You can also easily find powers of decimals with your calculator by using a similar procedure. To find $(2.35)^3$, you can enter

$2.35 \boxed{\times} 2.35 \boxed{\times} 2.35 \boxed{=}$

The display should read 12.977875.

Some calculators have keys that will find powers more quickly. Look for keys marked $\boxed{x^2}$ or $\boxed{y^x}$. Other calculators have a power key marked $\boxed{\wedge}$. To find $(2.35)^3$, enter $2.35 \boxed{\wedge} 3 \boxed{=}$ or $2.35 \boxed{y^x} 3 \boxed{=}$.

Again, the result is 12.977875.

Use your calculator for each exercise.

61. $127.85 \times 0.055 \times 15.84$

62. $18.28 \times 143.45 \times 0.075$

63. $(3.95)^3$

64. $(0.521)^2$

65. **GEOMETRY** Find the area of a rectangle with length 3.75 in. and width 2.35 in.

66. **BUSINESS AND FINANCE** Mark works 38.4 h in a given week. If his hourly rate of pay is $7.85, what will he be paid for the week?

67. **BUSINESS AND FINANCE** If fuel oil costs $1.879 per gallon, what will 150.4 gal cost?

68. **BUSINESS AND FINANCE** To find the simple interest on a loan for 1 year at 12.5%, multiply the amount of the loan by 0.125. What simple interest will you pay on a loan of $1,458 at 12.5% for 1 year?

69. **BUSINESS AND FINANCE** You are the office manager for Dr. Rogers. The increasing cost of making photocopies is a concern to Dr. Rogers. She wants to examine alternatives to the current financing plan. The office currently leases a copy machine for $110 per month and pays $0.025 per copy. A 3-year payment plan is available that costs $125 per month and $0.015 per copy.

(a) If the office expects to run 100,000 copies per year, which is the better plan?

(b) How much money will the better plan save over the other plan?

70. **BUSINESS AND FINANCE** In a bottling company, a machine can fill a 2-liter (L) bottle in 0.5 second (s) and move the next bottle into place in 0.1 s. How many 2-L bottles can be filled by the machine in 2 h?

What happens when your calculator wants to display an answer that is too big to fit in the display? Suppose you want to evaluate 10^{10}. If you enter 10 $\boxed{\wedge}$ 10 $\boxed{=}$, your calculator will probably display 1 E 10 or perhaps 1^{10}, both of which mean 1×10^{10}. Answers that are displayed in this way are said to be in **scientific notation.** This is a topic that you will study later. For now, we can use the calculator to see the relationship between numbers written in scientific notation and in decimal notation. For example, 3.485×10^4 is written in scientific notation. To write it in decimal notation, use your calculator to enter

3.485 $\boxed{\times}$ 10 $\boxed{y^x}$ 4 $\boxed{=}$ or 3.485 $\boxed{\times}$ 10 $\boxed{\wedge}$ 4 $\boxed{=}$

The result should be 34,850. Note that the decimal point has moved four places (the power of 10) to the right.

Write each number in decimal notation.

71. 3.365×10^3 **72.** 4.128×10^3 **73.** 4.316×10^5

74. 8.163×10^6 **75.** 7.236×10^8 **76.** 5.234×10^7

Answers

1. 7.82 **3.** 43.68 **5.** 184.32 **7.** 1.794 **9.** 36.895 **11.** 0.2832
13. 16.6296 **15.** 2.51424 **17.** 0.581256 **19.** 0.0004592 **21.** 1.9008
23. 1.38825 **25.** $39.92 **27.** 20.85 lb **29.** $142.50 **31.** $1,266.30
33. 604.8 cm^2 **35.** True **37.** False **39.** False **41.** 58.9 **43.** 2,379
45. 0.45 **47.** 43.1 **49.** 47.1 **51.** 712.5 **53.** 425 **55.** 34,500
57. $138 **59.** 2,200 g **61.** 111.38292 **63.** 61.629875 **65.** 8.8125 in.^2
67. $282.60 **69. (a)** 3-year plan; current plan: $11,460; 3-year lease: $9,000;
(b) $2,460 **71.** 3,365 **73.** 431,600 **75.** 723,600,000

Answers

65.

66.

67.

68.

69.

70.

71.

72.

73.

74.

75.

76.

chapter
4
> Make the Connection

Activity 11 ::
Safe Dosages?

Chemotherapy drug dosages are generally calculated based on a patient's body surface area (BSA) in square meters (m^2). A patient's BSA is calculated using a nomogram and is based on the patient's height and weight. Print out the adult nomogram from the Science Museum of Minnesota's website www.smm.org/heart/lessons/nomogram_adult.htm. Draw a line connecting the patient's height with his or her weight. The point where this line crosses the middle column denotes the patient's BSA.

Dosages are then calculated using the formula

$$\text{Dose} = \text{recommended dose} \times \text{BSA}$$

In each case, determine if the prescribed dose falls within the recommended dose range for the given patient.

1. The doctor has prescribed Blenoxane to treat an adult male patient with testicular cancer. The patient is 70 inches (in.) tall and weighs 260 pounds (lb). According to the RxList website (www.rxlist.com), the recommended dose of Blenoxane in the treatment of testicular cancer should be between 10 to 20 units per square meter (units/m^2). The ordered dose is 50 units once per week.

2. The doctor has prescribed BiCNU to treat an adult patient with a brain tumor. The patient is 65 in. tall and weighs 150 lb. According to the RxList website, the recommended dose of BiCNU should be between 150 to 200 milligrams per square meter (mg/m^2). The ordered dose is 300 mg once every 6 weeks.

3. The doctor has prescribed Cisplatin to treat an adult patient with advanced bladder cancer. The patient is 73 in. tall and weighs 275 lb. According to the RxList website, the recommended dose of Cisplatin should be between 50 to 70 mg/m^2. The ordered dose is 150 mg once every 3 to 4 weeks.

4.5 Dividing Decimals

< **4.5 Objectives** >

1 > Divide a decimal by a whole number
2 > Use division of decimals to solve application problems
3 > Divide a decimal by a decimal
4 > Divide a decimal by a power of 10

Division of decimals is very similar to our earlier work dividing whole numbers. The only difference is in learning to place the decimal point in the quotient. Let's start with the case of dividing a decimal by a whole number. Here, placing the decimal point is easy. You can apply the following rule.

Step by Step

To Divide a Decimal by a Whole Number

Step 1 Place the decimal point in the quotient *directly above* the decimal point of the dividend.

Step 2 Divide as you would with whole numbers.

Example 1 Dividing a Decimal by a Whole Number

< **Objective 1** >

NOTE

Do the division just as if you were dealing with whole numbers. Just remember to place the decimal point in the quotient *directly above* the one in the dividend.

Divide 29.21 by 23.

$$\begin{array}{r}
1.27\\
23\,\overline{)\,29.21}\\
\underline{23}\\
6\,2\\
\underline{4\,6}\\
1\,61\\
\underline{1\,61}\\
0
\end{array}$$

The quotient is 1.27.

Check Yourself 1

Divide 80.24 by 34.

Here is another example of dividing a decimal by a whole number.

Example 2 Dividing a Decimal by a Whole Number

NOTE

Again place the decimal point of the quotient above that of the dividend.

Divide 122.2 by 52.

$$\begin{array}{r}
2.3\\
52\,\overline{)\,122.2}\\
\underline{104}\\
18\,2\\
\underline{15\,6}\\
2\,6
\end{array}$$

RECALL

Affixing a zero at the end of a decimal does not change the value of the dividend. It simply allows us to complete the division process in this case.

We normally do not use a remainder when dealing with decimals. Add a 0 to the dividend and continue.

$$
\begin{array}{r}
2.35 \\
52\overline{)122.20} \\
\underline{104} \\
18\,2 \\
\underline{15\,6} \\
2\,60 \\
\underline{2\,60} \\
0
\end{array}
\quad \longleftarrow \text{Add a zero.}
$$

So 122.2 ÷ 52 = 2.35. The quotient is 2.35.

Check Yourself 2

Divide 234.6 by 68.

Often you will be asked to give a quotient to a certain place value. In this case, continue the division process to *one digit past* the indicated place value. Then round the result back to the desired accuracy.

When working with money, for instance, we normally give the quotient to the nearest hundredth of a dollar (the nearest cent). This means carrying the division out to the thousandths place and then rounding.

Example 3 Dividing a Decimal by a Whole Number and Rounding the Result

NOTE

Find the quotient to *one place past* the desired place and then round the result.

Find the quotient of 25.75 ÷ 15 to the nearest hundredth.

$$
\begin{array}{r}
1.716 \\
15\overline{)25.750} \\
\underline{15} \\
10\,7 \\
\underline{10\,5} \\
25 \\
\underline{15} \\
100 \\
\underline{90} \\
10
\end{array}
$$

Add a zero to carry the division to the thousandths place.

So 25.75 ÷ 15 = 1.72 (to the nearest hundredth).

Check Yourself 3

Find 99.26 ÷ 35 to the nearest hundredth.

As we mentioned, problems similar to the one in Example 3 often occur when we are working with money. Example 4 is one of the many applications of this type of division.

Example 4 An Application Involving the Division of a Decimal

< Objective 2 >

A carton of 144 items costs $56.10. What is the price per item to the nearest cent?

To find the price per item, divide the total price by 144.

NOTE

You might want to review the rules for rounding decimals in Section 4.1.

$$\begin{array}{r}
0.389\\
144\overline{)56.100}\\
\underline{43\ 2}\\
12\ 90\\
\underline{11\ 52}\\
1\ 380\\
\underline{1\ 296}\\
84
\end{array}$$

Carry the division to the thousandths place and then round.

The cost per item is rounded to \$0.39, or 39¢.

Check Yourself 4

An office paid \$26.55 for 72 pens. What was the cost per pen, to the nearest cent?

We now want to look at division *by* decimals. Here is an example using a fractional form.

Example 5 Rewriting a Problem That Requires Dividing by a Decimal

< Objective 3 >

NOTE

It is always easy to rewrite a division problem so that you are dividing by a whole number. Dividing by a whole number makes it easy to place the decimal point in the quotient.

RECALL

Multiplying by any whole-number power of 10 greater than 1 is just a matter of shifting the decimal point to the right.

Divide.

$$2.57 \div 3.4 = \frac{2.57}{3.4}$$

Write the division as a fraction.

$$= \frac{2.57 \times 10}{3.4 \times 10}$$

We multiply the numerator and denominator by 10 so the divisor is a whole number. This *does not change* the value of the fraction.

$$= \frac{25.7}{34}$$

Multiplying by 10, shift the decimal point in the numerator and denominator *one place to the right*.

$$= 25.7 \div 34$$

Our division problem is rewritten so that the divisor is a whole number.

So

$$2.57 \div 3.4 = 25.7 \div 34$$

After we multiply the numerator and denominator by 10, we see that $2.57 \div 3.4$ is the same as $25.7 \div 34$.

Check Yourself 5

Rewrite the division problem so that the divisor is a whole number.

$3.42 \div 2.5$

Do you see the rule suggested by example 5? We multiplied the numerator and the denominator (the dividend and the divisor) by 10. We made the divisor a whole number without altering the actual digits involved. All we did was shift the decimal point in the divisor and dividend the same number of places. This leads us to the following rule.

Step by Step

To Divide by a Decimal

Step 1 Move the decimal point in the divisor to the *right,* making the divisor a whole number.

Step 2 Move the decimal point in the dividend to the right *the same number of places.* Add zeros if necessary.

Step 3 Place the decimal point in the quotient directly above the decimal point of the dividend.

Step 4 Divide as you would with whole numbers.

Now look at an example of the use of our division rule.

Example 6 Rounding the Result of Dividing by a Decimal

Divide 1.573 by 0.48 and give the quotient to the nearest tenth.

Write

$$0.48\overline{)1.573}$$

Shift the decimal points two places to the right to make the divisor a whole number.

Now divide:

$$\begin{array}{r} 3.27\\ 48\overline{)157.30}\\ \underline{144}\\ 13\,3\\ \underline{9\,6}\\ 3\,70\\ \underline{3\,36}\\ 34 \end{array}$$

Note that we add a zero to carry the division to the hundredths place. In this case, we want to find the quotient to the nearest tenth.

NOTE

Once the division statement is rewritten, place the decimal point in the quotient above that in the dividend.

Round 3.27 to 3.3. So

$1.573 \div 0.48 = 3.3$ (to the nearest tenth)

Check Yourself 6

Divide, rounding the quotient to the nearest tenth.

$3.4 \div 1.24$

Many applications involve decimal division.

Example 7 Solving an Application Involving the Division of Decimals

Andrea worked 41.5 hours in a week and earned $488.87. What was her hourly rate of pay?

To find the hourly rate of pay we use division. We divide the number of hours worked into the total pay.

$$\begin{array}{r} 1\,1.78\\ 41.5\overline{)488.8\,70}\\ \underline{415}\\ 73\,8\\ \underline{41\,5}\\ 32\,3\,7\\ \underline{29\,0\,5}\\ 3\,3\,20\\ \underline{3\,3\,20}\\ 0 \end{array}$$

NOTE

We add a zero to the dividend to complete the division process.

Andrea's hourly rate of pay was $11.78.

Check Yourself 7

A developer wants to subdivide a 12.6-acre piece of land into 0.45-acre lots. How many lots are possible?

Example 8 Solving an Application Involving the Division of Decimals

At the start of a trip the odometer read 34,563. At the end of the trip, it read 36,235. If 86.7 gallons (gal) of gas were used, find the number of miles per gallon (to the nearest tenth).

First, find the number of miles traveled by subtracting the initial reading from the final reading.

$$\begin{array}{rl}
36{,}235 & \text{Final reading}\\
\underline{-\ 34{,}563} & \text{Initial reading}\\
1{,}672 & \text{Miles traveled}
\end{array}$$

Next, divide the miles traveled by the number of gallons used. This will give us the miles per gallon.

$$\begin{array}{r}
19.28\\
86{.}\underset{\curvearrowright}{7}\,\overline{)1{,}672.\underset{\curvearrowright}{0}\;00}\\
\underline{867}\\
805\;0\\
\underline{780\;3}\\
24\;7\;\;0\\
\underline{17\;3\;\;4}\\
7\;3\;\;60\\
\underline{6\;9\;\;36}\\
4\;\;24
\end{array}$$

Round 19.28 to 19.3 mi/gal.

Check Yourself 8

John starts his trip with an odometer reading of 15,436 and ends with a reading of 16,238. If he used 45.9 gallons (gal) of gas, find the number of miles per gallon (mi/gal) (to the nearest tenth).

Recall that you can multiply decimals by powers of 10 by simply shifting the decimal point to the right. A similar approach works for division by powers of 10.

Example 9 Dividing a Decimal by a Power of 10

< Objective 4 >

(a) Divide.

$$\begin{array}{r}
3.53\\
10\,\overline{)35.30}\\
\underline{30}\\
5\;3\\
\underline{5\;0}\\
30\\
\underline{30}\\
0
\end{array}$$

The dividend is 35.3. The quotient is 3.53. The decimal point has been shifted *one place to the left*. Note also that the divisor, 10, has *one* zero.

(b) Divide.

$$\begin{array}{r}
3.785\\
100\,\overline{)378.500}\\
\underline{300}\\
78\;5\\
\underline{70\;0}\\
8\;50\\
\underline{8\;00}\\
500\\
\underline{500}\\
0
\end{array}$$

Here the dividend is 378.5, whereas the quotient is 3.785. The decimal point is now shifted *two places to the left*. In this case the divisor, 100, has *two* zeros.

Check Yourself 9

Perform each division.

(a) $52.6 \div 10$ **(b)** $267.9 \div 100$

Example 9 suggests the following rule.

Property

To Divide a Decimal by a Power of 10

Move the decimal point *to the left* the same number of places as there are zeros in the power of 10.

Example 10 Dividing a Decimal by a Power of 10

NOTE

We may add zeros to correctly place the decimal point.

RECALL

10^4 is a 1 followed by *four zeros.*

Divide.

(a) $27.3 \div 10 = 2_{\wedge}7.3$ Shift one place to the left.

$= 2.73$

(b) $57.53 \div 100 = 0_{\wedge}57.53$ Shift two places to the left.

$= 0.5753$

(c) $39.75 \div 1{,}000 = 0_{\wedge}039.75$ Shift three places to the left.

$= 0.03975$

(d) $85 \div 1{,}000 = 0_{\wedge}085.$ The decimal after the 85 is implied.

$= 0.085$

(e) $235.72 \div 10^4 = 0_{\wedge}0235.72$ Shift four places to the left.

$= 0.023572$

Check Yourself 10

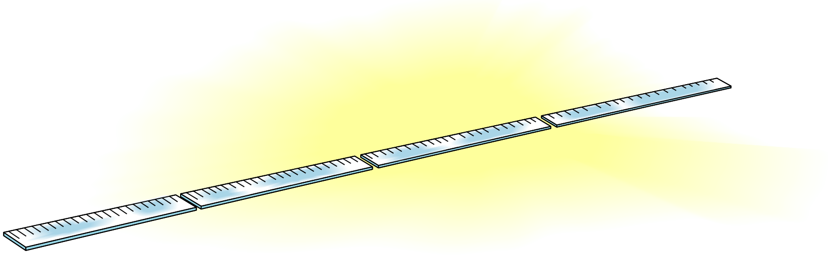
Divide.

(a) $3.84 \div 10$ **(b)** $27.3 \div 1{,}000$

Now, look at an application of our work in dividing by powers of 10.

Example 11 Solving an Application Involving a Power of 10

To convert from millimeters (mm) to meters (m), we divide by 1,000. How many meters does 3,450 mm equal?



3,450 mm $= 3_{\wedge}450.$ m Shift three places to the left to divide by 1,000.

$= 3.450$ m

Check Yourself 11

A shipment of 1,000 notebooks cost a stationery store \$658. What was the cost per notebook to the nearest cent?

Recall that the order of operations is always used to simplify a mathematical expression with several operations. You should recall the following order of operations.

Property

The Order of Operations

1. Perform any operations enclosed in **parentheses.**
2. Evaluate any **exponents.**
3. Do any **multiplication** and **division,** in order from left to right.
4. Do any **addition** and **subtraction,** in order from left to right.

Example 12 Applying the Order of Operations

Simplify each expression.

(a) $4.6 + (0.5 \times 4.4)^2 - 3.93$	Parentheses
$= 4.6 + (2.2)^2 - 3.93$	Exponent
$= 4.6 + 4.84 - 3.93$	Add (left of the subtraction)
$= 9.44 - 3.93$	Subtract
$= 5.51$	
(b) $16.5 - (2.8 + 0.2)^2 + 4.1 \times 2$	Parentheses
$= 16.5 - (3)^2 + 4.1 \times 2$	Exponent
$= 16.5 - 9 + 4.1 \times 2$	Multiply
$= 16.5 - 9 + 8.2$	Subtraction (left of the addition)
$= 7.5 + 8.2$	Add
$= 15.7$	

Check Yourself 12

Simplify each expression.

(a) $6.35 + (0.2 \times 8.5)^2 - 3.7$ **(b)** $2.5^2 - (3.57 - 2.14) + 3.2 \times 1.5$

Check Yourself ANSWERS

1. 2.36 **2.** 3.45 **3.** 2.84 **4.** \$0.37, or 37¢ **5.** 34.2 ÷ 25
6. 2.7 **7.** 28 lots **8.** 17.5 mi/gal **9.** **(a)** 5.26; **(b)** 2.679
10. **(a)** 0.384; **(b)** 0.0273 **11.** 66¢ **12.** **(a)** 5.54; **(b)** 9.62

Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 4.5

(a) When dividing decimals, place the decimal point in the quotient directly ________________ the decimal point of the dividend.

(b) When asked to give a quotient to a certain place value, continue the division process to one digit ________________ the indicated place value.

(c) When dividing by a decimal, first move the decimal point in the divisor to the right, making the divisor a ________________ ________________.

(d) To divide a decimal by a power of 10, move the decimal point to the ________________ the same number of places as there are zeros in the power of 10.

4.5 exercises

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Name ____________________

Section ________ Date ________

< Objectives 1–3 >

Divide.

1. $16.68 \div 6$ > Videos

2. $43.92 \div 8$

3. $1.92 \div 4$

4. $5.52 \div 6$

5. $5.48 \div 8$

6. $2.76 \div 8$

7. $\frac{13.89}{6}$ > Videos

8. $\frac{21.92}{5}$

9. $\frac{185.6}{32}$

10. $\frac{165.6}{36}$

11. $79.9 \div 34$

12. $179.3 \div 55$

13. $52\overline{)13.78}$

14. $76\overline{)26.22}$

15. $0.6\overline{)11.07}$

16. $0.8\overline{)10.84}$

17. $\frac{7.22}{3.8}$

18. $\frac{13.34}{2.9}$

19. $\frac{11.622}{5.2}$

20. $\frac{3.616}{6.4}$

21. $0.27\overline{)1.8495}$ > Videos

22. $0.038\overline{)0.8132}$

23. $0.046\overline{)1.587}$

24. $0.52\overline{)3.2318}$

25. $0.658 \div 2.8$

26. $0.882 \div 0.36$

< Objective 4 >

Divide by moving the decimal point.

27. $5.8 \div 10$

28. $5.1 \div 10$

29. $4.568 \div 100$

30. $3.817 \div 100$

31. $24.39 \div 1{,}000$ > Videos

32. $8.41 \div 100$

33. $6.9 \div 1{,}000$

34. $7.2 \div 1{,}000$

Answers

1. ________ 2. ________

3. ________ 4. ________

5. ________ 6. ________

7. ________ 8. ________

9. ________ 10. ________

11. ________ 12. ________

13. ________ 14. ________

15. ________ 16. ________

17. ________ 18. ________

19. ________ 20. ________

21. ________ 22. ________

23. ________ 24. ________

25. ________ 26. ________

27. ________ 28. ________

29. ________ 30. ________

31. ________ 32. ________

33. ________ 34. ________

Answers

35. ______ 36. ______

37. ______ 38. ______

39. ______ 40. ______

41. ______ 42. ______

43. ______ 44. ______

45. ______ 46. ______

47. ______ 48. ______

49. ______ 50. ______

51. ______ 52. ______

53. ______

54. ______

55. ______

56. ______

57. ______

58. ______

35. $7.8 \div 10^2$ **36.** $3.6 \div 10^3$

37. $45.2 \div 10^5$ **38.** $57.3 \div 10^4$

Divide and round the quotient to the indicated decimal place.

39. $23.8 \div 9$ tenths **40.** $5.27 \div 8$ hundredths

41. $38.48 \div 46$ hundredths > Videos **42.** $3.36 \div 36$ thousandths

43. $125.4 \div 52$ tenths **44.** $2.563 \div 54$ thousandths

45. $0.7\overline{)1.642}$ hundredths **46.** $0.6\overline{)7.695}$ tenths

47. $4.5\overline{)8.415}$ tenths **48.** $5.8\overline{)16}$ hundredths

49. $3.12\overline{)4.75}$ hundredths **50.** $64.2\overline{)16.3}$ thousandths

Basic Skills | **Challenge Yourself** | Calculator/Computer | Career Applications | Above and Beyond

< Objective 2 >

Solve each application.

51. **Business and Finance** Marv paid \$40.41 for three CDs on sale. What was the cost per CD?

52. **Business and Finance** Seven employees of an office donated a total of \$172.06 during a charity drive. What was the average donation?

53. **Business and Finance** A shipment of 72 paperback books cost a store \$190.25. What was the average cost per book to the nearest cent?

SEA VIEW
Dream Horizons
Pure Romance
Skippy's Adventure
Skippy's Adventure

54. **Business and Finance** A restaurant bought 50 glasses at a cost of \$39.90. What was the cost per glass to the nearest cent?

55. **Business and Finance** The cost of a box of 48 pens is \$28.20. What is the cost of an individual pen to the nearest cent?

56. **Business and Finance** An office bought 18 handheld calculators for \$284. What was the cost per calculator to the nearest cent?

57. **Business and Finance** Al purchased a new refrigerator that cost \$736.12 with interest included. He paid \$100 as a down payment and agreed to pay the remainder in 18 monthly payments. What amount will he be paying per month?

58. **Business and Finance** The cost of a television set with interest is \$490.64. If you make a down payment of \$50 and agree to pay the balance in 12 monthly payments, what will be the amount of each monthly payment?

ONLY $50 DOWN!!
on entire
12 MONTHS
TO PAY!!

Simplify each expression.

59. $79 - 28.2 + 13.7$

60. $63.1 - 4.8 + 5.2$

61. $29.64 - (4.2 + 12.39)$

62. $53.6 - (14 + 6.21)$

63. $8.2 \div 0.25 \times 3.6$

64. $7.14 \div 0.3 \times 5.1$

65. $\dfrac{7.8 + 4.2}{9.1 - 6.6}$

66. $\dfrac{6.08 + 3.58}{7.65 - 3.45}$

67. $6.4 + 1.3^2$

68. $(6.4 + 1.3)^2$

69. $15.9 - 4.2 \times 3.5$

70. $23.7 - 8.6 \times 0.8$

71. $6.1 + 2.3 \times (8.08 - 5.9)$

72. $2.09 + 4.5 \times (12.37 - 7.27)$

73. $5.2 - 3.1 \times 1.5 + (3.1 + 0.4)^2$

74. $150 + 4.1 \times 1.5 - (2.5 \times 1.6)^3 \times 2.4$

75. $17.9 \times 1.1 - (2.3 \times 1.1)^2 + (13.4 - 2.1 \times 4.6)$

76. $6.89^2 - 3.14 \times 2.5 + (3.2 \times 1.6 - 4.1)^2$

Basic Skills | Challenge Yourself | **Calculator/Computer** | Career Applications | Above and Beyond

Using your calculator to divide decimals is a good way to check your work and is also a reasonable way to solve applications. However, when using it for applications, we generally round off our answers to an appropriate place value.

Using your calculator, divide and round to the indicated place.

77. $2.546 \div 1.38$ hundredths

78. $45.8 \div 9.4$ tenths

79. $0.5782 \div 1.236$ thousandths

80. $1.25 \div 0.785$ hundredths

81. $1.34 \div 2.63$ two decimal places

82. $12.364 \div 4.361$ three decimal places

Basic Skills | Challenge Yourself | Calculator/Computer | **Career Applications** | Above and Beyond

83. **ALLIED HEALTH** Since people vary in body size, the cardiac index is used to normalize cardiac output measurements. The cardiac index, in liters per minute per square meters L/(min · m^2), is calculated by dividing a patient's cardiac output, in liters per minute (L/min), by his or her body surface area, in m^2. Calculate the cardiac index for a male patient whose cardiac output is 4.8 L/min if his body surface area is 2.03 m^2. Round your answer to the nearest hundredth.

84. **ALLIED HEALTH** The specific concentration of a radioactive drug, or radiopharmaceutical, is defined as the activity, in millicuries (mCi), divided by the volume, in milliliters (mL). Determine the specific concentration of a vial containing 7.3 mCi of I131 sodium iodide in 0.25 mL. Round your answer to the nearest tenth.

Answers

59. ______ 60. ______

61. ______ 62. ______

63. ______ 64. ______

65. ______ 66. ______

67. ______ 68. ______

69. ______ 70. ______

71. ______ 72. ______

73. ______ 74. ______

75. ______ 76. ______

77. ______ 78. ______

79. ______ 80. ______

81. ______ 82. ______

83. ______

84. ______

Answers

85. ____________

86. ____________

87. ____________

88. ____________

89. ____________

90. ____________

85. **Information Technology** A Web developer is responsible for designing a Web application for a customer. She uses a program called FTP to transmit pages from her local machine to a Web server. She needs to transmit 2.5 megabytes (Mbytes) of data. She notices it takes 10.2 seconds (s) to transmit the data. How fast is her connection to the Web server in Mbits/s? Round your answer to the nearest hundredth.

86. **Information Technology** After creating a presentation for a big customer, Joe sees that the size of the file is 1.6 Mbytes. Joe has a special application that allows him to save files across multiple disks. How many floppy disks will he need to store the file (a floppy disk can handle 1.4 Mbytes of data)?

Basic Skills | Challenge Yourself | Calculator/Computer | Career Applications | **Above and Beyond**

87. The blood alcohol content (BAC) of a person who has been drinking is determined by the Widmark formula. Find that formula using a search engine and use it to solve the following.

A 125-lb person is driving and is stopped by a policewoman on suspicion of driving under the influence (DUI). The driver claims that in the past 2 h he only consumed six 12-oz bottles of 3.9% beer. If he undergoes a breathalyzer test, what will his BAC be? Will this amount be under the legal limit for your state?

88. Four brands of soap are available in a local store.

Brand	Ounces	Total Price	Unit Price
Squeaky Clean	5.5	$0.36	
Smell Fresh	7.5	0.41	
Feel Nice	4.5	0.31	
Look Bright	6.5	0.44	

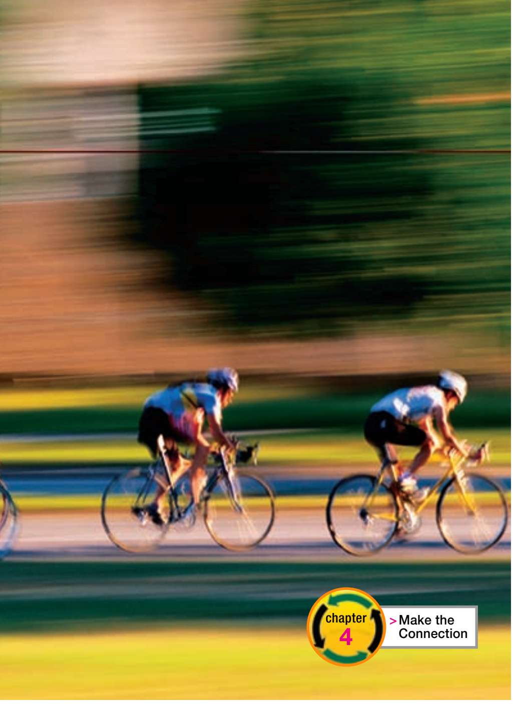
Compute the unit price and decide which brand is the best buy.

89. Sophie is a quality control expert. She inspects boxes of number 2 pencils. Each pencil weighs 4.4 grams (g). The contents of a box of pencils weigh 66.6 g. If a box is labeled CONTENTS: 16 PENCILS, should Sophie approve the box as meeting specifications? Explain your answer.

90. Write a plan to determine the number of miles per gallon (mi/gal) your car (or your family car) gets. Use this plan to determine your car's actual mileage.

Answers

1. 2.78 **3.** 0.48 **5.** 0.685 **7.** 2.315 **9.** 5.8 **11.** 2.35
13. 0.265 **15.** 18.45 **17.** 1.9 **19.** 2.235 **21.** 6.85 **23.** 34.5
25. 0.235 **27.** 0.58 **29.** 0.04568 **31.** 0.02439 **33.** 0.0069
35. 0.078 **37.** 0.000452 **39.** 2.6 **41.** 0.84 **43.** 2.4 **45.** 2.35
47. 1.9 **49.** 1.52 **51.** $13.47 **53.** $2.64 **55.** $0.59, or 59¢
57. $35.34 **59.** 64.5 **61.** 13.05 **63.** 118.08 **65.** 4.8
67. 8.09 **69.** 1.2 **71.** 11.114 **73.** 12.8 **75.** 17.0291
77. 1.84 **79.** 0.468 **81.** 0.51 **83.** 2.36 L/(min · m^2)
85. 0.25 Mbits/s **87.** Above and Beyond **89.** Above and Beyond



Activity 12 ::
The Tour de France

The Tour de France is perhaps the most grueling of all sporting events. It is a bicycle race that spans 22 days (including 2 rest days), and involves 20 stages of riding in a huge circuit around the country of France. This includes several stages that take the riders through two mountainous regions, the Alps and the Pyrenees. The following table presents the winners of each stage in the 2005 race, along with the winning time in hours and the length of the stage expressed in miles. For each stage, compute the winner's average speed by dividing the miles traveled by the winning time. Round your answers to the nearest tenth of a mile per hour.

Stage	Winner	Length (mi)	Time (h)	Speed (mi/h)
Prologue	Zabriskie	11.8	0.35	
1	Boonen	112.8	3.86	
2	Boonen	132	4.60	
3	Discovery Channel	41.9	1.18	
4	McEwen	113.7	3.77	
5	Bernucci	123.7	4.21	
6	McEwen	142	5.06	
7	Weening	143.8	5.07	
8	Rasmussen	106.3	4.14	
9	Valverde	110.9	4.84	
10	Vinokourov	107.5	4.79	
11	Moncoutie	116.2	4.34	
12	McEwen	107.8	3.72	
13	Totsehnig	137	5.73	
14	Hincapie	127.7	6.11	
15	Pereiro	112.2	4.64	
16	Savoldelli	148.8	5.69	
17	Serrano	117.4	4.63	
18	Guerini	95.4	3.55	
19	Armstrong	34.5	1.20	
20	Vinokourov	89.8	3.68	

Find the total number of miles traveled.

By examining the speeds of the stages, can you identify which stages occurred in the mountains?

The overall tour winner was Lance Armstrong with a total winning time of 86.25 hours. Compute his average speed for the entire race, again rounding to the nearest tenth of a mile per hour.

summary :: chapter 4

Definition/Procedure	Example	Reference
Place Value and Rounding		**Section 4.1**
Decimal Fraction A fraction whose denominator is a power of 10. We call decimal fractions *decimals*.	$\frac{7}{10}$ and $\frac{47}{100}$ are decimal fractions.	*p.* 249
Decimal Place Each position for a digit to the right of the decimal point. Each decimal place has a place value that is $\frac{1}{10}$ the value of the place to its left.	2.3456 Ten-thousandths Thousandths Hundredths Tenths	*p.* 249
Reading and Writing Decimals in Words		
Step 1 Read the digits *to the left* of the decimal point as a whole number. **Step 2** Read the decimal point as the word *and*. **Step 3** Read the digits *to the right* of the decimal point as a whole number followed by the place value of the rightmost digit.	Hundredths $8.\overset{\frown}{15}$ is read “eight and fifteen hundredths.”	*p.* 250
Rounding Decimals		
Step 1 Find the place to which the decimal is to be rounded. **Step 2** If the next digit to the right is 5 or more, increase the digit in the place you are rounding to by 1. Discard any remaining digits to the right. **Step 3** If the next digit to the right is less than 5, just discard that digit and any remaining digits to the right.	To round 5.87 to the nearest tenth: ↓ 5.87 is rounded to 5.9 To round 12.3454 to the nearest thousandth: ↓ 12.3454 is rounded to 12.345.	*p.* 252
Converting Between Fractions and Decimals		**Section 4.2**
To Convert a Common Fraction to a Decimal		
Step 1 Divide the numerator of the common fraction by its denominator. **Step 2** The quotient is the decimal equivalent of the common fraction.	To convert $\frac{1}{2}$ to a decimal: $\begin{array}{r} 0.5 \\ 2\overline{)1.0} \\ \underline{1\ 0} \\ 0 \end{array}$	*p.* 258
To Convert a Terminating Decimal Less Than 1 to a Common Fraction		
Step 1 Write the digits of the decimal without the decimal point. This will be the numerator of the common fraction. **Step 2** The denominator of the fraction is a 1 followed by as many zeros as there are places in the decimal.	To convert 0.275 to a common fraction: $0.275 = \frac{275}{1,000} = \frac{11}{40}$	*p.* 261

Definition/Procedure	Example	Reference
Adding and Subtracting Decimals		**Section 4.3**
To Add Decimals		
Step 1 Write the numbers being added in column form with their decimal points in a vertical line. **Step 2** Add just as you would with whole numbers. **Step 3** Place the decimal point of the sum in line with the decimal points of the addends.	To add 2.7, 3.15, and 0.48: $\begin{array}{r} 2.7 \\ 3.15 \\ +\ 0.48 \\ \hline 6.33 \end{array}$	*p.* 268
To Subtract Decimals		
Step 1 Write the numbers being subtracted in column form with their decimal points in a vertical line. You may have to place zeros to the right of the existing digits. **Step 2** Subtract just as you would with whole numbers. **Step 3** Place the decimal point of the difference in line with the decimal points of the numbers being subtracted.	To subtract 5.875 from 8.5: $\begin{array}{r} 8.500 \\ -\ 5.875 \\ \hline 2.625 \end{array}$	*p.* 270
Multiplying Decimals		**Section 4.4**
To Multiply Decimals		
Step 1 Multiply the decimals as though they were whole numbers. **Step 2** Count the number of decimal places in the factors. **Step 3** Place the decimal point in the product so that the number of decimal places in the product is the sum of the number of decimal places in the factors.	To multiply 2.85 × 0.045: $\begin{array}{rl} 2.85 & \longleftarrow \text{Two places} \\ \times\ 0.045 & \longleftarrow \text{Three places} \\ \hline 1425 & \\ 1140 & \\ \hline 0.12825 & \longleftarrow \text{Five places} \end{array}$	*p.* 281
Multiplying by Powers of 10		
Move the decimal point to the right the same number of places as there are zeros in the power of 10.	$2.37 \times 10 = 23.7$ $0.567 \times 1{,}000 = 567$	*p.* 284
Dividing Decimals		**Section 4.5**
To Divide by a Decimal		
Step 1 Move the decimal point in the divisor to the right, making the divisor a whole number. **Step 2** Move the decimal point in the dividend to the right the same number of places. Add zeros if necessary. **Step 3** Place the decimal point in the quotient directly above the decimal point of the dividend. **Step 4** Divide as you would with whole numbers.	To divide 16.5 by 5.5, move the decimal points: $\begin{array}{r} 3 \\ 5.5_{\wedge}\overline{)\,16.5_{\wedge}} \\ 16\ 5 \\ \hline 0 \end{array}$	*p.* 293
To Divide by a Power of 10		
Move the decimal point to the left the same number of places as there are zeros in the power of 10.	$25.8 \div 10 = 2_{\wedge}5.8 = 2.58$	*p.* 296

summary exercises :: chapter 4

This summary exercise set is provided to give you practice with each of the objectives of this chapter. Each exercise is keyed to the appropriate chapter section. When you are finished, you can check your answers to the odd-numbered exercises against those presented in the back of the text. If you have difficulty with any of these questions, go back and reread the examples from that section. The answers to the even-numbered exercises appear in the *Instructor's Solutions Manual.* Your instructor will give you guidelines on how to best use these exercises in your instructional setting.

4.1 *Find the indicated place values.*

1. 7 in 3.5742

2. 3 in 0.5273

Write the fractions in decimal form.

3. $\frac{37}{100}$

4. $\frac{307}{10{,}000}$

Write the decimals in words.

5. 0.071

6. 12.39

Write the fractions in decimal form.

7. Four and five tenths

8. Four hundred and thirty-seven thousandths

Complete each statement, using the symbol $<$, $=$, *or* $>$.

9. 0.79 ______ 0.785

10. 1.25 ______ 1.250

11. 12.8 ______ 13

12. 0.832 ______ 0.83

Round to the indicated place.

13. 5.837 hundredths

14. 9.5723 thousandths

15. 4.87625 three decimal places

Write each number as a common fraction or a mixed number.

16. 0.0067

17. 21.857

4.2 *Find the decimal equivalents.*

18. $\frac{7}{16}$

19. $\frac{3}{7}$ (round to the thousandth)

20. $\frac{4}{15}$ (use bar notation)

21. $3\frac{3}{4}$

Write as common fractions or mixed numbers. Simplify your answers.

22. 0.21

23. 0.084

24. 5.28

4.3 *Add.*

25.
$$\begin{array}{r} 2.58 \\ +\ 0.89 \\ \hline \end{array}$$

26.
$$\begin{array}{r} 3.14 \\ 0.8 \\ 2.912 \\ +\ 12 \\ \hline \end{array}$$

27. 1.3, 25, 5.27, and 6.158

28. Add eight, forty-three thousandths, five and nineteen hundredths, and seven and three tenths.

Subtract.

29.
$$\begin{array}{r} 29.21 \\ -\ \ 5.89 \\ \hline \end{array}$$

30.
$$\begin{array}{r} 6.73 \\ -\ 2.485 \\ \hline \end{array}$$

31. 1.735 from 2.81

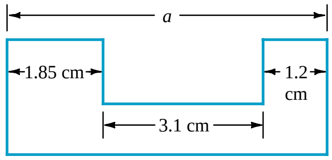
32. 12.38 from 19

Solve each application.

33. **GEOMETRY** Find the perimeter (to the nearest hundredth of a centimeter) of a rectangle that has dimensions 5.37 cm by 8.64 cm.

34. **SCIENCE AND MEDICINE** Janice ran 4.8 miles (mi) on Sunday, 5.3 mi on Tuesday, 3.9 mi on Thursday, and 8.2 mi on Saturday. How far did she run during the week?

35. **GEOMETRY** Find dimension *a* in the figure.



36. **BUSINESS AND FINANCE** A stereo system that normally sells for \$499.50 is discounted (or marked down) to \$437.75 for a sale. Find the savings.

37. **BUSINESS AND FINANCE** If you cash a \$50 check and make purchases of \$8.71, \$12.53, and \$9.83, how much money do you have left?

4.4 *Multiply.*

38.
$$\begin{array}{r} 22.8 \\ \times\ 0.72 \\ \hline \end{array}$$

39.
$$\begin{array}{r} 0.0045 \\ \times\ 0.058 \\ \hline \end{array}$$

40. 1.24 × 56

41. 0.0025 × 0.491

42. 0.052 × 1,000

43. 0.045×10^4

Solve each application.

44. **Business and Finance** Neal worked for 37.4 hours (h) during a week. If his hourly rate of pay was \$7.25, how much did he earn?

45. **Business and Finance** To find the simple interest on a loan at $11\frac{1}{2}\%$ for 1 year, we must multiply the amount of the loan by 0.115. Find the simple interest on a \$2,500 loan at $11\frac{1}{2}\%$ for 1 year.

46. **Business and Finance** A television set has an advertised price of \$499.50. You buy the set and agree to make payments of \$27.15 per month for 2 years. How much extra are you paying by buying with this installment plan?

47. **Business and Finance** A stereo dealer buys 100 portable radios for a promotion sale. If she pays \$57.42 per radio, what is her total cost?

4.5 *Divide. Round answers to the nearest hundredth.*

48. $8\overline{)3.08}$

49. $58\overline{)269.7}$

50. $55\overline{)17.69}$

Divide. Round answers to the nearest thousandth.

51. $0.7\overline{)1.865}$

52. $3.042 \div 0.37$

53. $5.3\overline{)6.748}$

54. $0.2549 \div 2.87$

Divide.

55. $7.6 \div 10$

56. $80.7 \div 1{,}000$

57. $457 \div 10^4$

Solve each application.

58. **Business and Finance** During a charity fund-raising drive 37 employees of a company donated a total of \$867.65. What was the donation per employee?

59. **Business and Finance** Faith always fills her gas tank as soon as the gauge hits the $\frac{1}{4}$ full mark. In six readings, Faith's gas mileage was 38.9, 35.3, 39.0, 41.2, 40.5, and 40.8 miles per gallon (mi/gal). What was the average mileage to the nearest tenth of a mile per gallon? (*Hint:* First find the sum of the mileages. Then divide the sum by 6, because there are 6 mileage readings.)

60. **Construction** A developer is planning to subdivide an 18.5-acre piece of land. She estimates that 5 acres will be used for roads and wants individual lots of 0.25 acre. How many lots are possible?

61. **Business and Finance** Paul drives 949 mi, using 31.8 gal of gas. What is his mileage for the trip (to the nearest tenth of a mile per gallon)?

62. **Business and Finance** A shipment of 1,000 videotapes cost a dealer \$7,090. What was the cost per tape to the dealer?

self-test 4

The purpose of this self-test is to help you assess your progress so that you can find concepts that you need to review before the next exam. Allow yourself about an hour to take this test. At the end of that hour, check your answers against those given in the back of this text. If you miss any, go back to the appropriate section to reread the examples until you have mastered that particular concept.

Complete each statement, using the symbols $<$ or $>$.

1. 0.889 ________ 0.89

2. 0.531 ________ 0.53

Find the decimal equivalents of the common fractions. When indicated, round to the given place value.

3. $\frac{7}{16}$

4. $\frac{3}{7}$ thousandths

5. $\frac{7}{11}$ use bar notation

Divide.

6. $4.983 \div 1{,}000$

7. $523 \div 10^5$

Write the decimals as common fractions or mixed numbers. Simplify your answer.

8. 0.072

9. 4.44

10. Find the place value of 8 in 0.5248.

11. Write 2.53 in words.

Round to the indicated place.

12. 0.5977 thousandths

13. 23.5724 two decimal places

14. 36,139.0023 thousands

15. Write $\frac{49}{1{,}000}$ in decimal form.

16. Write twelve and seventeen thousandths in decimal form.

Perform the indicated operations.

17.
$$\begin{array}{r} 3.45 \\ 0.6 \\ +\ 12.59 \\ \hline \end{array}$$

18.
$$\begin{array}{r} 18.32 \\ -\ 7.78 \\ \hline \end{array}$$

19. $4.1\overline{)10.455}$

20. 2.75×0.53

21. $27\overline{)63.45}$

22. Add:
2.4, 35, 4.73, and 5.123

23.
$$\begin{array}{r} 32.9 \\ \times\ 0.53 \\ \hline \end{array}$$

24.
$$\begin{array}{r} 40 \\ -\ 15.625 \\ \hline \end{array}$$

Name ____________________

Section __________ Date __________

Answers

1. ____________________
2. ____________________
3. ____________________
4. ____________________
5. ____________________
6. ____________________
7. ____________________
8. ____________________
9. ____________________
10. ____________________
11. ____________________
12. ____________________
13. ____________________
14. ____________________
15. ____________________
16. ____________________
17. ____________________
18. ____________________
19. ____________________
20. ____________________
21. ____________________
22. ____________________
23. ____________________
24. ____________________

Answers

25. ____________

26. ____________

27. ____________

28. ____________

29. ____________

30. ____________

31. ____________

32. ____________

33. ____________

34. ____________

35. ____________

36. ____________

37. ____________

38. ____________

39. ____________

40. ____________

41. ____________

42. ____________

25. **Business and Finance** A college bookstore purchases 1,000 pens at a cost of 54.3 cents per pen. Find the total cost of the order in dollars.

26. **Business and Finance** On a business trip, Martin bought the following amounts of gasoline: 14.4, 12, 13.8, and 10 gallons (gal). How much gasoline did he purchase on the trip?

27. **Construction** A 14-acre piece of land is being developed into home lots. If 2.8 acres of land will be used for roads and each home site is to be 0.35 acre, how many lots can be formed?

28. Insert < or > to form a true statement.

$$0.168 \;\underline{\qquad}\; \frac{3}{25}$$

29. Add: seven, seventy-nine hundredths, and five and thirteen thousandths

30. Find the area of a rectangle with length 3.5 inches (in.) and width 2.15 in.

Multiply.

31. $0.735 \times 1{,}000$

32. 1.257×10^4

33. **Construction** A street improvement project will cost $57,340, and that cost is to be divided among the 100 families in the area. What will be the cost to each individual family?

34. **Business and Finance** You pay for purchases of $13.99, $18.75, $9.20, and $5 with a $50 bill. How much cash will you have left?

Perform the indicated operation.

35. Subtract: 1.742 from 5.63

36. $8\overline{)3.72}$

37.

$$\begin{array}{r} 0.049 \\ \times\ 0.57 \\ \hline \end{array}$$

38. $0.6\overline{)1.431}$

39. A baseball team has a winning percentage of 0.458. Write this as a fraction in simplest form.

Divide. When indicated, round to the given place value.

40. $3.969 \div 0.54$

41. $2.72 \div 53$ thousandths

42. $0.263 \div 3.91$ three decimal places

cumulative review chapters 1-4

Name ____________________

Section __________ Date __________

The following exercises are presented to help you review concepts from earlier chapters. This is meant as review material and not as a comprehensive exam. The answers are presented in the back of the text. Before each answer is a section reference for the concept. If you have difficulty with any of these exercises, be certain to at least read through the summary related to that section.

1. Write 286,543 in words.

2. What is the place value of 5 in the number 343,563?

In exercises 3 to 8, perform the indicated operations.

3.
$$\begin{array}{r} 2{,}340 \\ 685 \\ +\ 31{,}569 \\ \hline \end{array}$$

4.
$$\begin{array}{r} 75{,}363 \\ -\ 26{,}475 \\ \hline \end{array}$$

5. 83×61

6. 231×305

7. $21\overline{)357}$

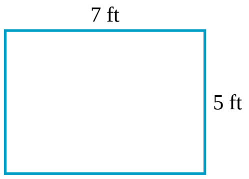
8. $463\overline{)16{,}216}$

9. Evaluate the expression $18 \div 2 + 4 \times 2^3 - (18 - 6)$.

10. Round each number to the nearest hundred and find an estimated sum.

$294 + 725 + 2{,}321 + 689$

11. Find the perimeter and area of the given figure.



12. Write the fraction $\frac{15}{51}$ in simplest form.

Answers

1. ____________________
2. ____________________
3. ____________________
4. ____________________
5. ____________________
6. ____________________
7. ____________________
8. ____________________
9. ____________________
10. ____________________
11. ____________________
12. ____________________

Answers

13. ____________________

14. ____________________

15. ____________________

16. ____________________

17. ____________________

18. ____________________

19. ____________________

20. ____________________

21. ____________________

22. ____________________

23. ____________________

24. ____________________

25. ____________________

26. ____________________

27. ____________________

28. ____________________

29. ____________________

30. ____________________

31. ____________________

Perform the indicated operations.

13. $\frac{2}{3} \times \frac{9}{8}$

14. $1\frac{2}{3} \times 1\frac{5}{7}$

15. $\frac{3}{4} \div \frac{17}{12}$

16. $\frac{6}{7} - \frac{3}{7} + \frac{2}{7}$

17. $\frac{4}{5} - \frac{7}{10} + \frac{4}{30}$

18. $6\frac{3}{5} - 2\frac{7}{10}$

19. $35.218 - 22.75$

20. $2.262 \div 0.58$

21. $523.8 \div 10^5$

22. 2.53×0.45

23. 1.53×10^4

24. Write 0.43 as a fraction.

25. Write the decimal equivalent of each number. Round to the given place value when indicated.

(a) $\frac{5}{8}$

(b) $\frac{9}{23}$ hundredths

26. Sam has had 15 hits in his last 35 at bats. That is, he has had a hit in $\frac{15}{35}$ of his times at bat. Write this as a decimal, rounding to the nearest thousandth.

Evaluate each expression.

27. $18.4 - 3.16 \times 2.5 + 6.71$

28. $17.6 \div 2.3 \times 3.4 + 13.812$ (Round to the nearest thousandth.)

Solve each application.

29. **GEOMETRY** If the perimeter of a square is 19.2 cm, how long is each side?

30. **BUSINESS AND FINANCE** In 1 week, Tom earned $356.60 by working 36.25 hours (h). What was his hourly rate of pay to the nearest cent?

31. **CONSTRUCTION** An 80.5-acre piece of land is being subdivided into 0.35-acre lots. How many lots are possible in the subdivision?

chapter
5
> Make the Connection

CHAPTER 5

Ratios and Proportions

INTRODUCTION

Sandra is a community college math instructor who loves teaching math and helping her students learn difficult concepts. When she is not teaching her classes, she spends a lot of time working out in the gym.

One day, during a particularly grueling yoga class, one of the participants asked Sandra to show him the right way to do the mountain pose. While Sandra was demonstrating this pose, she realized that she could become a personal trainer.

Sandra did some research and learned that she needed some coursework and that by passing an exam she could obtain a certificate through the American Fitness Professionals & Associates (AFPA). Some of the classes that personal trainers can teach include senior fitness, nutrition, group fitness, strength training, weight loss, and rehabilitative training.

Sandra learned some interesting facts about burning calories and weight loss such as the fact that a person has to burn off a lot more calories than he or she takes in just to lose one pound. In Activity 15 on page 355, we use ratios to determine the number of calories burned by various activities.

CHAPTER 5 OUTLINE

Chapter 5 :: Prerequisite Test 314
5.1 **Ratios** 315
5.2 **Rates and Unit Pricing** 324
5.3 **Proportions** 335
5.4 **Solving Proportions** 342
Chapter 5 :: Summary / Summary Exercises / Self-Test / Cumulative Review :: Chapters 1–5 356

Name ____________

Section ________ Date ________

This prerequisite test provides some exercises requiring skills that you will need to be successful in the coming chapter. The answers for these exercises can be found in the back of this text. This prerequisite test can help you identify topics that you will need to review before beginning the chapter.

Answers

1. ____________
2. ____________
3. ____________
4. ____________
5. ____________
6. ____________
7. ____________
8. ____________
9. ____________
10. ____________
11. ____________

Simplify each fraction.

1. $\frac{24}{32}$

2. $\frac{280}{525}$

3. $\frac{\frac{45}{2}}{30}$

Rewrite the mixed number as an improper fraction.

4. $5\frac{3}{5}$

Perform the indicated operations.

5. $\frac{45}{2} \times \frac{1}{30}$

6. 5.25×100

7. Find the GCF of 280 and 525.

Write each fraction as a decimal. Round to the nearest hundredth.

8. $\frac{50}{9}$

9. $\frac{260}{36}$

Determine whether the two given fractions are equivalent.

10. $\frac{6}{17}$ and $\frac{4}{11}$

11. $\frac{7}{15}$ and $\frac{84}{180}$

5.1

Ratios

< **5.1 Objectives** >

1 > Write the ratio of two quantities

2 > Write the ratio of two quantities in simplest form

In Chapter 2, you learned two meanings for a fraction:

1. A fraction can name a certain number of parts of a whole. The fraction $\frac{3}{5}$ names 3 parts of a whole that has been divided into 5 equal parts.
2. A fraction can indicate division. The fraction $\frac{3}{5}$ is the quotient $3 \div 5$.

We now want to turn to a third meaning for a fraction—a fraction can be a **ratio.**

Definition

Ratio A **ratio** is a comparison of two numbers or like quantities.

NOTE

In this text, we write ratios as simplified fractions.

The ratio a to b can also be written as $a : b$ and $\frac{a}{b}$. Ratios are always written in simplest form.

Example 1 Writing a Ratio as a Fraction

< **Objective 1** >

NOTE

Alternatively, the ratio of 3 to 5 can be written as 3:5.

Write the ratio 3 to 5 as a fraction.

To compare 3 to 5, we write the ratio of 3 to 5 as $\frac{3}{5}$.

So, $\frac{3}{5}$ also means "the ratio of 3 to 5."

Check Yourself 1

Write the ratio of 7 to 12 as a fraction.

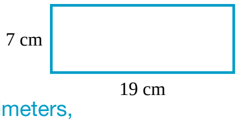
RECALL

Numbers with units attached are called *denominate numbers*.

Ratios are often used to compare *like quantities* such as quarts to quarts, centimeters to centimeters, and apples to apples. In this case, we can simplify the fraction by "canceling" the units. In its simplest form, a ratio is always written without units.

Example 2 Ratios of Denominate Numbers

A rectangle measures 7 cm wide and 19 cm long.



(a) Write the ratio of its width to its length, as a fraction.

$$\frac{7\text{ cm}}{19\text{ cm}} = \frac{7\cancel{\text{ cm}}}{19\cancel{\text{ cm}}} = \frac{7}{19}$$

We are comparing centimeters to centimeters, so we can simplify the fraction.

NOTE

Ratios are *never* written as mixed numbers. Ratios are always written as improper fractions, in simplest terms, when necessary.

(b) Write the ratio of its length to its width, as a fraction.

We need to write the ratio in the order requested by the example, rather than in the order given in the preceding description.

$$\frac{19\text{ cm}}{7\text{ cm}} = \frac{19\ \cancel{\text{cm}}}{7\ \cancel{\text{cm}}} = \frac{19}{7}$$

Check Yourself 2

A basketball team wins 17 of its 29 games in a season.

(a) Write the ratio of wins to games played.
(b) Write the ratio of wins to losses.

Because a ratio is a fraction, we can simplify it, as in Example 3.

Example 3 Simplifying a Ratio

< Objective 2 >

RECALL

This ratio may also be written as 2:3.

Write the ratio of 20 to 30 in simplest terms.

Begin by writing the fraction that represents the ratio: $\frac{20}{30}$. Now, simplify this fraction.

$$\frac{20}{30} = \frac{2}{3}$$

Simplify the fraction by dividing both the numerator and denominator by 10.

Check Yourself 3

Write the ratio of 24 to 32 in simplest terms.

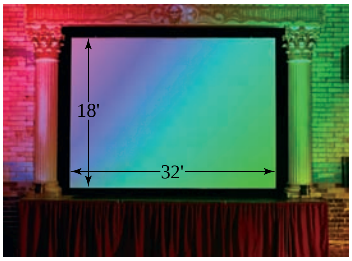
Because ratios are used to compare like quantities, a simplified ratio has no units. In Example 4, we need to simplify both the numbers and the units.

Example 4 Simplifying the Ratio of Denominate Numbers

NOTE

On DVDs, widescreen movies are often in 16:9 format.

A common size for a movie screen is 32 ft by 18 ft. Write this ratio in simplest form.



$$\frac{32\text{ ft}}{18\text{ ft}} = \frac{32\ \cancel{\text{ft}}}{18\ \cancel{\text{ft}}} = \frac{32}{18} = \frac{16}{9}$$

The GCF of 32 and 18 is 2.

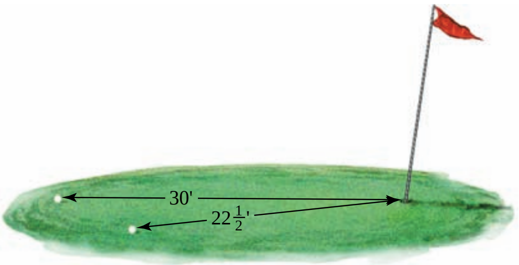
Check Yourself 4

A common computer display mode is 640 pixels (picture elements) by 480 pixels. Write this as a ratio in simplest terms.

Often, the quantities in a ratio are given as fractions or decimals. In either of these cases, the ratio should be rewritten as an equivalent ratio comparing whole numbers.

Example 5 Simplifying Ratios

(a) Loren sank a $22\frac{1}{2}$-ft putt and Carrie sank a 30-ft putt. Express the ratio of the two distances as a ratio of whole numbers.



We begin by writing the ratio of the two distances: $\dfrac{22\frac{1}{2}\text{ ft}}{30\text{ ft}}$. Then, we cancel the units and rewrite the mixed number as an improper fraction.

RECALL

In Section 2.3, you learned to rewrite $22\frac{1}{2}$ as $\frac{45}{2}$.

$$\frac{22\frac{1}{2}\text{ ft}}{30\text{ ft}} = \frac{\frac{45}{2}}{30}$$

RECALL

In Section 2.6, you learned to simplify complex fractions.

In order to simplify this complex fraction, we rewrite it as a division problem.

$$\begin{aligned}\frac{\frac{45}{2}}{30} &= \frac{45}{2} \div 30\\ &= \frac{45}{2} \times \frac{1}{30} && \text{Invert, and multiply.}\\ &= \frac{\overset{3}{\cancel{45}} \times 1}{2 \times \underset{2}{\cancel{30}}} && \text{The GCF of 45 and 30 is 15.}\\ &= \frac{3}{4}\end{aligned}$$

The ratio 3 to 4 is equivalent to the ratio $22\frac{1}{2}$ ft to 30 ft.

(b) The diameter of a 20-oz bottle is 2.8 in. The diameter of a 2-L bottle is 5.25 in. Express the ratio of the two diameters as a ratio of whole numbers.

$$\frac{2.8\text{ in.}}{5.25\text{ in.}} = \frac{2.8}{5.25}$$

In order to simplify this fraction, we need to rewrite it as an equivalent fraction of whole numbers, that is, without the decimals.

If we multiply the numerator by 10, it would be a whole number. However, we need to multiply the denominator by 100 in order to make it a whole number.

RECALL

In Section 4.4, you learned to multiply a decimal by a power of 10 by moving the decimal point.

Because we want to write an equivalent fraction, we multiply it by $1 = \frac{100}{100}$.

$$\begin{aligned}\frac{2.8}{5.25} \cdot \frac{100}{100} &= \frac{280}{525}\\ &= \frac{8}{15}\end{aligned}$$

Divide numerator and denominator by 5, and then again by 7. Or, we can determine that the GCF of 280 and 525 is 35, and simplify with one division.

The ratio of the bottle diameters is 8 to 15.

Check Yourself 5

(a) One morning Rita jogged $3\frac{1}{2}$ mi, while Yi jogged $4\frac{1}{4}$ mi. Express the ratio of the two distances as a ratio of whole numbers.

(b) A standard newspaper column is 2.625 in. wide and 19.5 in. long. Express the ratio of the two measurements as a ratio of whole numbers.

NOTE

We will learn to convert measurements in more depth in Chapter 7.

Sometimes, we use a ratio to compare the same type of measurement using different units. In Example 6, both quantities are measures of time. In order to construct and simplify the ratio, we must express both quantities in the same units.

Example 6 Rewriting Denominate Numbers to Find a Ratio

Joe took 2 hours (h) to complete his final exam. Jamie finished hers in 75 minutes (min). Write the ratio of the two times in simplest terms.

To find the ratio, both quantities must have the same units. Therefore, we rewrite 2 h as 120 min. This way, both quantities use minutes as the unit.

$$\frac{2\text{ h}}{75\text{ min}} = \frac{120\text{ min}}{75\text{ min}}$$

2 h = 120 min.

$$= \frac{120}{75} = \frac{8}{5}$$

The GCF of 120 and 75 is 15.

Check Yourself 6

Find the ratio of whole numbers that is equivalent to the ratio of 15 ft to 9 yd.

Check Yourself ANSWERS

1. $\frac{7}{12}$ **2. (a)** $\frac{17}{29}$; **(b)** $\frac{17}{12}$ **3.** $\frac{3}{4}$ **4.** $\frac{4}{3}$ **5. (a)** $\frac{14}{17}$; **(b)** $\frac{7}{52}$ **6.** $\frac{5}{9}$

Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 5.1

(a) A ______________ can indicate division.

(b) A ratio is a means of comparing two ______________ quantities.

(c) Because a ratio is a fraction, we can write it in ______________ terms.

(d) Ratios are never written as ______________ numbers.

5.1 exercises

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Name ____________________

Section __________ Date __________

< Objectives 1–2 >

Write each ratio in simplest form.

1. The ratio of 9 to 13

2. The ratio of 5 to 4

3. The ratio of 9 to 4

4. The ratio of 5 to 12

5. The ratio of 10 to 15 > Videos

6. The ratio of 16 to 12

7. The ratio of $3\frac{1}{2}$ to 14

8. The ratio of $5\frac{3}{5}$ to $2\frac{1}{10}$

9. The ratio of 10.5 to 2.7 > Videos

10. The ratio of 2.2 to 0.6

11. The ratio of 12 miles (mi) to 18 mi

12. The ratio of 100 centimeters (cm) to 90 cm

13. The ratio of 40 ft to 65 ft

14. The ratio of 12 oz to 18 oz

15. The ratio of $48 to $42

16. The ratio of 20 ft to 24 ft

Solve each application.

17. **Social Science** An algebra class has 7 men and 13 women. Write the ratio of men to women. Write the ratio of women to men.

18. **Statistics** A football team wins 9 of its 16 games with no ties. Write the ratio of wins to games played. Write the ratio of wins to losses.

19. **Social Science** In a school election 4,500 yes votes were cast, and 3,000 no votes were cast. Write the ratio of yes to no votes. > Videos

20. **Business and Finance** One car has an $11\frac{1}{2}$-gal tank and another has a $17\frac{3}{4}$-gal tank. Express the ratio of the capacities as a ratio of whole numbers.

21. **Business and Finance** One refrigerator holds $2\frac{2}{3}$ cubic feet (ft^3) of food, and another holds $5\frac{3}{4}$ ft^3 of food. Express the ratio of the capacities as a ratio of whole numbers. > Videos

22. **Science and Medicine** The price of an antibiotic in one drugstore is $12.50 although the price of the same antibiotic in another drugstore is $8.75. Write the ratio of the prices as a ratio of whole numbers.

23. **Social Science** A company employs 24 women and 18 men. Write the ratio of men to women employed by the company.

24. **Geometry** If a room is 30 ft long by 6 yd wide, write the ratio of the length to the width of the room.

Answers

1. ________ 2. ________

3. ________ 4. ________

5. ________ 6. ________

7. ________ 8. ________

9. ________ 10. ________

11. ________ 12. ________

13. ________ 14. ________

15. ________ 16. ________

17. ________ 18. ________

19. ________ 20. ________

21. ________ 22. ________

23. ________ 24. ________

Answers

25. ____________

26. ____________

27. ____________

28. ____________

29. ____________

30. ____________

31. ____________

32. ____________

33. ____________

34. ____________

35. ____________

36. ____________

37. ____________

38. ____________

39. ____________

40. ____________

Determine whether each statement is **true** *or* **false.**

25. We use ratios to compare like quantities.

26. We use ratios to compare different types of measurements.

Fill in each blank with **always, sometimes,** *or* **never.**

27. A ratio should __________ be written in simplest form.

28. A ratio is __________ written as a mixed number.

Basic Skills | **Challenge Yourself** | Calculator/Computer | Career Applications | Above and Beyond

Write each ratio in simplest form.

29. The ratio of 75 seconds (s) to 3 minutes (min)

30. The ratio of 7 oz to 3 lb

31. The ratio of 4 nickels to 5 dimes

32. The ratio of 8 in. to 3 ft

33. The ratio of 2 days to 10 h > Videos

34. The ratio of 4 ft to 4 yd

35. The ratio of 5 gallons (gal) to 12 quarts (qt)

36. The ratio of 7 dimes to 3 quarters

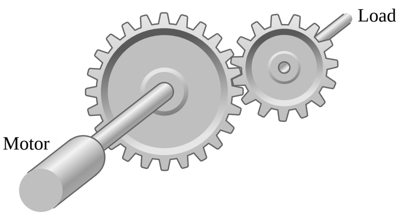
Basic Skills | Challenge Yourself | Calculator/Computer | **Career Applications** | Above and Beyond

37. **INFORMATION TECHNOLOGY** Millicent can fix 5 cell phones per hour. Tyler can fix 4 cell phones per hour. Express the ratio of the number of cell phones that Millicent can fix to the number that Tyler can fix as a fraction.

38. **ALLIED HEALTH** In preparing specimens for testing, it is often necessary to dilute the original solution. The dilution ratio is the ratio of the volume of original solution to the total volume. Each of these volumes is usually measured in milliliters (mL).

Determine the dilution ratio when 2.8 mL of blood serum is diluted with 47.2 mL of water.

39. **MECHANICAL ENGINEERING** A gear ratio is the ratio of the number of teeth on the driven gear to the number of teeth on the driving gear. In general, the driving gear is attached to the power source or motor. Write the gear ratio for the system shown.



40. **MANUFACTURING OPERATIONS TECHNOLOGY** Of the 384 parts manufactured during a shift, 26 were defective.

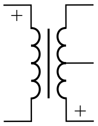
(a) Write the ratio of defective parts to total parts.

(b) Write the ratio of defective parts to good parts.

41. **Agricultural Technology** A soil test indicates that a field requires a fertilizer containing 400 lb of nitrogen and 500 lb of phosphorus. Write the ratio of nitrogen to phosphorus needed.

42. **Information Technology** Shakira connects to the Internet from home using a phone line. Her modem generally connects at a speed of 56,000 bits per second (bits/s). Her brother, Carl, connects with an older computer at 28,800 bits/s. Write the ratio of Shakira's connection speed to Carl's speed.

Basic Skills | Challenge Yourself | Calculator/Computer | Career Applications | **Above and Beyond**



The accompanying image is a common symbol on a schematic (electrical diagram) for a transformer. A transformer uses electromagnetism to change voltage levels. Commonly, two coils of wire (or some conductor) are located in close proximity but kept from directly touching or conducting. Some sort of ferromagnetic core (such as iron) is typically used. When alternating current (AC) is applied to one conductor or coil, referred to as the primary winding, current is induced on the second or secondary winding.

There is a relationship between the voltage supplied to the primary winding and the open-voltage induced in the secondary winding, based on the number of turns in each winding. This relationship is called the turns ratio (a):

$$a = \frac{N_p}{N_s}$$

in which N_p represents the number of turns in the primary winding and N_s represents the number of turns in the secondary winding.

Theoretically, the turns ratio is also equal to the voltage ratio:

$$a = \frac{V_p}{V_s}$$

in which V_p represents the voltage supplied to the primary winding and V_s represents the voltage induced in the secondary winding.

After setting the two ratios equal to one another and performing a multiplication manipulation to isolate V_p, this relationship can be expressed as

$$V_p = \frac{N_p}{N_s}V_s$$

43. Give three combinations of turns of the primary and secondary windings that achieve a turns ratio for a transformer of 3.2.

44. Using a turns ratio of 3.2 and a secondary voltage of 35 volts (V) AC, calculate the voltage supplied to the primary winding.

45. If the turns ratio is 4.5 and the primary voltage is 28 V AC, what is the induced open-voltage on the secondary winding?

46. If the turns ratio from exercise 45 is doubled, how will that affect the voltage on the secondary winding?

Answers

41. ____________________

42. ____________________

43. ____________________

44. ____________________

45. ____________________

46. ____________________

Answers

47. ____________________

48. ____________________

47. **(a)** Buy a 1.69-oz (medium-size) bag of M&M's. For each color, determine the ratio of M&M's that are that color to the total number of M&M's in the bag.

(b) Compare your ratios from part **(a)** to those of a classmate.

(c) Use the information from parts **(a)** and **(b)** to estimate the correct ratios for all the different color M&M's in a bag.

(d) Go to the M&M's manufacturer's website (Mars, Inc.) and see how your ratios compare to their claimed color distribution.

48. Sarah is a field service technician for ABC Networks, Inc. She has been asked to design a wireless home-network for a customer. This customer wants to have the fastest throughput for the wireless network in his home.

From your experience, you know that wireless networks come in different varieties: 802.11a, b, and g. The standard for most coffeehouses and restaurants is 802.11b.

If it takes 1 second to transmit a packet on 802.11b, how long does it take to transmit a packet on 802.11a and g?

Which technology will you recommend and why?

Answers

1. $\frac{9}{13}$ **3.** $\frac{9}{4}$ **5.** $\frac{2}{3}$ **7.** $\frac{1}{4}$ **9.** $\frac{35}{9}$ **11.** $\frac{2}{3}$ **13.** $\frac{8}{13}$ **15.** $\frac{8}{7}$

17. $\frac{7}{13}$; $\frac{13}{7}$ **19.** $\frac{3}{2}$ **21.** $\frac{32}{69}$ **23.** $\frac{3}{4}$ **25.** True **27.** always

29. $\frac{5}{12}$ **31.** $\frac{2}{5}$ **33.** $\frac{24}{5}$ **35.** $\frac{5}{3}$ **37.** $\frac{5}{4}$ **39.** $\frac{5}{8}$ **41.** $\frac{4}{5}$

43. Answers will vary **45.** $6\frac{2}{9}$ V AC **47.** Above and Beyond

Activity 13 ::
Working with Ratios Visually

To solve the following problems, we think that you and your group members will find one of these three approaches useful: (1) You may wish to use actual black and white markers; (2) you may wish to make sketches of such markers; or (3) you may wish to simply imagine the necessary markers. Each line in the following table is a new (and different) problem, and you are to fill in the missing parts in a given line. Be sure to express a ratio by using the smallest possible whole numbers.

	Ratio of Black to White Markers	Number of Black Markers	Number of White Markers	Total Number of Markers
1.	to		15	20
2.	to	12		30
3.	to		9	21
4.	to	15		33
5.	2 to 5	6		
6.	5 to 3		24	
7.	4 to 1	28		
8.	3 to 7		21	
9.	1 to 3			36
10.	3 to 5			40
11.	7 to 2			360
12.	4 to 7			550
13.				
14.				

For problems 13 and 14, create (and solve!) your own problems of the same sort. You might challenge another group with these.

5.2 Rates and Unit Pricing

< **5.2 Objectives** >

1 > Write a rate as a unit rate

2 > Interpret and compare unit rates

3 > Find a unit price

4 > Use unit prices to compare the cost of two items

In Section 5.1, we used ratios to compare two like quantities. For instance, the ratio of 9 seconds to 12 seconds is $\frac{3}{4}$.

$$\frac{9\ \cancel{\text{sec}}}{12\ \cancel{\text{sec}}} = \frac{\overset{3}{\cancel{9}}}{\underset{4}{\cancel{12}}} = \frac{3}{4}$$

RECALL

When simplified, a ratio has no units.

Because the units in the numerator and denominator are the same, we can "cancel" them and simplify the fraction.

We also learned that as long as the two quantities represented the same type of measurement, we could compare them using ratios. For example, if both quantities are measurements of length, then we can convert one of the measurements so that they are like quantities.

In Chapter 7, you will study measurement conversions in more depth. For now, we can do straightforward conversions. For example, we can use a ratio to compare 4 in. and 3 ft by converting 3 ft to 36 in.

$$\begin{aligned}\frac{4\text{ in.}}{3\text{ ft}} &= \frac{4\text{ in.}}{36\text{ in.}} \qquad 3\text{ ft} = 36\text{ in.}\\ &= \frac{4}{36}\\ &= \frac{1}{9}\end{aligned}$$

RECALL

Denominate numbers have units "attached."

Often, we want to compare denominate numbers with different types of units. For example, we might be interested in the gas mileage that a car gets. In such a case, we are comparing the miles driven (distance) and the gas used (volume). We make this comparison in part **(b)** of Example 1.

When we compare denominate numbers with different types of units, we get a *rate.*

Definition

Rate	A **rate** is a comparison of two denominate numbers with different types of units.

For example, if an animal moves 3 feet in 4 seconds, we can express the rate as:

$$\frac{3\text{ ft}}{4\text{ s}}$$

We read this rate as "3 feet per 4 seconds." In general, rates are presented in simplified form as *unit rates.*

Definition

Unit Rate

A **unit rate** is a *rate* that is simplified so that it compares a denominate number with a *single unit* of a different denominate number.

A **unit rate** is written so that the numerical value is given followed by the units, written as a fraction. For example, to express the rate $\frac{3\text{ ft}}{4\text{ s}}$ as a unit rate, we write it as $\frac{3}{4}\frac{\text{ft}}{\text{s}}$. We read this as "$\frac{3}{4}$ feet per second."

Example 1 Finding a Unit Rate

< Objective 1 >

NOTE

We read the rate $20\frac{\text{mi}}{\text{gal}}$ as "twenty miles *per* gallon."

Express each rate as a unit rate.

(a) $\frac{12\text{ feet}}{16\text{ seconds}} = \frac{12}{16}\frac{\text{ft}}{\text{s}} = \frac{3}{4}\frac{\text{ft}}{\text{s}}$

(b) $\frac{200\text{ miles}}{10\text{ gallons}} = \frac{200}{10}\frac{\text{mi}}{\text{gal}} = 20\frac{\text{mi}}{\text{gal}}$

(c) $\frac{10\text{ gallons}}{200\text{ miles}} = \frac{10}{200}\frac{\text{gal}}{\text{mi}} = \frac{1}{20}\frac{\text{gal}}{\text{mi}}$

Check Yourself 1

Express each rate as a unit rate.

(a) $\frac{250\text{ miles}}{10\text{ hours}}$ (b) $\frac{\$60{,}000}{2\text{ years}}$ (c) $\frac{2\text{ years}}{\$60{,}000}$

Consider part (a) in Example 1. We begin with 12 ft (length) compared to 16 seconds (time). We simplify the rate so that we know the number of feet, $\frac{3}{4}$, per 1 second. In general, we simplify a rate so that we are comparing the quantity of the numerator's units per one of the denominator's units.

In Example 2, we consider parts (b) and (c) of Example 1.

Example 2 Comparing Unit Rates

< Objective 2 >

NOTE

We could also write this rate as a decimal,

$\frac{1}{20}\frac{\text{gal}}{\text{mi}} = 0.05\frac{\text{gal}}{\text{mi}}$

(a) Write the rate $20\frac{\text{mi}}{\text{gal}}$ in words.

We write this rate as "twenty miles per gallon."

(b) Write the rate $\frac{1}{20}\frac{\text{gal}}{\text{mi}}$ in words.

We write this rate as "one-twentieth of a gallon per mile."

(c) Describe the difference between the rates in parts (a) and (b).

The rate $20\frac{\text{mi}}{\text{gal}}$ states that 20 miles can be traveled on a single gallon of fuel. The rate $\frac{1}{20}\frac{\text{gal}}{\text{mi}}$ states that $\frac{1}{20}$ of a gallon of fuel is used when traveling 1 mile.

We can also interpret these as 20 miles are traveled for each gallon of fuel used and $\frac{1}{20}$ of a gallon of fuel is used for each mile traveled, respectively.

Check Yourself 2

(a) Write the rate $30{,}000\ \frac{\text{dollars}}{\text{yr}}$ in words.

(b) Write the rate $\frac{1}{30{,}000}\ \frac{\text{yr}}{\text{dollar}}$ in words.

(c) Describe the difference between the rates in parts (a) and (b).

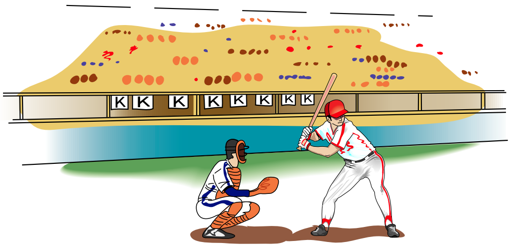
Sometimes, we need to find the appropriate rate within a written statement, as in Example 3.

Example 3 Finding a Unit Rate

RECALL

In Section 5.1 we stated that mixed numbers were inappropriate for ratios. When we write unit rates, mixed numbers and decimals are not only appropriate, they are preferred.

Randy Johnson had 320 strikeouts in 280 innings. What was his strikeout per inning rate?



$$\frac{320 \text{ strikeouts}}{280 \text{ innings}} = \frac{320}{280}\ \frac{\text{strikeouts}}{\text{inning}}$$

$$= 1\frac{1}{7}\ \frac{\text{strikeouts}}{\text{inning}}$$

Check Yourself 3

Chamique Holdsclaw scored 450 points in 15 games. What was her points per game rate?

One purpose for computing a rate is for comparison. As we see in Example 4, it is often convenient to write a rate in decimal form.

Example 4 Comparing Rates

Player A scores 50 points in 9 games and player B scores 260 points in 36 games. Which player scored at a higher rate?

$$\text{Player A's rate was } \frac{50 \text{ points}}{9 \text{ games}} = \frac{50}{9}\ \frac{\text{points}}{\text{game}}$$

$$\approx 5.56\ \frac{\text{points}}{\text{game}} \qquad \frac{50}{9} = 50 \div 9 \approx 5.56$$

$$\text{Player B's rate was } \frac{260 \text{ points}}{36 \text{ games}} = \frac{260}{36}\ \frac{\text{points}}{\text{game}}$$

$$= \frac{65}{9}\ \frac{\text{points}}{\text{game}}$$

$$\approx 7.22\ \frac{\text{points}}{\text{game}} \qquad 65 \div 9 \approx 7.22$$

Player B scored at a higher rate.

Check Yourself 4

Hassan scored 25 goals in 8 games and Lee scored 52 goals in 18 games. Which player scored at a higher rate?

Unit pricing represents one of the most common uses of rates. Posted on nearly every item in a supermarket or grocery store is the price of the item as well as its *unit price*.

Definition

Unit Price The **unit price** relates a price to some common unit.

A unit price is a price *per unit*. The unit used may be ounces, pints, pounds, or some other unit.

Example 5 Finding a Unit Price

< Objective 3 >

Find the unit price for each item.

(a) 8 ounces (oz) of cream cost $1.53.

$$\frac{\$1.53}{8\text{ oz}} = \frac{153\text{ cents}}{8\text{ oz}} = \frac{153}{8}\,\frac{\text{cents}}{\text{oz}}$$

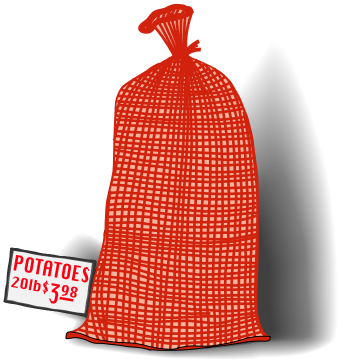
$$\approx 19\,\frac{\text{cents}}{\text{oz}}$$

(b) 20 pounds (lb) of potatoes cost $3.98.

$$\frac{\$3.98}{20\text{ lb}} = \frac{398\text{ cents}}{20\text{ lb}}$$

$$= \frac{398}{20}\,\frac{\text{cents}}{\text{lb}}$$

$$\approx 20\,\frac{\text{cents}}{\text{lb}}$$



Check Yourself 5

Find the unit price for each item.

(a) **12 soda cans cost $2.98.**
(b) **25 pounds (lb) of dog food cost $9.99.**

As with ratios, rates are most often used for comparisons. For instance, unit pricing allows people to compare the cost of different size items.

In Example 6, we use unit prices to determine whether a glass of milk is less expensive when poured from a 128-oz container (gallon) or a 32-oz container (quart).

Example 6 Using Unit Prices to Compare Cost

< Objective 4 >

NOTE

Usually, we round money to the nearest cent. When comparing unit prices, however, we may round to four decimal places (or more, if necessary).

A store sells a 1-gallon carton (128 oz) of organic whole milk for $4.89. They sell a 1-quart carton (32 oz) for $1.29. Which is the better buy?

We begin by determining the unit price of each item. To do this, we compute the cost per ounce for each carton of milk. We choose to use cents instead of dollars to make the decimal easier to work with.

Gallon

$$\frac{\$4.89}{128\text{ oz}} = \frac{489\text{ cents}}{128\text{ oz}} = \frac{489}{128}\,\frac{\text{cents}}{\text{oz}} \approx 3.8203\,\frac{\text{cents}}{\text{oz}}$$

Quart

$$\frac{\$1.29}{32\text{ oz}} = \frac{129\text{ cents}}{32\text{ oz}} = \frac{129}{32}\,\frac{\text{cents}}{\text{oz}} \approx 4.0313\,\frac{\text{cents}}{\text{oz}}$$

At these prices, the gallon of milk is the better buy.

Check Yourself 6

A store sells a 5-lb bag of Valencia oranges for $2.29. A 12-lb case sells for $5.69. Which is the better buy?

If we compare the two unit prices in Example 6, we see that both items round to 4¢ per ounce. However, milk sold by the gallon is about 0.211¢ cheaper per ounce than milk sold by the quart. We need to consider this small fraction of a cent because we are not buying 1 ounce of milk. Rather, we are buying cartons of milk, and for a whole quart of milk, the fraction adds up to nearly 7¢ (it adds up to 27¢ for a whole gallon).

Check Yourself ANSWERS

1. **(a)** $25\,\frac{\text{mi}}{\text{h}}$; **(b)** $30{,}000\,\frac{\text{dollars}}{\text{yr}}$; **(c)** $\frac{1}{30{,}000}\,\frac{\text{yr}}{\text{dollar}}$

2. **(a)** Thirty-thousand dollars per year. **(b)** One thirty-thousandth of a year per dollar. **(c)** The rate in part **(a)** describes the amount of money for each year. The rate in part **(b)** describes the amount of time per dollar.

3. $30\,\frac{\text{points}}{\text{game}}$ **4.** Hassan had a higher rate.

5. **(a)** $\approx 25\,\frac{\text{cents}}{\text{can}}$; **(b)** $\approx 40\,\frac{\text{cents}}{\text{pound}}$

6. The 5-lb bag is the better buy at 45.8¢ per pound compared to approximately 47.4¢ per pound for the case.

Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 5.2

(a) Ratios are used to compare ________________ quantities.

(b) When we compare measurements with different types of units, we get a ________________.

(c) ________________ numbers are preferable to improper fractions, when simplifying a rate.

(d) ________________ prices are used to compare the cost of items in different size packages.

5.2 exercises

< Objective 1 >

Write each rate as a unit rate.

1. $\frac{300 \text{ mi}}{4 \text{ h}}$ > Videos

2. $\frac{95 \text{ cents}}{5 \text{ pencils}}$

3. $\frac{\$10{,}000}{5 \text{ yr}}$

4. $\frac{680 \text{ ft}}{17 \text{ s}}$

5. $\frac{7{,}200 \text{ revolutions}}{16 \text{ mi}}$

6. $\frac{57 \text{ oz}}{3 \text{ cans}}$

7. $\frac{\$2{,}000{,}000}{4 \text{ yr}}$ > Videos

8. $\frac{150 \text{ cal}}{3 \text{ oz}}$

9. $\frac{240 \text{ lb of fertilizer}}{6 \text{ lawns}}$

10. $\frac{192 \text{ diapers}}{32 \text{ babies}}$

< Objective 2 >

Write each rate in words.

11. $120{,}000 \frac{\text{dollars}}{\text{yr}}$

12. $32 \frac{\text{ft}}{\text{s}}$

13. $\frac{1}{120{,}000} \frac{\text{yr}}{\text{dollar}}$

14. $\frac{1}{32} \frac{\text{s}}{\text{ft}}$

< Objective 3 >

Find the unit price of each item.

15. $57.50 for 5 shirts > Videos

16. $104.93 for 7 CDs

17. $5.16 for a dozen oranges

18. $10.44 for 18 bottles of water

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Name ____________________

Section ________ Date ________

Answers

1. ________ 2. ________

3. ________ 4. ________

5. ________ 6. ________

7. ________

8. ________ 9. ________

10. ________

11. ________

12. ________

13. ________

14. ________

15. ________

16. ________

17. ________

18. ________

Answers

19. ____________________

20. ____________________

21. ____________________

22. ____________________

23. ____________________

24. ____________________

< Objective 4 >

Find the best buy in each exercise.

19. Dishwashing liquid:

(a) 12 fl oz for $1.58
(b) 22 fl oz for $2.58

ULTRA
Brightdish
DETERGENT

20. Canned corn:

(a) 10 oz for 42¢
(b) 17 oz for 78¢

Green Grocer
Whole Kernel Sweet Corn

21. Syrup:

(a) 12 oz for $1.98
(b) 24 oz for $3.18
(c) 36 oz for $4.38

Pure
Maple
Syrup

22. Shampoo:

(a) 4 oz for $2.32
(b) 7 oz for $3.04
(c) 15 oz for $6.78

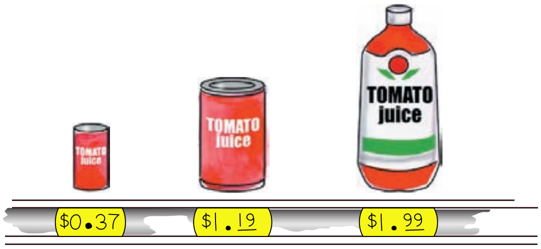
Extra Body
Shampoo

23. Salad oil (1 qt is 32 oz):

(a) 18 oz for $1.78
(b) 1 qt for $2.78
(c) 1 qt 16 oz for $4.38

24. Tomato juice (1 pt is 16 oz):

(a) 8 oz for 74¢
(b) 1 pt 10 oz for $2.38
(c) 1 qt 14 oz for $3.98


TOMATO juice
TOMATO juice
TOMATO juice
$0.37
$1.19
$1.99

25. Peanut butter (1 lb is 16 oz):

(a) 12 oz for $2.50
(b) 18 oz for $3.44
(c) 1 lb 12 oz for $5.08
(d) 2 lb 8 oz for $7.52

Peanut
Butter

26. Laundry detergent:

(a) 1 lb 2 oz for $3.98
(b) 1 lb 12 oz for $5.78
(c) 2 lb 8 oz for $8.38
(d) 5 lb for $15.98

Bright
Laundry
DETERGENT

Solve each application.

27. Trac uses 8 gallons of gasoline on a 256-mile drive. How many miles per gallon does his car get?

28. Seven pounds of fertilizer covers 1,400 square feet. How many square feet are covered by 1 pound of fertilizer?

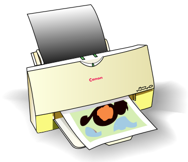
29. A local college has 6,000 registered vehicles for 2,400 campus parking spaces. How many vehicles are there for each parking space?

> Videos

30. A water pump can produce 280 gallons in 24 hours. How many gallons per hour is this?

31. The sum of $5,992 was spent for 214 shares of stock. What was the cost per share?

32. A printer produces 4 pages of print in 6 seconds. How many pages are produced per second?



33. A 12-ounce can of tuna costs $4.80. What is the cost of tuna per ounce?

34. The fabric for a dress costs $76.45 for 9 yards. What is the cost per yard?

35. Gerry laid 634 bricks in 35 minutes and his friend Matt laid 515 bricks in 27 minutes. Who is the faster bricklayer?

36. Mike drove 135 miles (mi) in 2.5 hours (h). Sam drove 91 mi in 1.75 h. Who drove faster?

37. Luis Gonzalez had 137 hits in 387 at bats. Larry Walker had 119 hits in 324 at bats. Who had the higher batting average?

38. Which is the better buy: 5 lb of sugar for $4.75 or 20 lb of sugar for $19.92?

Answers

25. ____
26. ____
27. ____
28. ____
29. ____
30. ____
31. ____
32. ____
33. ____
34. ____
35. ____
36. ____
37. ____
38. ____

Basic Skills | **Challenge Yourself** | Calculator/Computer | Career Applications | Above and Beyond

Determine whether each statement is **true** *or* **false.**

39. We use rates to compare like quantities.

40. We use rates to compare different types of measurements.

Fill in each blank with **always, sometimes,** *or* **never.**

41. The units __________ cancel in a rate.

42. The units __________ cancel in a ratio.

Find each rate.

43. $\frac{69\text{ ft}}{3\text{ s}}$

44. $\frac{3\text{ s}}{69\text{ ft}}$

45. $\frac{5\text{ yr}}{\$10{,}000}$

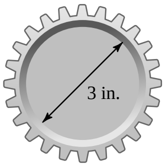
46. $\frac{480\text{ mi}}{15\text{ gal}}$

47. $\frac{15\text{ gal}}{480\text{ mi}}$

48. $\frac{657{,}200\text{ library books}}{5{,}200\text{ students}}$

Basic Skills | Challenge Yourself | Calculator/Computer | **Career Applications** | Above and Beyond

49. **MECHANICAL ENGINEERING** The pitch of a gear is given by the quotient of the number of teeth on the gear and the diameter of the gear (distance from end to end, through the center). Calculate the pitch of the gear shown.



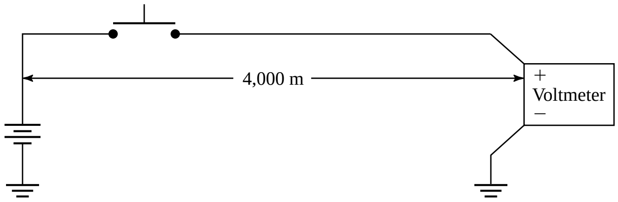
50. **ALLIED HEALTH** A patient's tidal volume, in milliliters (mL) per breath, is the quotient of his or her minute volume (mL/min) and his or her respiratory rate (breaths/min).

Report the tidal volume for an adult, female patient whose minute volume is 7,500 mL/min if her respiratory rate is 12 breaths/min.

chapter 5 > Make the Connection

51. **BUSINESS AND FINANCE** Determine the unit price of a 1,000-ft cable that costs $99.99.

52. **ELECTRICAL ENGINEERING** A 20-volt (V) DC pulse is sent down a 4,000-meter (m) length of conductor (see the figure). Because of resistance, when the pulse reaches the other end, the voltmeter measures the voltage as 4 V. What is the rate of voltage drop per meter of conductor?



Answers

39. ______________

40. ______________

41. ______________

42. ______________

43. ______________

44. ______________

45. ______________

46. ______________

47. ______________

48. ______________

49. ______________

50. ______________

51. ______________

52. ______________

53. **Business and Finance** A 200-bushel load of soybeans sells for $1,780. What is the price per bushel?

54. **Mechanical Engineering** Stress is calculated as the rate of force applied compared to the cross-sectional area of a post. What is the stress on a post that supports 13,475 lb and has a cross-sectional area of 12.25 square inches?

Basic Skills | Challenge Yourself | Calculator/Computer | Career Applications | **Above and Beyond**

55. Describe the difference between the rates $\frac{\$120{,}000}{\text{yr}}$ and $\frac{1}{120{,}000}\frac{\text{yr}}{\text{dollar}}$.

56. Describe the difference between the rates $32\frac{\text{ft}}{\text{s}}$ and $\frac{1}{32}\frac{\text{s}}{\text{ft}}$.

57. In your own words, explain the difference between a ratio and a rate.

58. Find several real-world examples of ratios and of rates.

59. Explain why unit pricing is useful.

60. Go to a supermarket or grocery store. Choose five items that have price and unit price listed. Check to see if the unit price given for each item is accurate.

Answers

1. $75\frac{\text{mi}}{\text{h}}$ **3.** $2{,}000\frac{\text{dollars}}{\text{yr}}$ **5.** $450\frac{\text{rev}}{\text{mi}}$ **7.** $500{,}000\frac{\text{dollars}}{\text{yr}}$
9. $40\frac{\text{lb}}{\text{lawn}}$ **11.** One hundred twenty thousand dollars per year
13. One one-hundred twenty thousandth of a year per dollar **15.** $11.50\frac{\text{dollars}}{\text{shirt}}$
17. $0.43\frac{\text{dollars}}{\text{orange}}$ **19.** (b) **21.** (c) **23.** (b) **25.** (c)
27. $32\frac{\text{mi}}{\text{gal}}$ **29.** $2.5\frac{\text{vehicles}}{\text{space}}$ **31.** $28\frac{\text{dollars}}{\text{share}}$ **33.** $40\frac{\text{cents}}{\text{oz}}$
35. Matt **37.** Larry Walker **39.** False **41.** never **43.** $23\frac{\text{ft}}{\text{s}}$
45. $0.0005\frac{\text{yr}}{\text{dollar}}$ **47.** $\frac{1}{32}\frac{\text{gal}}{\text{mi}}$ **49.** $8\frac{\text{teeth}}{\text{in.}}$ **51.** $\approx 0.10\frac{\text{dollars}}{\text{ft}}$
53. $8.90\frac{\text{dollars}}{\text{bushel}}$ **55.** Above and Beyond **57.** Above and Beyond
59. Above and Beyond

Answers

53. ____

54. ____

55. ____

56. ____

57. ____

58. ____

59. ____

60. ____

Activity 14 ::
Baseball Statistics

There are many statistics in the sport of baseball that are expressed in decimal form. Two of these are batting average and earned run average. Both are actually examples of rates.

A batting average is a rate for which the units are "hits per at bat." To compute the batting average for a hitter, divide the number of hits (H) by the number of times at bat (AB). The result will be a decimal less than 1 (unless the batter always gets a hit!), and it is always expressed to the nearest thousandth. For example, if a hitter has 2 hits in 7 at bats, we divide 2 by 7, getting 0.285714. . . . The batting average is then rounded to 0.286.

Compute the batting average for each of the following major league players.

	Player	Hits	At Bats	Average
1	Pujols	139	373	
2	Helton	134	384	
3	Guillen	99	290	
4	Suzuki	142	419	
5	Mora	99	293	

The earned run average (ERA) for a pitcher is also a rate; its units are "earned runs per 9 innings." It represents the number of earned runs the pitcher gives up in 9 innings. To compute the ERA for a pitcher, multiply the number of earned runs allowed by the pitcher by 9, and then divide by the number of innings pitched. The result is always rounded to the nearest hundredth.

Compute the earned run average for each of the following major league players.

	Player	Earned Runs	Innings	ERA
6	Loaiza	33	$137\frac{1}{3}$	
7	Martinez	27	110	
8	Brown	31	$123\frac{1}{3}$	

Challenge: Suppose a hitter has 54 hits in 200 times at bat. How many hits in a row must the hitter get in order to raise his average to at least 0.300?

5.3 Proportions

< 5.3 Objectives >

1 > Write a proportion
2 > Determine whether two fractions are proportional
3 > Determine whether two rates are proportional

Definition

Proportion A statement that two fractions or rates are equal is called a **proportion.**

NOTES

This is the same as saying the fractions are equivalent. They name the same number.

We call a letter representing an unknown value a *variable.* Here *a*, *b*, *c*, and *d* are variables. We could have chosen any other letter.

Because the ratio of 1 to 3 is equal to the ratio of 2 to 6, we can write the proportion

$$\frac{1}{3} = \frac{2}{6}$$

The proportion $\frac{a}{b} = \frac{c}{d}$ is read "*a* is to *b* as *c* is to *d*." We read the proportion $\frac{1}{3} = \frac{2}{6}$ as "one is to three as two is to six."

Example 1 Writing a Proportion

< Objective 1 >

Write the proportion 3 is to 7 as 9 is to 21.

$$\frac{3}{7} = \frac{9}{21}$$

Check Yourself 1

Write the proportion 4 is to 12 as 6 is to 18.

When you write a proportion for two rates, placement of similar units is important.

Example 2 Writing a Proportion with Two Rates

Write a proportion that is equivalent to the statement: If it takes 3 hours to mow 4 acres of grass, it will take 6 hours to mow 8 acres.

$$\frac{3 \text{ hours}}{4 \text{ acres}} = \frac{6 \text{ hours}}{8 \text{ acres}}$$

Note that, in both fractions, the hours units are in the numerator and the acres units are in the denominator.

Check Yourself 2

Write a proportion that is equivalent to the statement: If it takes 5 rolls of wallpaper to cover 400 square feet, it will take 7 rolls to cover 560 square feet.

If two fractions form a true proportion, we say that they are **proportional.**

Property

The Proportion Rule

If $\frac{a}{b} = \frac{c}{d}$, then $a \cdot d = b \cdot c$.

We say that the fractions $\frac{a}{b}$ and $\frac{c}{d}$ are proportional.

Example 3 Determining Whether Two Fractions Are Proportional

< Objective 2 >

NOTE

Use the centered dot (·) for multiplication rather than the cross (×), so that the cross won't be confused with the letter *x*.

RECALL

We saw this same test in Section 2.4 called "Testing for Equivalence."

Determine whether each pair of fractions is proportional.

(a) $\frac{5}{6} \overset{?}{=} \frac{10}{12}$

Multiply:

$$\left.\begin{aligned}5 \cdot 12 &= 60\\ 6 \cdot 10 &= 60\end{aligned}\right\}\ \text{Equal}$$

Because $a \cdot d = b \cdot c$, $\frac{5}{6}$ and $\frac{10}{12}$ are proportional.

(b) $\frac{3}{7} \overset{?}{=} \frac{4}{9}$

Multiply:

$$\left.\begin{aligned}3 \cdot 9 &= 27\\ 7 \cdot 4 &= 28\end{aligned}\right\}\ \text{Not equal}$$

The products are not equal, so $\frac{3}{7}$ and $\frac{4}{9}$ are not proportional.

Check Yourself 3

Determine whether each pair of fractions is proportional.

(a) $\frac{5}{8} \overset{?}{=} \frac{20}{32}$ **(b)** $\frac{7}{9} \overset{?}{=} \frac{3}{4}$

The proportion rule can also be used when fractions or decimals are involved.

Example 4 Verifying a Proportion

Determine whether each pair of fractions is proportional.

(a) $\dfrac{3}{\frac{1}{2}} \overset{?}{=} \dfrac{30}{5}$

$$3 \cdot 5 = 15$$

$$\frac{1}{2} \cdot 30 = 15$$

Because the products are equal, the fractions are proportional.

(b) $\frac{0.4}{20} \overset{?}{=} \frac{3}{100}$

$$0.4 \cdot 100 = 40$$

$$20 \cdot 3 = 60$$

Because the products are *not* equal, the fractions are not proportional.

Check Yourself 4

Determine whether each pair of fractions is proportional.

(a) $\frac{0.5}{8} \stackrel{?}{=} \frac{3}{48}$ **(b)** $\frac{\frac{1}{4}}{6} \stackrel{?}{=} \frac{3}{80}$

The proportion rule can also be used to verify that rates are proportional.

Example 5 Determining Whether Two Rates are Proportional

< Objective 3 >

NOTE

Colones are the monetary unit of Costa Rica.

Is the rate $\frac{5 \text{ U.S. dollars}}{15{,}000 \text{ colones}}$ proportional to the rate $\frac{27 \text{ U.S. dollars}}{81{,}000 \text{ colones}}$?

We want to know if the following is true.

$$\frac{5}{15{,}000} \stackrel{?}{=} \frac{27}{81{,}000}$$

$$5 \cdot 81{,}000 = 405{,}000$$

$$27 \cdot 15{,}000 = 405{,}000$$

The rates are proportional.

US Dollars	Colones
$1.00	3000
$0.00033	1.0

Check Yourself 5

Is the rate $\frac{50 \text{ pages}}{45 \text{ minutes}}$ proportional to the rate $\frac{30 \text{ pages}}{25 \text{ minutes}}$?

In Section 5.4, we will use proportions to solve many applications. For instance, if a 12-ft piece of steel stock weighs 27.6 lb, how much would a 25-ft piece weigh? Here, we check the accuracy of such a proportion.

Example 6 An Application of Proportions

A 12-ft piece of steel stock weighs 27.6 lb. If 57.5 lb is the weight of a 25-ft piece, do the two pieces have the same density?

We check that the two rates are proportional.

RECALL

Be sure to align the units, regardless of the order in which the information appears in the problem.

$$\frac{12 \text{ ft}}{27.6 \text{ lb}} \stackrel{?}{=} \frac{25 \text{ ft}}{57.5 \text{ lb}}$$

$$12 \cdot 57.5 = 690$$

$$27.6 \cdot 25 = 690$$

Because the two products are equal, we have a true proportion, so the two pieces have the same density.

Check Yourself 6

One supplier sells a 200-lb lot of steel for $522.36. A second supplier charges $789.09 for a 300-lb lot of steel. Determine whether the suppliers are offering steel for the same price per pound.

Check Yourself ANSWERS

1. $\frac{4}{12} = \frac{6}{18}$ **2.** $\frac{5 \text{ rolls}}{400 \text{ square feet}} = \frac{7 \text{ rolls}}{560 \text{ square feet}}$ **3.** **(a)** Yes; **(b)** no
4. **(a)** Yes; **(b)** no **5.** No **6.** No

Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 5.3

(a) A statement that two rates are ________________ is called a proportion.

(b) A letter used to represent an unknown value is called a ________________.

(c) If two fractions form a true ________________, we say that they are proportional.

(d) When writing a proportion for two ________________, corresponding units must be similarly placed.

5.3 exercises

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Name ____________________

Section __________ Date __________

< Objective 1 >

Write each statement as a proportion.

1. 4 is to 9 as 8 is to 18.

2. 6 is to 11 as 18 is to 33.

3. 2 is to 9 as 8 is to 36.

4. 10 is to 15 as 20 is to 30.

5. 3 is to 5 as 15 is to 25.

6. 8 is to 11 as 16 is to 22.

7. 9 is to 13 as 27 is to 39.

8. 15 is to 21 as 60 is to 84.

< Objective 2 >

Determine whether each pair of fractions is proportional.

9. $\frac{3}{4} \overset{?}{=} \frac{9}{12}$ > Videos

10. $\frac{6}{7} \overset{?}{=} \frac{18}{21}$

11. $\frac{3}{4} \overset{?}{=} \frac{15}{20}$

12. $\frac{3}{5} \overset{?}{=} \frac{6}{10}$

13. $\frac{11}{15} \overset{?}{=} \frac{9}{13}$ > Videos

14. $\frac{9}{10} \overset{?}{=} \frac{2}{7}$

15. $\frac{8}{3} \overset{?}{=} \frac{24}{9}$

16. $\frac{5}{8} \overset{?}{=} \frac{15}{24}$

17. $\frac{6}{17} \overset{?}{=} \frac{9}{11}$

18. $\frac{5}{12} \overset{?}{=} \frac{8}{20}$

19. $\frac{7}{16} \overset{?}{=} \frac{21}{48}$

20. $\frac{2}{5} \overset{?}{=} \frac{7}{9}$

21. $\frac{10}{3} \overset{?}{=} \frac{150}{50}$

22. $\frac{5}{8} \overset{?}{=} \frac{75}{120}$

23. $\frac{3}{7} \overset{?}{=} \frac{18}{42}$

24. $\frac{12}{7} \overset{?}{=} \frac{96}{50}$

25. $\frac{7}{15} \overset{?}{=} \frac{84}{180}$

26. $\frac{76}{24} \overset{?}{=} \frac{19}{6}$

< Objective 3 >

Determine if the given rates are proportional.

27. $\frac{\text{7 cups of flour}}{\text{4 loaves of bread}} \overset{?}{=} \frac{\text{4 cups of flour}}{\text{3 loaves of bread}}$ > Videos

28. $\frac{\text{6 U.S. dollars}}{\text{50 Krone}} \overset{?}{=} \frac{\text{15 U.S. dollars}}{\text{125 Krone}}$

Answers

1. __________ 2. __________
3. __________ 4. __________
5. __________ 6. __________
7. __________ 8. __________
9. __________ 10. __________
11. __________ 12. __________
13. __________ 14. __________
15. __________ 16. __________
17. __________ 18. __________
19. __________ 20. __________
21. __________ 22. __________
23. __________ 24. __________
25. __________ 26. __________
27. __________ 28. __________

Answers

29. ______________

30. ______________

31. ______________

32. ______________

33. ______________

34. ______________

35. ______________

36. ______________

37. ______________

38. ______________

39. ________ 40. ________

41. ________ 42. ________

43. ________ 44. ________

45. ________ 46. ________

29. $\dfrac{22 \text{ miles}}{15 \text{ gallons}} \overset{?}{=} \dfrac{55 \text{ miles}}{35 \text{ gallons}}$

30. $\dfrac{46 \text{ pages}}{30 \text{ minutes}} \overset{?}{=} \dfrac{18 \text{ pages}}{8 \text{ minutes}}$

31. $\dfrac{9 \text{ inches}}{57 \text{ miles}} \overset{?}{=} \dfrac{6 \text{ inches}}{38 \text{ miles}}$

32. $\dfrac{12 \text{ yen}}{5 \text{ pesos}} \overset{?}{=} \dfrac{108 \text{ yen}}{45 \text{ pesos}}$

Write the proportion that is equivalent to the given statement.

33. If 15 pounds (lb) of string beans cost \$4, then 45 lb will cost \$12.

> Videos

34. If Maria hit 8 home runs in 15 softball games, then she should hit 24 home runs in 45 games.

35. If 3 credits at Bucks County Community College cost \$216, then 12 credits cost \$864.

36. If 16 pounds (lb) of fertilizer cover 1,520 square feet (ft^2), then 21 lb should cover 1,995 ft^2.

37. If Audrey travels 180 miles (mi) on interstate I-95 in 3 hours (h), then she should travel 300 mi in 5 h.

38. If 2 vans can transport 18 people, then 5 vans can transport 45 people.

Basic Skills | **Challenge Yourself** | Calculator/Computer | Career Applications | Above and Beyond

Determine whether each statement is **true** *or* **false.**

39. Two ratios must be equal in order for the ratios to be proportional.

40. If $\dfrac{a}{b} = \dfrac{c}{d}$, then $a \cdot c = b \cdot d$.

Fill in each blank with **always, sometimes,** *or* **never.**

41. Proportions are ______________ used to compare two rates.

42. When writing a proportion for two rates, the placement of units is ______________ important.

Determine whether each pair of fractions or rates is proportional.

43. $\dfrac{60}{36} \overset{?}{=} \dfrac{25}{15}$

44. $\dfrac{\frac{1}{2}}{4} \overset{?}{=} \dfrac{5}{40}$

45. $\dfrac{3}{\frac{1}{5}} \overset{?}{=} \dfrac{30}{6}$

46. $\dfrac{\frac{2}{3}}{6} \overset{?}{=} \dfrac{1}{12}$

47. $\dfrac{\frac{3}{4}}{12} \overset{?}{=} \dfrac{1}{16}$ > Videos

48. $\dfrac{0.3}{4} \overset{?}{=} \dfrac{1}{20}$

49. $\dfrac{3}{60} \overset{?}{=} \dfrac{0.3}{6}$

50. $\dfrac{0.6}{0.12} \overset{?}{=} \dfrac{2}{0.4}$

51. $\dfrac{0.6}{15} \overset{?}{=} \dfrac{2}{75}$

52. $\dfrac{\text{12 gallons of paint}}{8{,}329 \text{ ft}^2} \overset{?}{=} \dfrac{\text{9 gallons of paint}}{1{,}240 \text{ ft}^2}$

53. $\dfrac{\text{12 inches of snow}}{\text{1.4 inches of rain}} \overset{?}{=} \dfrac{\text{36 inches of snow}}{\text{7 inches of rain}}$

54. $\dfrac{\text{9 people}}{\text{2 cars}} \overset{?}{=} \dfrac{\text{11 people}}{\text{3 cars}}$

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55. **ALLIED HEALTH** Quinidine is an antiarrhythmic heart medication. It is available for injection as an 80 milligrams per milliliter (mg/mL) solution. A patient receives a prescription for 300 mg of quinidine dissolved in 3.75 mL of solution. Are these rates proportional?

56. **INFORMATION TECHNOLOGY** A computer transmits 5 Web pages in 2 seconds (s) to a Web server. A second computer transmits 20 pages in 10 s to the server. Are these two computers transmitting at the same speed?

57. **MECHANICAL ENGINEERING** A gear has a pitch diameter of 5 in. and 20 teeth. A second gear has a pitch diameter of 18 in. and 68 teeth. In order to mesh, the teeth to diameter rates must be proportional. Will these two gears mesh?

58. **AGRICULTURAL TECHNOLOGY** A 13-acre field requires 7,020 lb of fertilizer. Will 11,340 lb of fertilizer cover a 21-acre field?

Answers

1. $\dfrac{4}{9} = \dfrac{8}{18}$ **3.** $\dfrac{2}{9} = \dfrac{8}{36}$ **5.** $\dfrac{3}{5} = \dfrac{15}{25}$ **7.** $\dfrac{9}{13} = \dfrac{27}{39}$

9. Yes **11.** Yes **13.** No **15.** Yes **17.** No **19.** Yes

21. No **23.** Yes **25.** Yes **27.** No **29.** No **31.** Yes

33. $\dfrac{15 \text{ lb}}{\$4} = \dfrac{45 \text{ lb}}{\$12}$ **35.** $\dfrac{\text{3 credits}}{\$216} = \dfrac{\text{12 credits}}{\$864}$ **37.** $\dfrac{180 \text{ mi}}{3 \text{ h}} = \dfrac{300 \text{ mi}}{5 \text{ h}}$

39. True **41.** sometimes **43.** Yes **45.** No **47.** Yes

49. Yes **51.** No **53.** No **55.** Yes **57.** No

Answers

47. ____________ 48. ____________

49. ____________ 50. ____________

51. ____________ 52. ____________

53. ____________ 54. ____________

55. ____________ 56. ____________

57. ____________ 58. ____________

5.4

Solving Proportions

< **5.4 Objectives** >

1 > Solve a proportion for an unknown value

2 > Solve an application involving a proportion

NOTE

$\frac{?}{3} = \frac{10}{15}$ is a proportion in which the first value is unknown. Our work in this section involves learning how to find that unknown value.

A proportion consists of four values. If three of the four values of a proportion are known, you can always find the missing or unknown value.

In the proportion $\frac{a}{3} = \frac{10}{15}$, the first value is unknown. We have chosen to represent the unknown value with the letter *a*. Using the proportion rule, we can proceed as follows.

$$\frac{a}{3} = \frac{10}{15}$$

$$15 \cdot a = 3 \cdot 10 \qquad \text{or} \qquad 15 \cdot a = 30$$

The equal sign tells us that $15 \cdot a$ and 30 are just different names for the same number. This type of statement is called an **equation.**

Definition

Equation An **equation** is a statement that two expressions are equal.

One important property of an equation is that we can divide both sides by the same nonzero number. Here we divide by 15.

NOTE

We always divide by the number multiplying the variable. This is called the *coefficient* of the variable.

$$15 \cdot a = 30$$

$$\frac{15 \cdot a}{15} = \frac{30}{15}$$

$$\frac{\overset{1}{\cancel{15}} \cdot a}{\underset{1}{\cancel{15}}} = \frac{\overset{2}{\cancel{30}}}{\underset{1}{\cancel{15}}}$$

Divide by the coefficient of the variable. Do you see why we divided by 15? It leaves our unknown *a* by itself in the left term.

$$a = 2$$

You should always check your result. It is easy in this case. We found a value of 2 for *a*. Replace the unknown *a* with that value. Then verify that the fractions are proportional. We started with $\frac{a}{3} = \frac{10}{15}$ and found a value of 2 for *a*. So we write

NOTE

Replace *a* with 2 and multiply.

$$\frac{2}{3} \overset{?}{=} \frac{10}{15}$$

$$2 \cdot 15 \overset{?}{=} 3 \cdot 10$$

$$30 = 30$$

The value of 2 for *a* is correct.

The procedure for solving a proportion is summarized as follows.

Step by Step

To Solve a Proportion

Step 1 Use the proportion rule to write the equivalent equation $a \cdot d = b \cdot c$.

Step 2 Divide both terms of the equation by the coefficient of the variable.

Step 3 Use the value found to replace the unknown in the original proportion. Check that the ratios or the rates are proportional.

Example 1 Solving Proportions for Unknown Values

< Objective 1 >

NOTE

You are really using algebra to solve these proportions. In algebra, we write the product $6 \cdot x$ as $6x$, omitting the dot. Multiplication of the number and the variable is understood.

Find the unknown value.

(a) $\frac{8}{x} = \frac{6}{9}$

Step 1 Using the proportion rule, we have the following:

$$6 \cdot x = 8 \cdot 9$$

or $$6x = 72$$

Step 2 Locate the coefficient of the variable, 6, and divide both sides of the equation by that coefficient.

$$\frac{\overset{1}{\cancel{6}}x}{\underset{1}{\cancel{6}}} = \frac{\overset{12}{\cancel{72}}}{\underset{1}{\cancel{6}}}$$

$$x = 12$$

NOTE

This gives us the unknown value. Now check the result.

Step 3 To check, replace x with 12 in the original proportion.

$$\frac{8}{12} \stackrel{?}{=} \frac{6}{9}$$

Multiply:

$$12 \cdot 6 \stackrel{?}{=} 8 \cdot 9$$

$$72 = 72$$ The value of 12 checks for x.

(b) $\frac{3}{4} = \frac{c}{25}$

Step 1 Use the proportion rule.

$$4 \cdot c = 3 \cdot 25$$

or $$4c = 75$$

Step 2 Locate the coefficient of the variable, 4, and divide both sides of the equation by that coefficient.

$$\frac{\overset{1}{\cancel{4}}c}{\underset{1}{\cancel{4}}} = \frac{75}{4}$$

$$c = \frac{75}{4}$$

Step 3 To check, replace c with $\frac{75}{4}$ in the original proportion.

$$\frac{3}{4} \stackrel{?}{=} \frac{\left(\frac{75}{4}\right)}{25}$$

Multiply:

$$3 \cdot 25 = 75$$

$$4 \cdot \frac{75}{4} = 75$$ The products are the same, so the value of $\frac{75}{4}$ checks for c.

RECALL

In the proportion equation $\frac{a}{b} = \frac{c}{d}$, we check to see if $ad = bc$.

Check Yourself 1

Solve the proportions for *n*. Check your result.

(a) $\frac{4}{5} = \frac{n}{25}$ **(b)** $\frac{5}{9} = \frac{12}{n}$

In solving for a missing term in a proportion, we may find an equation involving fractions or decimals. Example 2 involves finding the unknown value in such cases.

Example 2 Solving Proportions for Unknown Values

(a) Solve the proportion for *x*.

$$\frac{\frac{1}{4}}{3} = \frac{4}{x}$$

$$\frac{1}{4}x = 12$$

$$\frac{\frac{1}{4}x}{\frac{1}{4}} = \frac{12}{\frac{1}{4}}$$

We divide by the coefficient of *x*.
In this case it is $\frac{1}{4}$.

$$x = \frac{12}{\frac{1}{4}}$$

Remember: $\frac{12}{\frac{1}{4}}$ is $12 \div \frac{1}{4}$. Invert the divisor and multiply.

$$x = 48$$

RECALL

The coefficient is the number multiplying the variable, in this case $\frac{1}{4}$.

To check, replace *x* with 48 in the original proportion.

$$\frac{\frac{1}{4}}{3} \overset{?}{=} \frac{4}{48}$$

$$\frac{1}{4} \cdot 48 \overset{?}{=} 3 \cdot 4$$

$$12 = 12$$

(b) Solve the proportion for *d*.

$$\frac{0.5}{2} = \frac{3}{d}$$

$$0.5d = 6$$

$$\frac{0.5d}{0.5} = \frac{6}{0.5}$$

Divide by the coefficient, 0.5.

$$d = 12$$

We leave it to you to confirm that $0.5 \cdot 12 = 2 \cdot 3$.

NOTE

Here we must divide 6 by 0.5 to find the unknown value. The steps of that division are shown here for review.

```
      1 2.
0.5 )6.0
     5
     1 0
     1 0
       0
```

Check Yourself 2

(a) Solve for *d*. **(b) Solve for *x*.**

$$\frac{\frac{1}{2}}{5} = \frac{3}{d} \qquad \frac{0.4}{x} = \frac{2}{30}$$

Now that we have learned how to find an unknown value in a proportion, we can solve many applications.

Step by Step

Solving Applications of Proportions

Step 1 Read the problem carefully to determine the given information.

Step 2 Write the proportion necessary to solve the problem. Use a letter to represent the unknown quantity. Be sure to include the units in writing the proportion.

Step 3 Solve, answer the question of the original problem, and check the proportion.

Example 3 Solve an Application Involving a Proportion

< Objective 2 >

(a) In a shipment of 400 parts, 14 are found to be defective. How many defective parts should be expected in a shipment of 1,000?

Assume that the ratio of defective parts to the total number remains the same.

$$\frac{14 \text{ defective}}{400 \text{ total}} = \frac{x \text{ defective}}{1{,}000 \text{ total}}$$

We decided to let *x* be the unknown number of defective parts.

Multiply:

$400x = 14{,}000$

Divide by the coefficient, 400.

$x = 35$

So 35 defective parts should be expected in the shipment.

Checking the original proportion, we get

$$14 \cdot 1{,}000 \overset{?}{=} 400 \cdot 35$$
$$14{,}000 = 14{,}000$$

(b) Jill works 4.2 h and receives $21. How much will she get if she works 10 h?

The rate comparing hours worked and the amount of pay remains the same.

$$\frac{4.2 \text{ h}}{\$21} = \frac{10 \text{ h}}{\$a}$$

Let *a* be the unknown amount of pay.

$$4.2a = 210$$

$$\frac{4.2a}{4.2} = \frac{210}{4.2}$$

Divide both sides by 4.2.

$$a = \$50$$

Check Yourself 3

(a) An investment of $3,000 earned $330 for 1 year. How much will an investment of $10,000 earn at the same rate for 1 year?

(b) A piece of cable 8.5 centimeters (cm) long weighs 68 grams (g). What does a 10-cm length of the same cable weigh?

Example 4 Using Proportions to Solve a Map-Scale Application

The scale on a map is given as $\frac{1}{4}$ inch (in.) = 3 miles (mi). The distance between two towns is 4 in. on the map. How far apart are the towns in miles?

NOTE

We could divide both sides by $\frac{1}{4}$:

$$\frac{\frac{1}{4}\cdot x}{\frac{1}{4}} = \frac{3\cdot 4}{\frac{1}{4}}$$

$$x = \frac{3\cdot 4}{\frac{1}{4}}$$

$$x = \frac{12}{\frac{1}{4}}$$

then invert and multiply.

$$x = \frac{12}{1}\cdot\frac{4}{1}$$

$$= 48$$

For this solution we use the fact that the ratio of inches (on the map) to miles remains the same. We also use another important property of an equation: We can multiply both sides of the equation by the same non-zero number.

$$\frac{\frac{1}{4}\text{ in.}}{3\text{ mi}} = \frac{4\text{ in.}}{x\text{ mi}}$$

$$\frac{1}{4}\cdot x = 3\cdot 4$$

$$4\cdot\frac{1}{4}\cdot x = 4\cdot 3\cdot 4$$

We multiply both sides of the equation by 4, since $4\cdot\frac{1}{4} = 1$.

$$1\cdot x = 4\cdot 3\cdot 4$$

$$x = 48\text{ (mi)}$$

Check Yourself 4

Jack drives 125 mi in $2\frac{1}{2}$ hours (h). At the same rate, how far will he be able to travel in 4 h? (Hint: Write $2\frac{1}{2}$ as an improper fraction.)

In Example 5 we must convert the units stated in the problem.

Example 5 Using Proportions to Solve an Application

A machine can produce 15 tin cans in 2 minutes (min). At this rate how many cans can it make in an 8-h period?

In writing a proportion for this problem, we must write the times involved in terms of the same units.

NOTE

Always check that your units are properly placed.

$$\frac{15\text{ cans}}{2\text{ min}} = \frac{x\text{ cans}}{480\text{ min}}$$

Because 1 h is 60 min, convert 8 h to 480 min.

$$15\cdot 480 = 2x$$

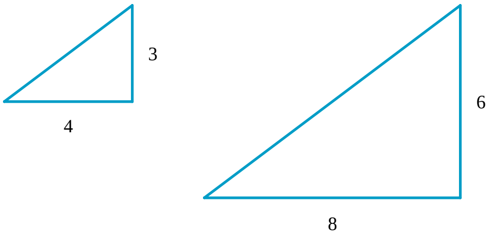
or $$7{,}200 = 2x$$

$$x = 3{,}600\text{ cans}$$

Check Yourself 5

Instructions on a can of film developer call for 2 ounces (oz) of concentrate to 1 quart (qt) of water. How much of the concentrate is needed to mix with 1 gallon (gal) of water? (4 qt = 1 gal.)

Proportions are important when working with *similar* geometric figures. These are figures that have the same shape and whose corresponding sides are proportional. For instance, in the similar triangles shown here, a proportion involving corresponding sides is



$$\frac{3}{4} = \frac{6}{8}$$

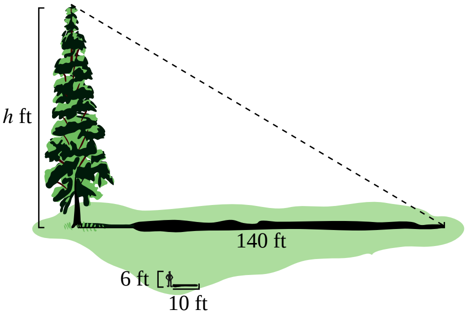
Example 6 Solving an Application Using Similar Triangles

NOTE

Connect the top of the tree to the end of the shadow to create a triangle. Connecting the top of the man to the end of his shadow creates a similar triangle.

If a 6-foot-tall man casts a shadow that is 10 feet (ft) long, how tall is a tree that casts a shadow that is 140 ft long?

Look at a picture of the two triangles involved.



From the similar triangles, we have the proportion

$$\frac{6}{10} = \frac{h}{140}$$

Using the proportion rule, we have $6 \cdot 140 = 10 \cdot h$, so

$$10 \cdot h = 840$$

$$\frac{10 \cdot h}{10} = \frac{840}{10}$$

$$h = 84$$

The tree is 84 ft tall.

Check Yourself 6

If a woman who is $5\frac{1}{2}$ ft tall casts a shadow that is 3 ft long, how tall is a building that casts a shadow that is 90 ft long?

Proportions are used in solving a variety of problems such as the allied health application in Example 7.

Example 7 An Application of Proportions

> CAUTION

Solving for *x* does not give us an answer directly. *x* represents the total volume, which includes both water and serum. We still need to subtract 8.5 from *x* to get a final answer.

In preparing specimens for testing, it is often necessary to dilute the original solution. The dilution ratio is the ratio of the volume, in milliliters (mL), of original solution to the total volume, also in mL, of the diluted solution. How much water is required to make a $\frac{1}{20}$ dilution from 8.5 mL of serum?

We set up a proportion equation. The original solution is the 8.5 mL of serum, so that should be in the numerator. We name the denominator *x*.

$$\frac{1}{20} = \frac{8.5}{x}$$

$$1 \cdot x = 20 \cdot 8.5$$

$$x = 170$$

Therefore, the total volume should be 170 mL. Because 8.5 mL of the total solution is serum, we need to add $170 - 8.5 = 161.5$ mL of water.

Check Yourself 7

How much water is required to make a $\frac{3}{50}$ dilution from 11.25 mL of serum?

Check Yourself ANSWERS

1. **(a)** $n = 20$; **(b)** $n = \frac{108}{5}$ **2.** **(a)** $d = 30$; **(b)** $x = 6$ **3.** **(a)** \$1,100; **(b)** 80 g **4.** 200 mi **5.** 8 oz **6.** 165 ft **7.** 176.25 mL

Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 5.4

(a) An ________________ is a statement that two expressions are equal.

(b) When a number and a variable are multiplied, the number is called a ________________.

(c) The first step to solving application problems is to ________________ the problem carefully.

(d) Two triangles are similar if corresponding sides are ________________.

5.4 exercises

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< Objective 1 >

Solve for the unknown in each proportion.

1. $\dfrac{x}{3} = \dfrac{6}{9}$ > Videos

2. $\dfrac{x}{6} = \dfrac{3}{9}$

3. $\dfrac{10}{n} = \dfrac{15}{6}$

4. $\dfrac{4}{3} = \dfrac{8}{n}$

5. $\dfrac{4}{7} = \dfrac{y}{14}$

6. $\dfrac{5}{8} = \dfrac{a}{16}$

7. $\dfrac{5}{7} = \dfrac{x}{35}$

8. $\dfrac{4}{15} = \dfrac{8}{n}$

9. $\dfrac{11}{a} = \dfrac{2}{44}$

10. $\dfrac{35}{40} = \dfrac{7}{n}$

11. $\dfrac{x}{8} = \dfrac{15}{24}$

12. $\dfrac{7}{12} = \dfrac{m}{24}$

13. $\dfrac{18}{12} = \dfrac{12}{p}$ > Videos

14. $\dfrac{20}{15} = \dfrac{100}{a}$

15. $\dfrac{5}{35} = \dfrac{a}{28}$

16. $\dfrac{20}{24} = \dfrac{p}{18}$

17. $\dfrac{12}{100} = \dfrac{3}{x}$

18. $\dfrac{b}{7} = \dfrac{21}{49}$

19. $\dfrac{p}{24} = \dfrac{25}{120}$

20. $\dfrac{5}{x} = \dfrac{20}{88}$

< Objective 2 >

Solve each application.

21. **BUSINESS AND FINANCE** If 12 books are purchased for $80, how much will you pay for 18 books at the same rate? > Videos

22. **CONSTRUCTION** If an 8-foot (ft) two-by-four costs $1.92, what should a 12-ft two-by-four cost?

23. **BUSINESS AND FINANCE** A box of 18 tea bags is marked $2.70. At that price, what should a box of 48 tea bags cost?

24. **BUSINESS AND FINANCE** A worker can complete the assembly of 15 MP3 players in 6 hours (h). At this rate, how many can the worker complete in a 40-h workweek?

25. **SOCIAL SCIENCE** The ratio of yes to no votes in an election was 3 to 2. How many no votes were cast if there were 2,880 yes votes?

Answers

1. ________ 2. ________

3. ________ 4. ________

5. ________ 6. ________

7. ________ 8. ________

9. ________ 10. ________

11. ________ 12. ________

13. ________ 14. ________

15. ________ 16. ________

17. ________ 18. ________

19. ________ 20. ________

21. ________ 22. ________

23. ____________________

24. ____________________

25. ____________________

Answers

26. ____________

27. ____________

28. ____________

29. ____________

30. ____________

31. ____________

32. ____________

33. ____________

34. ____________

35. ____________

36. ____________

26. **SOCIAL SCIENCE** The ratio of men to women at a college is 7 to 5. How many women students are there if there are 3,500 men?

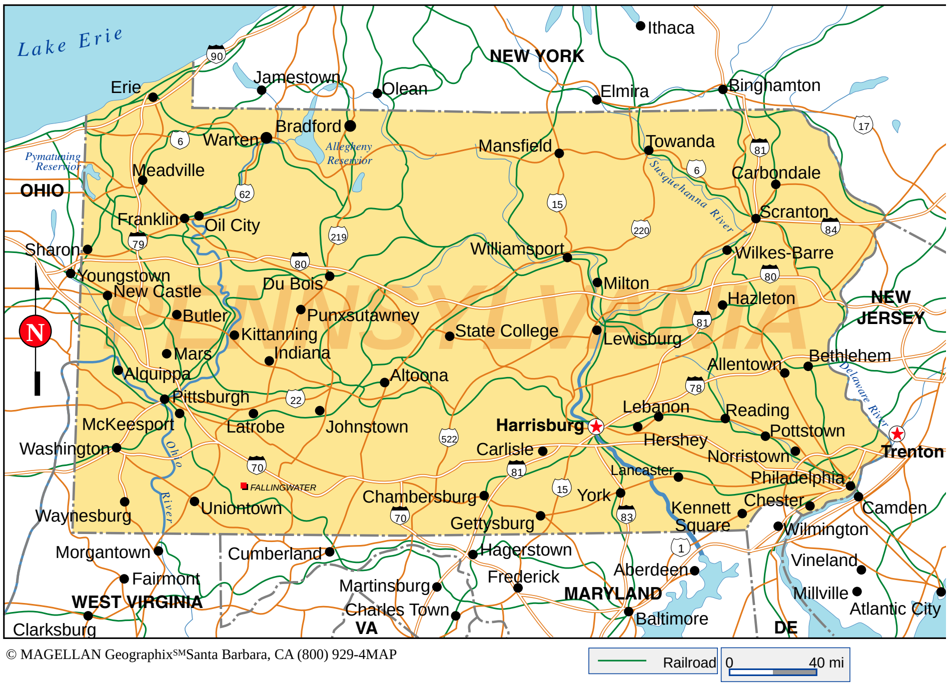
27. **CRAFTS** A photograph 5 inches (in.) wide by 6 in. high is to be enlarged so that the new width is 15 in. What will the height of the enlargement be?

28. **BUSINESS AND FINANCE** Christy can travel 110 miles (mi) in her new car on 5 gallons (gal) of gas. How far can she travel on a full (12 gal) tank?

29. **BUSINESS AND FINANCE** The Changs purchased a $120,000 home, and the property taxes were $2,100. If they make improvements and the house is now valued at $150,000, what will the new property tax be?

30. **BUSINESS AND FINANCE** A car travels 165 mi in 3 h. How far will it travel in 8 h if it continues at the same speed?

Using the given map, find the distances between the cities named in exercises 31 to 34. Measure distances to the nearest sixteenth of an inch.


Lake Erie
Ithaca
NEW YORK
Erie
Jamestown
Olean
Elmira
Binghamton
Warren
Bradford
Allegheny Reservoir
Mansfield
Towanda
Pymatuning Reservoir
Meadville
Carbondale
OHIO
Susquehanna River
Franklin
Oil City
Scranton
Sharon
Williamsport
Wilkes-Barre
Youngstown
New Castle
Du Bois
Milton
Hazleton
NEW JERSEY
Butler
Punxsutawney
State College
Lewisburg
N
Kittanning
Indiana
Mars
Alquippa
Altoona
Allentown
Bethlehem
Delaware River
Pittsburgh
Lebanon
Reading
McKeesport
Latrobe
Johnstown
Harrisburg
Hershey
Pottstown
Trenton
Washington
Carlisle
Norristown
Ohio River
FALLINGWATER
Lancaster
Philadelphia
Waynesburg
Uniontown
Chambersburg
York
Chester
Camden
Gettysburg
Kennett Square
Wilmington
Morgantown
Cumberland
Hagerstown
Vineland
Fairmont
Frederick
Aberdeen
Martinsburg
MARYLAND
Millville
WEST VIRGINIA
Charles Town
Atlantic City
Clarksburg
VA
Baltimore
DE
PENNSYLVANIA
90
6
62
79
80
219
15
220
6
81
84
80
81
78
22
522
70
81
15
83
70
1
17
Railroad
0
40 mi

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31. Find the distance from Harrisburg to Philadelphia.

32. Find the distance from Punxsutawney (home of the groundhog) to State College (home of the Nittany Lions).

33. Find the distance from Gettysburg to Meadville.

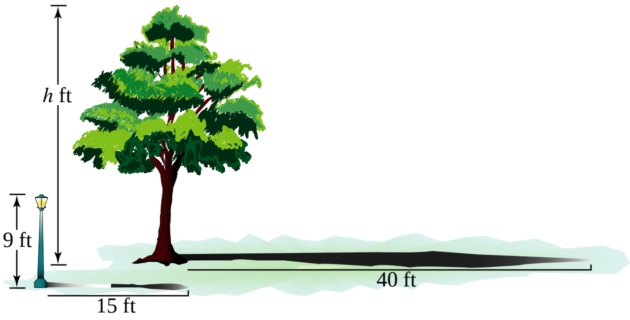
34. Find the distance from Scranton to Waynesburg.

35. **BUSINESS AND FINANCE** An inspection reveals 30 defective parts in a shipment of 500. How many defective parts should be expected in a shipment of 1,200?

36. **BUSINESS AND FINANCE** You invest $4,000 in a stock that pays a $180 dividend in 1 year. At the same rate, how much will you need to invest to earn $270?

37. **Construction** A 6-ft fence post casts a 9-ft shadow. How tall is a nearby pole that casts a 15-ft shadow?

38. **Construction** A 9-ft light pole casts a 15-ft shadow. Find the height of a nearby tree that is casting a 40-ft shadow.

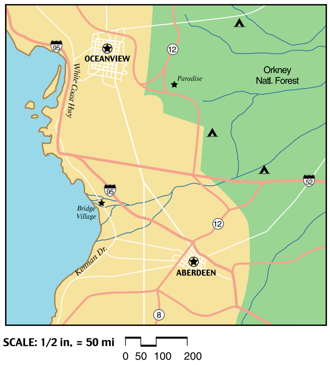

h ft
9 ft
40 ft
15 ft

39. **Construction** On the blueprint of the Wilsons' new home, the scale is 5 in. equals 7 ft. What will the actual length of a bedroom be if it measures 10 in. long on the blueprint?

> Videos

40. **Social Science** The scale on a map is $\frac{1}{2}$ in. = 50 mi. If the distance between two towns on the map is 6 in., how far apart are they in miles?

41. **Science and Medicine** A metal bar expands $\frac{1}{4}$ in. for each 12°F rise in temperature. How much will it expand if the temperature rises 48°F?


OCEANVIEW
12
White Coast Hwy
Paradise
Orkney Natl. Forest
95
52
Bridge Village
12
Kennan Dr.
ABERDEEN
8
SCALE: 1/2 in. = 50 mi
0 50 100 200

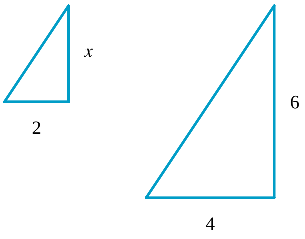
42. **Business and Finance** Your car burns $2\frac{1}{2}$ quarts (qt) of oil on a trip of 5,000 mi. How many quarts should you expect to use when driving 7,200 mi?

43. **Social Science** Approximately 7 out of every 10 people in the U.S. workforce drive to work alone. During morning rush hour there are 115,000 cars on the streets of a medium-sized city. How many of these cars have one person in them?

44. **Social Science** Approximately 15 out of every 100 people in the U.S. workforce carpool to work. There are an estimated 320,000 people in the workforce of a given city. How many of these people are in car pools?

Use a proportion to find the unknown side labeled x, in each pair of similar figures.

45.



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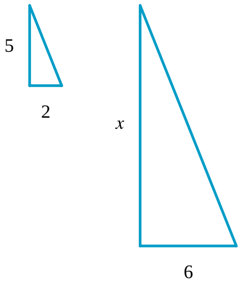
46.



Answers

37. ________

38. ________

39. ________

40. ________

41. ________

42. ________

43. ________

44. ________

45. ________

46. ________

Answers

47. ________ 48. ________

49. ________ 50. ________

51. ________ 52. ________

53. ________ 54. ________

55. ________ 56. ________

57. ________ 58. ________

59. ________ 60. ________

61. ________ 62. ________

63. ________ 64. ________

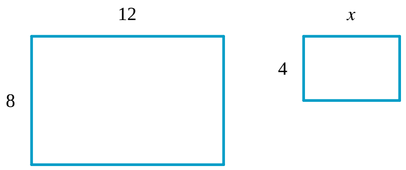
65. ________ 66. ________

67. ________ 68. ________

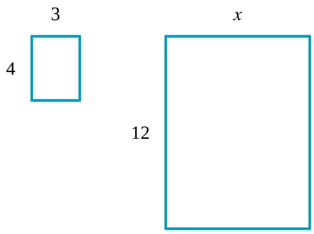
69. ________ 70. ________

71. ________ 72. ________

47.



48.



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Determine whether each statement is **true** *or* **false.**

49. Given the product of a number and a variable, the variable is called a coefficient.

50. Given the product of a number and a variable, the number is called a coefficient.

Fill in each blank with **always, sometimes,** *or* **never.**

51. In solving a proportion, we can ________ divide by the coefficient of the variable.

52. Two similar triangles ________ have corresponding sides that are proportional.

Solve for the unknown in each proportion.

53. $\frac{\frac{1}{2}}{2} = \frac{3}{a}$

54. $\frac{x}{5} = \frac{2}{\frac{1}{3}}$

55. $\frac{\frac{1}{4}}{12} = \frac{m}{40}$

56. $\frac{12}{\frac{1}{3}} = \frac{80}{y}$

57. $\frac{1}{6} = \frac{\frac{2}{x}}{18}$

58. $\frac{3}{4} = \frac{4}{\frac{x}{10}}$

59. $\frac{\frac{2}{5}}{8} = \frac{1.2}{n}$

60. $\frac{4}{a} = \frac{\frac{1}{4}}{0.8}$

61. $\frac{0.2}{2} = \frac{1.2}{a}$

62. $\frac{0.5}{x} = \frac{1.25}{5}$

63. $\frac{x}{3.3} = \frac{1.1}{6.6}$

64. $\frac{2.4}{5.7} = \frac{m}{1.1}$

65. $\frac{3}{2} = \frac{1}{x}$

66. $\frac{4}{2} = \frac{2}{x}$

67. $\frac{2}{t} = \frac{12}{5}$

68. $\frac{4}{x} = \frac{14}{15}$

69. $\frac{n}{5} = \frac{1}{20}$

70. $\frac{m}{2} = \frac{5}{24}$

71. $\frac{3}{4} = \frac{x}{6}$

72. $\frac{1}{14} = \frac{c}{7}$

Basic Skills | Challenge Yourself | **Calculator/Computer** | Career Applications | Above and Beyond

When dealing with practical applications, the numbers involved in a proportion may be large or contain inconvenient decimals. A calculator is likely to be the tool of choice for solving such proportions. Typically, we set up the solution without doing any calculating, and then we put the calculator to work, usually rounding the result to an appropriate place value. For example, suppose you drive 278 miles (mi) on 13.6 gallons (gal) of gas. If the gas tank holds 21 gal, and you want to know how far you can travel on a full tank of gas, you write

$$\frac{278 \text{ mi}}{13.6 \text{ gal}} = \frac{x \text{ mi}}{21 \text{ gal}}$$

Solving for x, you obtain

$$x = \frac{(278)(21)}{13.6}$$

With your calculator, you enter

278 $\boxed{\times}$ 21 $\boxed{\div}$ 13.6 $\boxed{=}$

The display shows 429.26471. Rounding to the nearest mile, you can travel 429 mi.

Use your calculator to solve each proportion.

73. $\frac{630}{1{,}365} = \frac{15}{a}$

74. $\frac{770}{1{,}988} = \frac{n}{71}$

75. $\frac{x}{4.7} = \frac{11.8}{16.9}$ (to nearest tenth)

76. $\frac{13.9}{8.4} = \frac{n}{9.2}$ (to nearest hundredth)

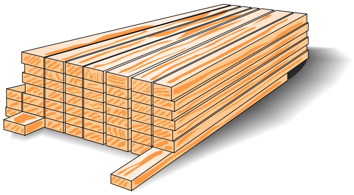
77. $\frac{2.7}{3.8} = \frac{5.9}{n}$ (to nearest tenth)

78. $\frac{12.2}{0.042} = \frac{x}{0.08}$ (to nearest hundredth)

Solve each application.

79. Business and Finance Bill earns $248.40 for working 34.5 hours (h). How much will he receive if he works at the same pay rate for 31.75 h?

80. Construction Construction-grade lumber costs $384.50 per 1,000 board-feet. What will be the cost of 686 board-feet? Round your answer to the nearest cent.



81. Science and Medicine A speed of 88 feet per second (ft/s) is equal to a speed of 60 miles per hour (mi/h). If the speed of sound is 750 mi/h, what is the speed of sound in feet per second?

82. Business and Finance A shipment of 75 parts is inspected, and 6 are found to be faulty. At the same rate, how many defective parts should be found in a shipment of 139? Round your result to the nearest whole number.

Answers

73. ____________

74. ____________

75. ____________

76. ____________

77. ____________

78. ____________

79. ____________

80. ____________

81. ____________

82. ____________

Answers

83. ______

84. ______

85. ______

86. ______

87. ______

Basic Skills | Challenge Yourself | Calculator/Computer | **Career Applications** | Above and Beyond

83. **INFORMATION TECHNOLOGY** A computer transmits 5 Web pages in 2 seconds to a Web server. How many pages can the computer transmit in 1 min?

84. **AUTOMOTIVE TECHNOLOGY** A tire shows $\frac{1}{8}$ in. of tread wear after 32,000 mi. Assuming that the rate of wear remains constant, how much tread wear would you expect the tire to show after 48,000 mi?

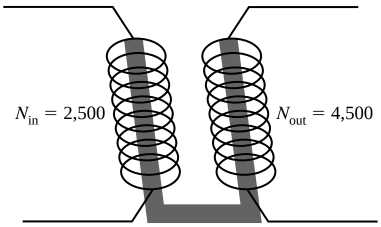
85. **MANUFACTURING TECHNOLOGY** Cutting 7 holes removes 0.322 pounds of material from a frame. How much weight would 43 holes remove?

86. **ELECTRICAL ENGINEERING** The voltage output V_{out} of a transformer is given by the proportion

$$\frac{N_{\text{out}}}{N_{\text{in}}} = \frac{V_{\text{out}}}{V_{\text{in}}}$$

in which N gives the number of turns in the coil (see the figure).

In the system shown, what voltage input is required in order for the output voltage to reach 630 volts?



Basic Skills | Challenge Yourself | Calculator/Computer | Career Applications | **Above and Beyond**

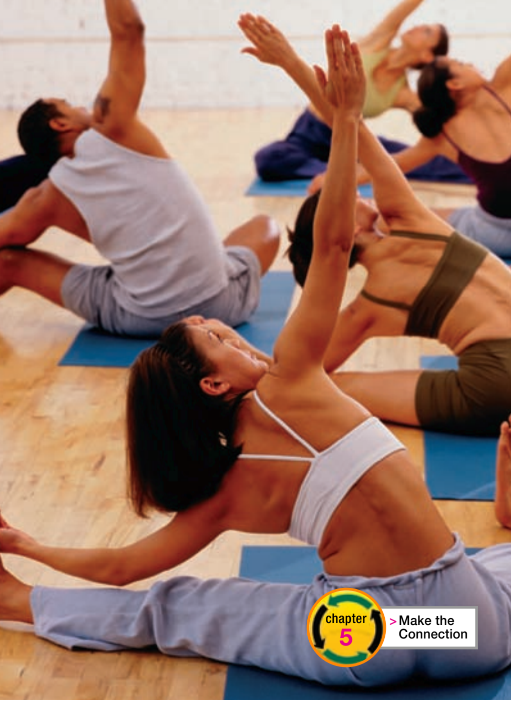
87. A recipe for 12 servings lists the following ingredients:

12 cups ziti	7 cups spaghetti sauce	4 cups ricotta cheese
$\frac{1}{2}$ cup parsley	1 teaspoon garlic powder	$\frac{1}{2}$ teaspoon pepper
4 cups mozzarella cheese	2 tablespoons parmesan cheese	

Determine the amount of ingredients necessary to serve 5 people.

Answers

1. $x = 2$ **3.** $n = 4$ **5.** $y = 8$ **7.** $x = 25$ **9.** $a = 242$
11. $x = 5$ **13.** $p = 8$ **15.** $a = 4$ **17.** $x = 25$ **19.** $p = 5$
21. \$120 **23.** \$7.20 **25.** 1,920 no votes **27.** 18 in.
29. \$2,625 **31.** 110 mi **33.** 215 mi **35.** 72 defective parts
37. 10 ft **39.** 14 ft **41.** 1 in. **43.** 80,500 cars with one person
45. 3 **47.** 6 **49.** False **51.** always **53.** $a = 12$
55. $m = \frac{5}{6}$ **57.** $x = \frac{2}{3}$ **59.** $n = 24$ **61.** $a = 12$ **63.** $x = 0.55$
65. $x = \frac{2}{3}$ **67.** $t = \frac{5}{6}$ **69.** $n = \frac{1}{4}$ **71.** $x = \frac{9}{2}$ **73.** $a = 32.5$
75. $x \approx 3.3$ **77.** $n \approx 8.3$ **79.** \$228.60 **81.** 1,100 ft/s
83. 150 Web pages **85.** 1.978 lb **87.** Above and Beyond



Activity 15 ::
Burning Calories

Many people are interested in losing weight through exercise. An important fact to consider is that a person needs to burn off 3,500 calories more than he or she takes in to lose 1 pound, according to the American Dietetic Association.

The table shows the number of calories burned per hour (cal/h) for a variety of activities, where the figures are based on a 150-pound person.

Activity	Cal/h	Activity	Cal/h
Bicycling 6 mi/h	240	Running 10 mi/h	1,280
Bicycling 12 mi/h	410	Swimming 25 yd/min	275
Cross-country skiing	700	Swimming 50 yd/min	500
Jogging $5\frac{1}{2}$ mi/h	740	Tennis (singles)	400
Jogging 7 mi/h	920	Walking 2 mi/h	240
Jumping rope	750	Walking 3 mi/h	320
Running in place	650	Walking $4\frac{1}{2}$ mi/h	440

Work with your group members to solve the following problems. You may find that setting up proportions is helpful.

For problems 1 through 4, assume a 150-pound person.

1. If a person jogs at a rate of $5\frac{1}{2}$ mi/h for $3\frac{1}{2}$ h in a week, how many calories will the person burn?

2. If a person runs in place for 15 minutes, how many calories will the person burn?

3. If a person cross-country skis for 35 minutes, how many calories will the person burn?

4. How many hours would a person have to jump rope in order to lose 1 pound? (Assume calorie consumption is just enough to maintain weight, with no activity.)

Heavier people burn more calories (for the same activity), and lighter people burn fewer. In fact, you can calculate similar figures for burning calories by setting up the appropriate proportion.

5. Assuming that the calories burned are proportional, at what rate would a 120-pound person burn calories while bicycling at 12 mi/h?

6. Assuming that the calories burned are proportional, at what rate would a 180-pound person burn calories while bicycling at 12 mi/h?

7. Assuming that the calories burned are proportional, how many hours of jogging at $5\frac{1}{2}$ mi/h would be needed for a 200-pound person to lose 5 pounds? (Again, assume calorie consumption is just enough to maintain weight, with no activity.)

summary :: chapter 5

Definition/Procedure	Example	Reference
Ratios		**Section 5.1**
Ratio A means of comparing two numbers or like quantities. A ratio can be written as a fraction.	$\frac{4}{7}$ can be thought of as "the ratio of 4 to 7."	*p.* 315
Rates and Unit Pricing		**Section 5.2**
Rate A fraction involving two unlike denominate numbers. **Unit Rate** A rate that has been simplified and read so that the denominator is one unit.	$\frac{50 \text{ home runs}}{150 \text{ games}} = \frac{1}{3}\,\frac{\text{home run}}{\text{game}}$	*pp.* 324–325
Unit Price The cost per unit.	$\frac{\$2}{5 \text{ rolls}} = \$0.40 \text{ per roll}$ $= 40\,\frac{\text{cents}}{\text{roll}}$	*p.* 327
Proportions		**Section 5.3**
Proportion A statement that two fractions or rates are equal.	$\frac{3}{5} = \frac{6}{10}$ is a proportion read "three is to five as six is to ten."	*p.* 335
The Proportion Rule If $\frac{a}{b} = \frac{c}{d}$, then $a \cdot d = b \cdot c$.	If $\frac{3}{5} = \frac{6}{10}$, then $3 \cdot 10 = 5 \cdot 6$	*p.* 336
Solving Proportions		**Section 5.4**
To Solve a Proportion **Step 1** Use the proportion rule to write the equivalent equation $a \cdot d = b \cdot c$. **Step 2** Divide both terms of the equation by the coefficient of the variable. **Step 3** Use the value found to replace the unknown in the original proportion. Check that the ratios or rates are proportional.	To solve: $\frac{x}{5} = \frac{16}{20}$ $20x = 5 \cdot 16$ $20x = 80$ $\frac{\overset{1}{\cancel{20}}x}{\underset{1}{\cancel{20}}} = \frac{80}{20}$ $x = 4$ Check: $\frac{4}{5} \overset{?}{=} \frac{16}{20}$ $4 \times 20 \overset{?}{=} 5 \times 16$ $80 = 80$	*p.* 342

Definition/Procedure	Example	Reference
To Solve a Problem by Using Proportions		
Step 1 Read the problem carefully to determine the given information. **Step 2** Write the proportion necessary to solve the problem, using a letter to represent the unknown quantity. Be sure to include the units in writing the proportion. **Step 3** Solve, answer the question of the original problem, and check the proportion as before.	A machine can produce 250 units in 5 min. At this rate, how many can it produce in a 12 h period? $\frac{250\text{ units}}{5\text{ min}} = \frac{x\text{ units}}{12\text{ h}}$ $\frac{250\text{ units}}{5\text{ min}} = \frac{x\text{ units}}{720\text{ min}}$ $250 \cdot 720 = 5x$ or $5x = 180{,}000$ $x = 36{,}000$ The machine can produce 36,000 units in 12 h.	*p.* 345

summary exercises :: chapter 5

This summary exercise set is provided to give you practice with each of the objectives of this chapter. Each exercise is keyed to the appropriate chapter section. When you are finished, you can check your answers to the odd-numbered exercises against those presented in the back of the text. If you have difficulty with any of these questions, go back and reread the examples from that section. The answers to the even-numbered exercises appear in the *Instructor's Solutions Manual.* Your instructor will give you guidelines on how to best use these exercises in your instructional setting.

5.1 *Write each ratio in simplest form.*

1. The ratio of 4 to 17

2. The ratio of 28 to 42

3. For a football team that has won 10 of its 16 games, the ratio of wins to games played

4. For a rectangle of length 30 inches and width 18 inches, the ratio of its length to its width

5. The ratio of $2\frac{1}{3}$ to $5\frac{1}{4}$

6. The ratio of 7.5 to 3.25

7. The ratio of 7 in. to 3 ft

8. The ratio of 72 h to 4 days

5.2 *Express each rate as a unit rate.*

9. $\frac{\text{600 miles}}{\text{6 hours}}$

10. $\frac{\text{270 miles}}{\text{9 gallons}}$

11. $\frac{\text{350 calories}}{\text{7 ounces}}$

12. $\frac{\text{36,000 dollars}}{\text{9 years}}$

13. $\frac{\text{5,000 feet}}{\text{25 seconds}}$

14. $\frac{\text{10,500 revolutions}}{\text{3 minutes}}$

15. A baseball team has had 117 hits in 18 games. Find the team's hits per game rate.

16. A basketball team has scored 216 points in 8 quarters. Find the team's points per quarter rate.

17. Taniko scored 246 points in 20 games. Marisa scored 216 points in 16 games. Which player has the highest points per game rate?

18. One shop will charge $306 for a job that takes $4\frac{1}{2}$ hours. A second shop can do the same job in 4 hours and will charge $290. Which shop has the higher cost per hour rate?

5.2 *Find the unit price for each item.*

19. A 32-oz bottle of dishwashing liquid costs \$2.88.

20. A 35-oz box of breakfast cereal costs \$5.60.

21. A 24-oz loaf of bread costs \$2.28.

22. Five large jars of fruit cost \$67.30.

23. Three CDs cost \$44.85.

24. Six tickets cost \$267.60.

5.3 *Write each proportion.*

25. 4 is to 9 as 20 is to 45.

26. 7 is to 5 as 56 is to 40.

27. If Jorge can travel 110 miles (mi) in 2 hours (h), he can travel 385 mi in 7 h.

28. If it takes 4 gallons (gal) of paint to cover 1,000 square feet (ft^2), it takes 10 gal of paint to cover 2,500 ft^2.

Determine whether the given fractions are proportional.

29. $\frac{4}{13} \overset{?}{=} \frac{7}{22}$

30. $\frac{8}{11} \overset{?}{=} \frac{24}{33}$

31. $\frac{9}{24} \overset{?}{=} \frac{12}{32}$

32. $\frac{7}{18} \overset{?}{=} \frac{35}{80}$

33. $\frac{5}{\frac{1}{6}} \overset{?}{=} \frac{120}{4}$

34. $\frac{0.8}{4} \overset{?}{=} \frac{12}{50}$

35. Is $\frac{\text{35 Euros}}{\text{40 dollars}}$ proportional to $\frac{\text{75.25 Euros}}{\text{86 dollars}}$?

36. Is $\frac{\text{188 words}}{\text{8 minutes}}$ proportional to $\frac{\text{121 words}}{\text{5 minutes}}$?

5.4 *Solve for the unknown in each proportion.*

37. $\frac{16}{24} = \frac{m}{3}$

38. $\frac{6}{a} = \frac{27}{18}$

39. $\frac{14}{35} = \frac{t}{10}$

40. $\frac{55}{88} = \frac{10}{p}$

41. $\frac{\frac{1}{2}}{18} = \frac{5}{w}$

42. $\frac{\frac{3}{2}}{9} = \frac{5}{a}$

43. $\frac{5}{x} = \frac{0.6}{12}$

44. $\frac{s}{2.5} = \frac{1.5}{7.5}$

Solve each application.

45. **Business and Finance** If 4 tickets to a civic theater performance cost $90, what is the price for 6 tickets?

46. **Social Science** The ratio of first-year to second-year students at a school is 8 to 7. If there are 224 second-year students, how many first-year students are there?

47. **Crafts** A photograph that is 5 inches (in.) wide by 7 in. tall is to be enlarged so that the new height will be 21 in. What will be the width of the enlargement?

48. **Business and Finance** Marcia assembles disk drives for a computer manufacturer. If she can assemble 11 drives in 2 hours (h), how many can she assemble in a workweek (40 h)?

49. **Business and Finance** A firm finds 14 defective parts in a shipment of 400. How many defective parts can be expected in a shipment of 800 parts?

50. **Social Science** The scale on a map is $\frac{1}{4}$ in. = 10 miles (mi). How many miles apart are two towns that are 3 in. apart on the map?

51. **Business and Finance** A piece of tubing that is 16.5 centimeters (cm) long weighs 55 grams (g). What is the weight of a piece of the same tubing that is 42 cm long?

52. **Crafts** If 1 quart (qt) of paint covers 120 square feet (ft^2), how many square feet does 2 gallons (gal) cover? (1 gal = 4 qt.)

self-test 5

Name ______________

Section ________ Date ________

Answers

1. ______________

2. ______________

3. ______________

4. ______________

5. ______________

6. ______________

7. ______________

8. ______________

9. ______________

10. ______________

11. ______________

12. ______________

13. ______________

14. ______________

15. ______________

16. ______________

The purpose of this self-test is to help you assess your progress so that you can find concepts that you need to review before the next exam. Allow yourself about an hour to take this test. At the end of that hour, check your answers against those given in the back of this text. If you miss any, go back to the appropriate section to reread the examples until you have mastered that particular concept.

Express each rate as a unit rate.

1. $\frac{840 \text{ miles}}{175 \text{ gallons}}$

2. $\frac{132 \text{ dollars}}{16 \text{ hours}}$

Solve for the unknown in each proportion.

3. $\frac{45}{75} = \frac{12}{x}$

4. $\frac{a}{26} = \frac{45}{65}$

5. Find the unit price, if 11 gallons of milk cost $28.16.

6. A basketball team wins 26 of its 33 games during a season. What is the ratio of wins to games played? What is the ratio of wins to losses?

Solve for the unknown in each proportion.

7. $\frac{\frac{1}{2}}{p} = \frac{5}{30}$

8. $\frac{3}{m} = \frac{0.9}{4.8}$

Write each ratio in simplest form.

9. The ratio of 7 to 19

10. The ratio of 75 to 45

11. The ratio of 8 ft to 4 yd

12. The ratio of 6 h to 3 days

Solve each application using a proportion.

13. **BUSINESS AND FINANCE** If ballpoint pens are marked 5 for 95¢, how much does a dozen cost?

14. **BUSINESS AND FINANCE** Your new compact car travels 324 miles on 9 gallons of gas. If the tank holds 16 usable gallons, how far can you drive on a tank of gas?

15. **BUSINESS AND FINANCE** An assembly line can install 5 car mufflers in 4 minutes. At this rate, how many mufflers can be installed in an 8-hour shift?

16. **CRAFTS** Instructions on a package of concentrated plant food call for 2 teaspoons (tsp) to 1 qt of water. We wish to use 3 gal of water. How much of the plant food concentrate should be added to the 3 gal of water?

Answers

17. ________________

18. ________________

19. ________________

20. ________________

Determine whether the given fractions are proportional.

17. $\frac{3}{9} \overset{?}{=} \frac{27}{81}$

18. $\frac{6}{7} \overset{?}{=} \frac{9}{11}$

19. $\frac{9}{10} \overset{?}{=} \frac{27}{30}$

20. $\frac{\frac{1}{2}}{5} \overset{?}{=} \frac{2}{18}$

cumulative review chapters 1-5

The following exercises are presented to help you review concepts from earlier chapters. This is meant as review material and not as a comprehensive exam. The answers are presented in the back of the text. Beside each answer is a section reference for the concept. If you have difficulty with any of these exercises, be certain to at least read through the summary related to that section.

1. Write 45,789 in words.

2. What is the place value of 2 in the number 621,487?

Perform the indicated operations.

3. $\begin{array}{r} 2{,}790 \\ 831 \\ +\ 22{,}683 \\ \hline \end{array}$

4. $\begin{array}{r} 84{,}793 \\ -\ 36{,}987 \\ \hline \end{array}$

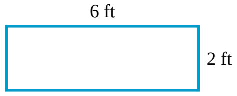
5. 76×58

6. $72\overline{)5{,}683}$

7. Luis owes \$815 on a credit card after a trip. He makes payments of \$125, \$80, and \$90. Interest amounting to \$48 is charged. How much does he still owe on the account?

8. Evaluate the expression: $48 \div 8 \times 2 - 3^2$.

9. Find the perimeter and area of the given figure.



10. A room that measures 6 yd by 8 yd is to be carpeted. The carpet costs \$23 per square yard. What will be the cost of the carpet?

11. Write the prime factorization of 924.

12. Find the greatest common factor (GCF) of 42 and 56.

13. Write the fraction $\frac{42}{168}$ in simplest form.

Perform the indicated operations.

14. $\frac{3}{4} \times \frac{24}{15}$

15. $2\frac{2}{3} \times 3\frac{3}{4}$

16. $\frac{6}{7} \div \frac{4}{21}$

17. $5\frac{1}{2} \div 3\frac{1}{4}$

18. $\frac{9}{11} - \frac{3}{4} + \frac{1}{2}$

19. $4\frac{5}{6} + 2\frac{3}{4}$

20. $8\frac{2}{7} - 3\frac{11}{14}$

Name

Section Date

Answers

1.
2.
3.
4.
5.
6.
7.
8.
9.
10.
11.
12.
13. **14.**
15. **16.**
17. **18.**
19. **20.**

Answers

21. ____________
22. ____________
23. ____________
24. ____________
25. ____________
26. ____________
27. ____________
28. ____________
29. ____________
30. ____________
31. ____________
32. ____________
33. ____________
34. ____________
35. ____________
36. ____________
37. ____________
38. ____________
39. ____________
40. ____________

21. Find the least common multiple (LCM) of 36 and 60.

22. Maria drove at an average speed of 55 mi/h for $3\frac{1}{5}$ h. How many miles did she travel?

23. Stefan drove 132 miles in $2\frac{3}{4}$ h. What was his average speed?

Perform the indicated operations.

24. 36.169 − 28.341

25. 3.1488 ÷ 2.56

26. 4.89 × 1.35

27. Write 0.36 as a fraction and simplify.

28. Write the decimal equivalent of $\frac{7}{22}$ (to the nearest hundredth).

29. Find the perimeter of a rectangle that has dimensions 4.23 m by 2.8 m.

30. Find the area of a rectangle with dimensions 8 cm by 6.28 cm.

Write each ratio in simplest form.

31. 12 to 26

32. 60 to 18

33. 6 dimes to 3 quarters

Determine whether the given fractions are proportional.

34. $\frac{5}{6} \stackrel{?}{=} \frac{20}{24}$

35. $\frac{3}{7} \stackrel{?}{=} \frac{9}{22}$

Solve for the unknown.

36. $\frac{x}{3} = \frac{8}{12}$

37. $\frac{5}{x} = \frac{4}{12}$

38. Give the unit price for an item that weighs 20 oz and costs $4.88.

39. On a map, 3 cm represents 250 km. How far apart are two cities if the distance between them on the map is 7.2 cm?

40. A company finds 15 defective items in a shipment of 600 items. How many defective items can be expected in a shipment of 2,000 items?

chapter
6
> Make the Connection

CHAPTER 6

Percents

INTRODUCTION

When she retired, Roberta decided that she wanted to study the stock market and investments. She learned to watch small companies and to purchase stocks whose value is greater than listed.

Roberta enrolled in continuing education courses covering business and finance at her local community college. She learned how to track data and where to find financial advice articles on the Internet.

Now, after purchasing stock in a company, Roberta continues to monitor the company through quarterly financial statements and tries to gauge the best time to sell her stocks. Her goal is to sell her stocks when she thinks their value has peaked.

A less risky way to invest is to put money in a savings account or some other vehicle with a fixed interest rate. In these accounts, money grows continually as the interest is added at regular intervals.

In Activity 17 on page 399, you will have the opportunity to conduct a more in-depth study of interest calculations and the growth of investments.

CHAPTER 6 OUTLINE

Chapter 6 :: Prerequisite Test 366
6.1 **Writing Percents as Fractions and Decimals** 367
6.2 **Writing Decimals and Fractions as Percents** 377
6.3 **The Three Types of Percent Problems** 389
6.4 **Applications of Percent Problems** 400
Chapter 6 :: Summary / Summary Exercises / Self-Test / Cumulative Review :: Chapters 1–6 414

6 prerequisite test

Name ____________________

Section __________ Date __________

Answers

1. ____________________
2. ____________________
3. ____________________
4. ____________________
5. ____________________
6. ____________________
7. ____________________
8. ____________________
9. ____________________
10. ____________________

This prerequisite test provides some exercises requiring skills that you will need to be successful in the coming chapter. The answers for these exercises can be found in the back of this text. This prerequisite test can help you identify topics that you will need to review before beginning the chapter.

Simplify the fraction.

1. $\frac{44}{100}$

Write the fraction as a decimal.

2. $\frac{5}{16}$

Evaluate.

3. $0.04 \times 1{,}040$

4. $\frac{2{,}300}{115}$

5. $\frac{1{,}800}{15}$

6. $\frac{5{,}250}{100}$

Solve each proportion.

7. $\frac{4}{5} = \frac{x}{60}$

8. $\frac{7}{12} = \frac{x}{100}$

9. $\frac{28}{x} = \frac{40}{100}$

10. $\frac{280}{x} = \frac{3.5}{100}$

The Streeter/Hutchison Series in Mathematics   Basic Mathematical Skills with Geometry

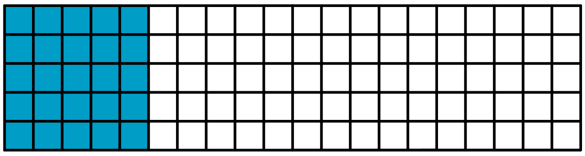
6.1

Writing Percents as Fractions and Decimals

< **6.1 Objectives** >

1 > Use percent notation
2 > Write a percent as a fraction or mixed number
3 > Write a percent as a decimal

When we considered parts of a whole in earlier chapters, we used fractions and decimals. *Percents* are another useful way of naming parts of a whole. We can think of percents as ratios whose denominators are 100. In fact, the word **percent** means "for each hundred." Consider the following figure:



RECALL

Multiplying by $\frac{1}{100}$ is the same as dividing by 100.

The symbol for percent, %, represents multiplication by the number $\frac{1}{100}$. In the figure, 25 of 100 squares are shaded. As a fraction, we write this as $\frac{25}{100}$.

$$\frac{25}{100} = 25\left(\frac{1}{100}\right) = 25\%$$

25 percent of the squares are shaded.

Example 1 Using Percent Notation

< **Objective 1** >

RECALL

You learned to solve proportions in Section 5.4.

(a) Four out of five geography students passed their midterm exams. Write this statement, using percent notation.

The ratio of passing students to all students is $\frac{4}{5}$, which we need to write as an equivalent fraction with a denominator of 100.

$$\frac{4}{5} = \frac{x}{100}$$

Using the proportion rule, we have

$$5 \cdot x = 4 \cdot 100$$

or $$5x = 400$$

The coefficient of the variable is 5, so we divide both sides of the equation by 5.

$$\frac{5x}{5} = \frac{400}{5}$$

$$x = 80 \qquad 400 \div 5 = 80$$

Therefore, we write

$$\frac{4}{5} = \frac{80}{100} = 80\left(\frac{1}{100}\right) = 80\%$$

and we say that 80% of the geography students passed their midterm exams.

NOTE

The ratio of compact cars to all cars is $\frac{35}{50}$.

(b) Of 50 automobiles sold by a dealer in 1 month, 35 were compact cars. Write this statement, using percent notation.

$$\frac{35}{50} = \frac{70}{100} = 70\left(\frac{1}{100}\right) = 70\%$$

We can say that 70% of the cars sold were compact cars.

Check Yourself 1

Rewrite the following statement using percent notation: 4 of the 50 parts in a shipment were defective.

Because there are different ways of naming the parts of a whole, you need to know how to change from one of these ways to another. First, we look at writing a percent as a fraction. Because a percent is a fraction or a ratio with denominator of 100, we can use the following rule.

Property

Writing a Percent as a Fraction

To change a percent to a fraction, replace the percent symbol with $\frac{1}{100}$ and multiply.

The use of this rule is shown in Example 2.

Example 2 Writing a Percent as a Fraction

< Objective 2 >

NOTE

You should write $\frac{25}{100}$ in simplest form.

Write each percent as a fraction.

(a) $7\% = 7\left(\frac{1}{100}\right) = \frac{7}{100}$

(b) $25\% = 25\left(\frac{1}{100}\right) = \frac{25}{100} = \frac{1}{4}$

Check Yourself 2

Write 12% as a fraction.

If a percent is *greater than 100*, the resulting fraction will be *greater than 1*, as shown in Example 3.

Example 3 Writing a Percent as a Mixed Number

Write 150% as a mixed number.

$$150\% = 150\left(\frac{1}{100}\right) = \frac{150}{100} = 1\frac{50}{100} = 1\frac{1}{2}$$

Check Yourself 3

Write 125% as a mixed number.

In Examples 2 and 3, we wrote percents as fractions by replacing the percent sign with $\frac{1}{100}$ and multiplying. How do we convert percents to decimal form? Since multiplying by $\frac{1}{100}$ is the same as dividing by 100, we just move the decimal point two places to the left. A shortened version of this process is given below.

Property

Writing a Percent as a Decimal

To write a percent as a decimal, remove the percent symbol and move the decimal point two places to the left.

Example 4 Writing a Percent as a Decimal

< Objective 3 >

NOTE

A percent greater than 100 gives a decimal greater than 1.

Change each percent to its decimal equivalent.

(a) 25% = 0.25 The decimal point in 25% is understood to be after the 5.

(b) 8% = 0.08 We must add a zero to place the decimal point.

(c) 130% = 1.30

Check Yourself 4

Write as decimals.

(a) 5% **(b)** 32% **(c)** 115%

Example 5 involves fractions of a percent, in this case, decimal fractions.

Example 5 Writing a Percent as a Decimal

Write as decimals.

(a) 4.5% = 0.045

(b) 0.5% = 0.005

Check Yourself 5

Write as decimals.

(a) 8.5% **(b)** 0.3%

There are many situations in which common fractions are involved in a percent. Example 6 illustrates this situation.

Example 6 Writing a Percent as a Decimal

NOTE

Write the common fraction as a decimal. Then remove the percent symbol by our earlier rule.

Write as decimals.

(a) $9\frac{1}{2}\% = 9.5\% = 0.095$

(b) $\frac{3}{4}\% = 0.75\% = 0.0075$

Check Yourself 6

Write as decimals.

(a) $7\frac{1}{2}\%$ **(b)** $\frac{1}{2}\%$

Writing a percent in fraction or decimal form is required in many applications. One such application is presented in Example 7.

Example 7 A Technology Application

A motor has an 86% efficiency rating. Express its efficiency rating as a fraction and as a decimal.

To express 86% as a fraction, we replace the % symbol with $\frac{1}{100}$ and simplify.

$$86\% = \frac{86}{100} = \frac{43}{50}$$

To express 86% as a decimal, we remove the % symbol and move the decimal point two places to the left.

$$86\% = 0.86$$

Check Yourself 7

An inspection of Carina's hard drive reveals that it is 60% full. Write the amount of hard drive capacity that is full as a fraction and as a decimal.

Check Yourself ANSWERS

1. 8% were defective **2.** 12% $= \frac{12}{100} = \frac{3}{25}$ **3.** $1\frac{1}{4}$

4. **(a)** 0.05; **(b)** 0.32; **(c)** 1.15 **5.** **(a)** 0.085; **(b)** 0.003

6. **(a)** 0.075; **(b)** 0.005 **7.** $\frac{3}{5}$; 0.6

Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 6.1

(a) The word *percent* means, "for each ______________."

(b) To rewrite a percent as a ______________, divide the number before the percent symbol by 100 and simplify.

(c) To write a percent in decimal form, remove the % symbol and move the decimal two places to the ______________.

(d) A percent that is larger than 100% is equivalent to a number ______________ than 1.

6.1 exercises

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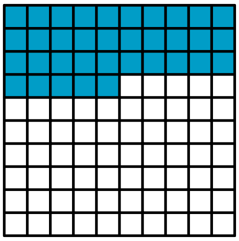
Name ______________

Section ________ Date ________

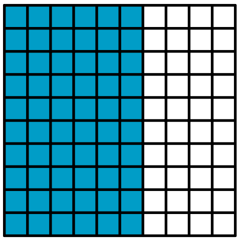
< Objective 1 >

Use percents to name the shaded portion of each drawing.

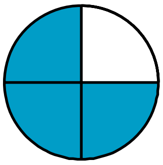
1.



2.

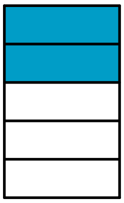


3.



> Videos

4.



Rewrite each statement, using percent notation.

5. Out of every 100 eligible people, 53 voted in a recent election. > Videos

6. You receive $5 in interest for every $100 saved for 1 year.

7. Out of every 100 entering students, 74 register for English composition.

8. Of 100 people surveyed, 29 watched a particular sports event on television.

9. Out of 10 voters in a state, 3 are registered as independents.

10. A dealer sold 9 of the 20 cars available during a 1-day sale.

11. Of 50 houses in a development, 27 are sold.

12. Of the 25 employees of a company, 9 are part-time.

13. Out of 50 people surveyed, 23 prefer decaffeinated coffee.

14. 17 out of 20 college students work at part-time jobs.

15. Of the 20 students in an algebra class, 5 receive a grade of A. > Videos

16. Of the 50 families in a neighborhood, 31 have children in public schools.

Answers

1. ______________

2. ______________

3. ______________

4. ______________

5. ______________

6. ______________

7. ______________

8. ______________

9. ______________

10. ______________

11. ______________

12. ______________

13. ______________

14. ______________

15. ______________

16. ______________

< Objective 2 >

Write as fractions.

17. 6% > Videos
18. 17%
19. 75%
20. 20%
21. 65%
22. 48%
23. 50%
24. 52%
25. 46%
26. 35%
27. 66%
28. 4%

< Objective 3 >

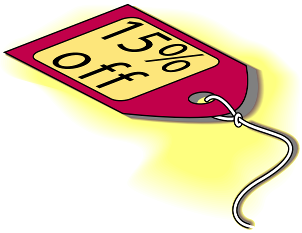
Write as decimals.

29. 20%
30. 70%
31. 35%
32. 75%
33. 39%
34. 27%
35. 5% > Videos
36. 7%
37. 135%
38. 250%
39. 240%
40. 160%

41. **SOCIAL SCIENCE** Automobiles account for 85% of the travel between cities in the United States. What fraction does this percent represent?

42. **SOCIAL SCIENCE** Automobiles and small trucks account for 84% of the travel to and from work in the United States. What fraction does this percent represent?

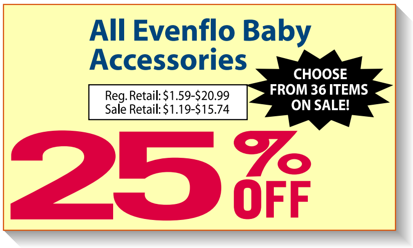
43. Convert the discount shown to a decimal and to a simplified fraction.



Answers

17. ______
18. ______
19. ______
20. ______
21. ______
22. ______
23. ______
24. ______
25. ______
26. ______
27. ______
28. ______
29. ______ 30. ______
31. ______ 32. ______
33. ______ 34. ______
35. ______ 36. ______
37. ______ 38. ______
39. ______ 40. ______
41. ______ 42. ______
43. ______

44. Convert the discount shown to a decimal and to a simplified fraction.



All Evenflo Baby Accessories

Reg. Retail: $1.59-$20.99
Sale Retail: $1.19-$15.74

CHOOSE FROM 36 ITEMS ON SALE!

25% OFF

45. The daily reference values (DRVs) for certain foods are given. They are based on a 2,000 calorie per day diet. Find decimal and fractional notation for the percent notation in each sentence.

(a) 1 ounce of Tostitos provides 9% of the DRV of fat.

Tostitos
TORTILLA CHIPS

(b) $\frac{1}{2}$ cup of B & M baked beans contains 15% of the DRV of sodium.

B&M
BAKED BEANS

(c) $\frac{1}{2}$ cup of Campbells' New England clam chowder provides 6% of the DRV of iron.

Campbells
NEW ENGLAND
CLAM
CHOWDER

(d) 2 ounces of Star Kist tuna provide 27% of the DRV of protein.

Star Kist
CHUNK LIGHT TUNA
IN WATER

(e) Four 4-in. Aunt Jemima pancakes provide 33% of the DRV of sodium.

Aunt Jemima
Pancake Mix

(f) 36 grams of Pop-Secret butter popcorn provide 2% of the DRV of sodium.

THIS SIDE UP
POP·SECRET

Answers

44.

45. (a)

(b)

(c)

(d)

(e)

(f)

Answers

46. ______

47. ______

48. ______

49. ______

50. ______

51. ______

52. ______

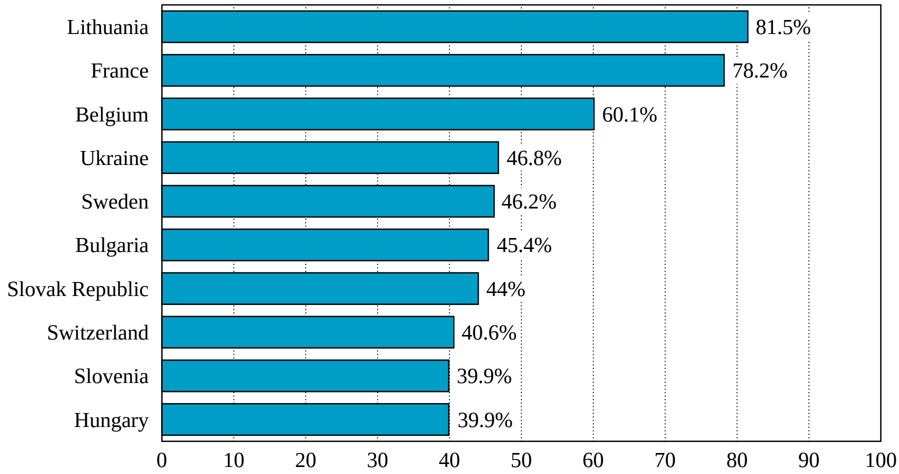
53. ______

54. ______

Basic Skills | **Challenge Yourself** | Calculator/Computer | Career Applications | Above and Beyond

46. Complete the chart for the percentages given in the bar graph.

Nations Most Reliant on Nuclear Energy, 1997
(Nuclear electricity generation as % of total)



Source: International Atomic Energy Agency, May 1998

Country	Decimal Equivalent	Fraction Equivalent
Lithuania		
France		
Belgium		
Ukraine		
Sweden		
Bulgaria		
Slovak Republic		
Switzerland		
Slovenia		
Hungary		

Rewrite each percent as a mixed number.

47. 150%

48. 140%

49. 225%

50. 450%

51. $166\frac{2}{3}\%$

52. $233\frac{1}{3}\%$

53. $212\frac{1}{2}\%$

54. $116\frac{2}{3}\%$

Rewrite each percent in decimal form.

55. 23.6%
56. 10.5%
57. 6.4%
58. 3.5%
59. 0.2%
60. 0.5%
61. 1.05%
62. 0.023%
63. $7\frac{1}{2}\%$
64. $8\frac{1}{4}\%$
65. $87\frac{1}{2}\%$
66. $16\frac{2}{5}\%$
67. $128\frac{3}{4}\%$
68. $220\frac{3}{20}\%$
69. $\frac{1}{2}\%$
70. $\frac{3}{4}\%$

Basic Skills | Challenge Yourself | Calculator/Computer | **Career Applications** | Above and Beyond

71. **INFORMATION TECHNOLOGY** An information technology project manager has a yearly budget of $49,000. She spent 10.2% of her budget in March. What fraction of her annual budget did she spend?

72. **MANUFACTURING TECHNOLOGY** A packaging company advertises that 87.5% of the machines that it produced in the last 20 years are still in operation. Express the proportion of machines still in service as a fraction.

73. **MANUFACTURING TECHNOLOGY** On an assembly line, 12% of all products are produced in the color red. Express this as a decimal and as a simplified fraction.

74. **CONSTRUCTION TECHNOLOGY** A board that starts with a width of 5.5 in. shrinks to 95% of its original width as it dries. What fraction is the dry width of the original width?

75. **MECHANICAL ENGINEERING** As a piece of metal cools, it shrinks 3.125%. What fraction of its original size is lost due to shrinkage?

76. **AGRICULTURAL TECHNOLOGY** 12.5% of a growing season's 52 in. of rain fell in August. Write the percent of rain that fell in August as a decimal and as a simplified fraction.

Basic Skills | Challenge Yourself | Calculator/Computer | Career Applications | **Above and Beyond**

77. Match each percent in column A with the equivalent fraction in column B.

Column A	Column B
(a) $37\frac{1}{2}\%$	(1) $\frac{3}{5}$
(b) 5%	(2) $\frac{5}{8}$
(c) $33\frac{1}{3}\%$	(3) $\frac{1}{20}$
(d) $83\frac{1}{3}\%$	(4) $\frac{3}{8}$
(e) 60%	(5) $\frac{5}{6}$
(f) $62\frac{1}{2}\%$	(6) $\frac{1}{3}$

Answers

55. ____________ 56. ____________
57. ____________ 58. ____________
59. ____________ 60. ____________
61. ____________ 62. ____________
63. ____________ 64. ____________
65. ____________ 66. ____________
67. ____________ 68. ____________
69. ____________ 70. ____________
71. ____________
72. ____________
73. ____________
74. ____________
75. ____________
76. ____________
77. ____________

Answers

78. ____________________

78. Explain the difference between $\frac{1}{4}$ of a quantity and $\frac{1}{4}$% of a quantity.

Answers

1. 35% **3.** 75% **5.** 53% **7.** 74% **9.** 30% **11.** 54%
13. 46% **15.** 25% **17.** $\frac{3}{50}$ **19.** $\frac{3}{4}$ **21.** $\frac{13}{20}$ **23.** $\frac{1}{2}$
25. $\frac{23}{50}$ **27.** $\frac{33}{50}$ **29.** 0.2 **31.** 0.35 **33.** 0.39 **35.** 0.05
37. 1.35 **39.** 2.4 **41.** $\frac{17}{20}$ **43.** 0.15; $\frac{3}{20}$ **45.** **(a)** 0.09; $\frac{9}{100}$;
(b) 0.15; $\frac{3}{20}$; **(c)** 0.06; $\frac{3}{50}$; **(d)** 0.27; $\frac{27}{100}$; **(e)** 0.33; $\frac{33}{100}$; **(f)** 0.02; $\frac{1}{50}$
47. $1\frac{1}{2}$ **49.** $2\frac{1}{4}$ **51.** $1\frac{2}{3}$ **53.** $2\frac{1}{8}$ **55.** 0.236 **57.** 0.064
59. 0.002 **61.** 0.0105 **63.** 0.075 **65.** 0.875 **67.** 1.2875
69. 0.005 **71.** $\frac{51}{500}$ **73.** 0.12; $\frac{3}{25}$ **75.** $\frac{1}{32}$
77. **(a)**-(4); **(b)**-(3); **(c)**-(6); **(d)**-(5); **(e)**-(1); **(f)**-(2)

6.2 Writing Decimals and Fractions as Percents

< **6.2 Objectives** >

1 > Write a decimal as a percent

2 > Write a fraction or mixed number as a percent

Writing a decimal as a percent is the opposite of writing a percent in decimal form. We simply reverse the process given in Section 6.1.

Property

Writing a Decimal as a Percent	To write a decimal as a percent, move the decimal point *two* places to the *right* and attach the percent symbol.

Example 1 Writing a Decimal as a Percent

< **Objective 1** >

NOTE

$$0.18 = \frac{18}{100} = 18\left(\frac{1}{100}\right) = 18\%$$

$$0.03 = \frac{3}{100} = 3\left(\frac{1}{100}\right) = 3\%$$

(a) Write 0.18 as a percent.

$0\underset{\curvearrowright}{.18} = 18\%$

(b) Write 0.03 as a percent.

$0\underset{\curvearrowright}{.03} = 3\%$

Check Yourself 1

Write each decimal in percent form.

(a) 0.27 **(b)** 0.05

There are many instances in which we use percents greater than 100%. For example, if a retailer marks goods up 25%, then the retail price is 125% of the wholesale price.

Example 2 Writing a Decimal as a Percent

Write 1.25 as a percent.

$1\underset{\curvearrowright}{.25} = 125\%$

NOTE

$$1.25 = \frac{125}{100} = 125\left(\frac{1}{100}\right)$$

$$= 125\%$$

Check Yourself 2

Write 1.3 as a percent.

If a percent includes numbers to the right of the decimal point after the decimal is moved two places to the right, the fractional portion can be written as a decimal or as a fraction.

Example 3 Writing a Decimal as a Percent

< Objective 1 >

Write as a percent.

(a) $0.045 = 4.5\%$ or $4\frac{1}{2}\%$

(b) $0.003 = 0.3\%$ or $\frac{3}{10}\%$

Check Yourself 3

Write 0.075 as a percent.

The following rule allows us to change fractions to percents.

Property

Writing a Fraction as a Percent

To write a fraction as a percent, write the decimal equivalent of the fraction. Then use the previous rule to change the decimal to a percent.

Example 4 Writing a Fraction as a Percent

< Objective 2 >

RECALL

We learned to write fractions as decimals in Section 4.2.

Write $\frac{3}{5}$ as a percent.

First write the decimal equivalent.

$\frac{3}{5} = 0.6$ To find the decimal equivalent, just divide the denominator into the numerator.

Now write the percent.

$\frac{3}{5} = 0.60 = 60\%$ Affix zeros to the right of the decimal if necessary.

Check Yourself 4

Write $\frac{3}{4}$ as a percent.

Again, you will find both decimals and fractions used in writing percents. Consider Example 5.

Example 5 Writing a Fraction as a Percent

Write $\frac{1}{8}$ as a percent.

$\frac{1}{8} = 0.125 = 12.5\%$ or $12\frac{1}{2}\%$

Check Yourself 5

Write $\frac{3}{8}$ as a percent.

Units ANALYSIS

We rarely express the units when computing percentages. When we say "percent" we are essentially saying, "numerator units per 100 denominator units."

EXAMPLES:

Of 800 students, 200 were boys. What percent of the students were boys?

$$\frac{200 \text{ boys}}{800 \text{ students}} = 0.25 = 25\%$$

But what happened to our units? At the decimal, the units boys/student (0.25 boy per student) wouldn't make much sense, but we can read the % as

25 "boys per 100 students"

and have a reasonable unit phrase.

Of 500 computers sold, 180 were equipped with a scanner. What percent were equipped with a scanner?

$$\frac{180}{500} = 0.36 = 36\%$$

36 were equipped with a scanner for each 100 computers sold.

To write a mixed number as a percent, we use exactly the same steps.

Example 6 Writing a Mixed Number as a Percent

NOTE

The resulting percent must be greater than 100% because the original mixed number is greater than 1.

Write $1\frac{1}{4}$ as a percent.

$$1\frac{1}{4} = 1.25 = 125\%$$

Check Yourself 6

Write $1\frac{2}{5}$ as a percent.

Some fractions have repeating-decimal equivalents. In writing these as percents, we either round to some indicated place or we write the percent as an exact fraction.

Example 7 Writing a Fraction as a Percent

(a) Write $\frac{1}{3}$ as a percent, using an exact fraction.

Dividing 1 by 3 gives:

$$\frac{1}{3} = 0.333333\cdots$$

Note that the 3's continue forever. Moving the decimal point two places to the right and attaching a percent symbol gives us:

$$\frac{1}{3} = 33.3333\cdots\%$$

Now we recognize that $0.3333\cdots$ is the decimal form for $\frac{1}{3}$, so we have:

$$\frac{1}{3} = 33\frac{1}{3}\%$$

(b) Write $\frac{5}{7}$ as a percent, rounding to the nearest tenth of a percent.

When we divide 5 by 7, we find:

$$\frac{5}{7} = 0.7142857\cdots$$

Then

$$\frac{5}{7} = 71.42857\cdots\%$$

Rounding to the nearest tenth of a percent, we have $\frac{5}{7} = 71.4\%$.

When rounding to a tenth of a percent, we need only divide far enough to round the decimal form to four places: 0.7142. We can then move the decimal point, round properly, and attach the percent symbol.

Check Yourself 7

(a) Write $\frac{2}{3}$ as a percent, using an exact fraction.

(b) Write $\frac{2}{9}$ as a percent, rounding to the nearest tenth of a percent.

Some students prefer to use the proportion method when writing a fraction as a percent. The proportion method gives an easier way of writing a fraction as an *exact value* percent instead of as an approximation. We demonstrate this method in Example 8.

Example 8 Using the Proportion Method

RECALL

You learned to solve proportions in Section 5.4.

(a) Write $\frac{7}{12}$ as a percent, using an exact fraction.

To use the proportion method, we write $\frac{7}{12}$ as an equivalent fraction whose denominator is 100.

$$\frac{7}{12} = \frac{x}{100}$$

Using the proportion rule, we have

$$12 \cdot x = 7 \cdot 100$$

$$12x = 700$$

Divide both sides by the coefficient of the variable, 12.

$$\frac{12x}{12} = \frac{700}{12} \qquad 700 \div 12 = 58\frac{4}{12} = 58\frac{1}{3}$$

$$x = 58\frac{1}{3}$$

Therefore,

$$\frac{7}{12} = \frac{58\frac{1}{3}}{100} = 58\frac{1}{3}\%$$

RECALL

$\dfrac{58\frac{1}{3}}{100}$ may be read as

"$58\frac{1}{3}$ per 100."

(b) Write $\frac{1}{7}$ as a percent, using an exact fraction.

Again, we look for an equivalent fraction with a denominator of 100.

$$\frac{1}{7} = \frac{x}{100}$$

$$7x = 100$$

$$x = \frac{100}{7} = 14\frac{2}{7}$$

Therefore, $\frac{1}{7} = 14\frac{2}{7}\%$.

Check Yourself 8

Write each fraction as a percent, using an exact fraction.

(a) $\frac{5}{6}$ **(b)** $\frac{10}{9}$ **(c)** $\frac{3}{7}$

Applications often require that we write fractions and decimals in percent form, as in Example 9.

Example 9 An Application of Percents

The hard drive in Emma's computer has a 74.5-gigabyte (GB) capacity. Currently, she is using 10.8 GB. What percent of her hard drive's capacity is Emma using?

We begin by constructing a fraction based on the given information.

$$\frac{\text{Used}}{\text{Capacity}} = \frac{10.8 \text{ GB}}{74.5 \text{ GB}}$$

$$= \frac{10.8 \cancel{\text{GB}}}{74.5 \cancel{\text{GB}}} \qquad \text{As with proportions, we can simply "cancel" the units.}$$

$$= \frac{108}{745} \qquad \text{Multiply the top and bottom by 10 to remove the decimal.}$$

$$\approx 0.145 \qquad \text{Divide and round the result to the nearest thousandth.}$$

$$= 14.5\%$$

Emma is using 14.5% of her hard drive's capacity.

Check Yourself 9

Three cylinders are not firing properly in an 8-cylinder engine. What percent of the cylinders are firing properly?

Certain percents appear frequently enough that you should memorize their fraction and decimal equivalents.

$100\% = 1$

$200\% = 2$

$25\% = 0.25 = \frac{1}{4}$

$50\% = 0.5 = \frac{1}{2}$

$75\% = 0.75 = \frac{3}{4}$

$5\% = 0.05 = \frac{1}{20}$

$10\% = 0.1 = \frac{1}{10}$

$20\% = 0.2 = \frac{1}{5}$

$30\% = 0.3 = \frac{3}{10}$

$40\% = 0.4 = \frac{2}{5}$

$60\% = 0.6 = \frac{3}{5}$

$70\% = 0.7 = \frac{7}{10}$

$80\% = 0.8 = \frac{4}{5}$

$90\% = 0.9 = \frac{9}{10}$

$12\frac{1}{2}\% = 0.125 = \frac{1}{8}$

$37\frac{1}{2}\% = 0.375 = \frac{3}{8}$

$62\frac{1}{2}\% = 0.625 = \frac{5}{8}$

$87\frac{1}{2}\% = 0.875 = \frac{7}{8}$

$33\frac{1}{3}\% = 0.\overline{3} = \frac{1}{3}$

$66\frac{2}{3}\% = 0.\overline{6} = \frac{2}{3}$

Check Yourself ANSWERS

1. **(a)** 27%; **(b)** 5%   **2.** 130%   **3.** 7.5% or $7\frac{1}{2}\%$ **4.** 75%

5. 37.5% or $37\frac{1}{2}\%$ **6.** 140%   **7.** **(a)** $66\frac{2}{3}\%$; **(b)** 22.2%

8. **(a)** $83\frac{1}{3}\%$; **(b)** $111\frac{1}{9}\%$; **(c)** $42\frac{6}{7}\%$ **9.** 62.5%

Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 6.2

(a) To write a decimal in percent form, move the decimal two places to the ______________ and add the % symbol.

(b) To write a fraction as a percent, first convert the fraction to a ______________.

(c) When writing a repeating decimal as a ______________, we round to some indicated place or we write the percent using an exact fraction.

(d) When writing a decimal as a percent, we may need to add ______________ to the right as placeholders.

6.2 exercises

< Objective 1 >

Write each number as a percent.

1. 0.08 > Videos

2. 0.09

3. 0.05

4. 0.13

5. 0.18

6. 0.63

7. 0.86

8. 0.45

9. 0.4

10. 0.3

11. 0.7

12. 0.6

13. 1.10 > Videos

14. 2.50

15. 4.40

16. 5

17. 0.065 > Videos

18. 0.095

19. 0.025

20. 0.085

< Objective 2 >

21. $\frac{1}{4}$

22. $\frac{4}{5}$

23. $\frac{2}{5}$

24. $\frac{1}{2}$

25. $\frac{1}{5}$ > Videos

26. $\frac{3}{4}$

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Name ______________________

Section __________ Date __________

Answers

1. ________ 2. ________

3. ________ 4. ________

5. ________ 6. ________

7. ________ 8. ________

9. ________ 10. ________

11. ________ 12. ________

13. ________ 14. ________

15. ________ 16. ________

17. ________________________

18. ________________________

19. ________________________

20. ________________________

21. ________ 22. ________

23. ________ 24. ________

25. ________ 26. ________

Answers

27.

28.

29.

30.

31.

32.

33.

34.

35.

36.

37.

38.

39.

40.

27. $\frac{5}{8}$

28. $\frac{7}{8}$

29. $\frac{5}{16}$

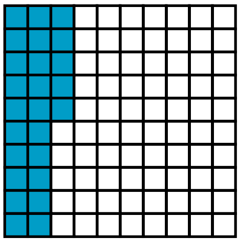
30. $1\frac{1}{5}$

31. $3\frac{1}{2}$ > Videos

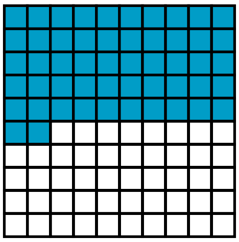
32. $\frac{2}{3}$

Express each partially shaded region as a decimal, a fraction, and a percent.

33.



34.



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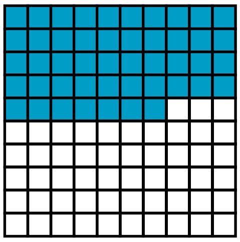

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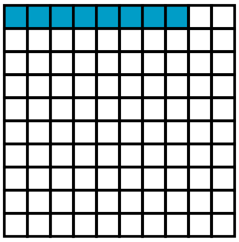

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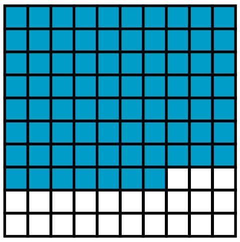

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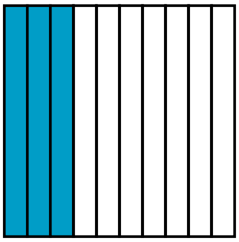

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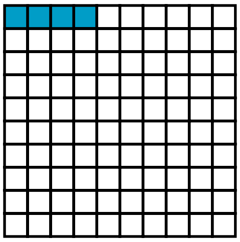
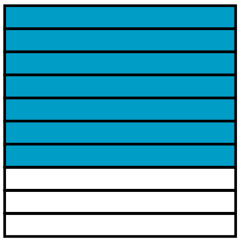
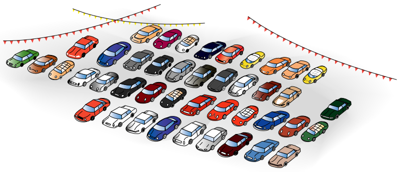


40.



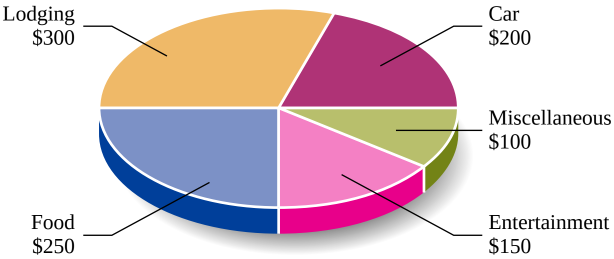
Between 1980 and 2001, the average fuel efficiency of new U.S. cars increased from 24.3 to 28.6 miles per gallon (mi/gal). During this same time, the average fuel efficiency for the entire fleet of U.S. cars rose from 16.0 to 22.1 mi/gal. For exercises 41 and 42, solve the applications involving fuel efficiency.



41. **BUSINESS AND FINANCE** The increase in fuel efficiency for new cars is given by the fraction $\frac{28.6 - 24.3}{24.3}$. Change this fraction to a percent. Round your answer to the nearest tenth of a percent.

42. **BUSINESS AND FINANCE** The increase in fuel efficiency for the fleet of U.S. cars is given by the fraction $\frac{22.1 - 16.0}{16.0}$. Change this fraction to a percent. Round your answer to the nearest tenth of a percent.

Business travelers were asked how much they spent on different items during a business trip. The circle graph shows the results for every $1,000 spent. Use this information to answer exercises 43 to 46.



43. What percent was spent on car expenses?

44. What percent was spent on food?

45. Where was the least amount of money spent? What percent was this?

46. What percent was spent on food and lodging together?

Basic Skills | **Challenge Yourself** ▲ | Calculator/Computer | Career Applications | Above and Beyond

Determine whether each statement is **true** *or* **false.**

47. To write a decimal as a percent, move the decimal two places to the right and add the % symbol.

48. To write a decimal as a percent, move the decimal two places to the left and add the % symbol.

Fill in each blank with **always, sometimes,** *or* **never.**

49. A decimal greater than 1 is ____________ equivalent to a percent greater than 100%.

50. A percent less than 100% can ____________ be written as a mixed number.

Answers

41. ______________________________

42. ______________________________

43. ______________________________

44. ______________________________

45. ______________________________

46. ______________________________

47. ______________________________

48. ______________________________

49. ______________________________

50. ______________________________

6.2 exercises

Answers

51. ____________

52. ____________

53. ____________

54. ____________

55. ____________

56. ____________

57. ____________

58. ____________

59. ____________

60. ____________

61. ____________

62. ____________

63. ____________

64. ____________

65. ____________

66. ____________

67. ____________

Write each number as a percent.

51. 0.002

52. 0.008

53. 0.004

54. 0.001

55. $\frac{1}{6}$ (exact value)

56. $\frac{3}{16}$

57. $\frac{7}{9}$ (to nearest tenth of a percent)

58. $\frac{5}{11}$ (to nearest tenth of a percent)

59. $\frac{7}{9}$ (exact value)

60. $\frac{5}{11}$ (exact value)

61. $5\frac{1}{4}$

62. $1\frac{3}{4}$

63. $4\frac{1}{3}$ (exact value)

64. $2\frac{11}{12}$ (exact value)

Basic Skills | Challenge Yourself | Calculator/Computer | **Career Applications** | Above and Beyond

65. **Automotive Technology** When an automobile engine is cold, its fuel efficiency is only $\frac{13}{16}$ of its maximum efficiency. Express this as a percent.

66. **Welding Technology** It was found that 165 of a total of 172 welds exceeded the required tensile strength. What percent of the welds exceeded the required tensile strength? Round your answer to the nearest whole percent.

67. **Electrical Engineering** The efficiency of an electric motor is defined as the output power divided by the input power. It is usually written as a percent. For a particular motor, the output power is measured to be 400 watts (W) given an input power level of 435 W. What is the efficiency of this motor? Round your answer to the nearest whole percent.

68. **MANUFACTURING TECHNOLOGY** A manufacturer determines that 2 out of every 27 products will be returned due to defects. What percent of the products will be returned? Round to the nearest tenth of a percent.

69. **AGRICULTURAL TECHNOLOGY** There were 52 in. of rainfall in one growing season. If 6.5 in. fell in August, what percent of the season's rainfall fell in August?

Basic Skills | Challenge Yourself | Calculator/Computer | Career Applications | **Above and Beyond** ▲

70. Complete the following table of equivalents. Round decimals to the nearest ten-thousandth. Round percents to the nearest hundredth of a percent.

Fraction	Decimal	Percent
$\frac{7}{12}$		
	0.08	
		35%
	0.265	
		$4\frac{3}{8}\%$
$\frac{11}{18}$		

71. When writing a fraction as a percent, explain when you should convert the fraction to a decimal and then write it as a percent, rather than using the proportion method.

72. When writing a fraction as a percent, explain when you should use the proportion method rather than converting the fraction to a decimal and then writing it as a percent.

Answers

1. 8% **3.** 5% **5.** 18% **7.** 86% **9.** 40% **11.** 70%
13. 110% **15.** 440% **17.** 6.5% **19.** 2.5% **21.** 25%
23. 40% **25.** 20% **27.** 62.5% **29.** 31.25% **31.** 350%
33. 0.25; $\frac{1}{4}$; 25% **35.** 0.47; $\frac{47}{100}$; 47% **37.** 0.77; $\frac{77}{100}$; 77%
39. 0.04; $\frac{1}{25}$; 4% **41.** 17.7% **43.** 20% **45.** Miscellaneous; 10%
47. True **49.** always **51.** 0.2% **53.** 0.4% **55.** $16\frac{2}{3}\%$
57. 77.8% **59.** $77\frac{7}{9}\%$ **61.** 525% **63.** $433\frac{1}{3}\%$ **65.** 81.25%
67. 92% **69.** 12.5% **71.** Above and Beyond

Answers

68. ____________________

69. ____________________

70. ____________________

71. ____________________

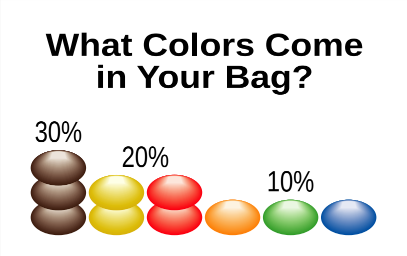
72. ____________________

Activity 16 ::

M&M’s

According to M&M’s/MARS, M&M’s are produced and packaged with approximately the following percentages.

Brown:	30%
Yellow:	20%
Red:	20%
Orange:	10%
Green:	10%
Blue:	10%



“M&M’s”® Milk Chocolate Candies Color Chart

A typical pack was opened, yielding the following counts:

Brown	Yellow	Red	Orange	Green	Blue
14	8	15	6	6	10

Calculate the percent for each color M&M in this pack. Round to the nearest percent.

Brown	Yellow	Red	Orange	Green	Blue

Do any of these seem to differ markedly from the percents named by the M&M’s/MARS company? If so, give reasons why this may have occurred.

Obtain your own package of M&M’s and determine the percents for each color. Comment on how closely your percents agree with the percents named by the company.

6.3

The Three Types of Percent Problems

< **6.3 Objectives** >

1 > Identify the rate in a percent problem
2 > Identify the base in a percent problem
3 > Identify the amount in a percent problem
4 > Solve the three types of percent problems

There are many practical applications of our work with percents. All of these problems have three basic parts that need to be identified. Here are some definitions that will help with that process.

Definition

Base, Amount, and Rate

The **base**, denoted *B*, is the whole in a problem. It is the standard used for comparison.

The **amount**, denoted *A*, is the part of the whole being compared to the base.

The **rate**, denoted *R*, is the ratio of the amount to the base. It is written as a percent.

The following examples provide some practice in determining the parts of a percent problem.

Example 1 Identifying Rates

< **Objective 1** >

NOTE

The *rate R* is the easiest of the terms to identify. The rate is written with the percent symbol (%) or the word *percent.*

Identify each rate.

(a) What is $\underset{\underset{R}{\uparrow}}{15\%}$ of 200?

Here 15% is the rate because it has the percent symbol attached.

(b) $\underset{\underset{R}{\uparrow}}{25\%}$ of what number is 50?

25% is the rate.

(c) 20 is $\underset{\underset{R}{\uparrow}}{\text{what percent}}$ of 40?

Here the rate is unknown.

Check Yourself 1

Identify the rate.

(a) 15% of what number is 75?
(b) What is 8.5% of 200?
(c) 200 is what percent of 500?

Instructor's note: In this section, we use proportions to solve percent problems. After introducing students to algebra, we take an equations approach in Section 11.5.

The *base B* is the whole, or 100%, in the problem. The base often follows the word *of*, as shown in Example 2.

Example 2 Identifying Bases

< Objective 2 >

Identify each base.

(a) What is 15% of $\underset{\substack{\uparrow \\ B}}{200}$?

200 is the base. It follows the word *of*.

(b) 25% of $\underset{\substack{\uparrow \\ B}}{\text{what number}}$ is 50?

Here the base is the unknown.

(c) 20 is what percent of $\underset{\substack{\uparrow \\ B}}{40}$?

40 is the base.

Check Yourself 2

Identify the base.

(a) 70 is what percent of 350? (b) What is 25% of 300?
(c) 14% of what number is 280?

The *amount A* is the part of the problem remaining once the rate and the base have been identified.

In many applications, the amount is found with the word *is*.

Example 3 Identifying Amounts

< Objective 3 >

Identify the amount.

(a) $\underset{\substack{\uparrow \\ A}}{\text{What}}$ is 15% of 200?

Here the amount is the unknown part of the problem. Note that the word *is* follows.

(b) 25% of what number is $\underset{\substack{\uparrow \\ A}}{50}$?

Here the amount, 50, follows the word *is*.

(c) $\underset{\substack{\uparrow \\ A}}{20}$ is what percent of 40?

Again the amount, here 20, can be found with the word *is*.

Check Yourself 3

Identify the amount.

(a) 30 is what percent of 600? (b) What is 12% of 5,000?
(c) 24% of what number is 96?

In Example 4, we identify all three parts in a percent problem.

Example 4 Identifying the Rate, Base, and Amount

Determine the rate, base, and amount in this problem:

12% of 800 is what number?

Finding the *rate* is not difficult. Just look for the percent symbol or the word *percent*. In this exercise, 12% is the rate.

The *base* is the whole. Here it follows the word *of*. 800 is the whole or the base.

The *amount* remains when the rate and the base have been found. Here the amount is the unknown. It follows the word *is*. "What number" asks for the unknown amount.

Check Yourself 4

Find the rate, base, and amount in the following statements or questions.

(a) 75 is 25% of 300. **(b) 20% of what number is 50?**

We use percents to solve a variety of applied problems. In all these situations, you have to identify the three parts of the problem. Example 5 is intended to help you build that skill.

Example 5 Identifying the Rate, Base, and Amount

(a) Determine the rate, base, and amount in the following application.

In an algebra class of 35 students, 7 received a grade of A. What percent of the class received an A?

The *base* is the whole in the problem, or the number of students in the class. 35 is the base.

The *amount* is the portion of the base, here the number of students that receive the A grade. 7 is the amount.

The *rate* is the unknown in this example. "What percent" asks for the unknown rate.

(b) Determine the rate, base, and amount in the following application:

Doyle borrows $2,000 for 1 year. If the interest rate is 12%, how much interest will he pay?

The *base* is again the whole, the size of the loan in this example. $2,000 is the base.

The *rate* is, of course, the interest rate. 12% is the rate.

The *amount* is the quantity left once the base and rate have been identified. Here the amount is the amount of interest that Doyle must pay. The amount is the unknown in this example.

Check Yourself 5

(a) Determine the rate, base, and amount in the following application: In a shipment of 150 parts, 9 of the parts were defective. What percent were defective?

(b) Determine the rate, base, and amount in the following application: Robert earned $120 interest from a savings account paying 8% interest. What amount did he have invested?

So far, you have learned that statements and problems about percents consist of three parts: the rate, base, and amount. In fact, every problem involving percents consists of these three parts.

In nearly every percent problem, one of these three parts is missing. Solving a percent problem is a matter of identifying and computing the missing part. To do this, we use the **percent proportion.**

Property

The Percent Proportion

$$\frac{A}{B} = \frac{r}{100}$$

where A is the amount, B is the base, and $\frac{r}{100}$ is the rate.

Previously, we said that the rate, R, is written as a percent. Note that R and r represent different things. For example, if $R = 54\%$ then $r = 54$. This means that r is the number before the percent symbol. The use of these symbols (A, B, r, and R) will become clear through the following examples.

Example 6 Finding an Unknown Amount

< Objective 4 >

What is 18% of 300?

We know that the rate, R, is 18%. This means that $r = 18$. We also know the base, B, equals 300. The amount, A, is unknown. Use the percent relationship to write the following proportion:

$$\frac{A}{300} = \frac{18}{100}$$

Applying the proportion rule that we learned in Section 5.3 gives

$$100 \cdot A = 18 \cdot 300$$

or

$$100A = 5{,}400$$

We identify the coefficient of the variable as 100 and divide by this number.

$$\frac{100A}{100} = \frac{5{,}400}{100}$$

$$A = 54$$

So, 54 is 18% of 300.

Check Yourself 6

Find 65% of 200.

The second type of percent problem requires us to find an unknown rate. We solve such a problem in the same way as before—we write the percent proportion and solve it.

Example 7 Finding an Unknown Rate

30 is what percent of 150?

We know that 30 is the amount, A, and 150 is the base, B. The rate, R, is the unknown.

$$\frac{30}{150} = \frac{r}{100}$$

$\frac{A}{B} = \frac{r}{100}$

$$150 \cdot r = 30 \cdot 100 = 3{,}000$$

Rewrite the equation using the proportion rule.
The coefficient of the variable is 150.

$$\frac{150r}{150} = \frac{3{,}000}{150}$$

Divide both sides by the coefficient of the variable.

$$r = 20$$

$3{,}000 \div 150 = 20$

Since $r = 20$, the rate R is 20%.

Therefore, 30 is 20% of 150.

Check Yourself 7

75 is what percent of 300?

The final type of percent problem is one with a missing base.

Example 8 Finding an Unknown Base

RECALL

$\frac{A}{B} = \frac{r}{100}$

28 is 40% of what number?

The amount, A, is 28 and the rate, R, is 40%. This means that $r = 40$. We use these to set up the percent proportion with a missing base.

$$\frac{28}{B} = \frac{40}{100}$$

As before, we use the proportion rule to solve this.

$$40B = 2{,}800 \qquad 28 \cdot 100 = 2{,}800$$

$$\frac{40B}{40} = \frac{2{,}800}{40} \qquad \text{40 is the coefficient of the variable.}$$

$$B = 70 \qquad 2{,}800 \div 40 = 70$$

Therefore, 28 is 40% of 70.

Check Yourself 8

70 is 35% of what number?

Remember that a percent (the rate) can be greater than 100.

Example 9 Finding an Unknown Rate

NOTE

The rate is 125%. The base is 300.

What is 125% of 300?

In the percent proportion, we have

$$\frac{A}{300} = \frac{125}{100} \qquad \text{Since } R = 125\%,\ r = 125.$$

So $100A = 300 \cdot 125$.

Dividing by 100 yields

$$\frac{100A}{100} = \frac{300 \cdot 125}{100} = \frac{37{,}500}{100} = 375$$

So $A = 375$. Therefore, 375 is 125% of 300.

NOTE

When the rate is greater than 100%, the amount will be *greater than* the base.

Check Yourself 9

Find 150% of 500.

We next look at an example of a percent problem involving a fraction of a percent.

Example 10 Finding an Unknown Base

NOTE

The amount is 34 and the rate is 8.5%. We want to find the base.

34 is 8.5% of what number?

Using the percent proportion yields

$$\frac{34}{B} = \frac{8.5}{100} \qquad \text{Since } R = 8.5\%,\ r = 8.5.$$

NOTE

Divide by 8.5.

Solving the proportion we have

$$8.5B = 34 \cdot 100$$

$$\frac{8.5B}{8.5} = \frac{34 \cdot 100}{8.5} = \frac{3{,}400}{8.5} = 400$$

So $B = 400$, and we have

34 is 8.5% of 400.

Check Yourself 10

12.5% of what number is 75?

Check Yourself ANSWERS

1. **(a)** 15%; **(b)** 8.5%; **(c)** "what percent" (the unknown) **2.** **(a)** 350; **(b)** 300; **(c)** "what number" (the unknown) **3.** **(a)** 30; **(b)** "What" (the unknown); **(c)** 96 **4.** **(a)** $R = 25\%$, $B = 300$, $A = 75$; **(b)** $R = 20\%$, $B =$ "What number," $A = 50$ **5.** **(a)** $R =$ "What percent" (the unknown), $B = 150$, $A = 9$; **(b)** $R = 8\%$, $B =$ "What amount," $A = \$120$ **6.** 130 **7.** 25% **8.** 200 **9.** 750 **10.** 600

Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 6.3

(a) In percent problems, the ________________ is the standard used for comparison.

(b) In percent problems, the ________________ is the ratio of the amount to the base.

(c) In percent problems, the ________________ is often written as a percent.

(d) In percent problems, the ________________ is the part of the whole being compared to the base.

6.3 exercises

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Name ____________________

Section _________ Date _________

< Objectives 1–3 >

Identify the rate, base, and amount in each statement or question. Do not solve *the exercise at this point.*

1. 23% of 400 is 92. > Videos

2. 150 is 20% of 750.

3. 40% of 600 is 240.

4. 200 is 40% of 500.

5. What is 7% of 325?

6. 80 is what percent of 400?

7. 16% of what number is 56? > Videos

8. What percent of 150 is 30?

9. 480 is 60% of what number?

10. What is 60% of 250?

11. What percent of 120 is 40? > Videos

12. 150 is 75% of what number?

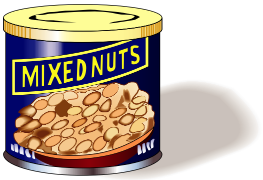
Identify the rate, base, and amount in each application. Do not solve *the application at this point.*

13. **Business and Finance** Jan has a 5% commission rate on all her sales. If she sells $40,000 worth of merchandise in 1 month, what commission will she earn? > Videos

14. **Business and Finance** 22% of Shirley's monthly salary is deducted for withholding. If those deductions total $209, what is her salary?

15. **Science and Medicine** In a chemistry class of 30 students, 5 received a grade of A. What percent of the students received A's?

16. **Business and Finance** A can of mixed nuts contains 80% peanuts. If the can holds 16 ounces, how many ounces of peanuts does it contain?



Answers

1. ____________________

2. ____________________

3. ____________________

4. ____________________

5. ____________________

6. ____________________

7. ____________________

8. ____________________

9. ____________________

10. ____________________

11. ____________________

12. ____________________

13. ____________________

14. ____________________

15. ____________________

16. ____________________

Answers

17. ____________

18. ____________

19. ____________

20. ____________

21. ____________

22. ____________

23. ________ 24. ________

25. ________ 26. ________

27. ________ 28. ________

29. ________ 30. ________

31. ________ 32. ________

33. ________ 34. ________

35. ________ 36. ________

37. ________ 38. ________

39. ________ 40. ________

41. ________ 42. ________

17. **BUSINESS AND FINANCE** The sales tax rate in a state is 5.5%. If you pay a tax of $3.30 on an item that you purchase, what is its selling price? > Videos

18. **BUSINESS AND FINANCE** In a shipment of 750 parts, 75 were found to be defective. What percent of the parts were faulty?

19. **SOCIAL SCIENCE** A college had 9,000 students at the start of a school year. If there is an enrollment increase of 6% by the beginning of the next year, how many additional students are there?

20. **BUSINESS AND FINANCE** Paul invested $5,000 in a time deposit. What interest will he earn for 1 year if the interest rate is 3.5%? chapter 6 > Make the Connection

Determine whether each statement is **true** *or* **false.**

21. The rate, in a percent problem, is never greater than 100%.

22. The base, in a percent problem, often follows the word *of*.

< Objective 4 >

Solve each percent problem.

23. What is 35% of 600? > Videos

24. 20% of 400 is what number?

25. 45% of 200 is what number?

26. What is 40% of 1,200?

27. Find 40% of 2,500.

28. What is 75% of 120?

29. What percent of 50 is 4? > Videos

30. 51 is what percent of 850?

31. What percent of 500 is 45?

32. 14 is what percent of 200?

33. What percent of 200 is 340?

34. 392 is what percent of 2,800?

35. 46 is 8% of what number? > Videos

36. 7% of what number is 42?

37. Find the base if 11% of the base is 55.

38. 16% of what number is 192?

39. 58.5 is 13% of what number?

40. 21% of what number is 73.5?

41. Find 110% of 800.

42. What is 115% of 600?

43. What is 108% of 4,000?

44. Find 160% of 2,000.

45. 210 is what percent of 120?

46. What percent of 40 is 52?

47. 360 is what percent of 90?

48. What percent of 15,000 is 18,000?

49. 625 is 125% of what number?

50. 140% of what number is 350?

51. Find the base if 110% of the base is 935.

52. 130% of what number is 1,170?

Basic Skills | **Challenge Yourself** | Calculator/Computer | Career Applications | Above and Beyond

53. Find 8.5% of 300.

54. $8\frac{1}{4}\%$ of 800 is what number?

55. Find $11\frac{3}{4}\%$ of 6,000.

56. What is 3.5% of 500?

57. What is 5.25% of 3,000?

58. What is 7.25% of 7,600?

59. 60 is what percent of 800?

60. 500 is what percent of 1,500?

61. What percent of 180 is 120?

62. What percent of 800 is 78?

63. What percent of 1,200 is 750?

64. 68 is what percent of 800?

65. 10.5% of what number is 420?

66. Find the base if $11\frac{1}{2}\%$ of the base is 46.

67. 58.5 is 13% of what number?

68. 6.5% of what number is 325?

69. 195 is 7.5% of what number?

70. 21% of what number is 73.5?

Basic Skills | Challenge Yourself | Calculator/Computer | **Career Applications** | Above and Beyond

Identify the rate, base, and amount in each statement or question. Do not solve *the problems at this point.*

71. **ALLIED HEALTH** How much 25% alcohol solution can be prepared using 225 milliliters (mL) of ethyl alcohol?

72. **INFORMATION TECHNOLOGY** An Ethernet network transmits a maximum packet size of 1,500 bytes. 1.7% of each packet is "overhead." How much information (in bytes) in a maximum-size packet is overhead?

73. **AGRICULTURAL TECHNOLOGY** Milk that is labeled "3.5%" is made up of 3.5% butterfat. How many grams of butterfat are in 1 liter (938 g) of 3.5% milk?

74. **ENVIRONMENTAL TECHNOLOGY** In some communities, "green" laws require that 40% of a lot remains green (covered in grass or other vegetation). How much green space is required in a 12,680-square-foot lot?

Answers

43. ________ 44. ________
45. ________ 46. ________
47. ________ 48. ________
49. ________ 50. ________
51. ________ 52. ________
53. ________ 54. ________
55. ________ 56. ________
57. ________ 58. ________
59. ________ 60. ________
61. ________ 62. ________
63. ________ 64. ________
65. ________ 66. ________
67. ________ 68. ________
69. ________ 70. ________

71. ________

72. ________

73. ________

74. ________

Answers

1. $\underset{\substack{\uparrow\\R}}{23\%}$ of $\underset{\substack{\uparrow\\B}}{400}$ is $\underset{\substack{\uparrow\\A}}{92}$ **3.** $\underset{\substack{\uparrow\\R}}{40\%}$ of $\underset{\substack{\uparrow\\B}}{600}$ is $\underset{\substack{\uparrow\\A}}{240}$ **5.** $\underbrace{\text{What}}_{\substack{\uparrow\\A}}$ is $\underset{\substack{\uparrow\\R}}{7\%}$ of $\underset{\substack{\uparrow\\B}}{325}$

7. $\underset{\substack{\uparrow\\R}}{16\%}$ of $\underbrace{\text{what number}}_{\substack{\uparrow\\B}}$ is $\underset{\substack{\uparrow\\A}}{56}$ **9.** $\underset{\substack{\uparrow\\A}}{480}$ is $\underset{\substack{\uparrow\\R}}{60\%}$ of $\underbrace{\text{what number}}_{\substack{\uparrow\\B}}$

11. $\underbrace{\text{What percent}}_{\substack{\uparrow\\R}}$ of $\underset{\substack{\uparrow\\B}}{120}$ is $\underset{\substack{\uparrow\\A}}{40}$

13. \$40,000 is the base. 5% is the rate. Her commission, the unknown, is the amount.

15. 30 is the base. 5 is the amount. The unknown percent is the rate.

17. 5.5% is the rate. The tax, \$3.30, is the amount. The unknown selling price is the base.

19. The base is 9,000. The rate is 6%. The unknown number of additional students is the amount.

21. False **23.** 210 **25.** 90 **27.** 1,000 **29.** 8% **31.** 9%

33. 170% **35.** 575 **37.** 500 **39.** 450 **41.** 880 **43.** 4,320

45. 175% **47.** 400% **49.** 500 **51.** 850 **53.** 25.5 **55.** 705

57. 157.5 **59.** 7.5% **61.** $66\frac{2}{3}\%$ **63.** 62.5% **65.** 4,000

67. 450 **69.** 2,600 **71.** $R = 25\%$, $B =$ unknown, $A = 225$ mL

73. $R = 3.5\%$, $B = 938$ g, $A =$ unknown

chapter
6
> Make the Connection

Activity 17 ::
A Matter of Interest

When you put your money into savings for investment purposes, you will often leave it for some time, allowing it to earn interest and grow. Typically, the interest your investment earns is computed periodically as a percent of your investment. The investment is referred to as the **principal,** and we use the following **interest** formula:

Interest = principal × rate × time

or $I = P \times R \times T$. If we use 1 year for time, the formula becomes $I = P \times R$. Note the following (for time = 1 year):

Interest = principal × rate

Suppose that your original investment is $1,000 and that your money earns interest at a rate of 4% (per year). Let us further suppose that the interest is computed at the end of each year and then added onto the principal. The principal then grows, and the amount of interest in the following year will be greater. To see how this works, complete the following table.

Year	Principal	Interest	Investment at End of Year
1	$1,000	$40	$1,040
2	1,040		
3			
4			
5			
6			
7			

After 7 years, how much has your investment grown?

For how many years must the money sit to earn total interest that is 50% of your original investment? (*Hint:* Continue the calculations in the table.)

6.4

Applications of Percent Problems

< **6.4 Objectives** >

1 > Solve applications of percents

2 > Solve applications that involve percent increase and decrease

3 > Solve percent applications involving interest

The concept of percent is perhaps the most frequently encountered arithmetic idea considered in this text. In this section, we show some of the many applications of percent and the special terms that are used in these applications.

In Example 1, we solve a percent application in which the amount is the unknown quantity.

If the rate is *less than* 100%, then the amount will be *less than* the base.

20 is 40% of 50 and $20 < 50$

If the rate is *greater than* 100%, then the amount will be *greater than* the base.

75 is 150% of 50 and $75 > 50$

Example 1 Solving a Percent Application with an Unknown Amount

< Objective 1 >

A student needs to answer at least 70% of the questions correctly on a 50-question exam in order to pass. How many questions must the student get right?

In this case, the rate is 70%, so $r = 70$, and the base, or total, is 50 questions, so $B = 50$. The amount A is unknown.

We write the percent proportion and solve.

$$\begin{aligned} \frac{A}{50} &= \frac{70}{100} & \\ 100A &= 70 \cdot 50 & \text{If } \frac{a}{b} = \frac{c}{d}\text{, then } ad = bc. \\ 100A &= 3{,}500 & \text{The coefficient of the variable is 100.} \\ \frac{100A}{100} &= \frac{3{,}500}{100} & \text{Divide both sides by the coefficient of the variable.} \\ A &= 35 & 3{,}500 \div 100 = 35 \end{aligned}$$

The student must answer at least 35 questions correctly in order to pass the exam.

Check Yourself 1

Generally, 72% of the students in a chemistry course pass the class. If there are 150 students in the class, how many are expected to pass?

Let us now look at an application involving an unknown rate.

If the amount is *less than* the base, then the rate will be *less than* 100%.

If the amount is *greater than* the base, then the rate will be *greater than* 100%.

Instructor's note: In this section, we use proportions to solve percent problems. After introducing students to algebra, we use an equations approach in Section 11.5.

Example 2 Solving a Percent Application with an Unknown Rate

Simon waits tables at La Catalana, an upscale restaurant. A family left a $45 tip on a $250 meal. What percent of the bill did the family leave as a tip?

We are asked to find the rate given a base $B = \$250$ and an amount $A = \$45$. To find the rate, we use the percent proportion.

$$\frac{45}{250} = \frac{r}{100}$$

$$250r = 4{,}500 \qquad 45 \cdot 100 = 4{,}500$$

$$\frac{250r}{250} = \frac{4{,}500}{250}$$

$$r = 18 \qquad 4{,}500 \div 250 = 18$$

NOTE

Remember to write your answer as a percent.

Therefore, the family left a tip that was 18% of the bill.

Check Yourself 2

Last year, Xian reported an income of $27,500 on her tax return. Of that, she paid $6,600 in taxes. What percent of her income did she pay in taxes?

We can now look at an application involving an unknown base.

Example 3 Solving a Percent Application with an Unknown Base

A computer ran 60% of a scan in 120 seconds (s). How long should it take to complete an entire scan?

In this case, the rate is 60% and the amount is 120 s. We employ the percent proportion again, and solve.

$$\frac{120}{B} = \frac{60}{100}$$

$$60B = 12{,}000$$

$$B = \frac{12{,}000}{60} \qquad \text{Divide by the coefficient of the variable, 60.}$$

$$B = 200$$

The entire scan should take 200 s.

Check Yourself 3

An indexing program takes 4 minutes to check 30% of the files on a laptop computer. How long should it take to index all the files (to the nearest second)?

Percents are used in too many ways for us to list. Notice the variety in the following examples, which illustrate some additional situations in which you will find percents.

Example 4 Solving a Percent Application: Commission

A salesman sells a used car for $9,500. His commission rate is 4%. What is his commission for the sale?

NOTE

A **commission** is the amount that a person is paid for a sale.

The base is the total of the sale, in this problem, \$9,500. The rate is 4%, and we want to find the commission. This is the amount. By the percent proportion

$$\frac{A}{9{,}500} = \frac{4}{100}$$
$$100A = 4 \cdot 9{,}500 = 38{,}000$$
$$A = \frac{38{,}000}{100}$$
$$A = 380$$

The salesman's commission is \$380.

Check Yourself 4

Jenny sells a \$36,000 building lot. If her real estate commission rate is 5%, what commission does she earn on the sale?

Example 5 Solving a Percent Application: Commission

A clerk sold \$3,500 in merchandise during 1 week. If he received a commission of \$140, what was the commission rate?

The base is \$3,500, and the amount is the commission of \$140. Using the percent proportion, we have

$$\frac{140}{3{,}500} = \frac{r}{100}$$
$$3{,}500r = 140 \cdot 100 = 14{,}000$$
$$r = \frac{14{,}000}{3{,}500}$$
$$r = 4$$

So $R = 4\%$. The commission rate is 4%.

Check Yourself 5

On a purchase of \$500 you pay a sales tax of \$21. What is the tax rate?

Example 6, involving a commission, shows how to find the total amount sold.

Example 6 Solving a Percent Application: Commission

A saleswoman has a commission rate of 3.5%. To earn \$280, how much must she sell?

The rate is 3.5%. The amount is the commission, \$280. We want to find the base. In this case, this is the amount that the saleswoman needs to sell.

By the percent proportion

$$\frac{280}{B} = \frac{3.5}{100} \quad \text{or} \quad 3.5B = 280 \cdot 100$$
$$B = \frac{28{,}000}{3.5}$$
$$B = 8{,}000$$

The saleswoman must sell \$8,000 to earn \$280 in commissions.

Check Yourself 6

Kerri earns a commission rate of 5.5%. If she wants to earn $825 in commissions, find the total sales that she must make.

Another common application of percents involves tax rates.

Example 7 Solving a Percent Application: Tax

A state taxes sales at 7%. If a 30-GB iPod is listed for $300, what is the total you will have to pay?

The tax you pay is the amount *A* (the part of the whole). Here the base *B* is the purchase price, $300. The rate *R* is 7%, so $r = 7$.

NOTE

In an application involving taxes, the tax paid is always the amount.

$$\frac{A}{300} = \frac{7}{100} \quad \text{or} \quad 100A = 300 \cdot 7$$

$$A = \frac{2{,}100}{100} = 21$$

The sales tax is $21. You will have to pay the sum: $300 + $21, or $321.

Check Yourself 7

Suppose that a state has a sales tax rate of $6\frac{1}{2}$%. If you buy a used car for $1,200, what is the total you will have to pay?

In applications that involve discounts or markups, the base *B* always represents the original price. This might be the price before the discount, or it might be the wholesale price prior to a markup. The amount *A* is usually the amount of the discount, or the amount of the markup.

Example 8 Solving a Percent Application: Discount

> CAUTION

The amount is not $92. The amount is the size of the discount.

A kitchen store offers an All-Clad saucepan on sale for $92. The saucepan normally sells for $115. What is the discount rate?

We begin by identifying the base as the original price, $115, and the rate as the unknown. The amount *A* is the amount of the discount:

$$A = \$115 - \$92 = \$23$$

Now we are ready to apply the percent proportion to solve this problem.

$$\frac{23}{115} = \frac{r}{100}$$

$$115r = 2{,}300 \qquad 23 \cdot 100 = 2{,}300$$

$$r = \frac{2{,}300}{115} \qquad \text{Divide both sides by the coefficient of the variable, 115.}$$

$$r = 20 \qquad 2{,}300 \div 115 = 20.$$

Therefore, the discount rate is 20%.

NOTE

The *markup* is the amount a store adds to the price of an item to cover expenses and profit. Retailers usually mark an item up by a percentage of its wholesale cost.

Check Yourself 8

An electronics store sells a certain Kicker amplifier for a car stereo system for $250. If the store pays $200 for the amplifier, what is the markup rate?

A common type of application involves *percent increase* or *percent decrease*. *Percent increase* and *percent decrease* always describe the change starting from the base.

Example 9 Solving a Percent Application: Percent Decrease

< Objective 2 >

NOTE

The base is the *original* population.

The population of a town decreased 15% in a 3-year period. If the original population was 12,000, what was the population at the end of the period?

Treynor
Welcomes you
population: 12,000

Since the original population was 12,000, this is the base B. The rate of decrease R is 15%. The amount of the decrease A is unknown.

$$\frac{A}{12{,}000} = \frac{15}{100} \qquad \text{so} \qquad 100A = 15 \cdot 12{,}000$$

$$A = \frac{180{,}000}{100}$$

$$A = 1{,}800$$

Since the amount of the decrease is 1,800, the population at the end of the period must be:

$$\underset{\text{Original population}}{12{,}000} - \underset{\text{Decrease}}{1{,}800} = \underset{\text{New population}}{10{,}200}$$

Check Yourself 9

A school's enrollment increased by 8% from a given year to the next. If the enrollment was 550 students the first year, how many students were enrolled the second year?

Example 10 Solving a Percent Application: Percent Increase

Enrollment at a school increased from 800 to 888 students from a given year to the next. What was the rate of increase?

First we must subtract to find the amount of the increase.

Increase: $888 - 800 = 88$ students

Now to find the rate, we have

NOTE

We use the *original* enrollment, 800, as our base. The size of the increase is the amount.

$$\frac{88}{800} = \frac{r}{100} \qquad \text{so} \qquad 800r = 88 \cdot 100$$

$$r = \frac{8{,}800}{800}$$

$$r = 11$$

The enrollment increased at a rate of 11%.

Check Yourself 10

Car sales at a dealership increased from 350 units one year to 378 units the next. What was the rate of increase?

Example 11 Solving a Percent Application: Percent Increase

A company hired 18 new employees in 1 year. If this was a 15% increase, how many employees did the company have before the increase?

The rate is 15%. The amount is 18, the number of new employees. The base in this problem is the number of employees *before the increase.* So

NOTE

The size of the increase is the amount. The original number of employees is the base.

$$\frac{18}{B} = \frac{15}{100}$$

$$15B = 18 \cdot 100$$

$$B = \frac{1{,}800}{15}$$

$$B = 120$$

The company had 120 employees before the increase.

Check Yourself 11

A school had 54 fewer students in one term than in the previous term. If this was a 12% decrease over the previous term, how many students were there before the decrease?

NOTE

The money borrowed or invested is called the **principal.**

Another common application of percent is interest. When you take out a home loan, you pay interest; when you invest money in a savings account, you earn interest. Interest is a percent of the whole, and the percent is called the **interest rate.**

When we work with interest on a certain amount of money (the principal) for a specific time period, the interest is called **simple interest.**

In the applications here, we will confine our study to interest earned or owed after one year. We will again put the percent proportion to use. In this case, the *interest* plays the same role as the "amount," and the *principal* takes the part of the "base."

Example 12 Solving a Percent Application: Simple Interest

< Objective 3 >

Find the interest you must pay if you borrow \$2,000 for 1 year at an interest rate of $9\frac{1}{2}\%$.

The principal, \$2,000, is the base B. The interest rate R is 9.5%, so $r = 9.5$. We want the amount of interest A.

$$\frac{A}{2{,}000} = \frac{9.5}{100}$$

$$100 \cdot A = 2{,}000 \cdot 9.5$$

$$\frac{100 \cdot A}{100} = \frac{19{,}000}{100}$$

$$A = 190$$

The amount of interest is \$190.

Check Yourself 12

You invest \$5,000 for 1 year at an annual rate of $8\frac{1}{2}\%$. How much interest will you earn?

As with other percent problems, you can find the principal or the rate using the percent proportion.

Example 13 Solving a Percent Application: Simple Interest

Ms. Hobson agrees to pay 11% interest on a loan for her new car. She is charged $2,200 interest on the loan for 1 year. How much did she borrow?

The rate is 11% and the interest (or amount) is $2,200. We need to find the principal (or base), which is the size of the loan.

As before, we set up the percent proportion to solve it.

$$\begin{aligned}
\frac{2{,}200}{B} &= \frac{11}{100} & & \\
11B &= 220{,}000 & & \text{2,200} \cdot 100 = 220{,}000 \\
\frac{11B}{11} &= \frac{220{,}000}{11} & & \text{The coefficient of the variable is 11.} \\
B &= 20{,}000 & & 220{,}000 \div 11 = 20{,}000
\end{aligned}$$

Ms. Hobson borrowed $20,000 to purchase her car.

Check Yourself 13

Sue pays $210 in interest for a 1-year loan at 10.5% interest. What is the size of her loan?

The true power of interest to earn money over time comes from the idea of **compound interest.** This means that once you earn (or are charged) interest, you start earning (or paying) interest on the interest. This is what is meant by compounding.

Compound interest is an exceptionally powerful idea. For many people, their retirement plans are not based on the amount of money invested, but rather on the fact that their retirement account grows with time.

The earnings on someone's retirement investments *compound* over decades and grow into much more than the original investments. This is why financial advisors always suggest that you should start saving for retirement as early as possible. The more time an investment has to grow, the larger it gets.

We conclude this section by looking at an example involving the compounding of interest.

Example 14 Solving a Percent Application: Compound Interest

Suppose you invest $1,000 at 5% (compounded annually) in a savings account for 2 years. How much will you have in the account at the end of the 2-year period?

After the first year, you earn 5% interest on your $1,000 principal.

$$\begin{aligned}
\frac{A}{1{,}000} &= \frac{5}{100} \\
100A &= 5{,}000 \\
A &= \frac{5{,}000}{100} \\
A &= 50
\end{aligned}$$

So, you earn $50 after 1 year. The compounding comes into play in the second year when your account begins with $1,000 + $50 = $1,050 in it. This is your new principal.

$$\underset{\text{Start}}{\$1{,}000} \xrightarrow{\text{At } 5\%} \underset{\text{Year 1}}{\$1{,}050}$$

In the second year, you earn 5% interest on this new principal. This process is called *compound interest.*

$$\begin{aligned} \frac{A}{1{,}050} &= \frac{5}{100} \\ 100A &= 5{,}250 && 5 \cdot 1{,}050 = 5{,}250 \\ A &= \frac{5{,}250}{100} \\ A &= 52.5 && 5{,}250 \div 100 = 52.5 \end{aligned}$$

You earn \$52.50 in the second year of your investment, therefore, your account balance will be \$1,050 + \$52.50 = \$1,102.50.

$$\begin{array}{ccccc} \$1{,}000 & \xrightarrow{\text{At } 5\%} & \$1{,}050 & \xrightarrow{\text{At } 5\%} & \$1{,}102.50 \\ \text{Start} & & \text{Year 1} & & \text{Year 2} \end{array}$$

Check Yourself 14

If you invest \$6,000 at 4% (compounded annually) for 2 years, how much will you have after 2 years?

Check Yourself ANSWERS

1. 108 students **2.** 24% **3.** 800 s **4.** \$1,800 **5.** 4.2%
6. \$15,000 **7.** \$1,278 **8.** 25% **9.** 594 students **10.** 8%
11. 450 students **12.** \$425 **13.** \$2,000 **14.** \$6,489.60

Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 6.4

(a) If the rate is greater than 100%, then the amount will be ______________ than the base.

(b) In an application involving taxes, the tax paid is always the ____________.

(c) The ______________ is the amount a store adds to the price of an item to cover expenses and profit.

(d) In an interest application, the money borrowed or invested is called the ______________.

6.4 exercises

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Name ______________________

Section __________ Date __________

Answers

1. ______________________

2. ______________________

3. ______________________

4. ______________________

5. ______________________

6. ______________________

7. ______________________

8. ______________________

9. ______________________

10. ______________________

11. ______________________

12. ______________________

13. ______________________

14. ______________________

15. ______________________

16. ______________________

< Objectives 1–3 >

Solve each application.

1. **BUSINESS AND FINANCE** What interest will you pay on a $3,400 loan for 1 year if the interest rate is 12%?

2. **SCIENCE AND MEDICINE** A chemist has 300 milliliters (mL) of solution that is 18% acid. How many milliliters of acid are in the solution?

3. **BUSINESS AND FINANCE** If a salesman is paid a $140 commission on the sale of a $2,800 sailboat, what is his commission rate?

4. **BUSINESS AND FINANCE** Ms. Jordan has been given a loan of $2,500 for 1 year. If the interest charged is $275, what is the interest rate on the loan?

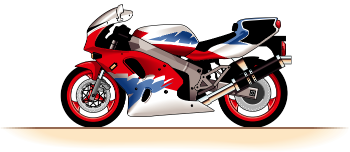
5. **STATISTICS** On a test, Alice had 80% of the problems right. If she had 20 problems correct, how many questions were on the test?

> Videos

6. **BUSINESS AND FINANCE** A state sales tax rate is 3.5%. If the tax on a purchase was $7, what was the price of the purchase?

7. **BUSINESS AND FINANCE** A state sales tax is levied at a rate of 6.4%. How much tax would one pay on a purchase of $260?

8. **BUSINESS AND FINANCE** Betty must make a $9\frac{1}{2}\%$ down payment on the purchase of a $6,000 motorcycle. What is her down payment?



9. **SOCIAL SCIENCE** A study has shown that 102 of the 1,200 people in the workforce of a small town are unemployed. What is the town's unemployment rate?

> Videos

10. **SOCIAL SCIENCE** A survey of 400 people found that 66 were left-handed. What percent of those surveyed were left-handed?

11. **STATISTICS** In a recent survey, 65% of those responding were in favor of a freeway improvement project. If 780 people were in favor of the project, how many people responded to the survey?

12. **SOCIAL SCIENCE** A college finds that 42% of the students taking a foreign language are enrolled in Spanish. If 1,512 students are taking Spanish, how many foreign language students are there?

13. **BUSINESS AND FINANCE** An appliance dealer marks up refrigerators 22% (based on cost). If the wholesale cost of one model is $1,200, what should its selling price be?

14. **SOCIAL SCIENCE** A school had 900 students at the start of a school year. If there is an enrollment increase of 7% by the beginning of the next year, what is the new enrollment?

15. **BUSINESS AND FINANCE** The price of a new van has increased $2,030, which amounts to a 14% increase. What was the price of the van before the increase?

16. **BUSINESS AND FINANCE** A television set is marked down $75, to be placed on sale. If this is a 12.5% decrease from the original price, what was the selling price before the sale?

17. **Business and Finance** Carlotta received a monthly raise of $162.50. If this represented a 6.5% increase, what was her monthly salary before the raise?

18. **Business and Finance** Mr. Hernandez buys stock for $15,000. At the end of 6 months, the stock's value has decreased 7.5%. What is the stock worth at the end of the period?

chapter 6 > Make the Connection

19. **Social Science** The population of a town increases 14% in 2 years. If the population was 6,000 originally, what is the population after the increase?

20. **Business and Finance** A store marks up merchandise 25% to allow for profit. If an item costs the store $11, what will its selling price be?

21. **Business and Finance** A virus scanning program is checking every file for viruses. It has completed checking 40% of the files in 300 seconds. How long should it take to check all the files?

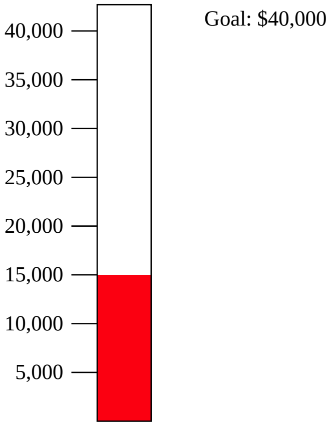
22. **Social Science** In 1990, there were an estimated 145.0 million passenger cars registered in the United States. The total number of vehicles registered in the United States for 1990 was estimated at 194.5 million. What percent of the vehicles registered were passenger cars? Round to the nearest tenth percent.

23. **Social Science** Gasoline accounts for 85% of the motor fuel consumed in the United States every day. If 8,882 thousand barrels (bbl) of motor fuel is consumed each day, how much gasoline is consumed each day in the United States? Round to the nearest thousand barrels.

24. **Social Science** In 1999, transportation accounted for 67% of U.S. petroleum consumption. If 13.3 million bbl of petroleum were used each day for transportation in the United States, what was the approximate total daily petroleum consumption by all sources in the United States?

25. **Science and Medicine** Each year, 540 million metric tons (t) of carbon dioxide is added to the atmosphere by the United States. Burning gasoline and other transportation fuels is responsible for 35% of the carbon dioxide emissions in the United States. How much carbon dioxide is emitted each year by the burning of transportation fuels in the United States?

26. **Social Science** The progress of the local Lions club fund drive is shown here. What percent of the goal has been achieved so far?



Answers

17. ____________________

18. ____________________

19. ____________________

20. ____________________

21. ____________________

22. ____________________

23. ____________________

24. ____________________

25. ____________________

26. ____________________

Answers

27. __________

28. __________

29. __________

30. __________

31. __________

32. __________

33. __________

34. __________

35. __________

36. __________

37. __________

38. __________

Basic Skills | **Challenge Yourself** | Calculator/Computer | Career Applications | Above and Beyond

The chart shows U.S. trade with Mexico from 1997 to 2002. Use this information for exercises 27 to 30.

U.S. Trade with Mexico, 1997–2002

	(millions of dollars) MEXICO		
Year	**Exports**	**Imports**	**Trade Balance[1]**
1997	71,389	85,938	−14,549
1998	78,773	94,629	−15,856
1999	86,909	109,721	−22,812
2000	111,349	135,926	−24,577
2001	101,297	131,338	−30,041
2002	97,470	134,616	−37,146

(1) Totals may not add due to rounding.
Source: U.S. Census Bureau.

27. What is the rate of increase (to the nearest whole percent) of exports from 1997 to 2002?

28. What is the rate of increase (to the nearest whole percent) of imports from 1997 to 2002?

29. By what percent did imports exceed exports in 1997?

30. By what percent did imports exceed exports in 2002?

In exercises 31 to 34, assume the interest is compounded annually (at the end of each year) and find the amount in an account with the given interest rate and principal.

31. \$4,000, 6%, 2 years

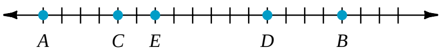
32. \$3,000, 7%, 2 years

chapter 6 > Make the Connection

33. \$4,000, 5%, 3 years

34. \$5,000, 6%, 3 years

Use the number line to complete exercises 35 to 38.



35. Length AC is what percent of length AB?

36. Length AD is what percent of AB?

37. Length AE is what percent of AB?

38. Length AE is what percent of AD?

Basic Skills | Challenge Yourself | **Calculator/Computer** | Career Applications | Above and Beyond

In many everyday applications of percent, the computations required can become quite messy. The calculator can be a great help. Whether we use the proportion approach or the equation approach in solving such an application, we typically set up the problem and isolate the desired variable before doing any calculations.

In some percent increase or percent decrease applications, we can set up the problem so it can be done in one step. (Previously, we did these as two-step problems.) For example, suppose that a store marks up an item 22.5%. If the original cost to the store was $36.40, we want to know what the selling price will be. Since the selling price is the cost to the store plus the markup, the selling price will be 122.5% of the store's cost (100% + 22.5%). We can restate the problem as "What is 122.5% of $36.40?" The base is $36.40 and the rate is 122.5%, and we want the amount.

$$\frac{A}{36.40} = \frac{122.5}{100} \qquad \text{so} \qquad A = \frac{36.40 \times 122.5}{100}$$

We enter

$36.40 \boxed{\times} 122.5 \boxed{\div} 100 \boxed{=}$

The selling price should be $44.59.

Suppose now that a certain card collection *decreases* 8.2% in value from $750. Note that 100% − 8.2% = 91.8%. To find the new value, we can restate the problem as: "What is 91.8% of $750?" We set up the problem accordingly:

$$\frac{A}{750} = \frac{91.8}{100} \qquad \text{so} \qquad A = \frac{750 \times 91.8}{100}$$

Entering $750 \boxed{\times} 91.8 \boxed{\div} 100 \boxed{=}$, we get $688.50.

Use your calculator to solve each application.

39. **Social Science** The population of a town increases 4.2% in 1 year. If the original population was 19,500, what is the population after the increase?

40. **Business and Finance** A store marks up items 42.5% to allow for profit. If an item costs a store $24.40, what will its selling price be?

41. **Business and Finance** A jacket that originally sold for $98.50 is marked down by 12.5% for a sale. Find its sale price (to the nearest cent).

Originally
$98.50 each
now
12.5% off!!

42. **Business and Finance** Jerry earned $18,500 one year and then received a 10.5% raise. What is his new yearly salary?

43. **Business and Finance** Carolyn's salary is $1,740 per month. If deductions average 24.6%, what is her take-home pay?

44. **Business and Finance** Yi Chen made a $6,400 investment at the beginning of a year. By the end of the year, the value of the investment had decreased by 8.2%. What was its value at the end of the year?

chapter 6 > Make the Connection

Basic Skills | Challenge Yourself | Calculator/Computer | **Career Applications** | Above and Beyond

Solve each application.

45. **Allied Health** How much 25% alcohol solution can be prepared using 225 milliliters (mL) of ethyl alcohol?

46. **Information Technology** An Ethernet network transmits a maximum packet size of 1,500 bytes. 1.7% of each packet is "overhead." How much information (in bytes) in a maximum-size packet is overhead?

Answers

39. ____________________

40. ____________________

41. ____________________

42. ____________________

43. ____________________

44. ____________________

45. ____________________

46. ____________________

Answers

47. ____________

48. ____________

49. ____________

50. ____________

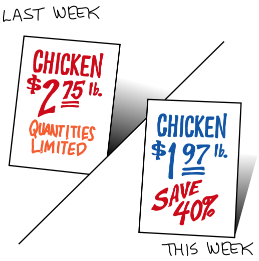
51. ____________

52. ____________

47. **AGRICULTURAL TECHNOLOGY** Milk that is labeled "3.5%" is made up of 3.5% butterfat. How many grams of butterfat are in 1 liter (938 g) of 3.5% milk?

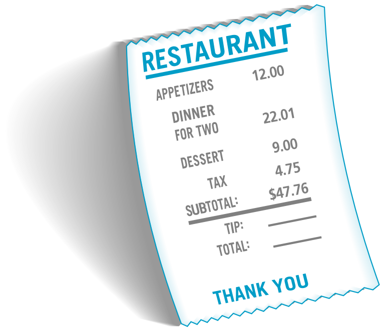
48. **ENVIRONMENTAL TECHNOLOGY** In some communities, "green" laws require that 40% of a lot remains green (covered in grass or other vegetation). How much green space is required in a 12,680-square-foot lot?

Basic Skills | Challenge Yourself | Calculator/Computer | Career Applications | **Above and Beyond**


LAST WEEK
CHICKEN
$2.75 lb.
QUANTITIES LIMITED
CHICKEN
$1.97 lb.
SAVE 40%
THIS WEEK

49. **BUSINESS AND FINANCE** The two ads pictured appeared last week and this week in the local paper. Is this week's ad accurate? Explain.

50. **BUSINESS AND FINANCE** At True Grip hardware, you pay \$10 in tax for a barbecue grill, which is 6% of the purchase price. At Loose Fit hardware, you pay \$10 in tax for the same grill, but it is 8% of the purchase price. At which store do you get the better buy? Why?


RESTAURANT
APPETIZERS 12.00
DINNER FOR TWO 22.01
DESSERT 9.00
TAX 4.75
SUBTOTAL: $47.76
TIP:
TOTAL:
THANK YOU

51. It is customary when eating in a restaurant to leave a 15% tip.

(a) Outline a method to do a quick approximation for the amount of tip to leave.

(b) Use this method to figure a 15% tip on a bill of \$47.76.

52. The dean of enrollment management at a college states, "Last year was not a good year. Our enrollments were down 25%. But this year we increased our enrollment by 30% over last year. I think we have turned the corner." Evaluate the dean's analysis.

Answers

1. \$408 **3.** 5% **5.** 25 questions **7.** \$16.64 **9.** 8.5%
11. 1,200 people **13.** \$1,464 **15.** \$14,500 **17.** \$2,500
19. 6,840 people **21.** 750 s **23.** 7,550 thousand bbl
25. 189 million t **27.** 37% **29.** 20% **31.** \$4,494.40
33. \$4,630.50 **35.** 25% **37.** 37.5% **39.** 20,319 people
41. \$86.19 **43.** \$1,311.96 **45.** 900 mL **47.** 32.83 g
49. Above and Beyond **51.** Above and Beyond

Activity 18 ::
Population Changes Revisited

The following table gives the population for the United States and each of the six largest states from both the 1990 census and the 2000 census. Use this table to answer the questions that follow. Round all computations of percents to the nearest tenth of a percent.

	1990 Population	2000 Population
United States	248,709,873	281,421,906
California	29,760,021	33,871,648
Texas	16,986,510	20,851,820
New York	17,990,455	18,976,457
Florida	12,937,926	15,982,378
Illinois	11,430,602	12,419,293
Pennsylvania	11,881,643	12,281,054

1. Find the percent increase in the U.S. population from 1990 to 2000.

2. By examining the table (no actual calculations yet!), predict which state had the greatest percent increase.

3. Predict which state had the smallest percent increase.

4. Now find the percent increase in population during this period for each state.

California: Texas:

New York: Florida:

Illinois: Pennsylvania:

5. Which state had the greatest percent increase during this period? Which had the smallest?

6. The population of the six largest states combined represented what percent of the U.S. population in 1990?

7. The population of the six largest states combined represented what percent of the U.S. population in 2000?

8. Determine the percent increase in population from 1990 to 2000 for the combined six largest states.

summary :: chapter 6

Definition/Procedure	Example	Reference
Writing Percents as Fractions and Decimals		**Section 6.1**
Percent Another way of naming parts of a whole. *Percent* means per hundred.	Fractions and decimals are other ways of naming parts of a whole. $21\% = 21\left(\frac{1}{100}\right) = \frac{21}{100} = 0.21$	*p.* 367
To write a percent as a fraction, replace the percent symbol with $\frac{1}{100}$ and then multiply and simplify.	$40\% = 40\left(\frac{1}{100}\right) = \frac{40}{100} = \frac{2}{5}$	*p.* 368
To write a percent as a decimal, remove the percent symbol and move the decimal point two places to the left.	$37\% = 0.37$	*p.* 369
Writing Decimals and Fractions as Percents		**Section 6.2**
To write a decimal as a percent, move the decimal point two places to the right and attach the percent symbol.	$0.581 = 58.1\%$	*p.* 377
There are two methods for writing a fraction as a percent. *Method 1 The Proportion Method* Use proportions to write an equivalent fraction with a denominator of 100. Then, write the fraction as a percent.	$\frac{3}{5} = \frac{x}{100} \Rightarrow 3 \cdot 100 = 5x$ $\frac{300}{5} = \frac{5x}{5} \Rightarrow 60 = x$ Therefore, $\frac{3}{5} = 60\%$.	*p.* 380
Method 2 The Decimal Method Write the decimal equivalent of the fraction, and then write that decimal as a percent.	$\frac{3}{5} = 0.6 = 60\%$	*p.* 379
The Three Types of Percent Problems		**Section 6.3**
Every percent problem has the following three parts: **1.** The *base, B.* This is the whole amount or starting amount in the problem. It is the standard used for comparison. **2.** The *amount, A.* This is the part of the whole being compared to the base. **3.** The *rate, R.* This is the ratio of the amount to the base. The rate is generally written as a percent.	$\underset{A}{\underset{\uparrow}{45}}$ is $\underset{R}{\underset{\uparrow}{30\%}}$ of $\underset{B}{\underset{\uparrow}{150}}$.	*p.* 389

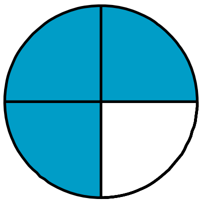
Definition/Procedure	Example	Reference
Percent problems are solved by identifying A, B, R, and r. The rate R is related to r as: $R = \frac{r}{100}$ The base B, the amount A, and r are related by the **percent proportion:** $\frac{A}{B} = \frac{r}{100}$	Suppose that $A = 45$, $B = 150$, $R = 30\%$ If the rate R is 30%, then $r = 30$. Then, the percent proportion is $\frac{45}{150} = \frac{30}{100}$	*p.* 392
Use the percent proportion to solve percent problems. **Step 1** Substitute the two known values into the proportion. **Step 2** Solve the proportion for the unknown value.	What is 24% of 300? $\frac{A}{300} = \frac{24}{100}$ $100A = 7{,}200$ $A = 72$ 72 is 24% of 300.	*p.* 392
Applications of Percent Problems		**Section 6.4**
Applications involving percentages are varied and include solving problems related to commissions, taxes, discounts, markups, and rates of increase or decrease.	An electronics store normally sells a DVD player for \$130. During a sale, they apply a 15% discount to the price. What is the sale price? $B = \$130$ and $R = 15\%$ (so $r = 15$): $\frac{A}{130} = \frac{15}{100}$ $100A = 15 \cdot 130 = 1{,}950$ $A = \frac{1{,}950}{100}$ $= 19.50$ The sale price is $\$130 - \$19.50 = \$110.50$	*p.* 400

summary exercises :: chapter 6

This summary exercise set is provided to give you practice with each of the objectives of this chapter. Each exercise is keyed to the appropriate chapter section. When you are finished, you can check your answers to the odd-numbered exercises against those presented in the back of the text. If you have difficulty with any of these questions, go back and reread the examples from that section. The answers to the even-numbered exercises appear in the *Instructor's Solutions Manual.* Your instructor will give you guidelines on how to best use these exercises in your instructional setting.

6.1

1. Use a percent to name the shaded portion of the diagram.



Write each percent as a common fraction or a mixed number.

2. 2%

3. 20%

4. 37.5%

5. 150%

6. $233\frac{1}{3}\%$

7. 300%

Write each percent as a decimal.

8. 75%

9. 4%

10. 6.25%

11. 13.5%

12. 0.6%

13. 225%

6.2 *Write as percents.*

14. 0.06

15. 0.375

16. 2.4

17. 7

18. 0.035

19. 0.005

20. $\frac{43}{100}$

21. $\frac{7}{10}$

22. $\frac{2}{5}$

23. $1\frac{1}{4}$

24. $2\frac{2}{3}$

25. $\frac{3}{11}$ (to nearest tenth of a percent)

6.3 *Find the unknown.*

26. 80 is 4% of what number?

27. 70 is what percent of 50?

28. 11% of 3,000 is what number?

29. 24 is what percent of 192?

30. Find the base if 12.5% of the base is 625.

31. 90 is 120% of what number?

32. What is 9.5% of 700?

33. Find 150% of 50.

34. Find the base if 130% of the base is 780.

35. 350 is what percent of 200?

36. 28.8 is what percent of 960?

6.4 *Solve each application.*

37. **BUSINESS AND FINANCE** Joan works on a 4% commission basis. She sold $45,000 in merchandise during 1 month. What was the amount of her commission?

38. **BUSINESS AND FINANCE** David buys a dishwasher that is marked down $77 from its original price of $350. What is the discount rate?

39. **SCIENCE AND MEDICINE** A chemist prepares a 400-milliliter (mL) acid-water solution. If the solution contains 30 mL of acid, what percent of the solution is acid?

40. **BUSINESS AND FINANCE** The price of a new compact car has increased $819 over that of the previous year. If this amounts to a 4.5% increase, what was the price of the car before the increase?

41. **BUSINESS AND FINANCE** A store advertises, "Buy the red-tagged items at 25% off their listed price." If you buy a coat marked $136, what will you pay for the coat during the sale?

42. **Business and Finance** Tom has 6% of his salary deducted for a retirement plan. If that deduction is $168, what is his monthly salary?

43. **Social Science** A college finds that 35% of its science students take biology. If there are 252 biology students, how many science students are there altogether?

44. **Business and Finance** A company finds that its advertising costs increased from $72,000 to $76,680 in 1 year. What was the rate of increase?

45. **Business and Finance** A savings bank offers 5.25% interest on 1-year time deposits. If you place $3,000 in an account, how much will you have at the end of the year?

chapter 6 > Make the Connection

46. **Business and Finance** Maria's company offers her a 4% pay raise. This will amount to a $126 per month increase in her salary. What is her monthly salary before and after the raise?

47. **Business and Finance** A virus scanning program is checking every file for viruses. It has completed 30% of the files in 150 seconds. How long should it take to check all the files?

self-test 6

The purpose of this self-test is to help you assess your progress so that you can find concepts that you need to review before the next exam. Allow yourself about an hour to take this test. At the end of that hour, check your answers against those given in the back of this text. If you miss any, go back to the appropriate section to reread the examples until you have mastered that particular concept.

Write as percents.

1. 0.03

2. 0.042

3. $\frac{2}{5}$

4. $\frac{5}{8}$

Solve the percent problems.

5. What is 4.5% of 250?

6. What percent of 300 is 60?

7. $33\frac{1}{3}\%$ of 1,500 is what number?

8. Find 125% of 600.

Solve each application.

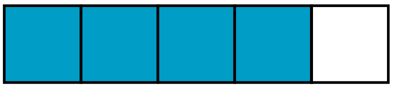
9. **Business and Finance** A state taxes sales at 6.2%. What tax will you pay on a sweater that costs $80?

10. **Statistics** You receive a grade of 75% on a test of 80 questions. How many questions did you have correct?

11. **Business and Finance** A shirt that costs a store $54 is marked up 30% (based on cost). Find its selling price.

12. **Business and Finance** Mrs. Sanford pays $300 in interest on a $2,500 loan for 1 year. What is the interest rate for the loan?

13. Use a percent to name the shaded portion of the following diagram.



Write as decimals.

14. 42%

15. 6%

16. 160%

In exercises 17 to 19, identify the rate, base, and amount. Do not solve *at this point.*

17. 50 is 25% of 200.

18. What is 8% of 500?

19. **Business and Finance** A state sales tax rate is 6%. If the tax on a purchase is $30, what is the amount of the purchase?

Name ____________________

Section ________ Date ________

Answers

1. ____________________
2. ____________________
3. ____________________
4. ____________________
5. ____________________
6. ____________________
7. ____________________
8. ____________________
9. ____________________
10. ____________________
11. ____________________
12. ____________________
13. ____________________
14. ____________________
15. ____________________
16. ____________________
17. ____________________
18. ____________________
19. ____________________

Answers

20. ____________________

21. ____________________

22. ____________________

23. ____________________

24. ____________________

25. ____________________

26. ____________________

27. ____________________

28. ____________________

29. ____________________

30. ____________________

Solve the percent problems.

20. 875 is what percent of 500?

21. 96 is 12% of what number?

22. 8.5% of what number is 25.5?

23. 4.5 is what percent of 60?

Solve each application.

24. **Business and Finance** A car is marked down $1,552 from its original selling price of $19,400. What is the discount rate?

25. **Business and Finance** Sarah earns $540 in commissions in one month. If her commission rate is 3%, what were her total sales?

26. **Social Science** A community college has 480 more students in fall 2008 than in fall 2007. If this is a 7.5% increase, what was the fall 2007 enrollment?

27. **Business and Finance** Shawn arranges financing for his new car. The interest rate for the financing plan is 12%, and he will pay $2,220 interest for 1 year. How much money did he borrow to finance the car?

28. **Business and Finance** Jovita's monthly salary is $2,200. If the deductions for taxes from her monthly paycheck are $528, what percent of her salary goes for these deductions?

Write as fractions.

29. 7%

30. 72%

cumulative review chapters 1-6

Name

Section Date

The following exercises are presented to help you review concepts from earlier chapters. This is meant as review material and not as a comprehensive exam. The answers are presented in the back of the text. Beside each answer is a section reference for the concept. If you have difficulty with any of these exercises, be certain to at least read through the summary related to that section.

1. What is the place value of 4 in the number 234,768?

Perform the indicated operations.

2. 56×203

3. $3{,}026 \div 34$

Evaluate each expression.

4. $8 - 5 + 2$

5. $15 - 3 \times 2$

6. $6 + 4 \times 3^2$

7. List the prime numbers between 50 and 70.

8. Write the prime factorization of 260.

9. Find the greatest common factor (GCF) of 84 and 140.

10. Find the least common multiple (LCM) of 18, 20, and 30.

Perform the indicated operations.

11. $3\frac{2}{5} \times 2\frac{1}{2}$

12. $5\frac{1}{3} \div 4$

13. $4\frac{3}{4} + 3\frac{5}{6}$

14. $7\frac{1}{6} - 2\frac{3}{8}$

15. A kitchen measures $5\frac{1}{2}$ yd by $3\frac{1}{4}$ yd. If vinyl flooring costs \$16 per yd^2, what will it cost to cover the floor?

16. If you drive 180 mi in $3\frac{1}{3}$ h, what is your average speed?

17. A bookshelf that is $54\frac{5}{8}$ in. long is cut from a board that is 8 ft long. If $\frac{1}{8}$ in. is wasted in the cut, what length board remains?

Find the indicated place values.

18. 8 in 4.2835

19. 4 in 6.09743

Answers

1. ______

2. ______

3. ______

4. ______

5. ______

6. ______

7. ______

8. ______

9. ______

10. ______

11. ______

12. ______

13. ______

14. ______

15. ______

16. ______

17. ______

18. ______

19. ______

Answers

20. ____________________
21. ____________________
22. ____________________
23. ____________________
24. ____________________
25. ____________________
26. ____________________
27. ____________________
28. ____________________
29. ____________________
30. ____________________
31. ____________________
32. ____________________
33. ____________________
34. ____________________
35. ____________________
36. ____________________
37. ____________________
38. ____________________
39. ____________________
40. ____________________

Complete each statement with $<$, $=$, or $>$.

20. 6.28 ________ 6.3

21. 3.75 ________ 3.750

Write as a common fraction or a mixed number. Simplify.

22. 0.36

23. 5.125

Perform the indicated operations.

24. 2.8×4.03

25. $54.528 \div 3.2$

26. A television set has an advertised price of $599.95. You buy the set and agree to make payments of $29.50 per month for 2 years. How much extra are you paying on this installment plan?

27. Find the area of a rectangle with length 3.4 m and width 1.85 m.

28. Find the volume of a box with dimensions 5.1 ft × 3.6 ft × 2.4 ft.

Write each ratio in simplest form.

29. $8\frac{1}{2}$ to $12\frac{3}{4}$

30. 34 feet to 8 yards

Solve for the unknown.

31. $\frac{3}{7} = \frac{8}{x}$

32. $\frac{1.9}{y} = \frac{5.7}{1.2}$

33. On a map the scale is $\frac{1}{4}$ in. $=$ 25 mi. How many miles apart are two towns that are $3\frac{1}{2}$ in. apart on the map?

34. Diane worked 23.5 hours on a part-time job and was paid $131.60. She is asked to work 25 hours the next week at the same pay rate. What salary will she receive?

35. Write 34% as a decimal and as a fraction.

36. Write $\frac{11}{20}$ as a decimal and as a percent.

37. Find 18% of 250.

38. 11% of what number is 55?

39. A company reduced the number of employees by 8% this year. 10 employees were laid off. How many were there last year?

40. The sales tax on an item priced at $72 is $6.12. What percent is the tax rate?

chapter
7
> Make the Connection

CHAPTER 7

Measurement

INTRODUCTION

Rachael is a college sophomore studying anthropology. Rachael works after school and during the summers, in a pharmacy, so she can make enough money to travel to other countries when she has time off from school. Rachael hopes to become a doctor someday and travel to less-developed countries to practice eye care. Rachael learned about a program called Doctors Without Borders, which provides humanitarian aid and medical assistance to countries that need it. The idea of helping people in need appeals to Rachael.

Rachael was surprised when she noticed that every country she visited uses the metric system. She is now becoming very adept at converting U.S. measurements to metric and back again. She can even convert Celsius temperatures to Fahrenheit. Rachael did some research on the metric system by going to USMA.com and found that the only countries in the world that do not use the metric system are the United States, Liberia (in western Africa), Yemen (on the Arabian Peninsula), and Myanmar (formerly Burma, in Southeast Asia). She also found that the metric system is a decimal-based system because it is based on multiples of 10. Just as English has become the global language for trade, the metric system has become the global language for measurement.

CHAPTER 7 OUTLINE

Chapter 7 :: Prerequisite Test 424
7.1 **The U.S. Customary System of Measurement** 425
7.2 **Metric Units of Length** 439
7.3 **Metric Units of Weight and Volume** 449
7.4 **Converting Between the U.S. Customary and Metric Systems of Measurement** 459
Chapter 7 :: Summary / Summary Exercises / Self-Test / Cumulative Review :: Chapters 1–7 471

7 prerequisite test

Name ______________________

Section __________ Date __________

Answers

1. ______________________

2. ______________________

3. ______________________

4. ______________________

5. ______________________

6. ______________________

7. ______________________

8. ______________________

9. ______________________

10. ______________________

11. ______________________

12. ______________________

This prerequisite test provides some exercises requiring skills that you will need to be successful in the coming chapter. The answers for these exercises can be found in the back of this text. This prerequisite test can help you identify topics that you will need to review before beginning the chapter.

Evaluate each expression.

1. 42.84×10^4

2. $42.84 \div 10^4$

Simplify each fraction.

3. $\frac{144}{108}$

4. $\frac{\frac{9}{28}}{\frac{27}{35}}$

Write each ratio in simplest form.

5. 15 to 6

6. 1.2 to 2

Write each rate as a unit rate.

7. $\frac{\$344}{32\text{ h}}$

8. $\frac{175\text{ mi}}{3.5\text{ h}}$

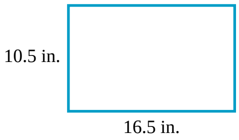
Find the unit price of each item.

9. A 64-fluid-ounce container of orange juice sells for $3.29 (round your result to the nearest hundredth of a cent).

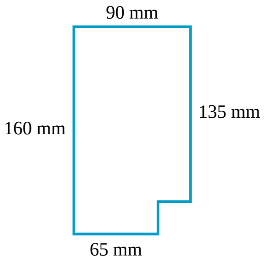
10. A case of 50 blank DVD-R disks sells for $22.50.

Find the perimeter of each figure.

11.



12.



7.1

The U.S. Customary System of Measurement

< 7.1 Objectives >

1 > Convert between two U.S. Customary units of measure

2 > Simplify denominate numbers

3 > Perform operations with denominate numbers

4 > Solve applications involving U.S. Customary units of measure

Many problems involve **units of measure.** When we measure an object, we describe some property it has by using a number and the appropriate unit. For instance, we might say that a board is 6 feet long to describe its length, or that a package weighs five pounds to describe its weight or mass. Feet and pounds are examples of units of measure.

The system you are probably most familiar with is the U.S. Customary system of measurement. The United States is the only industrialized nation in the world that has not switched to the *metric system*. We will introduce the metric system in Section 7.2.

In order to work with the U.S. Customary system of measurement, we need to know what the units are and how they relate to each other. The table below gives some commonly used units and their equivalents.

NOTE

The U.S. Customary system is also called the English system, though it is not used in England anymore.

U.S. Customary Units of Measure

Length	Weight/Mass
1 foot (ft) = 12 inches (in.)	1 pound (lb) = 16 ounces (oz)
1 yard (yd) = 3 ft	1 ton = 2,000 lb
1 mile (mi) = 5,280 ft	
Volume	**Time**
1 cup (c) = 8 fluid ounces (fl oz)	1 minute (min) = 60 seconds (s)
1 pint (pt) = 2 c	1 hour (h) = 60 min
1 quart (qt) = 2 pt	1 day = 24 h
1 gallon (gal) = 4 qt	1 week = 7 days

> CAUTION

Ounces and fluid ounces are different. An ounce is a measure of weight whereas fluid ounces measure volume.

We included time measures, though most of the world measures time the same way. That is, there is not a separate set of metric measures for time. We did not include *month* because a month can range from 28 to 31 days. We also do not include years. There are 365 days in most years, but every fourth year is called a *leap year* and contains 366 days.

NOTE

There are more subtle time issues. For instance, the year 1900 was not a leap year. You can find information about such details on the Internet.

In addition to the nonstandard units, like months, there are a number of other measures that we did not include. Some of these are application specific, whereas others are older and no longer in common use. For example, *teaspoons* and *tablespoons* are measures of volume commonly used in the kitchen; a *cord* is used to measure the volume of firewood. A *cubit* is an ancient measure of length that is no longer in use. Then there are newer measures such as *bits* and *bytes*, which are used in computer applications.

There are two good ways to use the equivalencies in the table to convert between units of measure. We will show you the *substitution method* first. It is the easier method and should be used when the problem is not too complex or complicated. It works especially well when converting from larger units to smaller units.

Step by Step

Converting Measures with the Substitution Method

Step 1 Rewrite the given measure, separating the number and the unit.
Step 2 Replace the unit of measure with an equivalent measure.
Step 3 Perform the arithmetic and simplify, if necessary.

Example 1 Converting Measures with the Substitution Method

< Objective 1 >

(a) How many feet are in 8 yards?

Begin by writing 8 yd as 8 × (1 yd).

From the table above, we know that 1 yd is equivalent to 3 ft, therefore we can replace 1 yd with 3 ft in the statement without changing the actual length.

$$\begin{aligned} 8 \text{ yd} &= 8 \times (1 \text{ yd}) \\ &= 8 \times (3 \text{ ft}) && \text{Replace 1 yd with its equivalent, in feet.} \\ &= 24 \text{ ft} && \text{Perform the arithmetic: } 8 \times 3 = 24. \end{aligned}$$

There are 24 ft in 8 yd.

(b) Find the number of hours in 4 days.

Again, we separate the number from the units (days) and then substitute in the equivalent number of hours in a day.

$$\begin{aligned} 4 \text{ days} &= 4 \times (1 \text{ day}) \\ &= 4 \times (24 \text{ h}) \\ &= 96 \text{ h} \end{aligned}$$

There are 96 hours in 4 days.

Check Yourself 1

Complete each statement.

(a) 4 ft = ______ in. **(b)** 12 qt = ______ pt
(c) 18 lb = ______ oz **(d)** 6.5 min = ______ s

In an effort not to let our conversion table get too large, we did not include some equivalences that we can compute using two or more steps. Some examples include the number of yards in a mile and the number of seconds in an hour. We can use the substitution method to make these conversions, as well.

Example 2 Using the Substitution Method in Multistep Conversions

(a) Find the number of seconds in 3 hours.

We do not have a direct conversion between hours and seconds in our table. However, we do have hour-minute and minute-second conversions, so we can still solve this problem.

$$\begin{aligned} 3 \text{ h} &= 3 \times (1 \text{ h}) \\ &= 3 \times (60 \text{ min}) && \text{Replace 1 h with its equivalent, in minutes.} \\ &= 180 \text{ min} && \text{Perform the arithmetic: } 3 \times 60 = 180. \\ &= 180 \times (1 \text{ min}) && \text{Repeat the process to convert minutes to seconds.} \\ &= 180 \times (60 \text{ s}) && \text{Replace 1 min with its equivalent, in seconds.} \\ &= 10{,}800 \text{ s} && \text{Perform the arithmetic: } 180 \times 60 = 10{,}800. \end{aligned}$$

There are 10,800 seconds in 3 hr.

(b) Convert gallons to cups.

$$\begin{aligned} 1 \text{ gal} &= 4 \text{ qt} \\ &= 4 \times (1 \text{ qt}) \\ &= 4 \times (2 \text{ pt}) \\ &= 8 \text{ pt} \\ &= 8 \times (2 \text{ c}) \\ &= 16 \text{ c} \end{aligned}$$

One gallon is equivalent to 16 cups.

RECALL

We defined ratios in Section 5.1.

Check Yourself 2

Convert, as indicated.

(a) Find the number of inches in 5 miles.
(b) Find the number of ounces in 3 tons.
(c) Find the number of seconds in 1 hour.

RECALL

Multiplying a quantity by 1 does not change the amount it describes.

When working with more complicated conversions, or when converting to larger units, it is often easier to use the second method: the *unit-ratio method*. The idea is to multiply the given measurement by a ratio equal to one. We choose the ratio that includes the unit of measure that we want in the numerator and the unit we want to "cancel" in the denominator. We then include the appropriate numbers so that the unit-ratio factor is equal to one.

Step by Step

Converting Measures with the Unit-Ratio Method

Step 1 Construct a ratio equal to one in which the numerator contains the desired unit of measure, or an intermediary measure, and the denominator contains the given unit of measure.

Step 2 Multiply the given measure by the unit measure and simplify.

Step 3 If necessary, repeat the process until the final unit is the one you want.

Example 3 Converting Measures with the Unit-Ratio Method

(a) Find the number of yards in 4 miles.

We want to convert miles to yards but our table does not have a direct conversion. We can work through feet (miles to feet to yards), but then we would be going from a smaller unit, feet, to a larger unit, yards. The unit-ratio method is the better method in this case.

First, we convert miles to feet. To do so, we construct a unit ratio with feet in the numerator and miles in the denominator.

$$\frac{5{,}280 \text{ ft}}{1 \text{ mi}} = 1$$

Because there are 5,280 feet in 1 mile, this ratio is equal to 1. Therefore, the distance described by 4 miles is unchanged if we multiply it by the ratio we just constructed.

NOTES

We are converting to feet so it goes in the numerator; we are converting from miles, so it goes in the denominator.

If it helps, you can think of 4 mi as $\frac{4 \text{ mi}}{1}$.

$$\begin{aligned} 4 \text{ mi} &= 4 \text{ mi} \times \frac{5{,}280 \text{ ft}}{1 \text{ mi}} \\ &= 4 \cancel{\text{mi}} \times \frac{5{,}280 \text{ ft}}{1 \cancel{\text{mi}}} \\ &= 4 \times 5{,}280 \text{ ft} \\ &= 21{,}120 \text{ ft} \end{aligned}$$

The unit ratio allows us to "cancel" miles?

To convert to yards, we construct a unit ratio with yards in the numerator and feet in the denominator.

$$\begin{aligned} 21{,}120 \text{ ft} &= 21{,}120 \text{ } \cancel{\text{ft}} \times \frac{1 \text{ yd}}{3 \text{ } \cancel{\text{ft}}} \\ &= \frac{21{,}120}{3} \text{ yd} \qquad \text{In this case, we have to divide.} \\ &= 7{,}040 \text{ yd} \end{aligned}$$

There are 7,040 yd in 4 mi.

(b) Find the number of minutes in 45 seconds.

We use the unit ratio that contains minutes in the numerator and seconds in the denominator.

$$\begin{aligned} 45 \text{ s} &= 45 \text{ } \cancel{\text{s}} \times \frac{1 \text{ min}}{60 \text{ } \cancel{\text{s}}} \\ &= \frac{45}{60} \text{ min} \\ &= \frac{3}{4} \text{ min} \end{aligned}$$

45 seconds are three-quarters of a minute.

Check Yourself 3

Convert, as indicated.

(a) Find the number of yards in 1 mile.
(b) Find the number of hours in 240 s.
(c) Find the number of feet in 8 in.
(d) Find the number of tons in 5,000 oz.

RECALL

You should always consider whether your answer to a problem is reasonable.

Here is a good way of determining whether your answer to a conversion problem is reasonable.

- If you are converting to larger units, your number should get smaller.
- If you are converting to smaller units, your number should get larger.

This is because it takes fewer of the larger unit to measure the same amount.

Consider Example 3 (a), above. We began with 4 miles and converted it to yards. Because yards are smaller than miles, we needed many more than 4 of them to measure a 4-mile distance, so our answer needs to be larger than 4. In this case, our answer was 4 mi = 7,040 yd. Since 7,040 is larger than 4 and yards are smaller than miles, our answer passes this test of reasonableness.

From our work so far, it should be clear that one big disadvantage of the U.S. Customary system is that the relationships between units are all different. One foot is 12 in., 1 lb is 16 oz, and so on. We will see in Sections 7.2 and 7.3 that this problem does not exist in the metric system.

NOTE

Historically, units were associated with various things. A foot was the length of a foot, of course. The yard was the distance from the end of a nose to the fingertips of an outstretched arm. Objects were weighed by comparing them with grains of barley.

In our units analysis features, we discussed the difference between denominate numbers (those with units attached) and abstract numbers.

A denominate number may involve two or more different units. We regularly combine feet and inches, pounds and ounces, and so on. The measures 5 lb 6 oz and 4 ft 7 in. are examples. When simplifying a denominate number with multiple units, the *largest unit should include as much of the measure as possible.* For example, 3 ft 2 in. is simplified, whereas 2 ft 14 in. is not.

Example 4 shows the steps used to simplify a denominate number with multiple units.

Example 4 Simplifying Denominate Numbers

< Objective 2 >

NOTE

18 in. is larger than 1 ft so it can be simplified.

(a) Simplify 4 ft 18 in.

$$4\text{ ft }18\text{ in.} = \underbrace{4\text{ ft} + 1\text{ ft}}_{\swarrow} + 6\text{ in.}, \quad \overbrace{1\text{ ft} + 6\text{ in.}}^{18\text{ in.}}$$

Write 18 in. as 1 ft 6 in. because 12 in. is 1 ft.

$$= 5\text{ ft }6\text{ in.}$$

(b) Simplify 5 h 75 min.

$$5\text{ h }75\text{ min} = \underbrace{5\text{ h} + 1\text{ h}}_{\swarrow} + 15\text{ min}, \quad \overbrace{1\text{ h} + 15\text{ min}}^{75\text{ min}}$$

Write 75 min as 1 h 15 min because 1 h is 60 min.

$$= 6\text{ h }15\text{ min}$$

Check Yourself 4

(a) **Simplify 5 lb 24 oz.** **(b)** **Simplify 7 ft 20 in.**

Denominate numbers with the same units are called *like numbers*. We can always add or subtract denominate numbers according to the following rule.

Step by Step

Adding Denominate Numbers

Step 1 Arrange the numbers so that the like units are in the same vertical column.

Step 2 Add in each column.

Step 3 Simplify if necessary.

Example 5 illustrates this rule for adding denominate numbers.

Example 5 Adding Denominate Numbers

< Objective 3 >

NOTES

The columns here represent inches and feet.

Be sure to simplify the results.

Add 5 ft 4 in., 6 ft 7 in., and 7 ft 9 in.

$$\begin{array}{rrl} & 5\text{ ft} & 4\text{ in.} \\ & 6\text{ ft} & 7\text{ in.} \\ + & 7\text{ ft} & 9\text{ in.} \\ \hline & 18\text{ ft} & 20\text{ in.} \\ = & 19\text{ ft} & 8\text{ in.} \end{array}$$

Arrange in a vertical column.

Add in each column.

Simplify as before.

Check Yourself 5

Add 3 h 15 min, 5 h 50 min, and 2 h 40 min.

To subtract denominate numbers, we have a similar rule.

Step by Step

Subtracting Denominate Numbers

Step 1 Arrange the numbers so that the like units are in the same vertical column.

Step 2 Subtract in each column. You may have to borrow from the larger unit at this point.

Step 3 Simplify if necessary.

Consider the following example of subtracting denominate numbers.

Example 6 Subtracting Denominate Numbers

Subtract 3 lb 6 oz from 8 lb 13 oz.

$$\begin{array}{rrr} & 8\text{ lb} & 13\text{ oz} \\ - & 3\text{ lb} & 6\text{ oz} \\ \hline & 5\text{ lb} & 7\text{ oz} \end{array}$$

Arrange vertically.

Subtract in each column.

Check Yourself 6

Subtract 5 ft 9 in. from 10 ft 11 in.

As step 2 points out, subtracting denominate numbers may involve borrowing.

Example 7 Subtracting Denominate Numbers

NOTES

Borrowing with denominate numbers is not the same as in the place-value system, in which we always borrow a power of 10.

9 ft becomes 8 ft 12 in. Combine the 12 in. with the original 3 in.

Subtract 5 ft 8 in. from 9 ft 3 in.

$$\begin{array}{rr} & 9\text{ ft }3\text{ in.} \\ - & 5\text{ ft }8\text{ in.} \\ \hline \end{array}$$

Do you see the problem? We cannot subtract in the inches column.

To complete the subtraction, we borrow 1 ft and rename. The "borrowed" number will depend on the units involved.

$$\begin{array}{rr} & 9\text{ ft }3\text{ in.} \\ - & 5\text{ ft }8\text{ in.} \\ \hline \end{array} \longrightarrow \begin{array}{rrr} & 8\text{ ft} & 15\text{ in.} \\ - & 5\text{ ft} & 8\text{ in.} \\ \hline & 3\text{ ft} & 7\text{ in.} \end{array}$$

Check Yourself 7

Subtract 3 lb 9 oz from 8 lb 5 oz.

Certain types of problems involve multiplying or dividing denominate numbers by abstract numbers, that is, numbers without a unit of measure attached. The following rule is used.

Step by Step

Multiplying or Dividing by Abstract Numbers

Step 1 Multiply or divide each part of the denominate number by the abstract number.

Step 2 Simplify if necessary.

The following examples illustrate this procedure.

Example 8 Multiplying Denominate Numbers

(a) Multiply 4×5 in.

4×5 in. $= 20$ in. or 1 ft 8 in.

(b) Multiply $3 \times (2$ ft 7 in.).

NOTE

Multiply each part of the denominate number by 3.

$$\begin{array}{lrr} & 2\text{ ft} & 7\text{ in.} \\ \times & & 3 \\ \hline & 6\text{ ft} & 21\text{ in.} \end{array}$$

Simplify. The product is 7 ft 9 in.

Check Yourself 8

Multiply 5 lb 8 oz by 4.

Division is illustrated in Example 9.

Example 9 Dividing Denominate Numbers

NOTE

Divide each part of the denominate number by 4.

Divide 8 lb 12 oz by 4.

$$\frac{8 \text{ lb } 12 \text{ oz}}{4} = 2 \text{ lb } 3 \text{ oz}$$

Check Yourself 9

Divide 9 ft 6 in. by 3.

We encounter the need to make such calculations in many applications.

Example 10 An Application Involving Denominate Numbers

< Objective 4 >

There were 482 lb 6 oz of steel in stock before a shipment of 219 lb 13 oz arrived. How much steel was in stock after the shipment?

We begin by lining up like numbers.

$$\begin{array}{rr} 482 \text{ lb} & 6 \text{ oz} \\ \underline{219 \text{ lb}} & \underline{13 \text{ oz}} \\ 701 \text{ lb} & 19 \text{ oz} \end{array}$$

Then, since 19 oz = 1 lb 3 oz, we simplify our result to 702 lb 3 oz.

Check Yourself 10

There are 6 gal 1 qt of aluminum sealer on hand. A run of parts will require 1 gal 2 qt of sealer. How much sealer will be left after the run?

RECALL

We studied rates in Section 5.2.

Earlier in this section, we showed that the unit-ratio method is better for complicated conversions. In many applications in science, engineering, and technology, speed is measured in feet per second (fps) rather than in miles per hour. We will need to construct several unit ratios in cases such as these.

Example 11 An Application Using the Unit-Ratio Method

Convert 65 miles per hour to feet per second.

We take the same approach converting miles to feet and hours to seconds. You can do this in one step, but we will use two steps to make it clearer.

RECALL

In Check Yourself 2, you showed that

1 h = 3,600 s,

so

$\frac{1 \text{ h}}{3{,}600 \text{ s}}$ is a unit ratio.

$$\frac{65 \text{ mi}}{1 \text{ h}} = \frac{65 \cancel{\text{mi}}}{1 \text{ h}} \times \frac{5{,}280 \text{ ft}}{1 \cancel{\text{mi}}}$$

We chose to convert miles to feet first.

$$= \frac{65 \times 5{,}280 \text{ ft}}{1 \text{ h}}$$

$65 \cdot 5{,}280 = 343{,}200$

$$= \frac{343{,}200 \text{ ft}}{1 \cancel{\text{h}}} \times \frac{1 \cancel{\text{h}}}{3{,}600 \text{ s}}$$

Now we convert hours to seconds.

$$= \frac{343{,}200}{3{,}600} \frac{\text{ft}}{\text{s}}$$

Perform the division.

$$= \frac{286}{3} \frac{\text{ft}}{\text{s}}$$

$\frac{286}{3} = 95\frac{1}{3}$

65 miles per hour is equivalent to $95\frac{1}{3}$ feet per second.

In the previous conversion, we wanted seconds in the denominator and we wanted to change from hours, so our unit ratio needed to have hours in the numerator so that it would "cancel."

Check Yourself 11

In an 8-h shift, a continuous cutting shear cuts 7,000 yd of steel. Calculate the rate of steel cut in feet per minute.

Check Yourself ANSWERS

1. **(a)** 48; **(b)** 24; **(c)** 288; **(d)** 390 **2.** **(a)** 316,800 in.; **(b)** 96,000 oz; **(c)** 3,600 s

3. **(a)** 1,760 yd; **(b)** 4 h; **(c)** $\frac{2}{3}$ ft; **(d)** 0.15625 tons or $\frac{5}{32}$ tons

4. **(a)** 6 lb 8 oz; **(b)** 8 ft 8 in. **5.** 11 h 45 min **6.** 5 ft 2 in.

7. Rename 8 lb 5 oz as 7 lb 21 oz. Then subtract for the result, 4 lb 12 oz.

8. 20 lb 32 oz, or 22 lb **9.** 3 ft 2 in. **10.** 4 gal 3 qt

11. $\frac{175}{4} \frac{\text{ft}}{\text{min}}$ or $43\frac{3}{4} \frac{\text{ft}}{\text{min}}$

Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 7.1

(a) The United States is the only industrialized nation that has not switched to the ________________ system.

(b) A fluid ounce is a measure of ________________.

(c) A unit ratio is equal to ________________.

(d) Denominate numbers with the same units are called ________________ numbers.

7.1 exercises

< **Objective 1** >

Complete each statement.

1. 8 ft = ________ in. > Videos

2. 9 gal = ________ qt

3. 3 lb = ________ oz > Videos

4. 300 s = ________ min

5. 360 min = ________ h > Videos

6. 5 pt = ________ fl oz

7. 4 days = ________ h

8. 6 h = ________ min

9. 16 qt = ________ gal

10. 11 min = ________ s

11. 10,000 lb = ________ tons

12. 5 mi = ________ ft

13. 30 pt = ________ qt

14. 64 fl oz = ________ pt

15. 64 oz = ________ lb > Videos

16. 540 min = ________ h

17. 7 yd = ________ ft

18. 24 qt = ________ gal

19. 39 ft = ________ yd

20. 192 oz = ________ lb

21. 8 min = ________ s

22. 18 qt = ________ pt

23. 192 h = ________ days

24. 360 h = ________ days

25. 16 qt = ________ pt

26. 7 days = ________ h

27. $7\frac{1}{4}$ h = ________ min

28. 43 pt = ________ qt

29. 56 oz = ________ lb

30. 20 fl oz = ________ pt

31. 225 s = ________ min

32. 44 in. = ________ ft

33. 1.55 lb = ________ oz

34. 4.72 ft = ________ in.

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Name ________________________

Section ________ Date ________

Answers

1. ________ 2. ________

3. ________ 4. ________

5. ________ 6. ________

7. ________ 8. ________

9. ________ 10. ________

11. ________ 12. ________

13. ________ 14. ________

15. ________ 16. ________

17. ________ 18. ________

19. ________ 20. ________

21. ________ 22. ________

23. ________ 24. ________

25. ________ 26. ________

27. ________ 28. ________

29. ________ 30. ________

31. ________ 32. ________

33. ________ 34. ________

Answers

35. ________ 36. ________

37. ________ 38. ________

39. ________ 40. ________

41. ________ 42. ________

43. ________ 44. ________

45. ________

46. ________

47. ________ 48. ________

49. ________ 50. ________

51. ________ 52. ________

53. ________

54. ________ 55. ________

56. ________ 57. ________

58. ________ 59. ________

60. ________ 61. ________

62. ________ 63. ________

64. ________

65. ________ 66. ________

67. ________ 68. ________

69. ________ 70. ________

35. 40 in. = _______ yd

36. 12 gal = _______ c

37. 16 gal = _______ fl oz

38. 16 fl oz = _______ gal

39. 1 ton = _______ oz

40. 4 mi = _______ in.

41. 6 weeks = _______ min

42. 15 s = _______ days

< Objective 2 >

Simplify.

43. 4 ft 18 in.

44. 6 lb 20 oz

45. 7 qt 5 pt

46. 7 yd 50 in.

47. 5 gal 9 qt

48. 3 min 110 s

49. 9 min 75 s

50. 9 h 80 min

< Objective 3 >

Add.

51. $\begin{array}{r} 8\text{ lb}\ \ 7\text{ oz} \\ \underline{+\ 6\text{ lb}\ 15\text{ oz}} \end{array}$

> Videos

52. $\begin{array}{r} 9\text{ ft}\ \ 7\text{ in.} \\ \underline{+\ 3\text{ ft}\ 10\text{ in.}} \end{array}$

53. $\begin{array}{r} 3\text{ h}\ 20\text{ min} \\ 4\text{ h}\ 25\text{ min} \\ \underline{+\ 5\text{ h}\ 35\text{ min}} \end{array}$

54. $\begin{array}{l} \ \ 5\text{ yd}\ 2\text{ ft} \\ \ \ 4\text{ yd} \\ \underline{+\ 6\text{ yd}\ 1\text{ ft}} \end{array}$

55. 4 lb 7 oz, 3 lb 11 oz, and 5 lb 8 oz

56. 7 ft 8 in., 8 ft 5 in., and 9 ft 7 in.

Subtract.

57. $\begin{array}{r} 9\text{ lb}\ 15\text{ oz} \\ \underline{-\ 5\text{ lb}\ \ 8\text{ oz}} \end{array}$

58. $\begin{array}{r} 7\text{ ft}\ 11\text{ in.} \\ \underline{-\ 4\text{ ft}\ \ 3\text{ in.}} \end{array}$

59. $\begin{array}{r} 6\text{ h}\ 30\text{ min} \\ \underline{-\ 3\text{ h}\ 50\text{ min}} \end{array}$

60. $\begin{array}{r} 7\text{ gal}\ 3\text{ qt} \\ \underline{-\ 1\text{ gal}\ 3\text{ qt}} \end{array}$

61. Subtract 2 yd 2 ft from 5 yd 1 ft.

62. Subtract 2 h 30 min from 7 h 25 min.

Multiply.

63. 4×13 oz

64. 4×10 in.

65. $3 \times$ (4 ft 5 in.)

66. $5 \times$ (4 min 20 s)

Divide.

67. $\dfrac{4\text{ ft } 6\text{ in.}}{2}$

68. $\dfrac{12\text{ lb } 15\text{ oz}}{3}$

69. $\dfrac{16\text{ min } 28\text{ s}}{4}$

70. $\dfrac{25\text{ h } 40\text{ min}}{5}$

< Objective 4 >

Solve each application.

71. **Science and Medicine** The United States emitted approximately 9 million tons of suspended particulates into the atmosphere in 1 year. How many pounds of (suspended) particulates did the United States emit that year?

72. **Science and Medicine** The United States emitted approximately 20 million tons of volatile organic compounds into the atmosphere in 1 year. How many pounds of volatile organic compounds did the United States emit that year?

73. **Construction** A railing for a deck requires pieces of cedar 4 ft 8 in., 11 ft 7 in., and 9 ft 3 in. long. What is the total length of material that is needed?

74. **Business and Finance** Ted worked 3 h 45 min on Monday, 5 h 30 min on Wednesday, and 4 h 15 min on Friday. How many hours did he work during the week?

75. **Crafts** A pattern requires a 2-ft 10-in. length of fabric. If a 2-yd length is used, what length remains?

76. **Crafts** You use 2 lb 8 oz of hamburger from a package that weighs 4 lb 5 oz. How much is left over?

77. **Crafts** A picture frame is to be 2 ft 6 in. long and 1 ft 8 in. wide. A 9-ft piece of molding is available for the frame. Will this be enough for the frame?

78. **Construction** A plumber needs two pieces of plastic pipe that are 6 ft 9 in. long and one piece that is 2 ft 11 in. long. He has a 16-ft piece of pipe. Is this enough for the job?

79. **Crafts** Mark uses 1 pt 9 fl oz and then 2 pt 10 fl oz from a container of film developer that holds 3 qt. How much of the developer remains?

80. **Business and Finance** Some flights limit passengers to 44 lb of checked-in luggage. Susan checks three pieces, weighing 20 lb 5 oz, 7 lb 8 oz, and 15 lb 7 oz. By how much was she under or over the limit?

81. **Business and Finance** Six packages weighing 2 lb 9 oz each are to be mailed. What is the total weight of the packages?

82. **Construction** A bookshelf requires four boards 3 ft 8 in. long and two boards 2 ft 10 in. long. How much lumber will be needed for the bookshelf?

83. **Business and Finance** You can buy three 12-oz cans of peanuts for $3 or one large can containing 2 lb 8 oz for the same price. Which is the better buy?

84. **Statistics and Mathematics** Rich, Susan, and Marc agree to share the driving on a 12-hour (12-h) trip. Rich has driven for 4 h 45 min, and Susan has driven for 3 h 30 min. How long must Marc drive to complete the trip?

Answers

71. ____________
72. ____________
73. ____________
74. ____________
75. ____________
76. ____________
77. ____________
78. ____________
79. ____________
80. ____________
81. ____________
82. ____________
83. ____________
84. ____________

Answers

85. ____

86. ____

87. ____

88. ____

89. ____

90. ____

91. ____

92. ____

93. ____

94. ____

95. ____

96. ____

97. ____

98. ____

99. ____

100. ____

101. ____

Basic Skills | **Challenge Yourself** | Calculator/Computer | Career Applications | Above and Beyond

Determine whether each statement is **true** *or* **false.**

85. A denominate number may have more than two units of measure attached.

86. It is impossible to divide a denominate number by an abstract number.

In each statement, fill in the blank with **always, sometimes,** *or* **never.**

87. We can __________ add or subtract denominate numbers if we arrange the numbers so that the like units are in the same vertical column.

88. Subtraction of denominate numbers __________ requires borrowing.

Evaluate and simplify.

89.
$$\begin{array}{r} 2\text{ gal } 3\text{ qt } 1\text{ pt} \\ +\ 3\text{ gal } 2\text{ qt } 1\text{ pt} \\ \hline \end{array}$$

90.
$$\begin{array}{r} 7\text{ weeks } 3\text{ days } 15\text{ hours} \\ +\ 3\text{ weeks } 9\text{ days } 10\text{ hours} \\ \hline \end{array}$$

91.
$$\begin{array}{r} 13\text{ yd } 15\text{ ft } 10\text{ in.} \\ -\ \ 9\text{ yd } 16\text{ ft } 15\text{ in.} \\ \hline \end{array}$$

92.
$$\begin{array}{r} 8\text{ gal } 3\text{ qt } 2\text{ pt} \\ -\ 5\text{ gal } 5\text{ qt } 3\text{ pt} \\ \hline \end{array}$$

93.
$$\begin{array}{r} 2\text{ weeks } 7\text{ days } 18\text{ h } 40\text{ min} \\ \times\qquad\qquad\qquad\qquad 2 \\ \hline \end{array}$$

94.
$$\begin{array}{r} 4\text{ gal } 5\text{ qt } 3\text{ pt } 10\text{ oz} \\ \times\qquad\qquad\qquad 2 \\ \hline \end{array}$$

Solve each application.

95. **Business and Finance** A half-gallon bottle of organic milk sells for $3.29. Find the cost per fluid ounce (to the nearest cent).

96. **Statistics** Colette's car has an average fuel efficiency of 30 mi/gal. How many yards can Colette travel on 1 fluid ounce of fuel?

97. **Statistics** In exercise 96, how many fluid ounces of fuel does Colette need in order to travel 1 yard?

98. **Construction** 220 gallons of water flow through a pipe every 3 hours. Find the flow rate, in fluid ounces per minute (round your result to the nearest whole fluid ounce).

99. **Statistics** A driver travels at a rate of 55 mi/h. How many seconds does it take the driver to travel 1,000 ft (to the nearest tenth of a second)?

100. **Science and Medicine** A greyhound can run at an average rate of 37 mi/h for $\frac{5}{16}$ of a mile. How long does it take the greyhound to run $\frac{5}{16}$ of a mile?

Basic Skills | Challenge Yourself | Calculator/Computer | **Career Applications** | Above and Beyond

101. **Mechanical Engineering** A piece of steel that is 13 ft 8 in long is to be sheared into four equal pieces. How long will each piece be?

102. **MANUFACTURING TECHNOLOGY** A part weighs 4 lb 11 oz. How much does a lot of 24 parts weigh?

103. **AUTOMOTIVE TECHNOLOGY** An engine block weighs 218 lb 12 oz. Each head weighs 36 lb 3 oz. There are two heads on the engine. What is the total weight of the block with the two heads?

104. **AUTOMOTIVE TECHNOLOGY** Each piston in an engine weighs 4 lb 13 oz. How much do the eight pistons add to the weight of an engine?

105. **MANUFACTURING TECHNOLOGY** A knee wall is to be constructed from 2 by 4 studs that are 4 ft 9 in. long. If the wall will use 13 studs, what is the total length of 2 by 4 required for the studs?

106. **ALLIED HEALTH** A premature, newborn baby boy weighs 98 oz. Determine his weight in pounds and ounces.

Basic Skills | Challenge Yourself | Calculator/Computer | Career Applications | **Above and Beyond**

107. **SCIENCE AND MEDICINE** On average, a human's heart beats at a rate of 72 beats per minute.

(a) Find the age at which the average person's heart has beaten 1 billion times (to the nearest whole day).

(b) Find the age at which the average person's heart has beaten 1 billion times (to the nearest whole year).

108. **SCIENCE AND MEDICINE** Each human heartbeat pumps about 1 fl oz of blood. How many gallons of blood does the human heart pump in a day? (See exercise 107.)

109. **SCIENCE AND MEDICINE**

(a) John is traveling at a speed of 60 mi/h. What is his speed in feet per second?

(b) Use the information in part **(a)** to develop a method to convert any speed from miles per hour to feet per second.

110. Refer to several sources and write a brief history of how the units that are currently used in the U.S. Customary system of measurement originated. Discuss some units that were previously used but are no longer in use today.

111. A unit of measurement used in surveying is the **chain.** There are 80 chains in a mile. If you measured the distance from your home to school, how many chains would you have traveled?

112. The average person takes about 17 breaths per minute. How many breaths have you taken in your lifetime?

Answers

102. ____________

103. ____________

104. ____________

105. ____________

106. ____________

107. ____________

108. ____________

109. ____________

110. ____________

111. ____________

112. ____________

Answers

113. ____________________

114. ____________________

113. **Science and Medicine** The average breath takes in about $1\frac{1}{2}$ pints of air. The air is about 20% oxygen. Of the oxygen that we breathe, about 25% makes its way into our bloodstream. How much oxygen does the average person take in every day? (See Exercise 112.)

114. What is your age in seconds? (Remember to consider leap years!)

Answers

1. 96 **3.** 48 **5.** 6 **7.** 96 **9.** 4 **11.** 5 **13.** 15
15. 4 **17.** 21 **19.** 13 **21.** 480 **23.** 8 **25.** 32
27. 435 **29.** 3.5 **31.** 3.75 **33.** 24.8 **35.** $\frac{10}{9}$ **37.** 2,048
39. 32,000 **41.** 60,480 **43.** 5 ft 6 in. **45.** 9 qt 1 pt **47.** 7 gal 1 qt
49. 10 min 15 s **51.** 15 lb 6 oz **53.** 13 h 20 min **55.** 13 lb 10 oz
57. 4 lb 7 oz **59.** 2 h 40 min **61.** 2 yd 2 ft **63.** 3 lb 4 oz
65. 13 ft 3 in. **67.** 2 ft 3 in. **69.** 4 min 7 s **71.** 18 billion lb
73. 25 ft 6 in. **75.** 3 ft 2 in. **77.** Yes, 8 in. will remain **79.** 1 pt 13 fl oz
81. 15 lb 6 oz **83.** The 2-lb 8-oz can **85.** True **87.** always
89. 6 gal 2 qt **91.** 3 yd 1 ft 7 in. **93.** 6 weeks 1 day 13 h 20 min
95. 5 ¢/fl oz **97.** $\frac{2}{825}$ fl oz/yd **99.** 12.4 s **101.** 3 ft 5 in.
103. 291 lb 2 oz **105.** 61 ft 9 in. **107.** **(a)** 9,645 days; **(b)** 26 years
109. Above and Beyond **111.** Above and Beyond **113.** 229.5 gal

7.2

Metric Units of Length

< 7.2 Objectives >

1 > Estimate metric units of length

2 > Convert between metric units of length

In Section 7.1 we studied the U.S. Customary system of measurement, which is used in the United States and a few other countries. We now concentrate on the **metric system,** used throughout the rest of the world.

The metric system is based on one unit of length, the **meter (m).** In the eighteenth century the meter was defined to be one ten-millionth of the distance from the north pole to the equator. Today the meter is scientifically defined in terms of the speed of light.

One big advantage of the metric system is that you can convert from one unit to another by simply multiplying or dividing by powers of 10. This advantage and the need for uniformity throughout the world have led to legislation that promotes the use of the metric system in the United States.

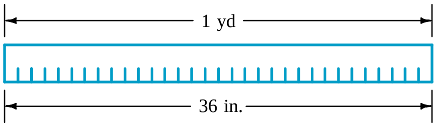
To see how the metric system works, we will start with measures of length and compare a basic U.S. Customary unit, the yard, with the meter.

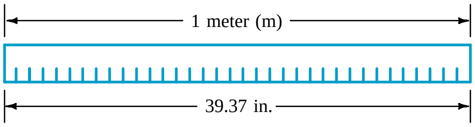
NOTES

Even in the United States, the metric system is used in science, medicine, the automotive industry, the food industry, and many other areas.

The basic unit of length in the metric system is also spelled *metre* (the British spelling).

The meter is one of the basic units of the International System of Units (abbreviated SI). This is a standardization of the metric system agreed to by scientists in 1960.





NOTE

There is a standard pattern of abbreviation in the metric system. We will introduce the abbreviation for each term as we go along. The abbreviation for meter is m (no period!).

As you can see, the meter is just slightly longer than the yard. It is used for measuring the same things you might measure in feet or yards. Look at Example 1 to get a feel for the size of the meter.

Example 1 Estimating Metric Length

< Objective 1 >

A room might be 6 meters (6 m) long.

A building lot could be 30 m wide.

A fence is 2 m tall.

Check Yourself 1

Try to estimate the following lengths in meters.

(a) A traffic lane is __________ m wide.
(b) A small car is __________ m long.
(c) You are __________ m tall.

For other units of length, the meter is multiplied or divided by powers of 10. One commonly used unit is the **centimeter (cm).**

Definition

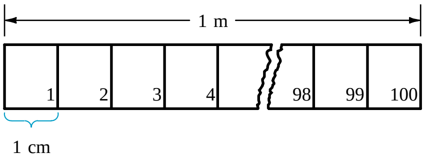
Comparing Centimeters (cm) to Meters (m)

1 centimeter (cm) = $\frac{1}{100}$ meter (m)

NOTE

The prefix *centi* means one–hundredth. This should be no surprise. What is our cent? It is one hundredth of a dollar.

The following drawing relates the centimeter and the meter:



There are 100 cm in 1 m.

Just to give you an idea of the size of the centimeter, it is about the width of your little finger. There are about $2\frac{1}{2}$ cm to 1 in., and the unit is used to measure small objects. Look at Example 2 to get a feel for the length of a centimeter.

Example 2 Estimating Metric Length

A small paperback book is 10 cm wide.

A playing card is 8 cm long.

A ballpoint pen is 16 cm long.

Check Yourself 2

Try to estimate each of the following. Then use a metric ruler to check your guess.

(a) This page is __________ cm long.
(b) A dollar bill is __________ cm long.
(c) The seat of the chair you are on is __________ cm from the floor.

To measure *very* small things, the **millimeter (mm)** is used. To give you an idea of its size, a millimeter is about the thickness of a new dime.

Definition

Comparing Millimeters (mm) to Meters (m)

1 millimeter (mm) = $\frac{1}{1,000}$ m

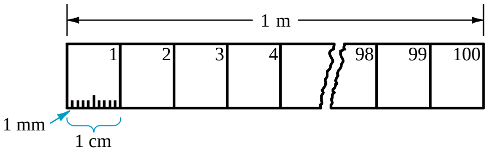
NOTES

The prefix *milli* means one–thousandth.

There are 10 mm to 1 cm.

The following diagram will help you see the relationships of the three units we have looked at.

To get used to the millimeter, consider Example 3.



Example 3 Estimating Metric Length

Standard camera film is 35 mm wide.

A small paper clip is 7 mm wide.

A water glass might be 2 mm thick.

Check Yourself 3

Try to estimate each of the following. Then use a metric ruler to check your guess.

(a) **Your pencil is __________ mm wide.**
(b) **The tabletop you are working on is __________ mm thick.**

The **kilometer (km)** is used to measure long distances. The kilometer is about $\frac{6}{10}$ of a mile.

NOTE

The prefix *kilo* means 1,000. You are already familiar with this. For instance, 1 kilowatt (kW) = 1,000 watts (W).

Definition

Comparing Kilometers (km) to Meters (m)

1 kilometer (km) = 1,000 m

Example 4 uses kilometers.

Example 4 Estimating Metric Length

The distance from New York to Boston is 338 km.

A popular distance for road races is 10 km.

Now that you have seen the four commonly used units of length in the metric system, you can review with the following Check Yourself exercise.

Check Yourself 4

Choose the most reasonable measure in each of the following statements.

(a) **The width of a doorway: 50 mm, 1 m, or 50 cm.**
(b) **The length of your pencil: 20 m, 20 mm, or 20 cm.**
(c) **The distance from your house to school: 500 km, 5 km, or 50 m.**
(d) **The height of a basketball center: 2.2 m, 22 m, or 22 cm.**

As we said earlier, to convert units of measure within the metric system, we multiply or divide by the appropriate power of 10. To accomplish this, we move the decimal point to the right or left the required number of places. This is the big advantage of the metric system.

Property

Converting Metric Measurements to Smaller Units

To convert to a *smaller* unit of measure, we *multiply* by a power of 10, moving the decimal point *to the right.*

Example 5 Converting Metric Length

< Objective 2 >

5.2 m = 520 cm   The *smaller* the unit, the *more* units it takes, so we multiply by 100 to convert from meters to centimeters.

8 km = 8,000 m   Multiply by 1,000.

6.5 m = 6,500 mm   Multiply by 1,000.

2.5 cm = 25 mm   Multiply by 10.

Check Yourself 5

Complete the following by moving the decimal point the appropriate number of places.

(a) 3 km = __________ m **(b)** 4.5 m = __________ cm

(c) 1.2 m = __________ mm **(d)** 6.5 cm = __________ mm

Property

Converting Metric Measurements to Larger Units

To convert to a *larger* unit of measure, we *divide* by a power of 10, moving the decimal point *to the left.*

Example 6 Converting Metric Length

NOTE

The *larger* the unit, the *fewer* units it takes, so *divide.*

$43 \text{ mm} = 4.3 \text{ cm}$ Divide by 10.

$3{,}000 \text{ m} = 3 \text{ km}$ Divide by 1,000.

$450 \text{ cm} = 4.5 \text{ m}$ Divide by 100.

Check Yourself 6

Complete the following statements.

(a) 750 cm = __________ m **(b)** 5,000 m = __________ km

(c) 78 mm = __________ cm **(d)** 3,500 mm = __________ m

We have introduced all the commonly used units of linear measure in the metric system. There are other prefixes that can be used to form other linear measures. The prefix *deci* means $\frac{1}{10}$, *deka* means 10, and *hecto* means 100. Their use is illustrated in the following table.

Definition

Using Metric Prefixes

1 *kilo*meter (km) = 1,000 m

1 *hecto*meter (hm) = 100 m

1 *deka*meter (dam) = 10 m

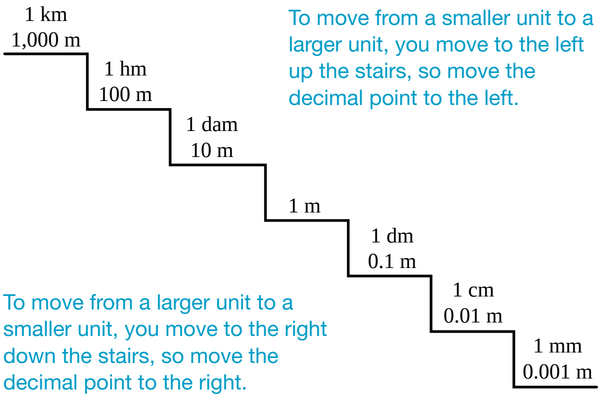
1 meter (m)

1 *deci*meter (dm) = $\frac{1}{10}$ m

1 *centi*meter (cm) = $\frac{1}{100}$ m

1 *milli*meter (mm) = $\frac{1}{1{,}000}$ m

You may find the following chart helpful when converting between metric units. Think of the chart as a set of stairs. Note that the largest unit is at the highest step on the stairs.



To move from a smaller unit to a larger unit, you move to the left up the stairs, so move the decimal point to the left.

To move from a larger unit to a smaller unit, you move to the right down the stairs, so move the decimal point to the right.

Example 7 Converting Between Metric Lengths

(a) 800 cm = ? m

To convert from centimeters to meters, you can see from the chart that you must move the decimal point *two places to the left.*

$800 \text{ cm} = 8_{\wedge}00 \text{ m} = 8 \text{ m}$

(b) 500 m = ? km

To convert from meters to kilometers, move the decimal point *three places to the left.*

$500 \text{ m} = {}_{\wedge}500 \text{ km} = 0.5 \text{ km}$

(c) 6 m = ? mm

To convert from meters to millimeters, move the decimal point *three places to the right.*

$6 \text{ m} = 6000_{\wedge} \text{ mm} = 6{,}000 \text{ mm}$

RECALL

When converting, if the unit gets larger, the amount must get smaller; if the unit gets smaller, the amount must get larger.

Check Yourself 7

Complete each statement.

(a) 300 cm = __________ m
(b) 370 mm = __________ m
(c) 4,500 m = __________ km

Example 8 A Manufacturing Application

A piece of steel that is 6 m long has a piece 372 cm long removed. How much is left? Express your answer in meters.

First, we convert the length 372 cm to meters. Since we move the decimal point two places to the left, we see

372 cm = 3.72 m

Now subtract:

$$\begin{array}{r} 6.00 \text{ m} \\ -3.72 \text{ m} \\ \hline 2.28 \text{ m} \end{array}$$

So the remaining piece of steel has length 2.28 m.

Check Yourself 8

A 2-m piece of wire is to be cut into five equal pieces. How long in centimeters will each piece be?

Check Yourself ANSWERS

1. **(a)** About 3 m; **(b)** perhaps 5 m; **(c)** you are probably between 1.5 and 2 m tall.
2. **(a)** About 28 cm; **(b)** almost 16 cm; **(c)** about 45 cm
3. **(a)** About 8 mm; **(b)** probably between 25 and 30 mm
4. **(a)** 1 m; **(b)** 20 cm; **(c)** 5 km; **(d)** 2.2 m
5. **(a)** 3,000 m; **(b)** 450 cm; **(c)** 1,200 mm; **(d)** 65 mm
6. **(a)** 7.5 m; **(b)** 5 km; **(c)** 7.8 cm; **(d)** 3.5 m
7. **(a)** 3 m; **(b)** 0.37 m; **(c)** 4.5 km **8.** 40 cm

Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 7.2

(a) The ____________ system is based on one unit of length, the meter.

(b) A meter is just slightly longer than a ____________.

(c) The prefix ____________ means one–hundredth.

(d) ____________ are used to measure long distances.

7.2 exercises

< **Objective 1** >

Choose the most reasonable measure.

1. The height of a ceiling
(a) 25 m
(b) 2.5 m
(c) 25 cm

> Videos

2. The diameter of a quarter
(a) 24 mm
(b) 2.4 mm
(c) 24 cm

3. The height of a kitchen counter
(a) 9 m
(b) 9 cm
(c) 90 cm

4. The diagonal measure of a television screen
(a) 50 mm
(b) 50 cm
(c) 5 m

5. The height of a two-story building
(a) 7 m
(b) 70 m
(c) 70 cm

6. An hour's drive on a freeway
(a) 9 km
(b) 90 m
(c) 90 km

7. The width of a roll of cellophane tape
(a) 1.27 mm
(b) 12.7 mm
(c) 12.7 cm

8. The width of a sheet of typing paper
(a) 21.6 cm
(b) 21.6 mm
(c) 2.16 cm

9. The thickness of window glass
(a) 5 mm
(b) 5 cm
(c) 50 mm

10. The height of a refrigerator
(a) 16 m
(b) 16 cm
(c) 160 cm

11. The length of a ballpoint pen
(a) 16 mm
(b) 16 m
(c) 16 cm

12. The width of a handheld calculator key
(a) 1.2 mm
(b) 12 mm
(c) 12 cm

Complete each statement, using a metric unit of length.

13. A playing card is 6 ___________ wide.

14. The diameter of a penny is 19 ___________.

15. A doorway is 2 ___________ high.

16. A table knife is 22 ___________ long.

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Name ______________________

Section __________ Date __________

Answers

1. ______________________

2. ______________________

3. ______________________

4. ______________________

5. ______________________

6. ______________________

7. ______________________

8. ______________________

9. ______________________

10. ______________________

11. ______________________

12. ______________________

13. ______________________

14. ______________________

15. ______________________

16. ______________________

Answers

17. ________ 18. ________

19. ________ 20. ________

21. ________ 22. ________

23. ________ 24. ________

25. ________ 26. ________

27. ________ 28. ________

29. ________ 30. ________

31. ________ 32. ________

33. ________

34. ________

35. ________

36. ________

37. ________

38. ________

39. ________

40. ________

41. ________

42. ________

43. ________

44. ________

17. A basketball court is 28 __________ long.

18. A commercial jet flies 800 __________ per hour.

19. The width of a nail file is 12 __________.

20. The distance from New York to Washington, D.C., is 387 __________.

21. A recreation room is 6 __________ long. > Videos

22. A ruler is 22 __________ wide.

23. A long-distance run is 35 __________.

24. A paperback book is 11 __________ wide.

< Objective 2 >

Complete each statement.

25. 3,000 mm = __________ m > Videos

26. 150 cm = __________ m

27. 8 m = __________ cm

28. 77 mm = __________ cm

29. 250 km = __________ cm > Videos

30. 500 cm = __________ m

31. 25 cm = __________ mm

32. 150 mm = __________ m

33. 7,000 m = __________ km > Videos

34. 9 m = __________ cm

35. 8 cm = __________ mm

36. 45 cm = __________ mm

37. 5 km = __________ m

38. 4,000 m = __________ km

39. 5 m = __________ mm

40. 7 km = __________ m

Basic Skills | **Challenge Yourself** | Calculator/Computer | Career Applications | Above and Beyond

Determine whether each statement is **true** *or* **false.**

41. To convert to a larger measure in the metric system, we divide by a power of 10.

42. To convert kilometers to meters, move the decimal point three decimal places to the left.

In each statement, fill in the blank with **always, sometimes,** *or* **never.**

43. When converting units of measure within the metric system, __________ multiply or divide by a power of 10.

44. If we are converting from a smaller unit to a larger unit, we ________ move the decimal point to the right.

Convert, as indicated.

45. 250 cm to m

46. 250 cm to km

47. 12 mm to cm

48. 12 mm to km

49. 3.4 m to mm

50. 3.4 mm to m

51. 132 km to m

52. 132 m to km

53. $90\ \frac{\text{km}}{\text{h}}$ to $\frac{\text{m}}{\text{s}}$

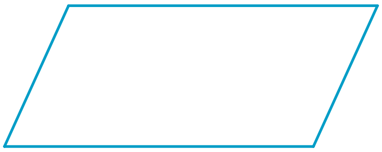
54. $2{,}500\ \frac{\text{m}}{\text{s}}$ to $\frac{\text{km}}{\text{h}}$

55. $800\ \frac{\text{km}}{\text{day}}$ to $\frac{\text{m}}{\text{h}}$

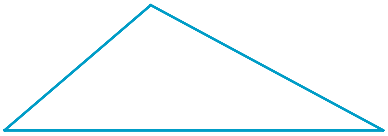
56. $5{,}000\ \frac{\text{m}}{\text{h}}$ to $\frac{\text{km}}{\text{day}}$

Use a metric ruler to measure the necessary dimensions and complete each statement.

57. The perimeter of the parallelogram is ___________ cm.

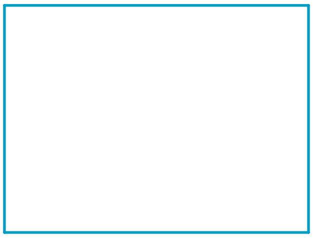


58. The perimeter of the triangle is ___________ mm.



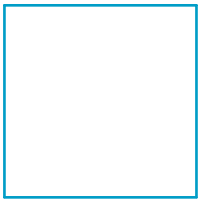
59. The perimeter of the rectangle is ___________ cm.

60. Its area is ___________ cm^2.



61. The perimeter of the square is ___________ mm.

62. The area of the square in exercise 61 is ___________ mm^2.



Basic Skills | Challenge Yourself | Calculator/Computer | **Career Applications** | Above and Beyond

63. **MANUFACTURING TECHNOLOGY** A 2 by 6 plank is 3 m long. If lengths of 86 cm, 9.3 dm, and 29 cm are cut from the plank, how long is the remaining piece? chapter 7 > Make the Connection

64. **MECHANICAL ENGINEERING** A piece of steel stock that is 2.5 m long has lengths of 82 cm, 2.4 dm, and 190 mm cut from it. How long is the remaining portion? chapter 7 > Make the Connection

65. **AGRICULTURE** In a barn, there are 40 stalls in a distance of 51.2 m. How many centimeters wide is each stall?

Answers

45. ___________

46. ___________

47. ___________

48. ___________

49. ___________

50. ___________

51. ___________

52. ___________

53. ___________

54. ___________

55. ___________

56. ___________

57. ___________

58. ___________

59. ___________

60. ___________

61. ___________

62. ___________

63. ___________

64. ___________

65. ___________

Answers

66. ____________

67. ____________

68. ____________

69. ____________

70. ____________

66. **ELECTRONICS** A printed circuit board (PCB) measures 45 mm by 67 mm. What is the area of the board in *square centimeters?*

67. **INFORMATION TECHNOLOGY** Radio waves travel at the speed of light, which is 300,000 km per second. What is the rate in meters per second?

68. **INFORMATION TECHNOLOGY** Satellites are located in geostationary orbit at an altitude of approximately 35,786 km above the equator. What is this distance in meters?

Basic Skills | Challenge Yourself | Calculator/Computer | Career Applications | **Above and Beyond**

69. **(a)** Determine the world record speed for both men and women in meters per second (m/s) for the following events: 100-, 400-, 1,500-, and 5,000-m run. The record times can be found at any one of several websites.

(b) Rank all the speeds obtained in order from fastest to slowest.

70. What units in the metric system would you use to measure each of the following quantities?

(a) Distance from Los Angeles to New York

(b) Your waist measurement

(c) Width of a hair

(d) Your height

Answers

1. (b) **3.** (c) **5.** (a) **7.** (b) **9.** (a) **11.** (c) **13.** cm
15. m **17.** m **19.** mm **21.** m **23.** km **25.** 3 **27.** 800
29. 25,000,000 **31.** 250 **33.** 7 **35.** 80 **37.** 5,000
39. 5,000 **41.** True **43.** always **45.** 2.5 m **47.** 1.2 cm
49. 3,400 mm **51.** 132,000 m **53.** 25 $\frac{m}{s}$ **55.** 33,333 $\frac{1}{3}$ $\frac{m}{h}$
57. 12 **59.** 14 **61.** 100 **63.** 92 cm **65.** 128 cm
67. 300,000,000 m/s **69.** Above and Beyond

7.3

Metric Units of Weight and Volume

< **7.3 Objectives** >

1 > Use appropriate metric units of weight/mass
2 > Convert metric units of weight/mass
3 > Use appropriate metric units of volume
4 > Convert metric units of volume

NOTE

Technically, the gram is a unit of *mass* rather than weight. Weight is the result of the force of gravity on an object. Thus, astronauts *weigh less* on the moon than on earth even though their masses are unchanged. For common use on the Earth, the terms *mass* and *weight* are still used interchangeably.

The basic unit of weight or mass in the metric system is a very small unit called the **gram.** Think of a paper clip. It weighs roughly 1 gram (g). About 28 g make 1 oz in the U.S. Customary system. Grams are most often used to measure items that are fairly light. For heavier items, a more convenient unit of weight is the **kilogram (kg).** From the prefix *kilo* you should be able to deduce that a kilogram is equal to 1,000 grams.

Definition

Comparing Kilograms (kg) to Grams (g)

1 kilogram (kg) = 1,000 grams (g)   (A kilogram is a bit more than 2 lb.)

Example 1 Using Appropriate Units of Metric Weight

< **Objective 1** >

The weight of a box of breakfast cereal is 320 g.

A woman might weigh 50 kg.

A nickel weighs 5 g.

Check Yourself 1

Choose the most reasonable measure.

(a) A penny: 30 g, 3 g, or 3 kg.
(b) A bar of soap: 120 g, 12 g, or 1.2 kg.
(c) A car: 5,000 kg, 1,000 kg, or 5,000 g.

Another metric unit of weight or mass that you will encounter is the **milligram.**

Definition

Comparing Milligrams (mg) to Grams (g)

$$1 \text{ milligram (mg)} = \frac{1}{1{,}000} \text{ g}$$

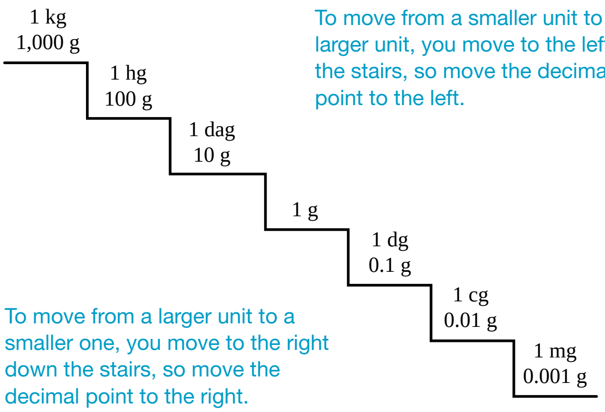
A milligram is an extremely small unit. It is used, for example, in medicine for measuring drug amounts. Thus, a pill might contain 300 mg of aspirin.

Just as with units of length, converting metric units of weight or mass is simply a matter of moving the decimal point. The following chart will help. Again, think of the chart as a set of stairs. Recall that the largest unit is at the highest step on the stairs.

NOTES

The prefix *milli* means one-thousandth.

kg, g, and mg are the units in common use.



To move from a smaller unit to a larger unit, you move to the left up the stairs, so move the decimal point to the left.

To move from a larger unit to a smaller one, you move to the right down the stairs, so move the decimal point to the right.

Example 2 Converting Metric Weight

< Objective 2 >

NOTES

We are converting to a *smaller* unit so the number gets larger.

We are converting to a *larger* unit so the number gets smaller.

Complete the following statements.

(a) 7 kg = ? g

7 kg = 7000 g = 7,000 g

Move the decimal point *three places to the right* (to multiply by 1,000).

(b) 5,000 mg = ? g

5,000 mg = 5 000 g = 5 g

Move the decimal point *three places to the left* (to divide by 1,000).

Check Yourself 2

(a) 3,000 g = ________ kg **(b)** 500 cg = ________ g

For massive objects, we use *metric tons* (t).

Definition

Metric Ton

1 metric ton (t) = 1,000 kg so $\frac{1\text{ t}}{1{,}000\text{ kg}} = 1$ and $\frac{1{,}000\text{ kg}}{1\text{ t}} = 1$

We will use this measure in Example 3.

Example 3 Converting to Metric Tons

Complete the following.

(a) 7,500 kg = ? t

$$7{,}500\text{ kg} = \frac{7{,}500\text{ kg}}{1} \times \frac{1\text{ t}}{1{,}000\text{ kg}} = 7.500\text{ t}$$

Move the decimal point three places to the left to divide by 1,000.

(b) 12.25 t = ? kg

$$12.25\text{ t} = \frac{12.25\text{ t}}{1} \times \frac{1{,}000\text{ kg}}{1\text{ t}}$$

$$= 12{,}250\text{ kg}$$

Move the decimal point three places to the right to multiply by 1,000.

NOTE

In both parts of Example 3, we multiply by a unit ratio (which equals 1).

Check Yourself 3

(a) 13,400 kg = ________ t **(b)** 0.76 t = ________ kg

In the metric system, the basic unit of volume is the **liter (L).** A liter is slightly more than a quart and is used for soft drinks, milk, oil, gasoline, and so on.

The metric unit used to measure smaller volumes is the **milliliter (mL).** From the prefix we know that it is one-thousandth of a liter.

Definition

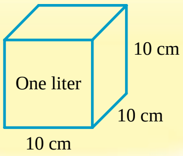
Comparing Liters (L) to Milliliters (mL)

1 liter (L) = 1,000 milliliters (mL)

NOTE

The liter is related to the meter. It is defined as the volume of a cube 10 cm on each edge, so

$1 \text{ L} = 1{,}000 \text{ cm}^3$



NOTE

This unit of volume is also spelled *litre* (the British spelling).

A milliliter contains the volume of a cube 1 cm on each edge. So 1 mL is equal to 1 cm^3. These units can be used interchangeably.

Scientists give measurements of volume in terms of cubic centimeters (cm^3).

Example 4 will help you get used to the metric units of volume. We will explore area and volume more in Chapter 8.

Example 4 Using Appropriate Units of Metric Volume

< Objective 3 >

A teaspoon is about 5 mL or 5 cm^3.

A 6-fl-oz cup of coffee is about 180 mL.

A quart of milk is 946 mL (just less than 1 L).

A gallon is just less than 4 L.

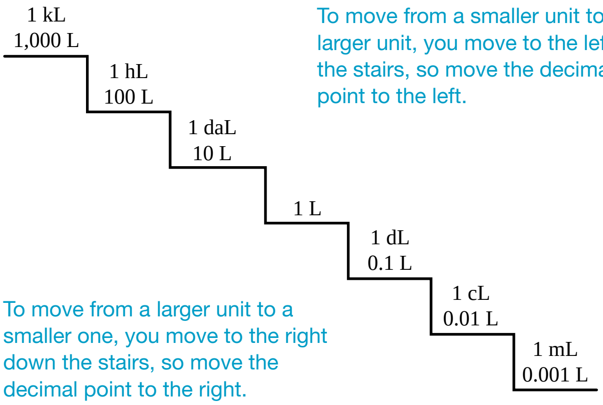
Now try these Check Yourself exercises.

Check Yourself 4

Choose the most reasonable measure.

(a) A can of soup: 3 L, 30 mL, or 300 mL.
(b) A pint of cream: 4.73 L, 473 mL, or 47.3 mL.
(c) A home-heating oil tank: 100 L, 1,000 L, or 1,000 mL.
(d) A tablespoon: 150 mL, 1.5 L, or 15 mL.

Converting metric units of volume is again just a matter of moving the decimal point. A chart similar to the ones you saw earlier may be helpful.



To move from a smaller unit to a larger unit, you move to the left up the stairs, so move the decimal point to the left.

To move from a larger unit to a smaller one, you move to the right down the stairs, so move the decimal point to the right.

NOTE

L, cL, and mL are the most commonly used units. We show the other units simply to indicate that the prefixes and abbreviations are used in a consistent fashion.

Example 5 Converting Metric Volume

< Objective 4 >

Complete the following statements.

(a) 4 L = ? mL

From the previous chart, we see that we should move the decimal point three places to the right (to multiply by 1,000).

4 L = 4 000 mL = 4,000 mL

(b) 3,500 mL = ? L

Move the decimal point three places to the left (to divide by 1,000).

3,500 mL = 3 500 L = 3.5 L

(c) 30 cL = ? mL

Move the decimal point one place to the right (to multiply by 10).

30 cL = 30 0 mL = 300 mL

NOTES

We are converting to a *smaller* unit.

We are converting to a *larger* unit.

We are converting to a *smaller* unit.

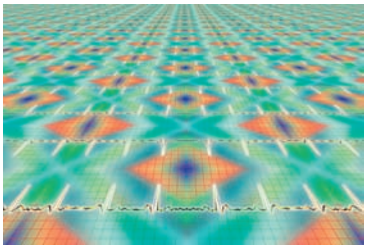
Check Yourself 5

Complete the following statements.

(a) 5 L = _______ mL **(b)** 7,500 mL = _______ L
(c) 550 mL = _______ cL

Consider the following application from the field of health sciences.

Example 6 An Allied Health Application



Cardiac output, measured in liters per minute (L/min), is the product of a patient's stroke volume, in liters per beat, times the heart rate, in beats per minute (beats/min). Determine the cardiac output for a patient with a stroke volume of 45 milliliters per beat (mL/beat) and a heart rate of 80 beats/min.

Since the stroke volume is 45 mL/beat and the patient's heart rate is 80 beats/min, the cardiac output is

$$45\frac{\text{mL}}{\text{beat}} \times 80\frac{\text{beats}}{\text{min}} = 3{,}600\frac{\text{mL}}{\text{min}}$$

Converting 3,600 mL to liters, the cardiac output is 3.6 L/min.

Check Yourself 6

Determine the cardiac output for a patient with a stroke volume of 68 mL/beat and a heart rate of 95 beats/min.

Check Yourself ANSWERS

1. **(a)** 3 g; **(b)** 120 g; **(c)** 1,000 kg **2.** **(a)** 3 kg; **(b)** 5 g
3. **(a)** 13.4 t; **(b)** 760 kg **4.** **(a)** 300 mL; **(b)** 473 mL; **(c)** 1,000 L; **(d)** 15 mL
5. **(a)** 5,000 mL; **(b)** 7.5 L; **(c)** 55 cL **6.** 6.46 L/min

Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 7.3

(a) The basic unit of weight or mass in the metric system is called the ___________.

(b) The ______________ is a small unit of weight that is used, for example, in measuring drug amounts.

(c) In the metric system, the basic unit of volume is the ______________.

(d) A ______________ contains the volume of a cube 1 cm on each edge.

7.3 exercises

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Name ____________________

Section ________ Date ________

Answers

1. ____________________

2. ____________________

3. ____________________

4. ____________________

5. ____________________

6. ____________________

7. ____________________

8. ____________________

9. ____________________

10. ____________________

11. ____________________

12. __________ 13. __________

14. __________ 15. __________

16. __________ 17. __________

< Objective 1 >

Choose the most reasonable measure of weight.

1. A nickel > Videos
(a) 5 kg
(b) 5 g
(c) 50 g

2. A portable television set
(a) 8 g
(b) 8 kg
(c) 80 kg

3. A flashlight battery
(a) 8 g
(b) 8 kg
(c) 80 g

4. A 10-year-old boy
(a) 30 kg
(b) 3 kg
(c) 300 g

5. A Volkswagen Rabbit
(a) 100 kg
(b) 1,000 kg
(c) 1,000 g

6. A 10-lb bag of flour
(a) 45 kg
(b) 4.5 kg
(c) 45 g

7. A dinner fork
(a) 50 g
(b) 5 g
(c) 5 kg

8. A can of spices
(a) 3 g
(b) 300 g
(c) 30 g

9. A slice of bread
(a) 2 g
(b) 20 g
(c) 2 kg

10. A house paintbrush
(a) 120 g
(b) 12 kg
(c) 12 g

11. A sugar cube
(a) 2 mg
(b) 20 g
(c) 2 g

12. A salt shaker
(a) 10 g
(b) 100 g
(c) 1 g

Complete each statement, using a metric unit of weight.

13. A marshmallow weighs 5 ________.

14. A toaster weighs 2 ________.

15. 1 ________ is $\frac{1}{1{,}000}$ g. > Videos

16. A bag of peanuts weighs 100 ________.

17. An electric razor weighs 250 ________.

18. A soupspoon weighs 50 ________.

19. A heavyweight boxer weighs 98 ________.

20. A vitamin C tablet weighs 500 ________.

21. A disposable lighter weighs 30 ________.

22. A clock radio weighs 1.5 ________.

23. A household broom weighs 300 ________.

24. A 60-watt light bulb weighs 25 ________.

< Objective 2 >

Complete each statement.

25. 8 kg = _________ g

> Videos

26. 5,000 mg = _________ g

27. 9,500 kg = _________ t

28. 3 kg = _________ g

29. 1.45 t = _________ kg

30. 12,500 kg = _________ t

31. 3 g = _________ mg

32. 2,000 g = _________ kg

< Objective 3 >

Choose the most reasonable measure of volume.

33. A bottle of wine

(a) 75 mL
(b) 7.5 L
(c) 750 mL

34. A gallon of gasoline

(a) 400 mL
(b) 4 L
(c) 40 L

35. A bottle of perfume

(a) 15 mL
(b) 150 mL
(c) 1.5 L

36. A can of frozen orange juice

(a) 1.5 L
(b) 150 mL
(c) 15 mL

37. A hot-water heater

(a) 200 mL
(b) 50 L
(c) 200 L

38. An oil drum

(a) 220 L
(b) 220 mL
(c) 22 L

Answers

18. ____________________
19. ____________________
20. ____________________
21. ____________________
22. ____________________
23. ____________________
24. ____________________
25. ____________________
26. ____________________
27. ____________________
28. ____________________
29. ____________________
30. ____________________
31. ____________________
32. ____________________
33. ____________________
34. ____________________
35. ____________________
36. ____________________
37. ____________________
38. ____________________

Answers

39. ____________________

40. ____________________

41. ____________________

42. ____________________

43. ____________________

44. ____________________

45. ____________________

46. ____________________

47. ____________________

48. ____________________

49. ____________________

50. ____________________

51. ____________________

52. ____________________

53. ____________________

54. ____________________

55. ____________________

56. ____________________

57. ____________________

58. ____________________

59. ____________________

60. ____________________

61. ____________________

62. ____________________

39. A bottle of ink

(a) 60 cm^3
(b) 6 cm^3
(c) 600 cm^3

40. A cup of tea

(a) 18 mL
(b) 180 mL
(c) 18 L

41. A jar of mustard

(a) 150 mL
(b) 15 L
(c) 15 mL

42. A bottle of aftershave lotion

(a) 50 mL
(b) 5 L
(c) 5 mL

43. A cream pitcher

(a) 12 mL
(b) 120 mL
(c) 1.2 L

44. One tablespoon

(a) 1.5 mL
(b) 1.5 L
(c) 15 mL

Complete each statement, using a metric unit of volume.

45. A can of tomato soup is 300 ________.

46. 1 ________ is $\frac{1}{100}$ L.

47. A saucepan holds 1.5 ________.

48. A thermos bottle contains 500 ________ of liquid.

49. A coffee pot holds 720 ________.

50. A garbage can will hold 120 ________.

51. A car's engine capacity is 2,000 cm^3. It is advertised as a 2.0 ________ model.

52. A bottle of vanilla extract contains 60 ________.

53. 1 ________ is $\frac{1}{10}$ cL.

54. A can of soft drink is 35 ________.

55. A garden sprinkler delivers 8 ________ of water per minute.

56. 1 kL is 1,000 ________.

< Objective 4 >

Complete each statement.

57. 7 L = ________ mL

> Videos

58. 4,000 cm^3 = ________ L

59. 4 hL = ________ L

60. 7 L = ________ cL

61. 8,000 mL = ________ L

62. 12 L = ________ mL

63. 5 L = ________ cm^3

64. 2 L = ________ cL

65. 75 cL = ________ mL

66. 5 kL = ________ L

67. 5 L = ________ cL

68. 400 mL = ________ cL

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Determine whether each statement is **true** *or* **false.**

69. A milliliter is the same as a cubic centimeter.

70. A kilogram is 1,000 times heavier than a milligram.

Solve each application.

71. BUSINESS AND FINANCE A caterer expects to serve 300-mL portions of soup to 70 people. How many liters of soup does the caterer need to prepare?

72. BUSINESS AND FINANCE A produce stand sells local walnuts for $9.98 per kilogram. How much does 750 grams cost?

73. BUSINESS AND FINANCE Dried oregano sells for 4¢ per gram, in bulk, at a local store. How much will 0.15 kilograms cost?

74. CONSTRUCTION If 788 g of mortar are required to mortar a brick into place, how many kg do you need to mortar 238 bricks into place (to the nearest whole kg)?

75. SCIENCE AND MEDICINE The United States emitted 67.3 million metric tons (t) of carbon monoxide (CO) into the atmosphere in 1999. One metric ton equals 1,000 kg. How many kilograms of CO were emitted to the atmosphere in the United States during 1999?

76. SCIENCE AND MEDICINE The United States emitted 19.5 million t of nitrogen oxides (NO) into the atmosphere in 1987. One metric ton (1 t) equals 1,000 kg. How many kilograms of NO were emitted to the atmosphere in the United States during 1987?

Basic Skills | Challenge Yourself | Calculator/Computer | **Career Applications** | Above and Beyond

77. ALLIED HEALTH Many medications are expressed as percent solutions where the percent indicates how many grams of the active ingredient are dissolved in 100 milliliters (mL) of diluting element (usually water or saline). Consider a 0.6% solution of metaproterenol, an oral inhalation medication used to treat bronchospasm.

(a) Determine the number of milligrams (mg) of metaproterenol per milliliter of diluting element in a 0.6% solution.

(b) Determine the volume (in milliliters) of the 0.6% solution to be administered if a dose of 15 mg of metaproterenol is ordered.

78. ALLIED HEALTH Many medications are expressed as percent solutions where the percent indicates how many grams of the active ingredient are dissolved in 100 milliliters (mL) of diluting element (usually water or saline). Consider a 2.5% solution of Demerol, a potent pain medication.

Answers

63. ________________

64. ________________

65. ________________

66. ________________

67. ________________

68. ________________

69. ________________

70. ________________

71. ________________

72. ________________

73. ________________

74. ________________

75. ________________

76. ________________

77. ________________

78. ________________

Answers

79. ____

80. ____

81. ____

82. ____

83. ____

84. ____

85. ____

86. ____

87. ____

(a) Determine the number of milligrams (mg) of Demerol per milliliter of diluting element in a 2.5% solution.

(b) Determine the volume (in milliliters) of the 2.5% solution to be administered if a dose of 120 mg of Demerol is ordered.

79. **MANUFACTURING TECHNOLOGY** There are 312.83 kg of steel in stock. A new part will use 1,600 mg. If 30 of these parts will be produced, how much steel will be left in stock?

80. **MANUFACTURING TECHNOLOGY** There are 8,370 mL of sealer in stock. A run of parts uses 2.4 L of sealer. How much is left after the run?

81. **AUTOMOTIVE TECHNOLOGY** In a 4-cylinder engine, the displacement of a single cylinder is 575 mL. What is the displacement of the engine?

82. **AUTOMOTIVE TECHNOLOGY** A quart of oil weighs 972 g. What is the weight, in kg, of the oil in a 4-qt oil pan (to the nearest tenth kilogram)?

83. **MANUFACTURING TECHNOLOGY** A green treated deck board weighs 4.19 kg. As it dries out, it loses 247 g of weight. How much does the board now weigh?

84. **AGRICULTURE** A pallet containing 65 bags of fertilizer weighs 1.482 metric tons. How many kilograms does each bag weigh?

Basic Skills | Challenge Yourself | Calculator/Computer | Career Applications | **Above and Beyond**

85. Mass (weight) and volume are connected in the metric system. The weight of water in a cube 1 cm on a side is 1 g. Does such a relationship exist in the U.S. Customary system of measurement? If so, what is it? If not, why not?

86. **(a)** Determine how many liters of gasoline your car will hold.

(b) Using current prices, determine what a liter of gasoline should cost to make it competitive.

(c) How much would it cost to fill your car?

87. Do the following doses of medicine seem reasonable or unreasonable?

(a) Take 5 L of Kaopectate every morning.

(b) Soak your feet in 5 L of epsom salt bath every evening.

(c) Inject yourself with $\frac{3}{4}$ L of insulin every day.

Answers

1. (b) **3.** (c) **5.** (b) **7.** (a) **9.** (b) **11.** (c) **13.** g
15. mg **17.** g **19.** kg **21.** g **23.** g **25.** 8,000 **27.** 9.5
29. 1,450 **31.** 3,000 **33.** (c) **35.** (a) **37.** (c) **39.** (a)
41. (a) **43.** (b) **45.** mL (or cm^3) **47.** L **49.** mL **51.** L
53. mL **55.** L **57.** 7,000 **59.** 400 **61.** 8 **63.** 5,000
65. 750 **67.** 500 **69.** True **71.** 21 L **73.** $6
75. 67.3 billion kg **77.** **(a)** 6 mg/mL; **(b)** 2.5 mL **79.** 312.782 kg
81. 2.3 L **83.** 3.943 kg **85.** Above and Beyond
87. **(a)** Unreasonable; **(b)** reasonable; **(c)** unreasonable

7.4

Converting Between the U.S. Customary and Metric Systems of Measurement

< 7.4 Objectives >

1 > Convert between U.S. Customary and metric units of length

2 > Convert between U.S. Customary and metric units of mass/weight

3 > Convert between U.S. Customary and metric units of volume

4 > Convert between Fahrenheit and Celsius temperature scales

Often, we need to convert between the U.S. Customary and metric systems of measurement. We usually do this with the help of conversion tables and a calculator.

The conversion table shown gives conversion factors between some U.S. Customary and metric units of length. Each factor is rounded to the nearest hundredth (two decimal places). Even though all of these factors are approximate, we use an equal sign for its convenience.

Property: Converting Length

U.S. to Metric	Metric to U.S.
1 in. = 2.54 cm	1 cm = 0.39 in.
1 ft = 0.30 m	1 m = 39.37 in.
1 yd = 0.91 m	1 m = 1.09 yd
1 mi = 1.61 km	1 km = 0.62 mi

When reading this table, remember that a mile is larger than a kilometer, an inch is larger than a centimeter, and that a yard is smaller than a meter.

In Section 7.1, we presented two methods for converting within the U.S. Customary system—the substitution method and the unit-ratio method. Both methods work well when converting between the U.S. Customary and metric systems.

We usually use the substitution method for straightforward conversions and the unit-ratio method for more complex conversions.

Example 1 Converting Between Measurement Systems—Substitution

< Objective 1 >

NOTE

To use the table, choose the conversion equation that has 1 of your given unit. In (a), we use 1 in. = 2.54 cm.

Convert, as indicated.

(a) 5 in. to cm

1 in. is 2.54 cm, so we can substitute as we did in Section 7.1.

5 in. = 5 × (1 in.) We write 5 in. as 5 × (1 in.).
= 5 × (2.54 cm) We substitute 2.54 cm for 1 in.
= 12.7 cm Finally, we perform the arithmetic.

(b) 12 mi to km

According to the table, 1 mi is 1.61 km.

12 mi = 12 × (1 mi)
= 12 × (1.61 km) We substitute 1.61 km for 1 mi.
= 19.32 km

(c) 3 m to in.

We begin with a metric measure, so we use the *Metric to U.S.* column in the Property table, with 1 m = 39.37 in.

$$
\begin{aligned}
3\text{ m} &= 3 \times (1\text{ m}) && \text{We write 3 m as } 3 \times (1\text{ m}). \\
&= 3 \times (39.37\text{ in.}) && \text{We substitute 39.37 in. for 1 m.} \\
&= 118.11\text{ in.} && \text{Finally, we perform the arithmetic.}
\end{aligned}
$$

RECALL

All conversions between the U.S. Customary and metric systems of measurement are approximations.

(d) 25 km to mi

$$
\begin{aligned}
25\text{ km} &= 25 \times (1\text{ km}) \\
&= 25 \times (0.62\text{ mi}) && \text{We substitute 0.62 mi for 1 km.} \\
&= 15.5\text{ mi}
\end{aligned}
$$

Check Yourself 1

Convert, as indicated.

(a) 8 in. to cm **(b)** 14 ft to m **(c)** 90 km to mi **(d)** 5 cm to in.

In order to prevent substitution tables from growing too large, we omit some conversions. For instance, the Property table does not have a direct meters-to-feet conversion. In this case, we could convert from meters to yards and then from yards to feet or we could use unit ratios to do the conversion directly.

In the next example, we choose to do the conversions directly with unit ratios.

Example 2 Converting Between Measurement Systems—Unit Ratios

RECALL

The unit you want to convert to should be in the numerator. The unit you want to "cancel" needs to be in the denominator.

Convert 6 m to ft.

The *U.S. to Metric* column in the Property table lists the conversion 1 ft = 0.30 m. We use this to form a unit ratio with feet in the numerator and meters in the denominator.

$$
\begin{aligned}
6\text{ m} &= \frac{6\ \cancel{\text{m}}}{1} \times \frac{1\text{ ft}}{0.30\ \cancel{\text{m}}} && 1 = \frac{1\text{ ft}}{0.30\text{ m}} \text{ is a unit ratio.} \\
&= \frac{6}{0.30}\text{ ft} && \text{After simplifying the units, we have a division problem.} \\
&= 20\text{ ft} && 6 \div 0.30 = 20
\end{aligned}
$$

Check Yourself 2

Convert 42 in. to m (round your result to the nearest hundredth meter).

Even with unit ratios, we may need to pass through one measurement to get to another. Unit ratios are still the better method for these conversions, which we demonstrate in the next example.

Example 3 Converting Between Measurement Systems—Unit Ratios

NOTE

When doing multiple conversions, it is a good idea to outline a plan before beginning.

Convert 2 km to in.

We can reach inches from kilometers in several ways. We might convert from kilometers to miles, miles to feet, and then feet to inches. Or, we can convert from kilometers to meters, and then meters to inches. We choose this second method in this example.

$$
\begin{aligned}
2\text{ km} &= \frac{2\ \cancel{\text{km}}}{1} \times \frac{1{,}000\ \cancel{\text{m}}}{1\ \cancel{\text{km}}} \times \frac{39.37\text{ in.}}{1\ \cancel{\text{m}}} && 1\text{ m} = 39.37\text{ in., so } \frac{39.37\text{ in.}}{1\text{ m}} \text{ is a unit ratio.} \\
&= 2 \times 1{,}000 \times 39.37\text{ in.} && \text{After simplifications, we are left with inches.} \\
&= 78{,}740\text{ in.}
\end{aligned}
$$

Check Yourself 3

Convert 1 km to ft.

When we need to convert rates between systems, we rely on the unit-ratio method, as we show in Example 4.

Example 4 Converting Rates Between Systems

Convert 55 $\frac{\text{mi}}{\text{h}}$ to $\frac{\text{m}}{\text{s}}$.

RECALL

1 mi = 1.61 km,
1 km = 1,000 m, and
1 h = 3,600 s.

We write 55 $\frac{\text{mi}}{\text{h}}$ as $\frac{55\ \text{mi}}{\text{h}}$ and use unit ratios to convert miles to meters (miles to kilometers, kilometers to meters) and hours to seconds.

$$\frac{55\ \cancel{\text{mi}}}{1\ \cancel{\text{h}}} \times \frac{1.61\ \cancel{\text{km}}}{1\ \cancel{\text{mi}}} \times \frac{1{,}000\ \text{m}}{1\ \cancel{\text{km}}} \times \frac{1\ \cancel{\text{h}}}{3{,}600\ \text{s}} = \frac{55 \times 1.61 \times 1{,}000}{3{,}600}\ \frac{\text{m}}{\text{s}}$$

$$= 24.60\ \frac{\text{m}}{\text{s}}$$

Check Yourself 4

Convert 85 $\frac{\text{ft}}{\text{s}}$ to $\frac{\text{km}}{\text{h}}$.

RECALL

Weight is the effect of gravity on an object's mass. We generally use the terms interchangeably when on Earth.

We use the same methods to convert between U.S. and metric units of weight/mass or volume.

Property: Converting Weight/Mass

U.S. to Metric	Metric to U.S.
1 oz = 28.35 g	1 g = 0.04 oz
1 lb = 0.45 kg	1 kg = 2.20 lb

As before, we can convert directly with substitution or use unit ratios for more complex conversions.

Example 5 Converting Mass/Weight

< Objective 2 >

(a) A roast beef weighs 3 kg. What is its weight, in pounds?

We have the conversion 1 kg = 2.20 lb.

$$\begin{aligned} 3\ \text{kg} &= 3 \times (1\ \text{kg}) && \text{Write 3 kg as } 3 \cdot (1\ \text{kg}). \\ &= 3 \times (2.2\ \text{lb}) && 1\ \text{kg} = 2.20\ \text{lb} \\ &= 6.6\ \text{lb} && 3 \cdot 2.2 = 6.6 \end{aligned}$$

(b) A package weighs 5 oz. Find its weight, in grams.

Use 1 oz = 28.35 g.

$$\begin{aligned} 5\ \text{oz} &= 5 \times (1\ \text{oz}) \\ &= 5 \times (28.35\ \text{g}) \\ &= 141.75\ \text{g} \end{aligned}$$

RECALL

1 lb = 16 oz

(c) How many ounces are in 3.5 kg?

We use unit ratios to convert from kilograms to pounds to ounces.

$$\begin{aligned} 3.5 \text{ kg} &= \frac{3.5 \cancel{\text{kg}}}{1} \times \frac{2.2 \cancel{\text{lb}}}{1 \cancel{\text{kg}}} \times \frac{16 \text{ oz}}{1 \cancel{\text{lb}}} \\ &= 3.5 \times 2.2 \times 16 \text{ oz} \\ &= 123.2 \text{ oz} \end{aligned}$$

Check Yourself 5

(a) A radio weighs 8 lb. Find its weight in kilograms.
(b) A box of cereal lists its contents at 375 g. Convert this to ounces.
(c) A medium sized package contains 10.5 oz of candy. How many kilograms does a case of 24 packages contain? Round your result to the nearest hundredth kilogram.

NOTE

We will convert square and cubic units in Chapter 8.

Property: Converting Volume

U.S. to Metric	Metric to U.S.
1 qt = 0.95 L	1 L = 1.06 qt
1 fl oz = 29.57 mL	1 mL = 0.03 fl oz

Example 6 Converting Units of Volume

< Objective 3 >

(a) How many liters does a 2-qt bottle of milk contain?

We use the conversion 1 qt = 0.95 L, and substitute.

$$\begin{aligned} 2 \text{ qt} &= 2 \times (1 \text{ qt}) && \text{Write 2 qt as } 2 \cdot (1 \text{ qt}). \\ &= 2 \times (0.95 \text{ L}) && 1 \text{ qt} = 0.95 \text{ L} \\ &= 1.9 \text{ L} && 2 \cdot 0.95 = 1.9 \end{aligned}$$

(b) How many liters of fuel does a 16-gal gas tank hold?

We do not have a direct conversion for gallons to liters, so we use unit ratios to convert gallons to quarts and quarts to liters.

$$\begin{aligned} 16 \text{ gal} &= \frac{16 \cancel{\text{gal}}}{1} \times \frac{4 \cancel{\text{qt}}}{1 \cancel{\text{gal}}} \times \frac{0.95 \text{ L}}{1 \cancel{\text{qt}}} \\ &= 16 \times 4 \times 0.95 \text{ L} \\ &= 60.8 \text{ L} \end{aligned}$$

Check Yourself 6

(a) How many liters are in a 40-gal hot-water tank?
(b) How many quarts does a 3-L bottle contain?

As you would expect, there are more important applications requiring us to convert between the systems.

Example 7 An Application Involving Conversions

An automobile has a 16-gal fuel tank and gets 32 mpg. How many km can the car travel on a single tank of gas?

There are a couple of ways to approach this problem. For instance, we could convert all measurements to their metric equivalent, and then compute the number of km that the car could travel.

Alternatively, we can find the total number of miles the car can go, and then convert that to km. In this example, we choose the second method as it only requires a single conversion.

To find the distance, we multiply 16 gal and 32 mpg.

$$16\ \cancel{\text{gal}} \times 32\ \frac{\text{mi}}{\cancel{\text{gal}}} = 512\text{ mi}$$

The car can drive 512 miles on a full tank of gas.

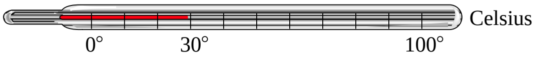
$$512\text{ mi} = 512(1.61\text{ km}) = 824.32\text{ km}$$

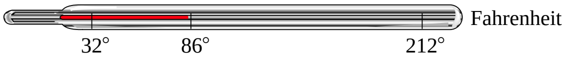
The car can drive 824.32 km on a single tank of gas.

Check Yourself 7

The load limit of a trailer is listed at 2,500 lb. It is to be loaded with pallets of concrete blocks. Each block weighs 3 kg and there are 50 blocks to the pallet; each pallet weighs 10 kg. What is the maximum number of pallets that can be loaded onto the trailer?

Temperature is expressed in **degrees Celsius** in the metric system, whereas **degrees Fahrenheit** is the unit of temperature used in the U.S. Customary system. The boiling point of water (at sea level) is 100 degrees Celsius, written 100°C, while it is 212 degrees Fahrenheit, written 212°F. The freezing point of water (at sea level) is 0°C, which corresponds to 32°F. The temperature on a hot day might be 30°C, corresponding to 86°F.





To convert between units of temperature, use the following formulas.

Property

Converting Between U.S. Customary and Metric Units of Temperature

To convert from degrees Celsius (°C) to degrees Fahrenheit (°F), multiply by 9, divide by 5, and then add 32. A formula that describes this is

$$F = \frac{9C}{5} + 32$$

To convert from degrees Fahrenheit (°F) to degrees Celsius (°C), subtract 32, multiply by 5, and then divide by 9. A formula that describes this is

$$C = \frac{5(F - 32)}{9}$$

Example 8 shows the use of these formulas.

Example 8 Converting Between Fahrenheit and Celsius Temperatures

< Objective 4 >

To what temperature Fahrenheit does 18°C correspond?

Following the first formula given,

$$F = \frac{9(18)}{5} + 32 = \frac{162}{5} + 32 = 32.4 + 32 = 64.4$$

So the temperature is 64.4°F.

To what temperature Celsius does 77°F correspond?

Using the second rule,

$$C = \frac{5(77 - 32)}{9} = \frac{5(45)}{9} = \frac{225}{9} = 25$$

The temperature is 25°C.

Check Yourself 8

Complete the following:

(a) 12°C = ______ °F

(b) 83°F = ______ °C

Example 9 A Temperature Conversion Application

The melting point of a cast-iron block is approximately 2,300°F. What is this temperature in degrees Celsius?

Using the second formula given,

$$C = \frac{5(2{,}300 - 32)}{9} = \frac{5(2{,}268)}{9} = \frac{11{,}340}{9} = 1{,}260$$

The corresponding temperature is 1,260°C.

Check Yourself 9

The thermostat of a car opens at a temperature of 165°F. Convert this temperature to degrees Celsius. Round your result to the nearest tenth degree Celsius.

We close this section with a brief table of the conversions between units of length, mass/weight, volume, and temperature.

Length	
U.S. to Metric	**Metric to U.S.**
1 in. = 2.54 cm	1 cm = 0.39 in.
1 ft = 0.30 m	1 m = 39.37 in.
1 yd = 0.91 m	1 m = 1.09 yd
1 mi = 1.61 km	1 km = 0.62 mi
Weight/Mass	
U.S. to Metric	**Metric to U.S.**
1 oz = 28.35 g	1 g = 0.04 oz
1 lb = 0.45 kg	1 kg = 2.20 lb
Volume	
U.S. to Metric	**Metric to U.S.**
1 qt = 0.95 L	1 L = 1.06 qt
1 fl oz = 29.57 mL	1 mL = 0.03 fl oz
Temperature	
U.S. to Metric	**Metric to U.S.**
$C = \frac{5}{9}(F - 32)$	$F = \frac{9}{5}C + 32$

Check Yourself ANSWERS

1. **(a)** 20.32 cm; **(b)** 4.2 m; **(c)** 55.8 mi; **(d)** 1.95 in. **2.** 1.07 m
3. 3,273.6 ft **4.** $91.8\ \frac{\text{km}}{\text{h}}$ **5.** **(a)** 3.6 kg; **(b)** 15 oz; **(c)** 7.14 kg
6. **(a)** 152 L; **(b)** 3.18 qt **7.** 7 pallets **8.** **(a)** 53.6°F; **(b)** $28\frac{1}{3}$ °C
9. 73.9°C

Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 7.4

(a) For straightforward conversions, we usually use the ________________ method.

(b) When we need to convert rates between systems, we rely on the ________________ method.

(c) Weight is the effect of ________________ on an object's mass.

(d) In the metric system, temperature is expressed in degrees ____________.

7.4 exercises

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Name ______________________

Section ________ Date ________

Answers

1. __________ 2. __________

3. __________ 4. __________

5. __________ 6. __________

7. __________ 8. __________

9. __________ 10. __________

11. __________ 12. __________

13. __________ 14. __________

15. __________ 16. __________

17. __________ 18. __________

19. __________ 20. __________

21. __________ 22. __________

23. __________ 24. __________

25. __________ 26. __________

27. __________ 28. __________

29. __________ 30. __________

< Objectives 1–4 >

Complete each statement. (Round to the nearest hundredth.)

1. 250 km = __________ mi > Videos

2. 9 cm = __________ in.

3. 150 mi = __________ km

4. 9 yd = __________ m

5. 2.6 m = __________ in. > Videos

6. 72 in. = __________ cm

7. 3 ft = __________ mm

8. 16 km = __________ ft

9. 19.4 mm = __________ yd

10. 45 ft = __________ km

11. $75 \frac{\text{km}}{\text{h}} = \_\_\_\_\_\_\_\_\_\_ \frac{\text{ft}}{\text{s}}$

12. $50 \frac{\text{mi}}{\text{h}} = \_\_\_\_\_\_\_\_\_\_ \frac{\text{m}}{\text{s}}$

13. 6 lb = __________ kg

14. 8 oz = __________ g

15. 0.25 kg = __________ oz

16. 5 lb = __________ g

17. 12 kg = __________ lb

18. 450 g = __________ oz

19. 8,000 lb = __________ t (metric tons)

20. 4,500 kg = __________ U.S. tons

21. $14.5 \frac{\text{oz}}{\text{week}} = \_\_\_\_\_\_\_\_\_\_ \frac{\text{g}}{\text{day}}$

22. $2.3 \frac{\text{kg}}{\text{day}} = \_\_\_\_\_\_\_\_\_\_ \frac{\text{oz}}{\text{h}}$

23. 4 qt = __________ L

24. 7 L = __________ qt

25. 8 fl oz = __________ mL

26. 15.9 gal = __________ L

27. 760 mL = __________ qt

28. 15 L = __________ gal

29. 72 mL = __________ fl oz

30. 450 fl oz = __________ L

31. $12\ \frac{\text{gal}}{\text{h}} = \underline{\hspace{2cm}}\ \frac{\text{L}}{\text{min}}$

32. $3.8\ \frac{\text{L}}{\text{min}} = \underline{\hspace{2cm}}\ \frac{\text{fl oz}}{\text{s}}$

33. 52°F = ________ °C > Videos

34. 6°C = ________ °F

35. 24°C = ________ °F > Videos

36. 95°F = ________ °C

37. 86°F = ________ °C

38. 10°C = ________ °F

39. 20°C = ________ °F

40. 72°F = ________ °C

41. 100°F = ________ °C

42. 27°C = ________ °F

43. 37°C = ________ °F

44. 98.6°F = ________ °C

Solve each application.

45. **SCIENCE AND MEDICINE** A football team's fullback weighs 250 lb. How many kilograms does he weigh?

46. **SCIENCE AND MEDICINE** Samantha's speedometer reads in kilometers per hour. If the legal speed limit is 55 mi/h, how fast can she drive?

SCIENCE AND MEDICINE A Boeing 747 can travel 8,336 mi on one 57,285-gal tank of airplane fuel.

47. How many liters of fuel can a Boeing 747 tank hold?

48. How many kilometers can the Boeing 747 travel on one fuel tank (to the nearest whole km)?

49. Express the fuel efficiency of the Boeing 747 in km/L (accurate to three decimal places).

50. How many liters of fuel does the Boeing 747 use per kilometer traveled (accurate to one decimal place)?

SCIENCE AND MEDICINE A Boeing 777 can travel 11,029 km on one 171,835 L tank of airplane fuel.

51. How many gallons of fuel can a Boeing 777 tank hold (to the nearest whole gallon)?

52. How many miles can the Boeing 777 travel on one fuel tank (to the nearest mile)?

53. Express the fuel efficiency of the Boeing 777 in mi/gal (accurate to three decimal places).

54. How many gallons of fuel does the Boeing 777 use per mile traveled (accurate to one decimal place)?

Answers

31. ________ **32.** ________

33. ________ **34.** ________

35. ________ **36.** ________

37. ________ **38.** ________

39. ________ **40.** ________

41. ________ **42.** ________

43. ________ **44.** ________

45. ________ **46.** ________

47. ________ **48.** ________

49. ________ **50.** ________

51. ________ **52.** ________

53. ________ **54.** ________

Answers

55. ______________________

56. ______________________

57. ______________________

58. ______________________

59. ______________________

60. ______________________

61. ______________________

62. ______________________

63. ______________________

64. ______________________

65. ______________________

66. ______________________

67. ______________________

68. ______________________

69. ______________________

70. ______________________

Basic Skills | **Challenge Yourself** | Calculator/Computer | Career Applications | Above and Beyond

Determine whether each statement is **true** *or* **false.**

55. A mile is shorter than a kilometer.

56. An inch is larger than a centimeter.

57. A quart is smaller then a liter.

58. A pound is more than half of a kilogram.

Basic Skills | Challenge Yourself | Calculator/Computer | **Career Applications** | Above and Beyond

59. **Manufacturing Technology** The label for a primer states that it should not be applied to surfaces at less than 10°C. What is this temperature in degrees Fahrenheit?

60. **Manufacturing Technology** A construction adhesive should not be exposed to temperatures above 140°F. What is this temperature in degrees Celsius?

61. **Agriculture** In order for a corn seed to germinate, a soil temperature of at least 12.5°C is required. What is this temperature in degrees Fahrenheit?

62. **Manufacturing Technology** A copper-nickel alloy at 40% nickel becomes liquid at 1,280°C and becomes solid at 1,240°C. Convert these temperatures into degrees Fahrenheit.

63. **Allied Health** An infant measures 51 cm long at birth. Determine the baby's length at birth in inches. Round to the nearest inch.

chapter 7 > Make the Connection

64. **Allied Health** An adult female patient is 5 ft 4 in. tall. Determine her height in centimeters. Round to the nearest cm.

chapter 7 > Make the Connection

65. **Manufacturing Technology** A machine is listed with a gross weight of 1,200 kg. The load limit of the trailer is listed at 2,500 lb. Can the machine be hauled on the trailer?

66. **Manufacturing Technology** A piece of steel rod stock is 5 m long. For a prototype, 6 ft 2 in. is used. How much is left (in meters)? Round to the nearest hundredth meter.

chapter 7 > Make the Connection

67. **Automotive Technology** The oil pan of a diesel engine calls for 2 gal of oil. How many liters of oil would this be?

68. **Manufacturing Technology** Which is larger: a metric ton of aluminum or a standard net ton of aluminum?

Basic Skills | Challenge Yourself | Calculator/Computer | Career Applications | **Above and Beyond**

69. In exercise 49, you expressed the fuel efficiency of the Boeing 747 in terms of km/L. In exercise 50, you expressed its rate in L/km. Use complete sentences to describe how these two rates differ.

70. In exercise 53, you expressed the fuel efficiency of the Boeing 777 in mi/gal. In exercise 54, you expressed its rate in gal/mi. Use complete sentences to describe how these two rates differ.

71. Complete the puzzle.

Across

1. 6,000 milliliters
6. 1,760 yards
7. paradise
8. LV + LV
9. extraterrestrial
10. out of whack
13. tome
14. seven thousandths g

Down

1. 600 centimeters
2. top
3. ______ de France
4. ______ dm in 1 m
5. sixty thousand g
10. Presidential nickname
11. didn't lose
12. read-only memory

MATHWORK PUZZLE

1	2	3	4			5
6				■	■	
7				■	8	
	■	■	■	■	■	
9		■	10	11	12	
	■	■	13			
14						

Answers

71. ____________________

Answers

1. 155 **3.** 240 **5.** 102.36 **7.** 900 **9.** 0.02 **11.** 68.2
13. 2.7 **15.** 8.8 **17.** 26.4 **19.** 3.6 **21.** 58.73 **23.** 3.8
25. 240 **27.** 0.81 **29.** 2.16 **31.** 0.76 **33.** 11.11 **35.** 75.2
37. 30 **39.** 68 **41.** 37.78 **43.** 98.6 **45.** 112.5 kg
47. 217,683 L **49.** $0.062 \frac{\text{km}}{\text{L}}$ **51.** 45,536 gal **53.** $0.150 \frac{\text{mi}}{\text{gal}}$
55. False **57.** True **59.** 50°F **61.** 54.5°F **63.** 20 in.
65. No **67.** 7.6 L **69.** Above and Beyond
71.

6	L	I	T	E	R	S
M	I	L	E	■	■	I
E	D	E	N	■	C	X
T	■	■	■	■	■	T
E	T	■	A	W	R	Y
R	■	■	B	O	O	K
S	E	V	E	N	M	G

Activity 19 ::
Tool Sizes

chapter
7
> Make the Connection

Perhaps you work in construction, remodeling, or automotive repair. Or maybe you simply enjoy working on your own car or doing projects around the house. You have probably found the need to use both U.S. Customary and metric tool sizes.

The following drill bits came in a typical set. Convert each bit size to millimeters (mm), rounding to the nearest tenth of a millimeter.

$\frac{1}{16}$ in.	$\frac{5}{64}$ in.	$\frac{3}{32}$ in.	$\frac{7}{64}$ in.	$\frac{1}{8}$ in.
$\frac{9}{64}$ in.	$\frac{5}{32}$ in.	$\frac{3}{16}$ in.	$\frac{7}{32}$ in.	$\frac{1}{4}$ in.

A set of wrenches with metric unit sizes consists of those listed in the following chart. Convert each size to a corresponding U.S. Customary unit wrench. In each case, find a U.S. Customary size accurate to the nearest $\frac{1}{32}$ of an inch.

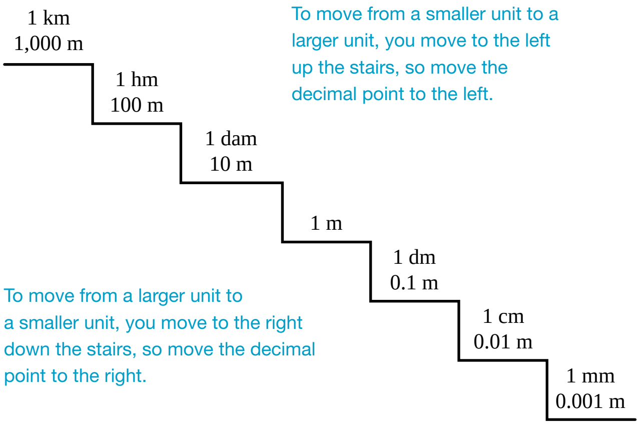
8 mm	**10 mm**	**12 mm**	**13 mm**	**14 mm**	**17 mm**

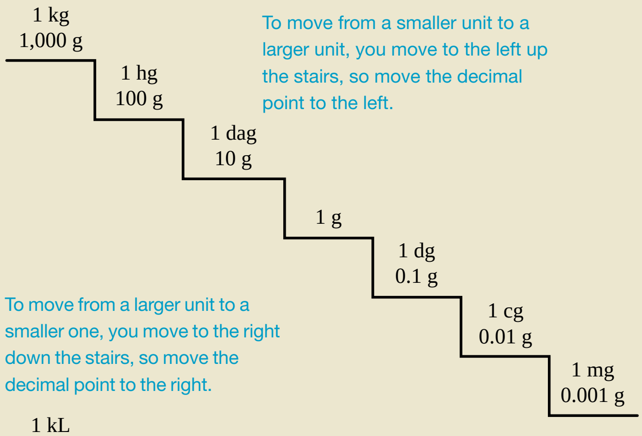
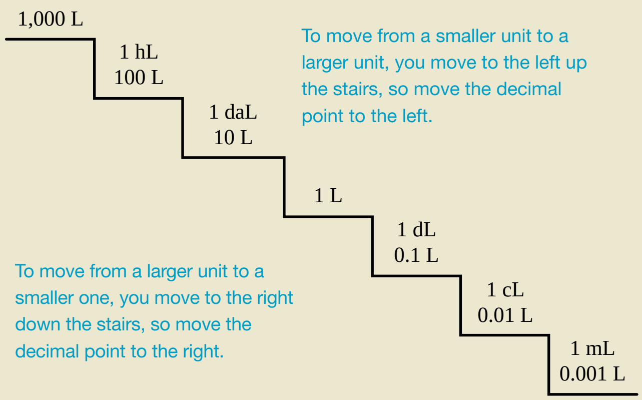
Locate at least three other tool sizes in your home or apartment and make an appropriate conversion, either U.S. Customary to metric or metric to U.S. Customary.

summary :: chapter 7

Definition/Procedure	Example	Reference
The U.S. Customary System		**Section 7.1**
U.S. Customary Units of Measure *Length* 1 foot (ft) = 12 inches (in.) 1 yard (yd) = 3 ft 1 mile (mi) = 5,280 ft *Weight/Mass* 1 pound (lb) = 16 ounces (oz) 1 ton = 2,000 lb *Volume* 1 cup (c) = 8 fluid ounces (fl oz) 1 pint (pt) = 2 c 1 quart (qt) = 2 pt 1 gallon (gal) = 4 qt *Time* 1 minute (min) = 60 seconds (s) 1 hour (h) = 60 min 1 day = 24 h 1 week = 7 days		*p.* 425
To Convert Units in the U.S. System *Substitution Method* **Step 1** Rewrite the given measure, separating the number and the unit. **Step 2** Replace the unit of measure with an equivalent measure. **Step 3** Perform the arithmetic and simplify, if necessary.	$8\text{ yd} = 8 \times (1\text{ yd})$ $= 8 \times (3\text{ ft})$ $= 24\text{ ft}$	*p.* 426
Unit Ratios A fraction whose value is 1. Unit ratios can be used to convert units.	$\frac{12\text{ in.}}{1\text{ ft}}$ and $\frac{60\text{ min}}{1\text{ h}}$ are unit ratios.	*p.* 427
To Convert Units in the U.S. System *Unit-Ratio Method* **Step 1** Construct a ratio equal to one in which the numerator contains the desired unit of measure, or an intermediary measure, and the denominator contains the given unit of measure. **Step 2** Multiply the given measure by the unit measure and simplify. **Step 3** If necessary, repeat the process until the final unit is the one you want.	$12\text{ pt} = \frac{12\,\cancel{\text{pt}}}{1} \times \frac{1\text{ qt}}{2\,\cancel{\text{pt}}}$ $= \frac{12}{2}\text{ qt}$ $= 6\text{ qt}$	*p.* 427
To Add Like Denominate Numbers **Step 1** Arrange the numbers so that the like units are in the same column. **Step 2** Add in each column. **Step 3** Simplify if necessary.	To add 4 ft 7 in. and 5 ft 10 in.: $\begin{array}{r} 4\text{ ft} \quad 7\text{ in.} \\ +\ 5\text{ ft} \quad 10\text{ in.} \\ \hline 9\text{ ft} \quad 17\text{ in.} \end{array}$ or $10\text{ ft} \quad 5\text{ in.}$	*p.* 429

Continued

Definition/Procedure	Example	Reference
To Multiply or Divide Denominate Numbers by Abstract Numbers		
Step 1 Multiply or divide each part of the denominate number by the abstract number. **Step 2** Simplify if necessary.	$2 \times (3 \text{ yd } 2 \text{ ft}) = 6 \text{ yd } 4 \text{ ft}$, or 7 yd 1 ft	*p.* 430
Metric Units of Length **Metric units of length** are the meter (m), centimeter (cm), millimeter (mm), and kilometer (km).		**Section 7.2** *pp.* 439–441
Basic Metric Prefixes		
*milli** means $\frac{1}{1{,}000}$ *kilo** means 1,000 *centi** means $\frac{1}{100}$ *hecto* means 100 *deci* means $\frac{1}{10}$ *deka* means 10	*These are the most commonly used and should be memorized.	*pp.* 441–442
Converting Metric Units You can use the following chart.  1 km 1,000 m 1 hm 100 m 1 dam 10 m 1 m 1 dm 0.1 m 1 cm 0.01 m 1 mm 0.001 m To move from a smaller unit to a larger unit, you move to the left up the stairs, so move the decimal point to the left. To move from a larger unit to a smaller unit, you move to the right down the stairs, so move the decimal point to the right. To convert between metric units, just move the decimal point the same number of places to the left or right as indicated by the chart.	$500 \text{ cm} = ? \text{ m}$ To convert from centimeters to meters, move the decimal point two places to the *left*. $500 \text{ cm} = 5\underset{\curvearrowleft}{\,00.} \text{ cm} = 5 \text{ m}$	*p.* 443

Definition/Procedure	Example	Reference
Metric Units of Weight and Volume		**Section 7.3**
Conversions between units of volume (liters) or units of weight (grams) work in exactly the same fashion as those between units of length.  To move from a smaller unit to a larger unit, you move to the left up the stairs, so move the decimal point to the left. To move from a larger unit to a smaller one, you move to the right down the stairs, so move the decimal point to the right.  To move from a smaller unit to a larger unit, you move to the left up the stairs, so move the decimal point to the left. To move from a larger unit to a smaller one, you move to the right down the stairs, so move the decimal point to the right.	3 L = ? mL To convert from liters to milliliters, move the decimal point three places to the *right*. $3\text{ L} = 3\underset{\curvearrowright}{.000}{}_{\wedge}\text{ mL} = 3{,}000\text{ mL}$	*pp.* 450–451
Converting Between the U.S. and Metric Systems		**Section 7.4**
Length ***U.S. to Metric*** 1 in. = 2.54 cm 1 ft = 0.30 m 1 yd = 0.91 m 1 mi = 1.61 km ***Metric to U.S.*** 1 cm = 0.39 in. 1 m = 39.37 in. 1 m = 1.09 yd 1 km = 0.62 mi	$90\text{ km} = 90 \times (1\text{ km})$ $= 90 \times (0.62\text{ mi})$ $= 55.8\text{ mi}$	*pp.* 459–461

Continued

Definition/Procedure	Example	Reference
Weight/Mass ***U.S. to Metric*** 1 oz = 28.35 g 1 lb = 0.45 kg ***Metric to U.S.*** 1 g = 0.04 oz 1 kg = 2.20 lb ***Volume*** ***U.S. to Metric*** 1 qt = 0.95 L 1 fl oz = 29.57 mL ***Metric to U.S.*** 1 L = 1.06 qt 1 mL = 0.03 fl oz	$50\,\frac{\text{gal}}{\text{h}} = \frac{50\,\cancel{\text{gal}}}{1\text{ h}} \times \frac{4\,\cancel{\text{qt}}}{1\,\cancel{\text{gal}}} \times \frac{0.95\text{ L}}{1\,\cancel{\text{qt}}}$ $= 50 \times 4 \times 0.95\,\frac{\text{L}}{\text{h}}$ $= 190\,\frac{\text{L}}{\text{h}}$	
Temperature Conversions From degrees Celsius (°C) to degrees Fahrenheit (°F): $F = \frac{9C}{5} + 32$ From degrees Fahrenheit (°F) to degrees Celsius (°C): $C = \frac{5(F - 32)}{9}$	To convert 34°C, $F = \frac{9(34)}{5} + 32 = 93.2$ To convert 75°F, $C = \frac{5(75 - 32)}{9} \approx 23.9$	*p.* 463

summary exercises :: chapter 7

This summary exercise set is provided to give you practice with each of the objectives of this chapter. Each exercise is keyed to the appropriate chapter section. When you are finished, you can check your answers to the odd-numbered exercises against those presented in the back of the text. If you have difficulty with any of these questions, go back and reread the examples from that section. The answers to the even-numbered exercises appear in the *Instructor's Solutions Manual.* Your instructor will give you guidelines on how to best use these exercises in your instructional setting.

7.1 *Complete each statement.*

1. 11 ft = _______ in.

2. 72 h = _______ days

3. 6 gal = _______ qt

4. 80 fl oz = _______ pt

5. 4 lb = _______ oz

6. 5 mi = _______ ft

7. 8,000 lb = _______ tons

8. 16 pt = _______ qt

Simplify.

9. 3 ft 23 in.

10. 4 lb 20 oz

Add.

11.
$$\begin{array}{r} 3\text{ lb}\ \ \ 9\text{ oz} \\ \underline{+\ 5\text{ lb}\ 10\text{ oz}} \end{array}$$

12.
$$\begin{array}{r} 5\text{ h }20\text{ min} \\ 3\text{ h }40\text{ min} \\ \underline{+\ 2\text{ h }20\text{ min}} \end{array}$$

Subtract.

13.
$$\begin{array}{r} 7\text{ ft }11\text{ in.} \\ \underline{-\ 2\text{ ft}\ \ \ 4\text{ in.}} \end{array}$$

14.
$$\begin{array}{r} 3\text{ h }30\text{ min} \\ \underline{-\ 1\text{ h }50\text{ min}} \end{array}$$

Multiply.

15. $3 \times (1\text{ h }25\text{ min})$

Divide.

16. $\dfrac{10\text{ lb }12\text{ oz}}{2}$

17. **Business and Finance** John worked 6 h 15 min, 8 h, 5 h 50 min, 7 h 30 min, and 6 h during 1 week. What were the total hours worked?

18. **Construction** A room requires two pieces of floor molding 12 ft 8 in. long, one piece 6 ft 5 in. long, and one piece 10 ft long. Will 42 ft of molding be enough for the job?

7.2 *Choose the most reasonable measure.*

19. A marathon race

(a) 40 km
(b) 400 km
(c) 400 m

20. The distance around your wrist

(a) 15 mm
(b) 15 cm
(c) 1.5 m

21. The diameter of a penny

(a) 19 cm
(b) 1.9 mm
(c) 19 mm

22. The width of a portable television screen

(a) 28 mm
(b) 28 cm
(c) 2.8 m

Complete each statement, using a metric unit of length.

23. A matchbook is 39 _____________ wide.

24. The distance from San Francisco to Los Angeles is 618 _____________.

25. A 1-lb coffee can has a diameter of 10 _____________.

Complete each statement.

26. 2 km = _____________ m

27. 3 cm = _____________ mm

28. 3,000 mm = _____________ m

29. 8 m = _____________ mm

30. 6 cm = _____________ m

31. 8 m = _____________ km

7.3 *Choose the most reasonable measure of weight.*

32. A quarter

(a) 6 g
(b) 6 kg
(c) 60 g

33. A tube of toothpaste

(a) 20 kg
(b) 200 g
(c) 20 g

34. A refrigerator

(a) 120 kg
(b) 1,200 kg
(c) 12 kg

35. A paperback book

(a) 1.2 kg
(b) 120 g
(c) 12 g

Complete each statement, using a metric unit of weight.

36. A loaf of bread weighs 500 _______.

37. A compact car weighs 900 _______.

38. A television set weighs 25 _______.

7.3 *Complete each statement.*

39. 5 kg = _______ g

40. 2,000 g = _______ kg

41. 5 t = _______ kg

42. 2,000 mg = _______ g

Choose the most reasonable measure of volume.

43. The gas tank of your car

(a) 500 mL
(b) 5 L
(c) 50 L

44. A bottle of eye drops

(a) 18 cm^3
(b) 180 cm^3
(c) 1.8 L

45. A can of soft drink

(a) 3.5 L
(b) 350 mL
(c) 35 mL

46. A punch bowl

(a) 200 L
(b) 20 L
(c) 200 mL

Complete each statement, using a metric unit of volume.

47. The crankcase of an automobile takes 5.5 _______ of oil.

48. The correct dosage for a cough medicine is 40 _______.

49. A bottle of iodine holds 20 _______.

50. A large mixing bowl holds 6 _______.

Complete each statement.

51. 5 L = _______ mL

52. 6,000 cm^3 = _______ L

53. 9 L = _______ cm^3

54. 10 mL = _______ L

7.4 *Complete each statement. Round to the nearest hundredth.*

55. 8.3 m = _______ in.

56. 42 in. = _______ cm

57. 15 lb = _______ kg

58. 27.5 kg = _______ lb

59. 5.2 L = _______ qt

60. 18 gal = _______ L

61. 17°C = _______ °F

62. 41°F = _______ °C

63. 98.6°F = _______ °C

64. 6°C = _______ °F

65. 59°F = _______ °C

66. 30°C = _______ °F

67. 5°C = _______ °F

68. 35°F = _______ °C

self-test 7

Name ________________

Section ________ Date ________

Answers

1. ________ 2. ________

3. ________ 4. ________

5. ________ 6. ________

7. ________ 8. ________

9. ________ 10. ________

11. ________ 12. ________

13. ________________

14. ________________

15. ________________

16. ________________

17. ________________

18. ________________

19. ________________

20. ________________

21. ________________

22. ________________

23. ________________

24. ________________

25. ________________

The purpose of this self-test is to help you assess your progress so that you can find concepts that you need to review before the next exam. Allow yourself about an hour to take this test. At the end of that hour, check your answers against those given in the back of this text. If you miss any, go back to the appropriate section to reread the examples until you have mastered that particular concept.

Complete each statement. Round to the nearest tenth.

1. 8 ft = ________ in.

2. 3 pt = ________ fl oz

3. 5 m = ________ mm

4. 3 kg = ________ g

5. 300 cL = ________ L

6. 40 in. = ________ cm

7. 5.2 km = ________ mi

8. 14.5 gal = ________ L

9. 150 lb = ________ kg

10. 12 in. = ________ cm

11. 58°F = ________ °C

12. 24°C = ________ °F

Simplify.

13. 5 ft 21 in.

14. 2 days 47 h 72 min

Evaluate as indicated.

15.
$$\begin{array}{r} 7 \text{ ft } 9 \text{ in.} \\ +\ 3 \text{ ft } 8 \text{ in.} \\ \hline \end{array}$$

16.
$$\begin{array}{r} 7 \text{ lb } \ \ 3 \text{ oz} \\ -\ 4 \text{ lb } 10 \text{ oz} \\ \hline \end{array}$$

17. $4 \times (3 \text{ h } 50 \text{ min})$

18. $\dfrac{12 \text{ lb } 18 \text{ oz}}{3}$

Choose the most reasonable measure.

19. The width of your hand

(a) 50 cm
(b) 10 cm
(c) 1 m

20. The speed limit on a freeway

(a) 10 km/h
(b) 100 km/h
(c) 100 m/h

21. A football player

(a) 12 kg
(b) 120 kg
(c) 120 g

22. A small can of tomato juice

(a) 4 L
(b) 400 mL
(c) 40 mL

23. **CONSTRUCTION** The Martins are fencing in a rectangular yard that is 110 ft long by 40 ft wide. If the fencing costs $3.50 per linear foot, what is the total cost of the fencing?

24. **AUTOMOTIVE TECHNOLOGY** A quart of oil weighs 972 g. What is the weight, in lb, of the oil in a 4-qt oil pan (to the nearest tenth pound)?

25. **BUSINESS AND FINANCE** A gallon of orange juice is on sale for $4.79. Find the cost per mL (to the nearest hundredth cent).

cumulative review chapters 1-7

Name ______________

Section ________ Date ________

The following exercises are presented to help you review concepts from earlier chapters. This is meant as review material and not as a comprehensive exam. The answers are presented in the back of the text. Beside each answer is a section reference for the concept. If you have difficulty with any of these exercises, be certain to at least read through the summary related to that section.

1. A classroom is 7 yd wide by 8 yd long. If the room is to be recarpeted with material costing $16 per square yard, find the cost of the carpeting.

2. Michael bought a washer-dryer combination that, with interest, cost $959. He paid $215 down and agreed to pay the balance in 12 monthly payments. Find the amount of each payment.

3. Evaluate the expression: $8 + 16 \div 4 \times 2$

4. Write the prime factorization for 168.

5. Find the greatest common factor of 12 and 20.

6. Arrange in order from smallest to largest: $\frac{5}{8}, \frac{3}{5}, \frac{2}{3}$

7. Multiply: $\frac{2}{3} \times 1\frac{4}{5} \times \frac{5}{8}$

8. Divide: $4\frac{1}{6} \div 10$

9. Find the least common multiple of 6, 15, and 20.

10. Add: $\frac{3}{5} + \frac{1}{6} + \frac{4}{15}$

11. Subtract: $7\frac{3}{8} - 3\frac{5}{6}$

12. You pay for purchases of $14.95, $18.50, $11.25, and $7 with a $70 check. How much cash will you have left?

13. Find the area of a rectangle with length 6.4 centimeters (cm) and width 4.35 cm.

14. Find the perimeter of a rectangle with length 6.4 cm and width 4.35 cm.

15. Find the decimal equivalent of $\frac{9}{16}$.

16. Write the decimal form of $\frac{7}{13}$. Round to the nearest thousandth.

Answers

1. ______________

2. ______________

3. ______________

4. ______________

5. ______________

6. ______________

7. ______________

8. ______________

9. ______________

10. ______________

11. ______________

12. ______________

13. ______________

14. ______________

15. ______________

16. ______________

Answers

17. ______

18. ______

19. ______

20. ______

21. ______

22. ______

23. ______

24. ______

25. ______

26. ______

27. ______

28. ______

29. ______

30. ______

31. ______

32. ______

33. ______

34. ______ 35. ______

36. ______ 37. ______

38. ______ 39. ______

17. Solve for the unknown in the following proportion: $\frac{15}{x} = \frac{10}{16}$

18. If the scale on a map is $\frac{1}{4}$ in. equals 20 mi, how far apart are two towns that are 5 in. apart on the map?

19. Felipe traveled 342 mi, using 19 gal of gas. At this rate, how far can he travel on 25 gal?

20. Write as a percent: 0.375

21. Write as a simplified fraction: 12.5%

22. What is 43% of 8,200?

23. 315 is what percent of 140?

24. 120% of what number is 180?

25. A home that was purchased for $125,000 increased in value by 14% over a 3-year period. What was its value at the end of that period?

26. Complete the statement: 5 days = ________ hours

27. Subtract:

$$\begin{array}{r} 4 \text{ min } 10 \text{ s} \\ -\ 2 \text{ min } 35 \text{ s} \\ \hline \end{array}$$

28. Find the sum of 8 lb 14 oz and 12 lb 13 oz.

29. Find the difference between 7 ft 2 in. and 4 ft 5 in.

30. Multiply and simplify: 8 × (2 h 40 min)

Complete each statement.

31. 43 cm = _______ m

32. 62 kg = _______ g

33. 740 mm = _______ cm

34. 14 L = _______ mL

35. 500 cm^3 = _______ L

Complete each statement. Round to the nearest tenth.

36. 8.3 mi = _______ km

37. 68 kg = _______ lb

Convert each temperature. Round to the nearest tenth.

38. 85°F = _______ °C

39. 9°C = _______ °F

chapter 8
> Make the Connection

CHAPTER 8

Geometry

INTRODUCTION

Since the Norman conquests in Europe, Norman architecture has been extremely popular. In the eleventh century (ca. 1066), institutions such as churches, castles, and universities began including Norman windows in their design, as shown in the photograph. Later designs, such as Gothic architecture, grew out of the Norman designs.

Norman windows are one example of composite geometric figures. These are figures that incorporate several simpler geometric figures. Another composite figure, a racetrack, is formed by attaching a semicircle to either end of a rectangle.

Whether determining the amount of fertilizer needed for the grass inside a racetrack, the amount of glass needed for a custom window, the amount of paint needed for the trim of a home design, or the amount of fluid that a bottle can hold, you will find that composite geometric figures are all around us.

The properties of geometric figures, such as perimeter and area, are critical for success in many trades. Architects, craftspeople, and contractors frequently work with two- and three-dimensional figures.

We examine the geometric properties of such figures throughout this chapter.

CHAPTER 8 OUTLINE

Chapter 8 :: Prerequisite Test 482
8.1 **Lines and Angles** 483
8.2 **Perimeter and Area** 498
8.3 **Circles and Composite Figures** 515
8.4 **Triangles** 526
8.5 **Square Roots and the Pythagorean Theorem** 540
Chapter 8 :: Summary / Summary Exercises / Self-Test / Cumulative Review :: Chapters 1–8 550

8 prerequisite test

Name ____________________

Section ________ Date ________

Answers

1. ____________________
2. ____________________
3. ____________________
4. ____________________
5. ____________________
6. ____________________
7. ____________________
8. ____________________
9. ____________________
10. ____________________
11. ____________________
12. ____________________

This prerequisite test provides some exercises requiring skills that you will need to be successful in the coming chapter. The answers for these exercises can be found in the back of this text. This prerequisite test can help you identify topics that you will need to review before beginning the chapter.

Evaluate, as indicated.

1. 5^2

2. 13^2

3. $3.14 \times (5)^2$

4. $\frac{1}{2}(8)(6 + 3)$

5. $2(5.6) + 2(3.1)$

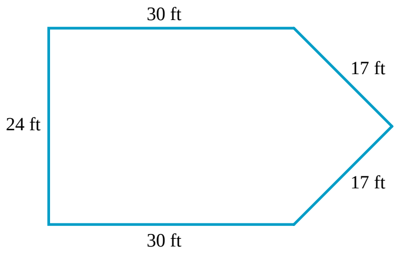
6. $\frac{1}{2}(9)(5)$

7. Evaluate 3.14×2.5^2 (round your result to the nearest hundredth).

8. Convert 18 cm to in. (round your result to the nearest tenth inch).

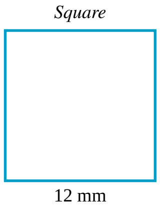
9. Convert 14 ft 8 in. to meters (round your result to the nearest tenth meter).

10. Find the perimeter of the figure shown.

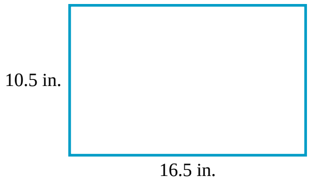


Find the area of each figure.

11.



12.



8.1

Lines and Angles

< **8.1 Objectives** >

1 > Recognize lines and line segments
2 > Recognize perpendicular and parallel lines
3 > Name an angle
4 > Determine whether an angle is right, acute, or obtuse
5 > Use a protractor to measure an angle
6 > Find the measures of angles using complements and supplements
7 > Find the measures of angles using vertical angles, alternate interior angles, and corresponding angles

NOTE

Geo means earth, just as it does in the words *geography* and *geology.*

Once the Egyptians and Babylonians had mastered the counting of their animals, they became interested in measuring their land. This is the foundation of geometry. Literally translated, *geometry* means earth measurement. Many of the topics we consider in geometry (topics such as angles, perimeter, and area) were first studied as part of surveying.

As is usually the case, we start the study of a new topic by learning some vocabulary. Most of the terms we will discuss are familiar to you. It is important that you understand what we mean when we use these words in the context of geometry.

We begin with the word *point.* A point is a location; it has no size and covers no area.

If we string points together forever, we create a *line.* In our studies we will consider only straight lines. We use arrowheads to indicate that a line goes on forever.

A piece of a line that has two endpoints is called a *line segment.*

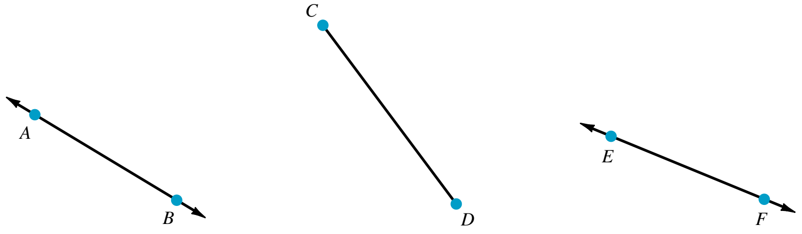
Example 1 — Recognizing Lines and Line Segments

< **Objective 1** >

NOTE

The capital letters are labels for points.

Label each figure as a line or a line segment.

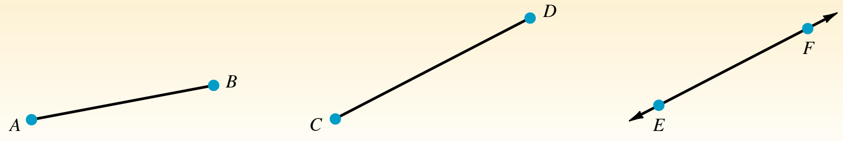


(*a*) (*b*) (*c*)

Both (*a*) and (*c*) continue forever in both directions. They are lines. Part (*b*) has two endpoints. It is a line segment.

Check Yourself 1

Label each figure as a line or a line segment.



(*a*) (*b*) (*c*)

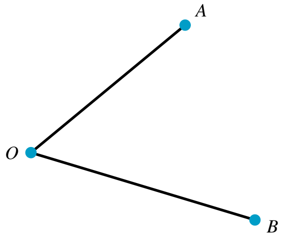
Definition

Angle

An **angle** is a geometric figure consisting of two line segments that share a common endpoint.

OA and *OB* are line segments. *O* is the vertex of the angle.

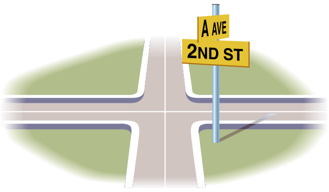
Surveyors use an instrument called a *transit*. A transit allows surveyors to measure angles so that, from a mathematical description, they can determine exactly where a property line is.



Definition

Perpendicular Lines

When two lines cross (or intersect), they form four angles. If the lines intersect such that four equal angles are formed, we say that the two lines are **perpendicular.**

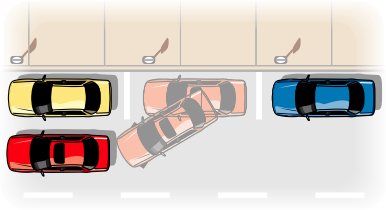


At most intersections, the two roads are perpendicular.

Definition

Parallel Lines

If two lines are drawn so that they never intersect (even if we extend the lines forever), we say that the two lines are **parallel.**

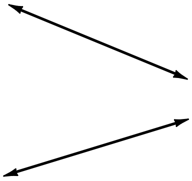


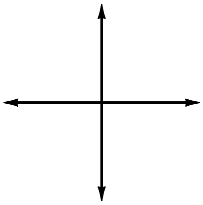
Parallel parking gets its name from the fact that the parking spot is parallel to the traffic lane.

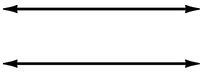
Example 2 Recognizing Parallel and Perpendicular Lines

< Objective 2 >

Label each pair of lines as parallel, perpendicular, or neither.


(*a*)

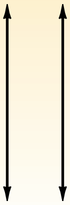

(*b*)


(*c*)

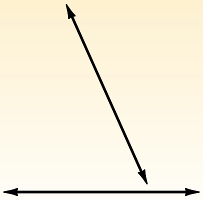
Although part (*a*) does not show the lines intersecting, if they were extended as the arrowheads indicate, they would. The lines of part (*b*) are perpendicular because the four angles formed are equal. Only the lines in part (*c*) are parallel.

Check Yourself 2

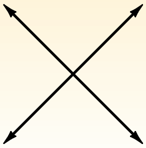
Label each pair of lines as parallel, perpendicular, or neither.



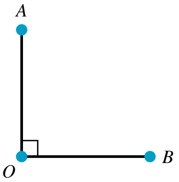
(*a*)



(*b*)



(*c*)



We call the angle formed by two perpendicular lines or line segments a *right angle.* We designate a right angle by forming a small square.

We can refer to a specific angle by naming three points. The middle point is the vertex of the angle.

Example 3 Naming an Angle

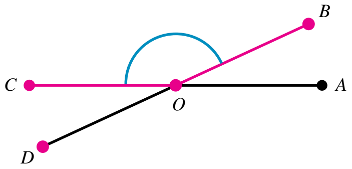
< Objective 3 >

NOTE

We could also call this angle ∠*BOC.*

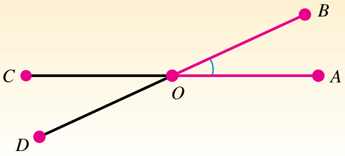
Name the angle highlighted in blue.

The vertex of the angle is *O*, and the angle begins at *C* and ends at *B*, so we would name the angle ∠*COB.*

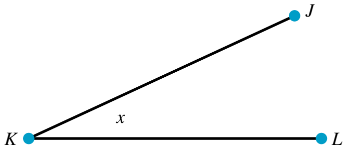


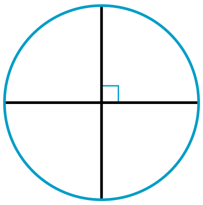
Check Yourself 3

Name the indicated angle.

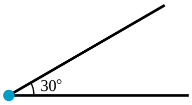


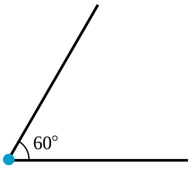
When there is no possibility of confusion, we may refer to an angle by naming only the label of the vertex, or perhaps using a symbol that appears in the angle. In the figure, the angle shown may be named ∠*JKL* or ∠*LKJ* (as noted earlier), or by simply writing ∠*K* or ∠*x.*

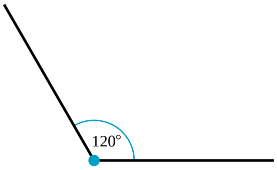


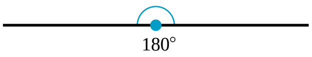


One way to measure an angle is to use a unit called a *degree.* There are 360 degrees (we write this as 360°) in a complete circle. You can see from the picture on the left that there are four right angles in a circle. If we divide 360° by 4, we find that each right angle measures 90°. Here are some other angles with their measurements.







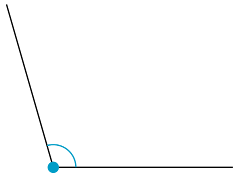


An *acute angle* measures between 0° and 90°. An *obtuse angle* measures between 90° and 180°. A *straight angle* measures 180°.

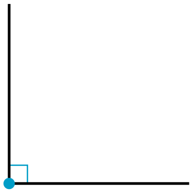
Example 4 Labeling Types of Angles

< Objective 4 >

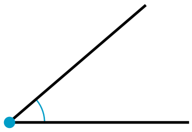
Label each angle as acute, obtuse, right, or straight.



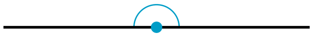
(*a*)



(*b*)



(*c*)

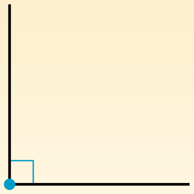


(*d*)

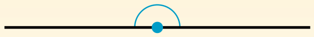
Part (a) is obtuse (the angle is more than 90°). Part (b) is a right angle (designated by the small square). Part (c) is an acute angle (it is less than 90°), and part (d) is a straight angle.

Check Yourself 4

Label each angle as an acute, obtuse, right, or straight angle.



(*a*)



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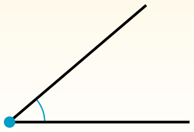
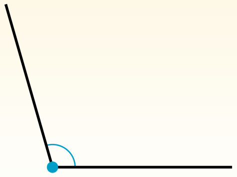


(*c*)

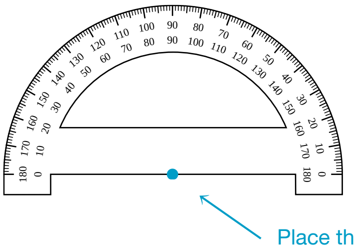


(*d*)

NOTE

Your protractor may show the degree measures in both directions.

When measuring an angle, we usually use a tool called a *protractor.*



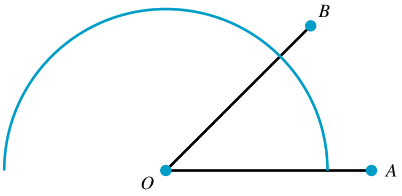
Place the protractor so that the vertex of the angle is here.

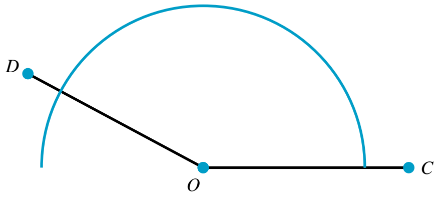
We read the protractor by placing one line segment of the angle at 0°. We then read the number that the other line segment passes through. This number represents the degree measurement of the angle. The point at the center of the protractor, the endpoint of the two line segments, is the vertex of the angle.

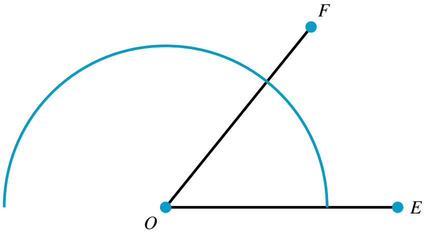
Example 5 Measuring an Angle

< Objective 5 >

Use a protractor to estimate the measurement for each angle.



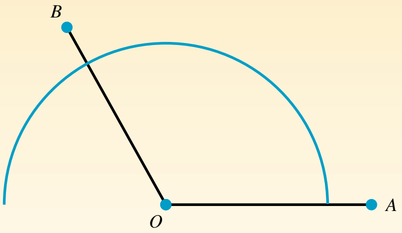




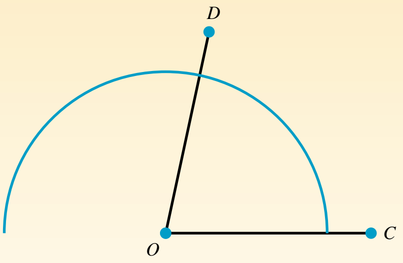
The measure of ∠*AOB* is 45°. The measure of ∠*COD* is 150°. The measure of ∠*EOF* is between 50° and 55°. We could estimate that it is a 52° angle.

Check Yourself 5

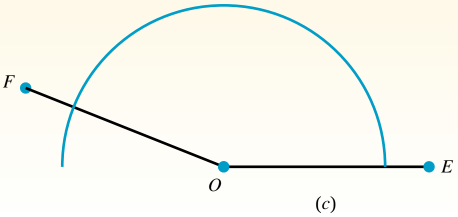
Use a protractor to estimate the measurement for each angle.



(*a*)



(*b*)



(*c*)

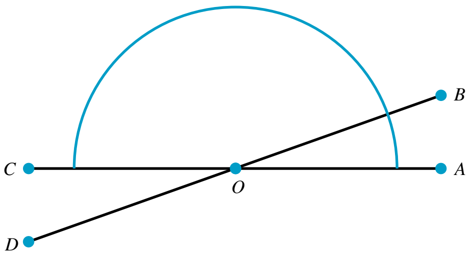
If we wish to refer to the degree measure of ∠*ABC*, we write *m*∠*ABC*.

Example 6 Measuring an Angle

Find *m*∠*AOB*.

NOTE

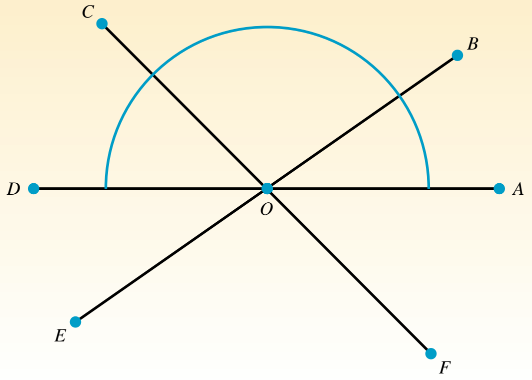
m∠*AOB* = 20° is read "the measure of angle *AOB* is 20 degrees."



Using a protractor, we find *m*∠*AOB* = 20°.

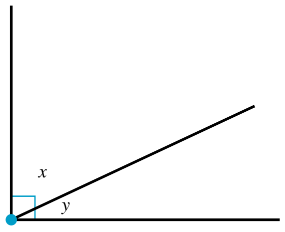
Check Yourself 6

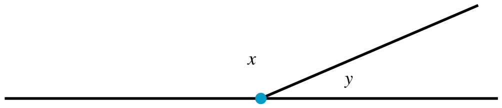
Find $m\angle AOC$.



If the sum of the measures of two angles is 90°, the two angles are said to be **complementary.** In the figure at the right, $\angle x$ and $\angle y$ are complementary angles.

If the sum of the measures of two angles is 180°, the two angles are said to be **supplementary.** In the figure below, $\angle x$ and $\angle y$ are supplementary angles.



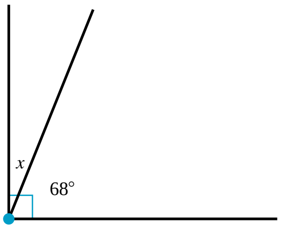


Example 7 Finding the Measure of an Angle

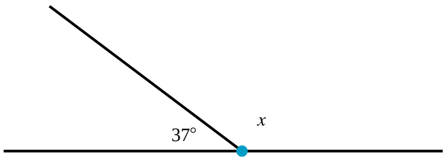
< Objective 6 >

In each case, find the measure of angle x.

(a)



(b)



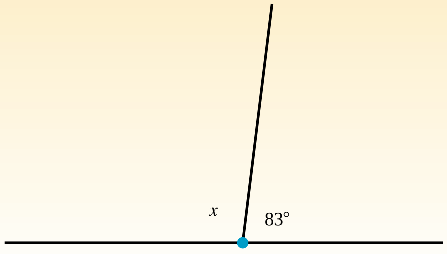
(a) Since $m\angle x + 68° = 90°$ (complementary angles), we must have

$$m\angle x = 90° - 68° = 22°$$

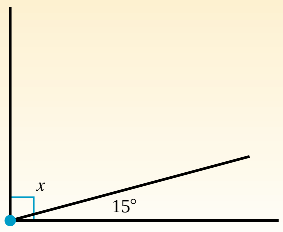
(b) Since $m\angle x + 37° = 180°$ (supplementary angles), we must have $m\angle x = 180° - 37° = 143°$

Check Yourself 7

In each case, find the measure of angle x.

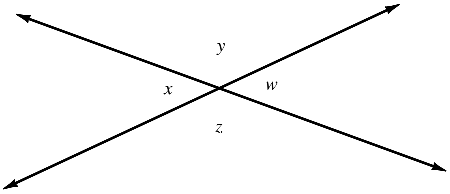


(a)



(b)

Suppose that two lines intersect, as shown in the figure, forming four angles.



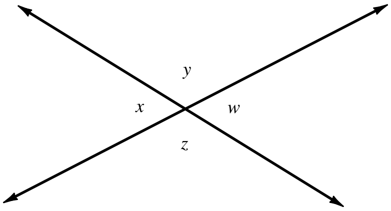
We say that $\angle x$ and $\angle w$ are **vertical angles.** Likewise, $\angle y$ and $\angle z$ are vertical angles. We have the following property:

Property

Vertical Angles Vertical angles have equal measure.

Example 8 Finding the Measures of Angles

< Objective 7 >



Suppose $m\angle w = 59°$. Find the measures of $\angle x$, $\angle y$, and $\angle z$.

Note that $m\angle x = 59°$, since $\angle x$ and $\angle w$ are vertical angles.

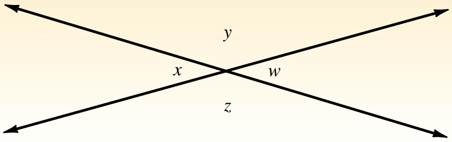
Since $\angle x$ and $\angle y$ are supplementary,

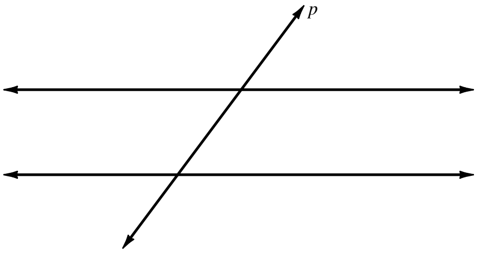
$$\begin{aligned} m\angle y &= 180° - m\angle x \\ &= 180° - 59° \\ &= 121° \end{aligned}$$

We note that $\angle y$ and $\angle z$ are vertical angles, so $m\angle z = m\angle y$. $m\angle z = 121°$.

Check Yourself 8

Find the measures of $\angle x$, $\angle y$, and $\angle z$, if $m\angle w = 32°$.

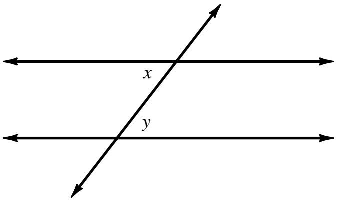




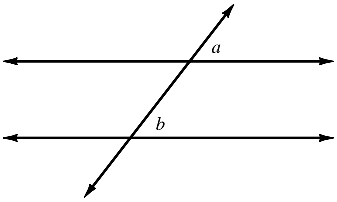
Suppose now that two parallel lines are intersected by a third line p as in the figure at the right.

The line p is called a **transversal.** Several angles are created in this situation.

In the figure below, the two indicated angles, $\angle x$ and $\angle y$, are called **alternate interior angles.**



In this figure, $\angle a$ and $\angle b$ are called **corresponding angles.**



Property

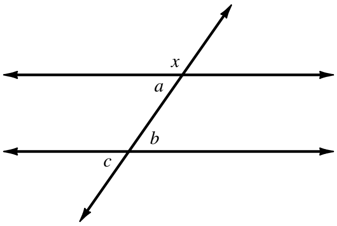
Parallel Lines and a Transversal

When two parallel lines are intersected by a transversal,

1. Alternate interior angles have equal measure.
2. Corresponding angles have equal measure.

Example 9 Finding the Measures of Angles

Given that $m\angle x = 125°$, find the measures of $\angle a$, $\angle b$, and $\angle c$.



$m\angle x + m\angle a = 180°$ Supplementary angles

So $m\angle a = 180° - 125° = 55°$.

$m\angle a = m\angle b$ Alternate interior angles

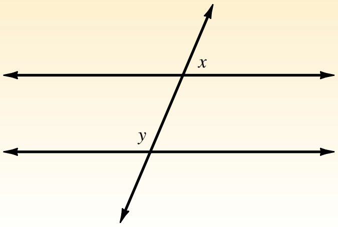
So $m\angle b = 55°$.

$m\angle a = m\angle c$ Corresponding angles

So $m\angle c = 55°$.

Check Yourself 9

Given that $m\angle x = 67°$, find $m\angle y$.



Check Yourself ANSWERS

1. **(a)** Line segment; **(b)** line segment; **(c)** line
2. **(a)** Parallel; **(b)** neither; **(c)** perpendicular
3. $\angle BOA$ or $\angle AOB$
4. **(a)** Right; **(b)** straight; **(c)** acute; **(d)** obtuse
5. **(a)** 120°; **(b)** 75°; **(c)** 160°
6. 135°
7. **(a)** 97°; **(b)** 75°
8. $m\angle x = 32°$; $m\angle y = 148°$; $m\angle z = 148°$
9. 113°

Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 8.1

(a) *Geo* means ________________, just as it does in the words *geography* and *geology*.

(b) If two lines intersect such that four right angles are formed, we say that the two lines are ________________.

(c) An ________________ angle measures between 90° and 180°.

(d) If the sum of the measures of two angles is 90°, the two angles are said to be ________________.

8.1 exercises

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Name ____________________

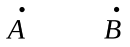
Section __________ Date __________

Answers

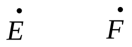
1. ____________________
2. ____________________
3. ____________________
4. ____________________
5. ____________________
6. ____________________
7. ____________________
8. ____________________
9. ____________________
10. ____________________
11. ____________________
12. ____________________
13. ____________________

< Objective 1 >

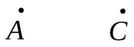
1. Draw line segment *AB*.



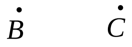
2. Draw line *EF*.



3. Draw line *AC*.

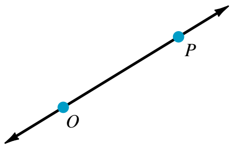


4. Draw line segment *BC*.



Identify each object as a line or line segment.

5.



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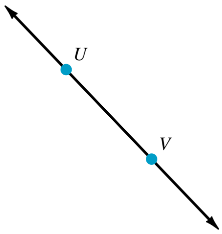

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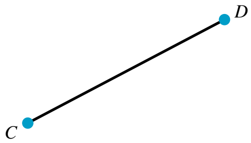

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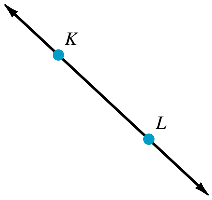

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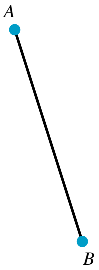

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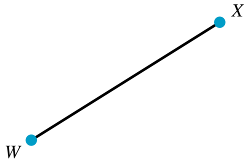

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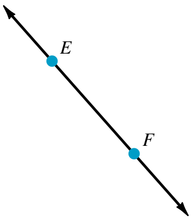

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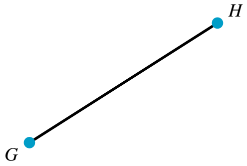
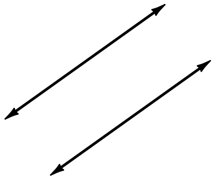
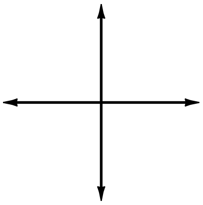


< Objective 2 >

13. Are the given lines parallel, perpendicular, or neither?



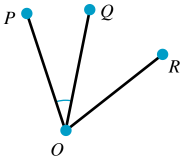
14. Are the given lines parallel, perpendicular, or neither?



< **Objective 3** >

Give an appropriate name for each indicated angle, using the given letters.

15.



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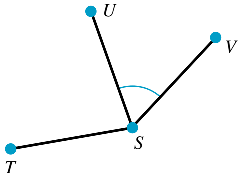
16.



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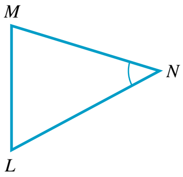

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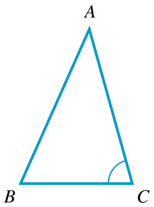

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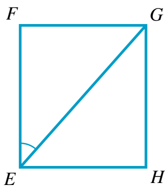

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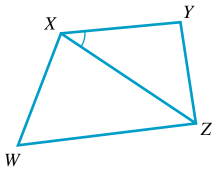
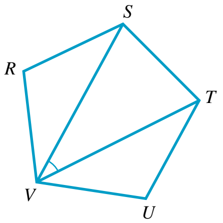


21.



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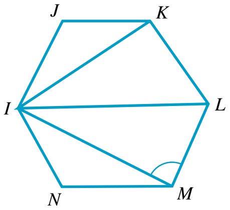
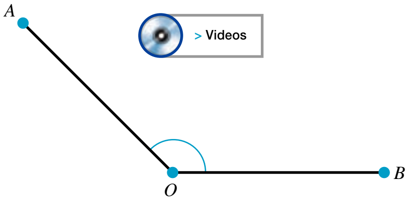


< **Objectives 4–5** >

Measure each angle with a protractor. Identify the angle as acute, right, obtuse, or straight.

23.



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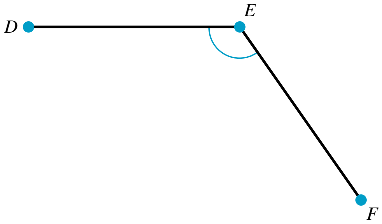


Answers

14. ________________

15. ________________

16. ________________

17. ________________

18. ________________

19. ________________

20. ________________

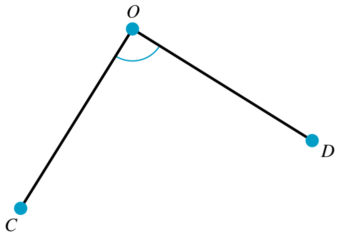
21. ________________

22. ________________

23. ________________

24. ________________

25.



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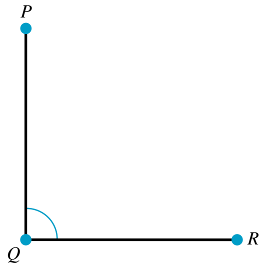

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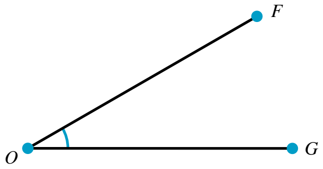

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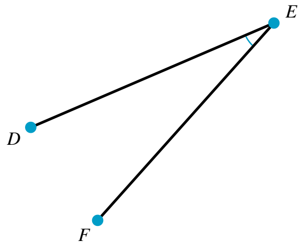


< Objective 6 >

Suppose that $m\angle x = 29°$.

29. Find the complement of $\angle x$.

30. Find the supplement of $\angle x$.

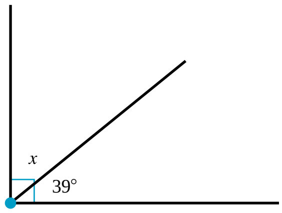
Suppose that $m\angle y = 53°$.

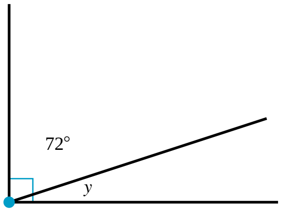
31. Find the supplement of $\angle y$.

32. Find the complement of $\angle y$.

33. Find $m\angle x$.

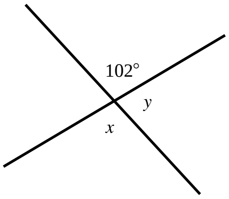
34. Find $m\angle y$.





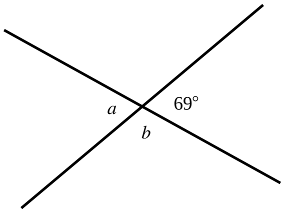
< Objective 7 >

35. Find $m\angle x$ and $m\angle y$.

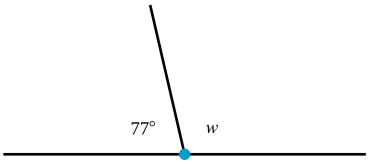


> Videos

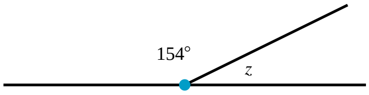
36. Find $m\angle a$ and $m\angle b$.



37. Find $m\angle w$.



38. Find $m\angle z$.



Answers

25. ____________

26. ____________

27. ____________

28. ____________

29. ____________

30. ____________

31. ____________

32. ____________

33. ____________

34. ____________

35. ____________

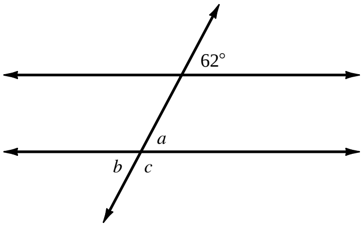
36. ____________

37. ____________

38. ____________

Find m∠a, m∠b, and m∠c.

39.



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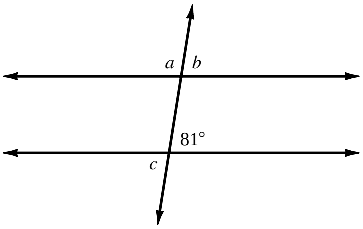

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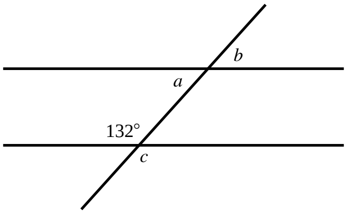
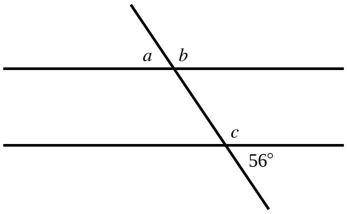


42.



Basic Skills | **Challenge Yourself** | Calculator/Computer | Career Applications | Above and Beyond

Label exercises 43 to 48 as **true** *or* **false.**

43. There are exactly two different line segments that can be drawn through two points.

44. There are exactly two different lines that can be drawn through two points.

45. Two opposite sides of a square are parallel line segments.

46. Two adjacent sides of a square are perpendicular line segments.

47. ∠*ABC* will always have the same measure as ∠*CAB*.

48. Two acute angles have the same measure.

For each angle described, give its measure in degrees. Sketch the angle.

49. ∠*A* represents $\frac{1}{6}$ of a complete circle.

50. ∠*B* represents $\frac{1}{3}$ of a complete circle.

51. ∠*C* represents $\frac{7}{12}$ of a complete circle.

52. ∠*D* represents $\frac{11}{12}$ of a complete circle.

Answers

39. ______

40. ______

41. ______

42. ______

43. ______

44. ______

45. ______

46. ______

47. ______

48. ______

49. ______

50. ______

51. ______

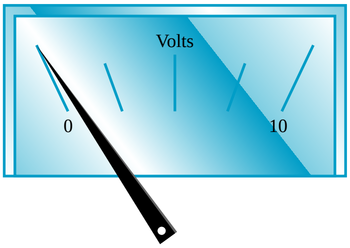
52. ______

Answers

53. ____________________

54. ____________________

53. Electronics This is a picture of an analog voltmeter. The needle rotates clockwise as the voltage increases. The total angular distance covered by the needle as it travels from 0 volts (V) to 10 V is 100°.

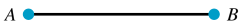


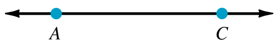
(a) Assuming that the angle is proportional to the voltage, what angular distance do you estimate the needle would travel from the initial 0-V position if 5 V are measured?

(b) If the angular distance traveled from the original 0-V location is 85°, estimate the voltage.

54. Automotive Technology In a 4-stroke engine, the crankshaft turns two complete turns for each cycle of a cylinder. How many degrees does the crankshaft turn for one cycle?

Answers

1. 

3. 

5. Line

7. Line segment **9.** Line segment **11.** Line **13.** Parallel

15. $\angle POQ$ or $\angle QOP$ **17.** $\angle MNL$ or $\angle LNM$ **19.** $\angle FEG$ or $\angle GEF$

21. $\angle SVT$ or $\angle TVS$ **23.** 135°; obtuse **25.** 90°; right

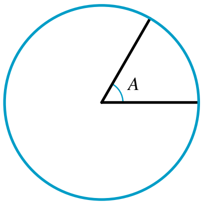
27. 30°; acute **29.** 61° **31.** 127° **33.** 51°

35. $m\angle x = 102°$; $m\angle y = 78°$ **37.** 103°

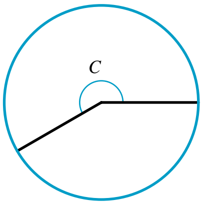
39. $m\angle a = 62°$; $m\angle b = 62°$; $m\angle c = 118°$

41. $m\angle a = 48°$; $m\angle b = 48°$; $m\angle c = 132°$ **43.** False **45.** True

47. False **49.** 60°;



51. 210°;



53. **(a)** 50°; **(b)** 8.5 V

Activity 20 ::
Know the Angles

Your group members might wish to share these tasks:

1. Draw any triangle, using a ruler. With your protractor, carefully measure the three interior angles, and find their sum.

$\angle A =$ $\angle B =$ $\angle C =$ Sum $=$

Do this again with another triangle of a different shape.

$\angle A =$ $\angle B =$ $\angle C =$ Sum $=$

What do you notice about the sums of the angles?

Make a conjecture about the sum of the angles of *any* triangle. Then test your conjecture on another triangle.

2. A **quadrilateral** is a four-sided polygon. Draw any quadrilateral and measure the four interior angles with a protractor. Record these and find their sum.

$\angle A =$ $\angle B =$ $\angle C =$ $\angle D =$ Sum $=$

Make a conjecture concerning the sum of the interior angles of *any* quadrilateral. Then test your conjecture on another quadrilateral.

3. A **pentagon** is a five-sided polygon. Draw any pentagon and measure the five interior angles with a protractor. Record these and find their sum.

$\angle A =$ $\angle B =$ $\angle C =$ $\angle D =$ $\angle E =$
Sum $=$

Make a conjecture concerning the sum of the interior angles of *any* pentagon. Then test your conjecture on another pentagon.

4. A **hexagon** is a six-sided polygon. Draw any hexagon and measure the six interior angles with a protractor. Record these and find their sum.

$\angle A =$ $\angle B =$ $\angle C =$ $\angle D =$ $\angle E =$
$\angle F =$ Sum $=$

Make a conjecture concerning the sum of the interior angles of *any* hexagon. Then test your conjecture on another hexagon.

5. Now try to generalize. Suppose we have a polygon having k sides. Give a formula for the sum of the interior angles. Sum $=$

8.2

Perimeter and Area

< **8.2 Objectives** >

1 > Identify polygons based on sides
2 > Find the perimeter of a polygon
3 > Find the area of a polygon
4 > Convert units of area

You have been working with many of the important concepts in geometry throughout this text. As early as Section 1.2, you were finding the perimeters of squares, rectangles, and other figures. By Section 1.5, you were even finding the areas of many of these objects.

In this section, we review these geometry concepts, add some new ones, and tie them together as a single unit. We begin by identifying some of the figures that we work with.

Definition

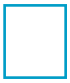
Polygon	A polygon is a simple closed figure with three or more sides in which each side is a line segment.

By this, we mean that there are no openings, that nonadjacent sides do not intersect, and that there are no curves.

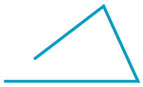
Example 1 Identifying Polygons

Determine whether each object is a polygon or not. Explain your answer.

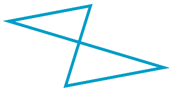
(a)



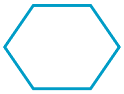
(b)



(c)



(d)



(a) This is a polygon because there are four straight sides and the figure is closed.

(b) This is not a polygon because the geometric figure is not closed.

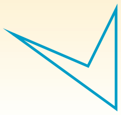
(c) This is not a polygon because the sides intersect (it is not simple); it is two polygons (triangles) that meet at a point.

(d) This is a polygon. The figure has six sides and it is closed and simple.

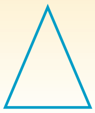
Check Yourself 1

Determine which figures are polygons. Explain your answers.

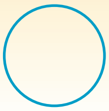
(a)



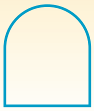
(b)



(c)



(d)



You have probably heard the names of the more common polygons.

Definition

Types of Polygons

NOTE

Other polygon names, based on the number of sides, include:
Pentagon: Five sides
Hexagon: Six sides
Octagon: Eight sides

Stop signs are typically octagons.

Triangle: Any polygon with exactly three sides is a triangle.

Quadrilateral: Any polygon with exactly four sides is a quadrilateral.

Parallelogram: A quadrilateral in which opposite sides are parallel is a parallelogram.

Equivalently, a parallelogram is a quadrilateral in which the opposite sides have the same length.

Rectangle: A quadrilateral in which the interior angles are right angles is a rectangle.

Square: A rectangle in which all four sides have the same length is a square.

A square is an example of a **regular polygon.** A regular polygon is one in which all of the sides have the same length, and all of the interior angles have the same measure.

Example 2 Identifying Polygons

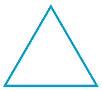
< Objective 1 >

NOTE

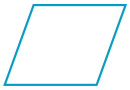
We will take a more in-depth look at triangles in Section 8.4.

Name the polygon and determine if it is regular. If it is not on the list, give the number of sides.

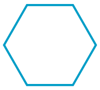
(a)



(b)



(c)



(a) This is a triangle because it has three sides.
The sides are all the same length, so it is regular.
When a triangle is regular, we call it equilateral.

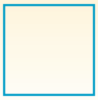
(b) This is a parallelogram, but it is not regular.

(c) This six-sided polygon (commonly called a hexagon) is regular.

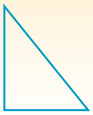
Check Yourself 2

Name the polygon and determine if it is regular. If it is not on the list, give the number of sides.

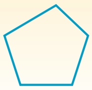
(a)



(b)



(c)



There are two important properties that we associate with polygons: perimeter and area. We will look at the perimeter of a polygon first.

Definition

The Perimeter of a Polygon

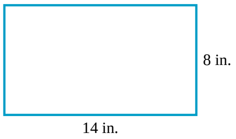
The perimeter of a polygon is the distance around the polygon.
It is found by taking the sum of the lengths of its sides.

Example 3 Finding the Perimeter of a Polygon

< Objective 2 >

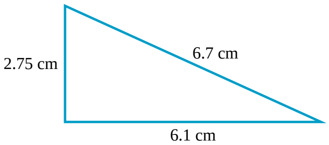
Find the perimeter of each figure.

(a)



The opposite sides of a rectangle have the same length, so its perimeter is $8 + 14 + 8 + 14 = 44$ in.

(b)

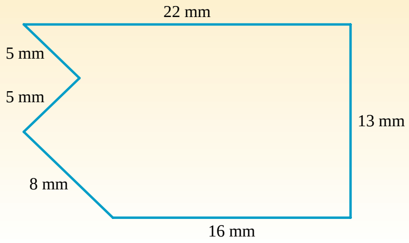


The perimeter of this triangle is $2.75 + 6.7 + 6.1 = 15.55$ cm.

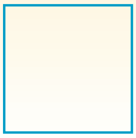
Check Yourself 3

Find the perimeter of each polygon.

(a)

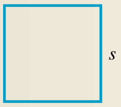


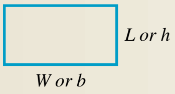
(b) Each side of the square measures $\frac{4}{5}$ in.



Property

Perimeter Formulas





Square

Each side has length s.

$P = 4 \cdot s$

The perimeter of a square is 4 times the length of a side.

Rectangle

If we label the sides as L and W (length and width),

$P = 2 \cdot L + 2 \cdot W = 2(L + W)$

In words, the perimeter of a rectangle is the sum of twice the length and twice the width. Equivalently, the perimeter is twice the sum of the length and width.

Adjacent sides have length b and h (base and height).

$P = 2 \cdot b + 2 \cdot h = 2(b + h)$

Note: All four ways of writing the rectangle formula give the same result.

Regular Polygon

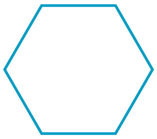
Each of the n sides has length s.

$P = n \cdot s$

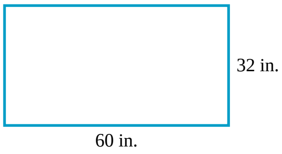
Example 4 Finding the Perimeter of a Polygon

Find the perimeter of each polygon.

(a) Each side measures 2 cm.



(b)



(a) There are six sides, so the perimeter is

$$\begin{aligned} P &= 6 \cdot s \\ &= 6(2) \\ &= 12 \end{aligned}$$

The perimeter is 12 cm.

(b) We use the rectangle formula with a 60-in. length and 32-in. width.

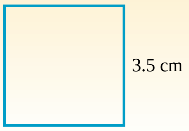
$$\begin{aligned} P &= 2 \cdot L + 2 \cdot W \\ &= 2(60) + 2(32) \\ &= 120 + 64 \\ &= 184 \end{aligned}$$

Its perimeter is 184 in.

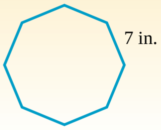
Check Yourself 4

Find the perimeter of each regular polygon.

(a)



(b)



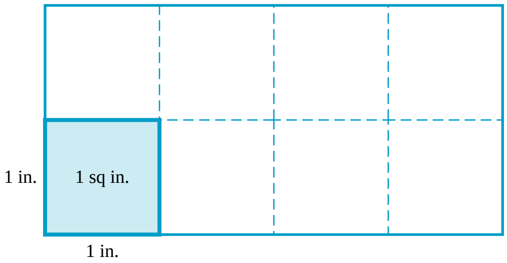
Another important property of a polygon is its **area.** By area we mean the amount of space that a two-dimensional object takes up, or how much it takes to fill it.

While perimeter is measured in one dimension because it is a length, area is a two-dimensional measure (space filled). If our units are inches, for example, then we measure area in the number of one-inch squares that it takes to "cover" the interior space of the figure. Example 5 illustrates this idea.

Example 5 Computing Area

< Objective 3 >

Find the area of the rectangle shown.



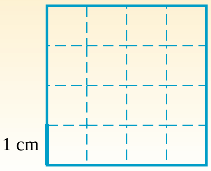
Because the rectangle holds 8 one-inch squares, its area is 8 sq in.

RECALL

We also use exponent notation: 8 in.^{2}.

Check Yourself 5

Find the area of the square shown.



As you learned in Section 1.5, we can find the area of a rectangle by taking the product of the lengths of its base and height. In Example 5, we had two rows of 4 1-inch squares, so we had

$A = 2 \text{ in.} \times 4 \text{ in.} = 8 \text{ in.}^2$

Property

The Area of a Rectangle

If the dimensions of the rectangle are given as length L and width W,

$A = L \cdot W$

In words, the area is the product of the length and the width.

The area of a rectangle with base b and height h is

$A = b \cdot h$

Because the base and height of a square have the same length, we describe its area differently.

Property

The Area of a Square

The area of a square with side s is

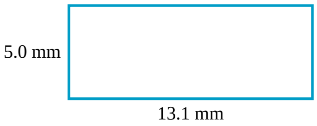
$A = s^2$

The area of a square is the square of the length of a side.

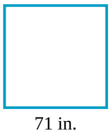
Example 6 Computing Area

Find the area of each figure.

(a)



(b)



(a) We find the product of the dimensions.

$$\begin{aligned} A &= L \cdot W \\ &= (13.1\text{ mm})(5.0\text{ mm}) \\ &= 65.5\text{ mm}^2 \end{aligned}$$

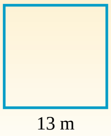
(b) According to the formula for the area of a square, we square the length of a side.

$$\begin{aligned} A &= s^2 \\ &= (71\text{ in.})^2 \\ &= 5{,}041\text{ in.}^2 \end{aligned}$$

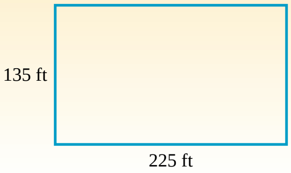
Check Yourself 6

Find the area of each figure.

(a)



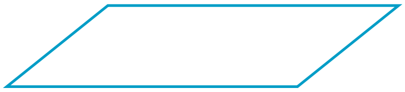
(b)



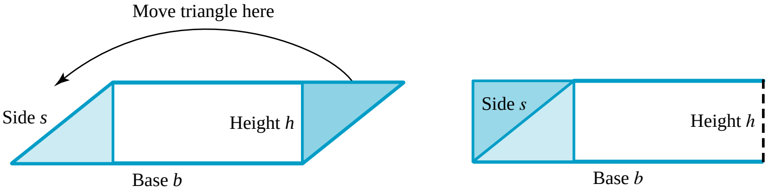
NOTE

Both rectangles and squares are parallelograms.

Two other important figures are parallelograms and triangles. The figure shown at the right is a **parallelogram.** Opposite sides are parallel and have the same length.



Because opposite sides have the same length, if we include the height, and "cut off" a triangle from one side, it fits perfectly onto the other side, giving us a rectangle.



You should see that even though we use the length of a side to compute the perimeter of a parallelogram, we use the height to find its area.

Property

The Perimeter and Area of a Parallelogram

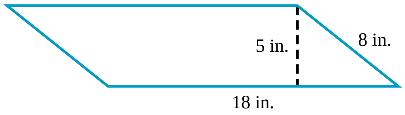
The **perimeter** of a parallelogram can be found by adding the lengths of the sides.

$P = 2 \cdot b + 2 \cdot s = 2(b + s)$

The **area** of a parallelogram can be found by taking the product of the length of its base and its height.

$A = b \cdot h$

Example 7 Parallelograms



Find the perimeter and area of the parallelogram shown.

The parallelogram's base is 18 in., its side measures 8 in., and its height is 5 in.

Perimeter

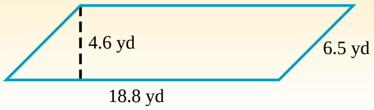
$$\begin{aligned} P &= 2 \cdot b + 2 \cdot s \\ &= 2(18 \text{ in.}) + 2(8 \text{ in.}) \\ &= 36 \text{ in.} + 16 \text{ in.} \\ &= 52 \text{ in.} \end{aligned}$$

Area

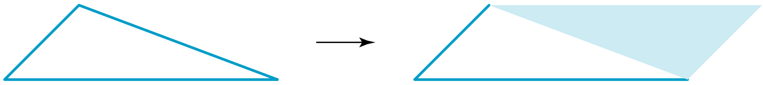
$$\begin{aligned} A &= b \cdot h \\ &= (18 \text{ in.})(5 \text{ in.}) \\ &= 90 \text{ in.}^2 \end{aligned}$$

Check Yourself 7

Find the perimeter and area of the parallelogram shown.



Triangles are three-sided polygons. Every triangle can be formed by halving some parallelogram along a diagonal.



Second copy of triangle rotated to fit; they form a parallelogram.

We can think of a triangle as half a parallelogram, giving the following formula.

Property

The Area of a Triangle

The **area** of a triangle can be found by taking half the product of the length of its base and its height.

$$A = \frac{1}{2} \cdot b \cdot h$$

Example 8 Triangles

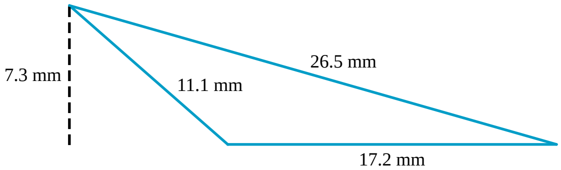
NOTES

The base and height of a **right triangle** are the sides that form the right angle.

We will look at triangles more closely in Section 8.4.

Find the perimeter and area of the triangle shown.

To find the perimeter of the triangle, we add the lengths of the sides.



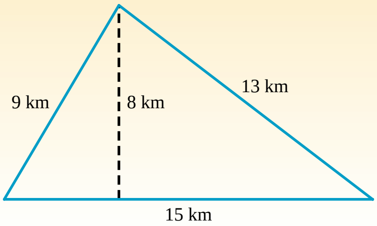
$$P = 11.1\text{ mm} + 17.2\text{ mm} + 26.5\text{ mm}$$
$$= 54.8\text{ mm}$$

We use the triangle's base, 17.2 mm, and its height, 7.3 mm, to find its area.

$$A = \frac{1}{2} b \cdot h$$
$$= \frac{1}{2}(17.2\text{ mm})(7.3\text{ mm})$$
$$= 62.78\text{ mm}^2$$

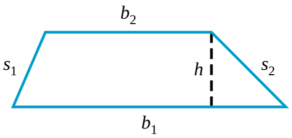
Check Yourself 8

Find the perimeter and area of the triangle shown.

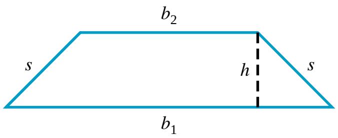


Moving on from triangles, we look at a quadrilateral called a **trapezoid.** A trapezoid is a four-sided polygon with exactly one pair of parallel sides.

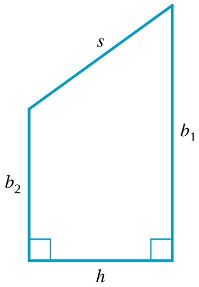
Several such figures are shown below. The first is a general trapezoid. The second is an *isosceles trapezoid* because the nonparallel sides are the same length. The third is a *right trapezoid* because one of its sides is perpendicular to the parallel sides.



Trapezoid



Isosceles Trapezoid



Right Trapezoid

We find a trapezoid's perimeter in the usual manner, by adding the lengths of all four sides. To find the area, we use the formula below.

Property

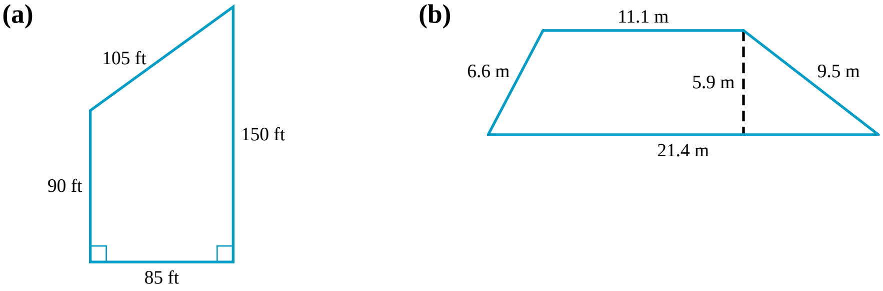
The Area of a Trapezoid

The area of a trapezoid is given by

$$A = \frac{1}{2}h(b_1 + b_2)$$

Example 9 Trapezoids

Find the perimeter and area of each trapezoid.


(a)
105 ft
150 ft
90 ft
85 ft
(b)
11.1 m
6.6 m
5.9 m
9.5 m
21.4 m

(a) The perimeter is simply the sum of the sides.

$$P = 90 \text{ ft} + 105 \text{ ft} + 150 \text{ ft} + 85 \text{ ft} = 430 \text{ ft}$$

To find the area, we identify the "height" as the 85-ft side.

$$\begin{aligned} A &= \frac{1}{2}h(b_1 + b_2) \\ &= \frac{1}{2}(85)(90 + 150) \\ &= 10{,}200 \text{ ft}^2 \end{aligned}$$

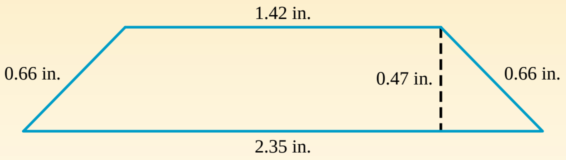
(b) $P = 6.6 \text{ m} + 11.1 \text{ m} + 9.5 \text{ m} + 21.4 \text{ m} = 48.6 \text{ m}$

$$\begin{aligned} A &= \frac{1}{2}h(b_1 + b_2) \\ &= \frac{1}{2}(5.9 \text{ m})(21.4 \text{ m} + 11.1 \text{ m}) \\ &= 95.875 \text{ m}^2 \end{aligned}$$

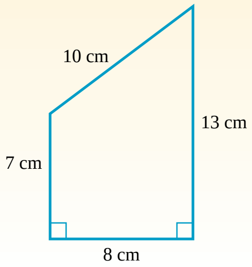
Check Yourself 9

Find the perimeter and area of each trapezoid (round to the nearest tenth, if necessary).

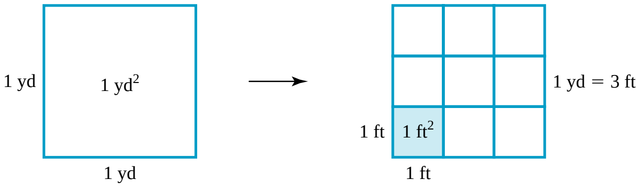
(a)


1.42 in.
0.66 in.
0.47 in.
0.66 in.
2.35 in.

(b)


10 cm
13 cm
7 cm
8 cm

You learned to convert measurements in Chapter 7. We need another step, when converting square units. Consider the square yards shown.



NOTE

$3^2 = 9$
$12^2 = 144$

Because 1 yd = 3 ft, we can see that 1 yd^2 = 9 ft^2. Similarly, 1 ft^2 = 144 in.^2 More generally, we need to square conversion factors when converting units of area (square units).

Property

Conversion Factors

U.S. System of Measurement	Metric System of Measurement	Conversions
$1\ \text{ft}^2 = 144\ \text{in.}^2$	$1\ \text{cm}^2 = 100\ \text{mm}^2$	$1\ \text{in.}^2 = (2.54)^2\ \text{cm}^2$
$1\ \text{yd}^2 = 9\ \text{ft}^2$	$1\ \text{m}^2 = 10{,}000\ \text{cm}^2$	$= 6.45\ \text{cm}^2$
$1\ \text{mi}^2 = 27{,}878{,}400\ \text{ft}^2$	$= 1{,}000{,}000\ \text{mm}^2$	$1\ \text{cm}^2 = (0.39)^2\ \text{in.}^2$
$= 3{,}097{,}600\ \text{yd}^2$	$1\ \text{km}^2 = 1{,}000{,}000\ \text{m}^2$	$= 0.15\ \text{in.}^2$
$= 640$ acres	1 hectare (ha) $= (100)^2\ \text{m}^2$	$1\ \text{mi}^2 = (1.61)^2\ \text{km}^2$
1 acre $= 43{,}560\ \text{ft}^2$	$= 10{,}000\ \text{m}^2$	$= 2.59\ \text{km}^2$
$= 4{,}840\ \text{yd}^2$		$1\ \text{km}^2 = (0.62)^2\ \text{mi}^2$
		$= 0.38\ \text{mi}^2$

Example 10 Converting Area Measurements

< Objective 4 >

(a) A room measures 12 ft by 15 ft. How many square yards of carpeting are needed to cover the floor?

Begin by finding the area of the floor.

$$\begin{aligned} A &= L \cdot W \\ &= (12)(15) \\ &= 180\ \text{ft}^2 \end{aligned}$$

We then convert to square yards by using the appropriate conversion factor.

$$\begin{aligned} 180\ \cancel{\text{ft}^2} \times \frac{1\ \text{yd}^2}{9\ \cancel{\text{ft}^2}} &= \frac{180}{9}\ \text{yd}^2 \\ &= 20\ \text{yd}^2 \end{aligned}$$

(b) A rectangular field is 220 yd long and 110 yd wide. Find its area, in acres.

Begin by finding the area, in square yards.

$$\begin{aligned} A &= L \cdot W \\ &= (220\ \text{yd})(110\ \text{yd}) \\ &= 24{,}200\ \text{yd}^2 \end{aligned}$$

Then, we use the conversion factor, above, to convert this to acres.

$$\begin{aligned} 24{,}200\ \cancel{\text{yd}^2} \times \frac{1\ \text{acre}}{4{,}840\ \cancel{\text{yd}^2}} &= \frac{24{,}200}{4{,}840}\ \text{acre} \\ &= 5\ \text{acres} \end{aligned}$$

Check Yourself 10

(a) A hallway is 27 ft long and 4 ft wide. How many square yards of linoleum are needed to cover the hallway?

(b) A proposed site for an elementary school is 200 yd by 198 yd. Find its area in acres (round to the nearest tenth acre).

Check Yourself ANSWERS

1. **(a)** Yes; **(b)** yes; **(c)** no; **(d)** no
2. **(a)** Square; **(b)** triangle, not regular; **(c)** 5 sides (pentagon), not regular
3. **(a)** 69 mm; **(b)** $\frac{16}{5}$ in. **4.** **(a)** 14 cm; **(b)** 56 in. **5.** 16 cm^2
6. **(a)** 169 m^2; **(b)** 30,375 ft^2 **7.** $P = 50.6$ yd^2; $A = 86.48$ yd^2
8. $P = 37$ km; $A = 60$ km^2
9. **(a)** $P = 5.1$ in.; $A = 0.9$ in.^2; **(b)** $P = 38$ cm; $A = 80$ cm^2
10. **(a)** 12 yd^2; **(b)** 8.2 acres

Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 8.2

(a) A ______________ is a polygon with exactly three sides.

(b) A polygon is ______________ if all of the sides have the same length and all of the interior angles have the same measure.

(c) You can find the perimeter of a polygon by taking the ______________ of the lengths of the sides of the polygon.

(d) One square yard is equal to ______________ square feet.

8.2 exercises

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Name ____________________

Section __________ Date __________

Answers

1. ____________________

2. ____________________

3. ____________________

4. ____________________

5. ____________________

6. ____________________

7. ____________________

8. ____________________

9. ____________________

10. ____________________

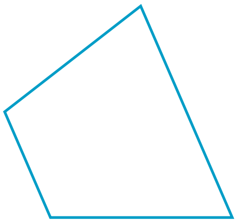
11. ____________________

12. ____________________

< Objective 1 >

Determine whether each object is a polygon or not. If an object is a polygon, determine its type and whether or not it is regular.

1.



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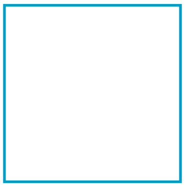

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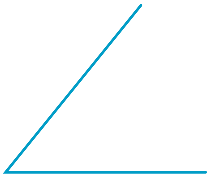

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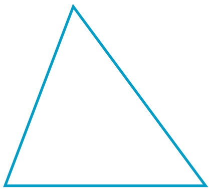

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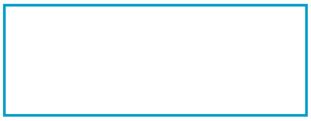
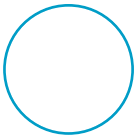


6.



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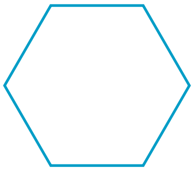

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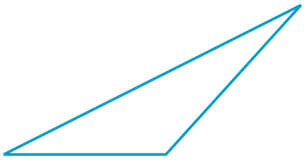
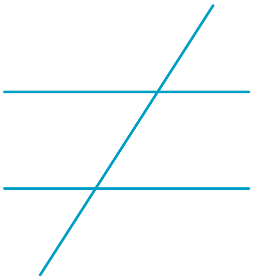


9.



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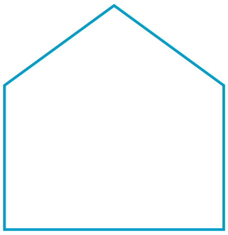
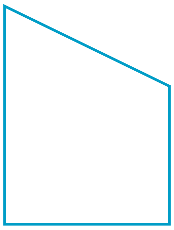


11.



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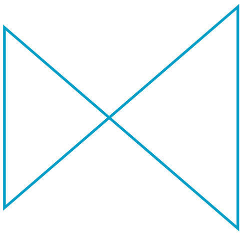

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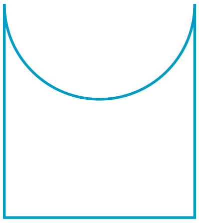

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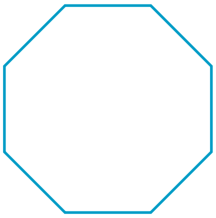

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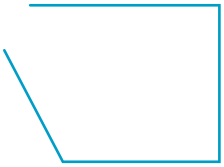
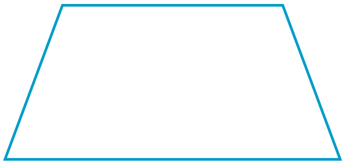


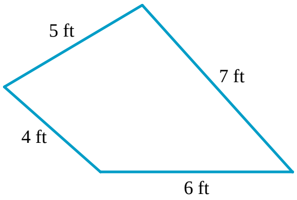
16.



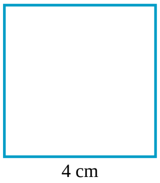
< **Objective 2** >

Find the perimeter of each figure.

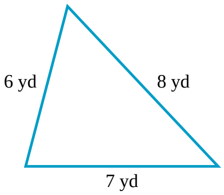
17.


5 ft
7 ft
4 ft
6 ft

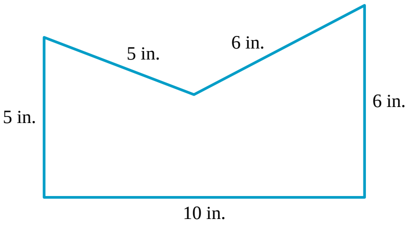
18.



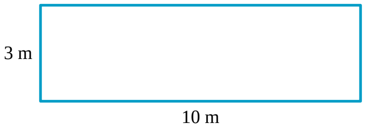
19.



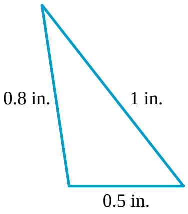
20.


5 in.
6 in.
5 in.
6 in.
10 in.

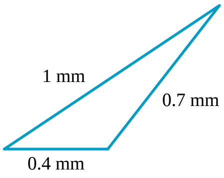
21.



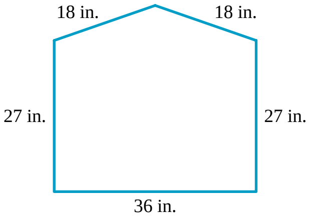
22.


0.8 in.
1 in.
0.5 in.

23.


1 mm
0.7 mm
0.4 mm

24.


18 in.
18 in.
27 in.
27 in.
36 in.

Answers

13.

14.

15.

16.

17.

18.

19.

20.

21.

22.

23.

24.

Answers

25. ______________________

26. ______________________

27. ______________________

28. ______________________

29. ______________________

30. ______________________

31. ______________________

32. ______________________

33. ______________________

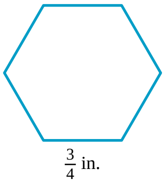
34. ______________________

35. ______________________

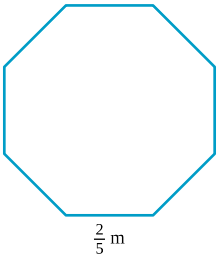
36. ______________________

37. ______________________

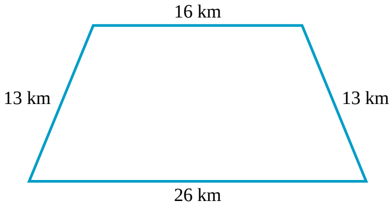
25.



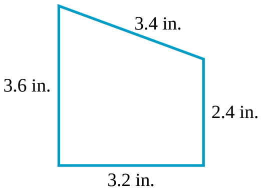
26.



27.


16 km
13 km
13 km
26 km

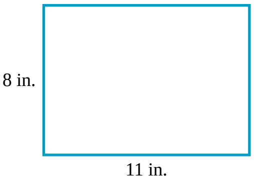
28.


3.4 in.
3.6 in.
2.4 in.
3.2 in.

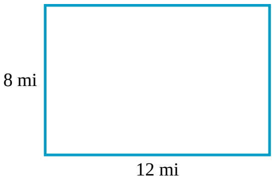
< **Objective 3** >

Find the area of each figure.

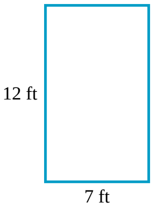
29.



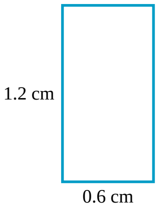
30.



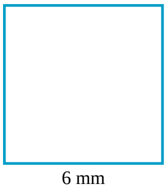
31.



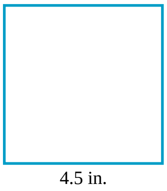
32.



33.



34.



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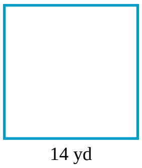
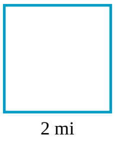
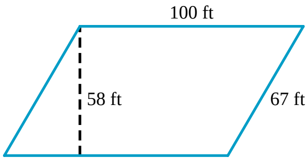


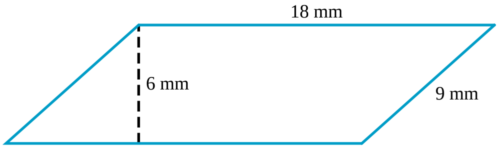
36.



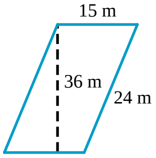
37.


100 ft
58 ft
67 ft

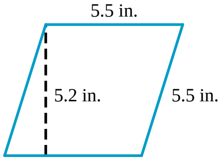
38.


18 mm
6 mm
9 mm

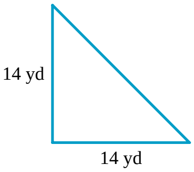
39.



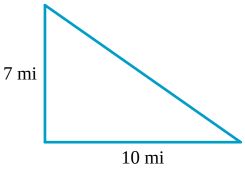
40.


5.5 in.
5.2 in.
5.5 in.

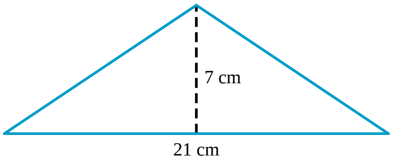
41.



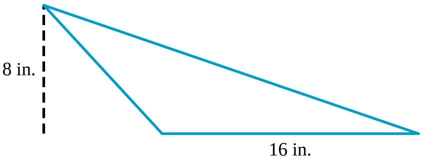
42.



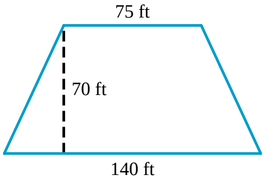
43.



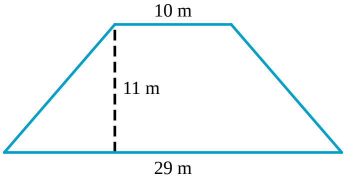
44.



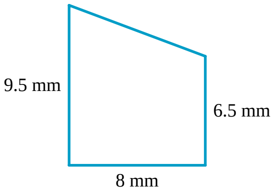
45.


75 ft
70 ft
140 ft

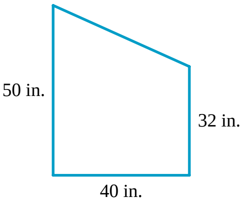
46.



47.


9.5 mm
6.5 mm
8 mm

48.


50 in.
32 in.
40 in.

< **Objective 4** >

Convert each measurement.

49. 15 ft^2 to in.^{2}

50. 1,050 in.^{2} to ft^2

51. 360 ft^2 to yd^2

52. 20 yd^2 to in.^{2}

53. 8 km^2 to m^2

54. 10,000,000 m^2 to km^2

55. 30 m^2 to cm^2

56. 4,200 mm^2 to cm^2

57. 16 in.^{2} to cm^2

58. 6 km^2 to mi^2

Answers

38.

39.

40.

41.

42.

43.

44.

45.

46.

47.

48.

49.

50.

51.

52.

53.

54.

55.

56.

57.

58.

Answers

59. ____________

60. ____________

61. ____________

62. ____________

63. ____________

64. ____________

65. ____________

66. ____________

67. ____________

68. ____________

69. ____________

70. ____________

71. ____________

72. ____________

73. ____________

74. ____________

75. ____________

76. ____________

Determine whether each statement is **true** *or* **false.**

59. A polygon can have as few as two sides.

60. All regular triangles are equilateral triangles.

61. All regular quadrilaterals are squares.

62. Circles are a type of polygon.

Fill in each blank with **always, sometimes,** *or* **never.**

63. Two copies of the same triangle can __________ be fit together to form a parallelogram.

64. One of the sides of a trapezoid __________ gives the height of the trapezoid.

Convert the following acre and hectare measurements (round your results to one decimal place).

65. 18,000 acres to mi^2

66. 4,200,000 ft^2 to acres

67. 12,000,000 m^2 to ha

68. 1,500 ha to km^2

69. 2,500 ha to acres

70. 2,500 acres to ha

Solve each application.

71. **CRAFTS** A Tetra-Kite uses 12 triangular pieces of plastic for its surface. Each triangle has a 12-in. base and a 12-in. height. How much material (sq in.) is needed for such a kite?

72. **CONSTRUCTION** How much does it cost to carpet a 12-ft by 18-ft recreation room with $15 per sq yd carpeting? *Hint:* You cannot purchase fractions of a square yard of carpeting.

73. **CONSTRUCTION** A triangular hole is to be punched into a piece of sheet metal. The base of the triangle is 1.6 in. and the height is 0.85 in. What is the area of the hole to be punched?

74. **CONSTRUCTION** A can of paint covers 600 sq ft. How many cans of paint are needed to paint a 12-ft by 14-ft room if it needs **two coats** and has 8-ft high ceilings (paint the four walls, but not the ceiling or floor)?

75. **CRAFTS** A vegetable garden measures 180 ft by 265 ft.

(a) How many acres make up the garden?

(b) How many feet of fencing does it take to enclose the garden?

76. **BUSINESS AND FINANCE** A shopping center sits on a rectangular lot with dimensions 550 yd by 440 yd. Find its size, in acres.

77. **CONSTRUCTION** A square lot measures 160 yd per side.

(a) What is its area, to the nearest tenth acre?

(b) What is its area, to the nearest tenth hectare?

78. **CONSTRUCTION** A lot measures 6.3 km by 9.1 km.

(a) What is its area, to the nearest hectare?

(b) What is its area, to the nearest tenth acre?

Basic Skills | Challenge Yourself | Calculator/Computer | **Career Applications** | Above and Beyond

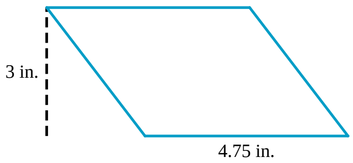
79. **MANUFACTURING TECHNOLOGY** A piece of rectangular stock is to be coated on the top side with sealer; the top of the piece measures 2.3 in. by 4.8 in.

(a) Calculate the area to be covered (to the nearest square inch).

(b) One can of sealer covers 500 sq ft. How many (whole) parts does one can of sealer cover?

80. **MECHANICAL ENGINEERING** The allowable compressive stress of steel is 2,900 pounds per sq in. (psi). Can a rectangular piece of stock measuring $\frac{3}{4}$ in. by $\frac{1}{2}$ in. handle a force of 1,100 lb?

81. A piece of 3-in.-wide strapping material is to be cut into parallelograms, each with a 4.75-in. base (see figure). What is the area of each piece?



82. **INFORMATION TECHNOLOGY** A printed circuit board (PCB) measures 45 mm by 67 mm.

(a) What is the area of the board?

(b) Report the area of the PCB in square centimeters.

83. **AUTOMOTIVE TECHNOLOGY** A rectangular tire patch is 4 cm by 7 cm. Find its area.

84. **AUTOMOTIVE TECHNOLOGY** The air filter in a car measures 6 in. by 9 in.

(a) Find the area of the filter.

(b) To wrap the filter in an aluminum frame, find the perimeter of the filter.

85. **MANUFACTURING TECHNOLOGY** A 4-in. by 14-in. steel plate weighs 0.048 pounds per sq in. What is the weight of the plate?

86. **CONSTRUCTION** How many feet of baseboard does it take to go around an 11-ft 4-in. by 13-ft 8-in. room?

Basic Skills | Challenge Yourself | Calculator/Computer | Career Applications | **Above and Beyond**

87. What is the effect on the area of a rectangle if one dimension is doubled? Construct some examples to demonstrate your answer.

88. What is the effect on the area of a rectangle if both dimensions are doubled? Construct some examples to demonstrate your answer.

Answers

77.

78.

79.

80.

81.

82.

83.

84.

85.

86.

87.

88.

Answers

89. ________________

90. ________________

89. What is the effect on the area of a triangle if the base is doubled and the height is halved? Construct some examples to demonstrate your answer.

90. What is the effect on the area of a triangle if both the base and height are doubled? Construct some examples to demonstrate your answer.

Answers

1. Yes; quadrilateral; not regular **3.** No **5.** Yes; rectangle; not regular
7. Yes; hexagon; regular **9.** No **11.** Yes; trapezoid; not regular
13. No **15.** No **17.** 22 ft **19.** 21 yd **21.** 26 m **23.** 2.1 mm
25. $\frac{9}{2}$ in. **27.** 68 km **29.** 88 in.^{2} **31.** 84 ft^2 **33.** 36 mm^2
35. 196 yd^2 **37.** 5,800 ft^2 **39.** 360 m^2 **41.** 98 yd^2
43. $\frac{147}{2}$ cm^2 **45.** 7,525 ft^2 **47.** 64 mm^2 **49.** 2,160 in.^{2}
51. 40 yd^2 **53.** 8,000,000 m^2 **55.** 300,000 cm^2 **57.** 103.2 cm^2
59. False **61.** True **63.** always **65.** 28.1 mi^2 **67.** 1,200 ha
69. 6,080 acres **71.** 864 in.^{2} **73.** 0.68 in.^{2}
75. **(a)** 1.1 acres; **(b)** 890 ft **77.** **(a)** 5.3 acres; **(b)** 2.1 ha
79. **(a)** 11 in.^{2}; **(b)** 6,545 parts **81.** 14.25 in.^{2} **83.** 28 cm^2
85. 2.688 lb **87.** Above and Beyond **89.** Above and Beyond

8.3 Circles and Composite Figures

< **8.3 Objectives** >

1 > Find the circumference of a circle

2 > Find the area of a circle

3 > Find the perimeter of a composite figure

4 > Find the area of a composite figure

We have already seen how to find the perimeter and area of several figures. In this section we study these concepts as applied to circles and composite figures. The distance around the outside of a circle is closely related to the concept of perimeter. We call the perimeter of a circle its **circumference.**

Definition

Circumference of a Circle

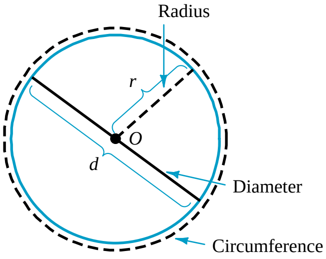
The *circumference* of a circle is the distance around that circle.

NOTE

The circumference formula comes from the ratio

$\frac{C}{d} = \pi$

We begin by defining some terms. In the circle, *d* represents the **diameter.** This is the distance across the circle through its center (labeled with the letter *O*, for **origin**). The **radius** *r* is the distance from the center to a point on the circle. The diameter is always twice the radius.



It was discovered long ago that the ratio of the circumference of a circle to its diameter always stays the same. The ratio has a special name. It is named by the Greek letter π (pi). Pi is approximately 3.14, rounded to two decimal places. We can write the following formula.

Property

Formula for the Circumference of a Circle

$C = \pi d$

Example 1 Finding the Circumference of a Circle

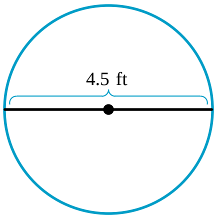
< **Objective 1** >

NOTE

Because 3.14 is an approximation for pi, we can only say that the circumference is approximately 14.1 ft. The symbol ≈ means "approximately."

A circle has a diameter of 4.5 ft. Find its circumference, using 3.14 for π. If your calculator has a $\boxed{\pi}$ key, use that key instead of a shorter decimal approximation for π.

By the formula,



$$C = \pi d$$
$$\approx 3.14 \times 4.5 \text{ ft}$$
$$\approx 14.1 \text{ ft} \quad \text{(rounded to one decimal place)}$$

Check Yourself 1

A circle has a diameter of $3\frac{1}{2}$ inches (in.). Find its circumference.

There is another useful formula for the circumference of a circle. Because $d = 2r$ (the diameter is twice the radius) and $C = \pi d$, we have $C = \pi(2r)$, or $C = 2\pi r$.

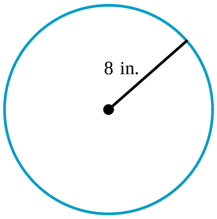
Property

Formula for the Circumference of a Circle

$$C = 2\pi r$$

Example 2 Finding the Circumference of a Circle

A circle has a radius of 8 in. Find its circumference, using 3.14 for π.



NOTE

If you want to approximate π, you do not need to worry about running out of decimal places. The value for pi has been calculated to over 51 billion decimal places on a computer (the printout would be over 150,000 pages long).

From the formula,

$$\begin{aligned} C &= 2\pi r \\ &\approx 2 \times 3.14 \times 8 \text{ in.} \\ &\approx 50.2 \text{ in.} \quad \text{(rounded to one decimal place)} \end{aligned}$$

Check Yourself 2

Find the circumference of a circle with a radius of 2.5 in.

The number pi (π), which we used to find circumference, is also used in finding the area of a circle. If r is the radius of a circle, we have the following formula.

Property

Formula for the Area of a Circle

$$A = \pi r^2$$

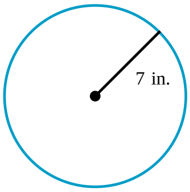
This is read, “Area equals pi r squared.” You can multiply the radius by itself and then by pi.

Example 3 Finding the Area of a Circle

< Objective 2 >

A circle has a radius of 7 inches (in.) (see figure). What is its area?

Use the area formula, using 3.14 for π and $r = 7$ in.



$$\begin{aligned} A &\approx 3.14 \times (7 \text{ in.})^2 \\ &\approx 153.86 \text{ in.}^2 \end{aligned}$$

Again, the area is an approximation because we use 3.14, an approximation for π.

Check Yourself 3

Find the area of a circle whose diameter is 4.8 centimeters (cm). Remember that the formula refers to the radius. Use 3.14 for π and round your result to the nearest tenth of a square centimeter.

Many objects have odd shapes—that is, they are not the basic shapes we have studied so far. In Chapter 1, we called these "oddly shaped figures." The proper name for these types of objects is **composite geometric figures** or just **composite figures.** More exactly, a composite figure is formed by joining simple geometric figures together.

Definition

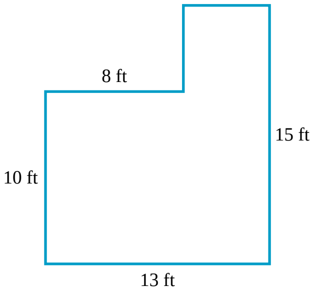
Composite Geometric Figures	A *composite figure* is a geometric figure that is formed by adjoining two or more basic geometric figures.

To help you to understand what we mean, consider the floor plan in the next example.

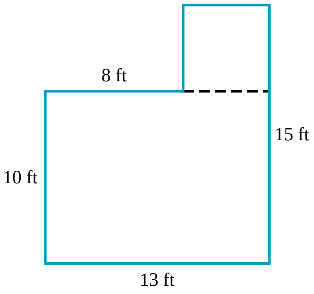
Example 4 Perimeter of a Composite Figure

< Objective 3 >

The floor plan shown is a living room adjoining a foyer. Find the length of molding (perimeter) needed for the room.



First, we subdivide the room into more basic shapes.



Then, we compute the missing lengths. For example, the top wall of the foyer is

$$13 \text{ ft} - 8 \text{ ft} = 5 \text{ ft}$$

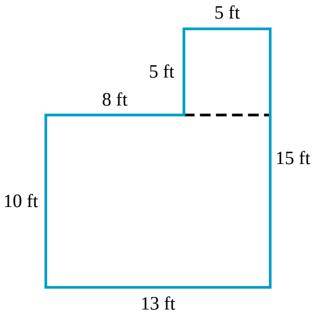
Similarly, the left wall of the foyer area is

$$15 \text{ ft} - 10 \text{ ft} = 5 \text{ ft}$$

As usual, the perimeter (or the amount of molding needed) is the sum of the lengths of the sides.

$$\begin{aligned} P &= 13 + 15 + 5 + 5 + 8 + 10 \\ &= 56 \end{aligned}$$

The room needs 56 ft of molding.

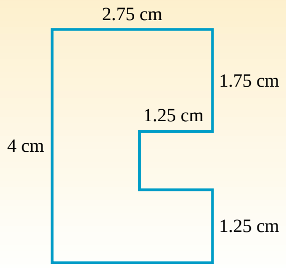


NOTE

We are simplifying this by omitting gaps such as doors.

Check Yourself 4

Find the perimeter of the figure shown.



To find the area of a composite figure, we usually consider the more basic shapes separately. That is, we find the area of each of the simple figures and combine them to produce the area of the larger, composite figure. We illustrate this in Example 5.

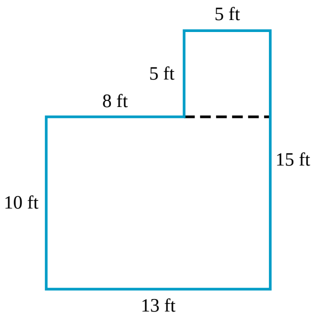
Example 5 Area of a Composite Figure

< Objective 4 >

Find the amount of carpeting (area) needed for the living room and foyer shown in Example 4.

We have already found the missing dimensions, so we do not need to repeat that calculation.

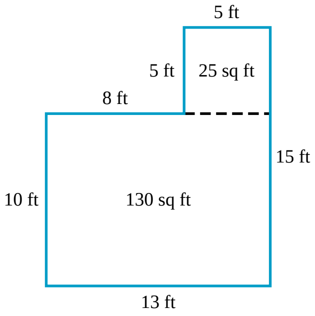
To compute the area, we consider the foyer (a square) and the living room (rectangle) separately. Compute the area of each part, and then add the areas together to get a total area.



$$A_{\text{Foyer}} = 5 \times 5 = 25 \text{ ft}^2$$

$$A_{\text{LvgRm}} = 10 \times 13 = 130 \text{ ft}^2$$

$$A = A_{\text{Foyer}} + A_{\text{LvgRm}} = 25 + 130 = 155 \text{ ft}^2$$



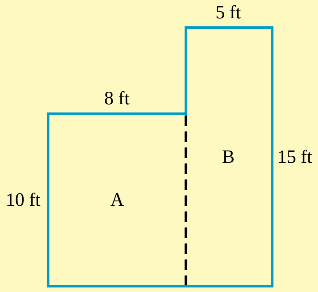
As a general rule, carpeting is sold by the square yard. There are 9 square feet to the square yard.

$$\frac{155}{9} = 17\frac{2}{9}$$

Because we are sold whole square yards, we must purchase 18 yd^2 of carpet.

NOTE

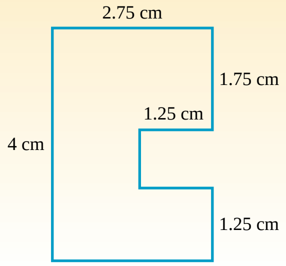
We could have divided the room as shown below. The result would be the same.



$$\begin{aligned} A_A &= 8 \times 10 \\ &= 80 \text{ ft}^2 \\ A_B &= 15 \times 5 \\ &= 75 \text{ ft}^2 \\ A_A + A_B &= 80 + 75 \\ &= 155 \text{ ft}^2 \end{aligned}$$

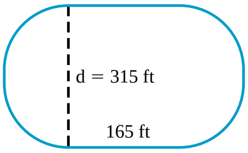
Check Yourself 5

Find the area of the figure shown.



Often, composite figures involve parts of circles coupled with polygons. Consider the track in the next example.

Example 6 Composite Figures



A high school track is shown at the right. Compute the distance around the track and the area contained by it.

There are two 165-ft lengths and two semicircles (making one full circle). To compute the distance around the track, we add the two lengths to the circumference of the circle.

$C = \pi d \approx 3.14 \times 315 \approx 989$ ft
$989 + 2 \times 165 = 1{,}319$

The track is approximately 1,319 ft long (about one-quarter mile).

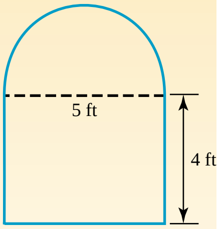
To compute the area, we compute the areas of the interior rectangle and the circle separately, and then add the total. Remember to compute the radius of the circle for the area ($d = 2r$).

$$r = \frac{315}{2} = 157.5$$
$$A_{\text{Circle}} = \pi r^2 \approx 3.14(157.5)^2 = 77{,}891.625$$
$$A_{\text{Rect}} = lw = 165 \times 315 = 51{,}975$$
$$A = A_{\text{Circle}} + A_{\text{Rect}} = 77{,}891.625 + 51{,}975 = 129{,}866.625$$

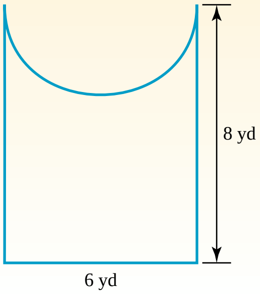
The area is approximately 129,867 sq ft.

Check Yourself 6

(a) We wish to build a wrought-iron gate frame according to the diagram below. How many feet of material will we need? Use 3.14 for π and round to the nearest foot. *Hint:* We are looking for the distance around the object.



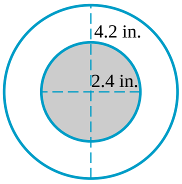
(b) Find the perimeter and area of the figure shown. Use 3.14 for π and round to the nearest tenth yard.



In manufacturing applications, it is frequently necessary to find the difference between two areas (or volumes) in order to calculate the area (or volume) of a production piece.

Example 7 A Manufacturing Technology Application

The figure represents a cross section of an O-ring. The diameters of the inner and outer circles are shown. Find the area of the O-ring shown (the white part in the reverse-color image of two circles). Report your result to the nearest hundredth sq in.



The area of the larger circle, which includes the O-ring, can be computed using its radius. When we halve the diameter, we get a radius of 2.1 in.

$$A_{\text{outer}} = \pi r^2 = \pi(2.1)^2 = 13.85 \text{ in.}^2$$

Similarly, we compute the area of the inner circle, which has a radius of 1.2 in.

$$A_{\text{inner}} = \pi r^2 = \pi(1.2)^2 = 4.52 \text{ in.}^2$$

The area of the O-ring is the difference between these two areas.

$$A_{\text{O-ring}} = A_{\text{outer}} - A_{\text{inner}} = 13.85 - 4.52 = 9.33 \text{ in.}^2$$

Check Yourself 7

Find the outer and inner circumferences of the O-ring pictured in Example 7. Use 3.14 for π and round to the nearest hundredth in.

Check Yourself ANSWERS

1. 11 in. **2.** 15.7 in. **3.** 31.4 yd **4.** 16 cm **5.** 9.75 cm^2
6. **(a)** 21 ft; **(b)** $P = 31.4$ yd; $A = 33.9$ yd^2 **7.** Outer: 13.19 in.; inner: 7.54 in.

Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 8.3

(a) We call the perimeter of a circle the ________________.

(b) The ________________ is the distance from the center to a point on the circle.

(c) Joining basic figures into a single shape forms a ________________ geometric figure.

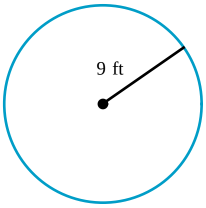
(d) You can find the ________________ of a composite figure by taking the sum of the areas of each of its basic-figure components.

8.3 exercises

< Objective 1 >

Find the circumference of each figure. Use 3.14 for π and round your answer to one decimal place.

1.



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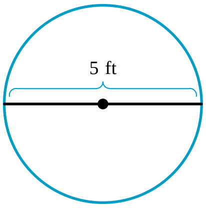

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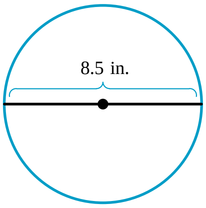

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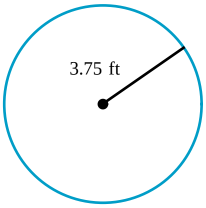

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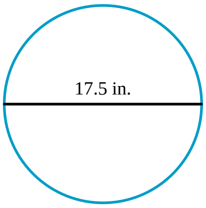

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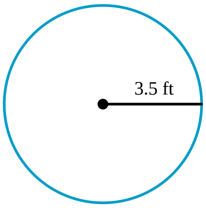
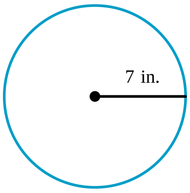


< Objective 2 >

Find the area of each figure. Use 3.14 for π and round your answer to one decimal place.

7.



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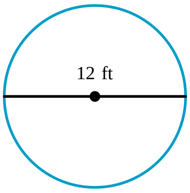
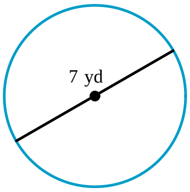


9.



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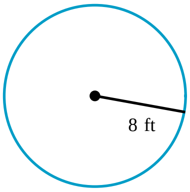

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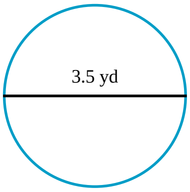

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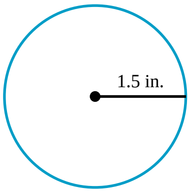
12.



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Name ______________________

Section ________ Date ________

Answers

1. ______________________
2. ______________________
3. ______________________
4. ______________________
5. ______________________
6. ______________________
7. ______________________
8. ______________________
9. ______________________
10. ______________________
11. ______________________
12. ______________________

Answers

13. ______________________

14. ______________________

15. ______________________

16. ______________________

17. ______________________

18. ______________________

19. ______________________

20. ______________________

21. ______________________

22. ______________________

23. ______________________

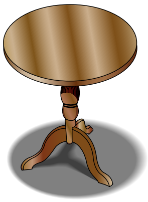
24. ______________________

Solve each application.

13. Science and Medicine A path runs around a circular lake with a diameter of 1,000 yards (yd). Robert jogs around the lake three times for his morning run. How far has he run?

14. Crafts A circular rug is 6 feet (ft) in diameter. Binding for the edge costs $1.50 per yard. What will it cost to bind the rug?

15. Business and Finance A circular piece of lawn has a radius of 28 ft. You have a bag of fertilizer that will cover 2,500 ft^2 of lawn. Do you have enough?



16. Crafts A circular coffee table has a diameter of 5 ft. What will it cost to have the top refinished if the company charges $3 per square foot for the refinishing?

17. Construction A circular terrace has a radius of 6 ft. If it costs $1.50 per square foot to pave the terrace with brick, what will the total cost be?

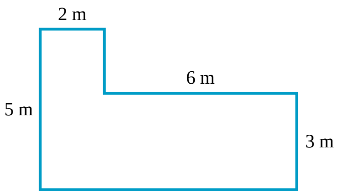
18. Construction A house addition is in the shape of a semicircle (a half-circle) with a radius of 9 ft. What is its area?

Basic Skills | **Challenge Yourself** | Calculator/Computer | Career Applications | Above and Beyond

< Objective 3 >

Find the perimeter of each figure. Use 3.14 for π and round answers to one decimal place, when appropriate.

19.



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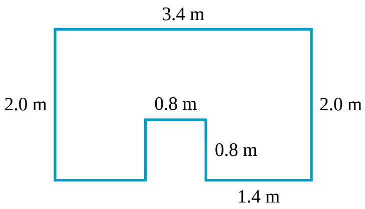

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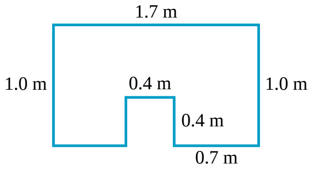

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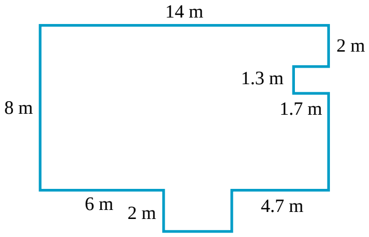

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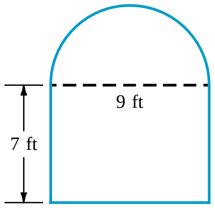
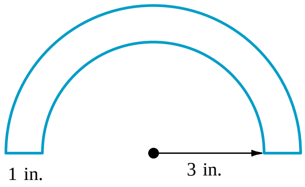


chapter 8 > Make the Connection

24.



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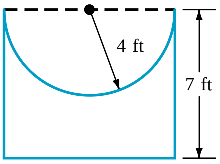
25.



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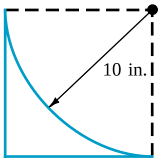
26.



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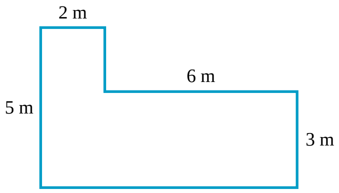
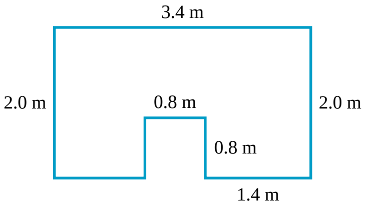
< Objective 4 >

Find the area of each figure. Use 3.14 for π and round your answers to one decimal place, when appropriate.

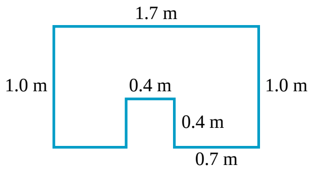
27.



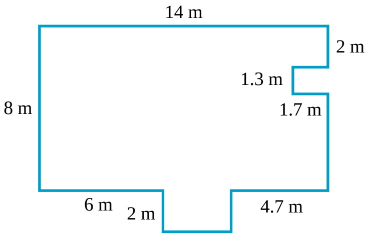
28.


3.4 m
2.0 m
0.8 m
2.0 m
0.8 m
1.4 m

29.

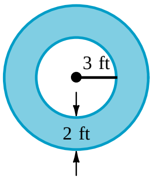

1.7 m
1.0 m
0.4 m
1.0 m
0.4 m
0.7 m

30.

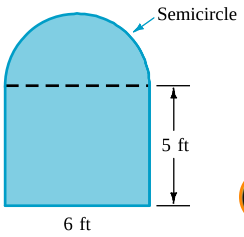

14 m
2 m
1.3 m
8 m
1.7 m
6 m
2 m
4.7 m

Find the area of the shaded part in each figure. Use 3.14 for π and round your answers to one decimal place, when appropriate.

31.

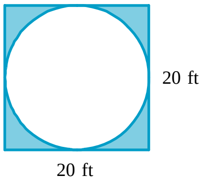


32.


Semicircle
5 ft
6 ft

chapter 8 > Make the Connection

33.



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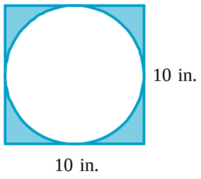
34.



chapter 8 > Make the Connection

Basic Skills | Challenge Yourself | Calculator/Computer | Career Applications | Above and Beyond

35. **ALLIED HEALTH** An ultrasound technician measures the biparietal (or head) diameter of a 14-week-old fetus as 2.5 cm. Determine the head circumference of the fetus.

36. **ALLIED HEALTH** An ultrasound technician measures the biparietal (or head) diameter of a 28-week-old fetus as 7.3 cm. Determine the head circumference of the fetus.

Answers

25. ____________________

26. ____________________

27. ____________________

28. ____________________

29. ____________________

30. ____________________

31. ____________________

32. ____________________

33. ____________________

34. ____________________

35. ____________________

36. ____________________

Answers

37. ____________________

38. ____________________

39. ____________________

40. ____________________

41. ____________________

42. ____________________

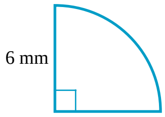
43. ____________________

44. ____________________

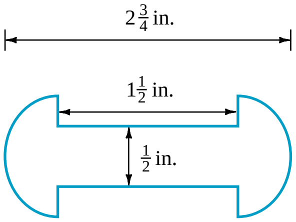
37. **ALLIED HEALTH** During a high-risk pregnancy, amniotic fluid levels are monitored. To measure fluid levels, the ultrasound technician looks for roughly circular areas of fluid on the ultrasound image. One such fluid pocket had a diameter of 1.7 cm. Determine the area (cross-sectional area) of this circular fluid pocket.

38. **ALLIED HEALTH** During a high-risk pregnancy, amniotic fluid levels are monitored. To measure fluid levels, the ultrasound technician will look for roughly circular areas of fluid on the ultrasound image. One such fluid pocket had a diameter of 0.9 cm. Determine the area (cross-sectional area) of this circular fluid pocket.

39. **MANUFACTURING TECHNOLOGY** The cross section of a shaft key takes the shape of a quarter-circle. Find the perimeter and area of this cross section of the shaft key.



40. **MANUFACTURING TECHNOLOGY** The surface of this piece needs to be coated with a nonstick coating. A container of the coating can cover 28 square feet. How many parts can be coated with one can?



chapter 8 > Make the Connection

Basic Skills | Challenge Yourself | Calculator/Computer | Career Applications | **Above and Beyond**

41. Papa Doc's delivers pizza. The 8-in.-diameter pizza is \$8.99, and the price of a 16-in.-diameter pizza is \$17.98. Write a plan to determine which is the better buy.

42. The distance from Philadelphia to Sea Isle City is 100 mi. A car was driven this distance using tires with a radius of 14 in. How many revolutions of each tire occurred on the trip?

43. Find the area and the circumference (or perimeter) of each object.

(a) A penny, **(b)** a nickel, **(c)** a dime, **(d)** a quarter, **(e)** a half-dollar, **(f)** a silver dollar, **(g)** a Susan B. Anthony dollar, **(h)** a dollar bill, and **(i)** one face of the pyramid on the back of a \$1 bill.

44. How would you determine the cross-sectional area of a Douglas fir tree (at, say, 3 ft above the ground), without cutting it down? Use your method to solve the following problem:

If the circumference of a Douglas fir is 6 ft 3 in., measured at a height of 3 ft above the ground, compute the cross-sectional area of the tree at that height.

Answers

1. 56.5 ft **3.** 26.7 in. **5.** 55.0 in. **7.** 153.9 in.^{2} **9.** 38.5 yd^2
11. 9.6 yd^2 **13.** 9,420 yd **15.** Yes **17.** \$169.56 **19.** 26 m
21. 6.2 m **23.** 37.1 ft **25.** 34.6 ft **27.** 28 m^2 **29.** 1.5 m^2
31. 50.2 ft^2 **33.** 86 ft^2 **35.** 7.9 cm **37.** 2.3 cm^2
39. $P = 21.42$ mm; $A = 28.27$ mm^2 **41.** Above and Beyond
43. Above and Beyond

Activity 21 ::
Exploring Circles

In this activity, we ask the question, How does multiplying the radius of a circle by a certain factor change the circumference? And how does this same action change the area? Begin by computing the following, using 3.14 for π. The first one is done for you.

	Radius	Circumference	Factor	New Radius	New Circumference
Circle 1	10 ft	62.8 ft	2	20 ft	125.6 ft
	10 ft		3		
	10 ft		$\frac{1}{2}$		
Circle 2					

For circle 2, you and your group members choose values for the radius and for the multiplying factor.

How does the new circumference compare to the original circumference in each case? Try to describe your observations with a general statement.

Now check the effect on the area of each circle. Again using 3.14 for π, complete the table.

	Radius	Area	Factor	New Radius	New Area
Circle 1	10 ft		2		
	10 ft		3		
	10 ft		$\frac{1}{2}$		
Circle 2					

How does the new area compare to the original area in each case? Try to describe your observations with a general statement.

Here is a challenge for your group: If you want to double the area of a circle, what should you multiply the radius by? (*Hint:* You will probably have to solve this by trial and error. And you will not find an exact answer, but you can determine an answer accurate to the nearest thousandth.)

8.4

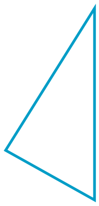
Triangles

< **8.4 Objectives** >

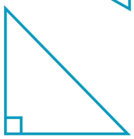
1 > Classify triangles by angles
2 > Classify triangles by sides
3 > Find the measures of the angles of a triangle
4 > Identify similar triangles
5 > Find the lengths of the sides of a triangle

Now that you know something about angles, it is interesting to look again at triangles. Why is this shape called a triangle?

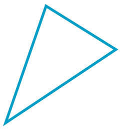
Literally, *triangle* means "three angles."



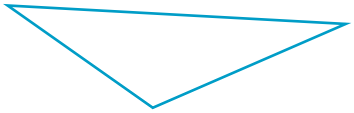
The same classifications we used for angles can be used for triangles. If a triangle has a right angle, we call it a *right triangle.*



If it has three acute angles, it is called an *acute triangle.*



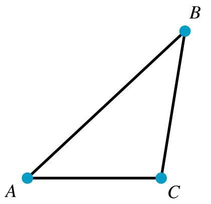
If it has an obtuse angle, it is called an *obtuse triangle.*

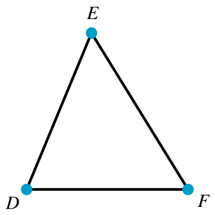


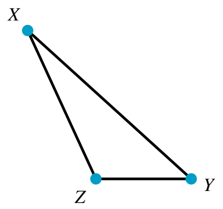
Example 1 — Identifying an Acute Triangle

< Objective 1 >

Which of the following triangles is acute?



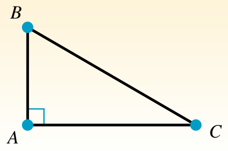


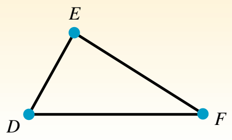


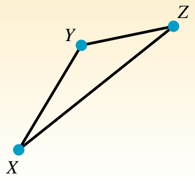
Only $\triangle DEF$ is an acute triangle. Both $\triangle ABC$ and $\triangle XYZ$ have one obtuse angle.

Check Yourself 1

Which of the following triangles is obtuse?





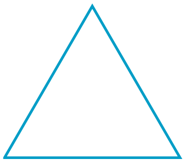


We can also classify triangles based on how many sides have the same length.

NOTE

All three angles of an equilateral triangle have the same measure.

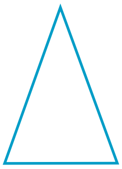
A triangle is called an *equilateral triangle* if all three sides have the same length.



NOTE

Exactly two angles of an isosceles triangle have equal measure.

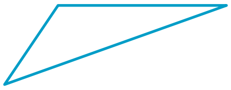
A triangle is called an *isosceles triangle* if exactly two sides have the same length.



NOTE

All three angles of a scalene triangle have different measures.

A triangle is called a *scalene triangle* if no two sides have the same length.



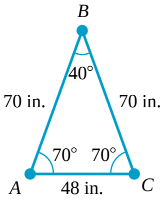
Example 2 Labeling Types of Triangles

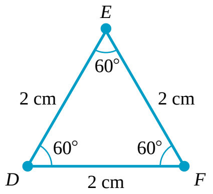
< Objective 2 >

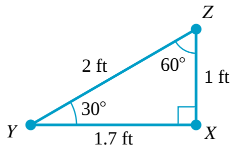
NOTE

Each of these triangles can be classified in different ways. *XYZ* is a right triangle, but it is also scalene.

Of these triangles, which are equilateral? Isosceles? Scalene?





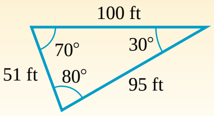


$\triangle ABC$ is an isosceles triangle because two of the angles have the same measure. And $\triangle DEF$ is an equilateral triangle because all three angles have the same measure. $\triangle XYZ$ is a scalene triangle because no two angles have the same measure.

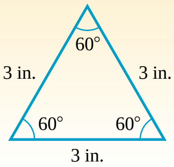
Check Yourself 2

Label each triangle as equilateral, isosceles, or scalene.

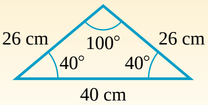
(a)



(b)



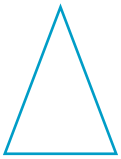
(c)



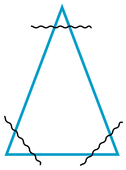
Go back and look at the sum of the angles inside each of the triangles in Example 2. You will note that they always add to 180°. No matter how we draw a triangle, the sum of the three angles inside the triangle will *always* be 180°.

Here is an experiment that might convince you that this is always the case.

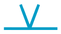
1. Using a straight edge, draw any triangle on a sheet of paper.



2. Use scissors to cut out the triangle.
3. Rip the three vertices off of the triangle.



4. Lay the three vertices (with the points of the triangle touching) together. They will always form a straight angle, which we saw in Section 8.1 has a measure of 180°.



Property

Angles of a Triangle

For any triangle *ABC*,

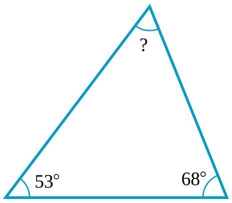
$$m\angle A + m\angle B + m\angle C = 180^\circ$$

We say that the sum of the (interior) angles of every triangle is 180°.

Example 3 Finding an Angle Measure

< Objective 3 >

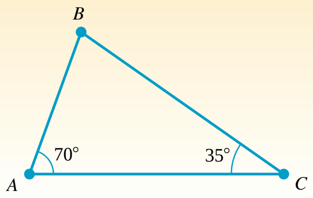
Find the measure of the third angle in this triangle.



We need the three measurements to have a sum of 180°, so we add the two given measurements (53° + 68° = 121°). Then we subtract that from 180° (180° − 121° = 59°). This gives us the measure of the third angle, 59°.

Check Yourself 3

Find the measure of $\angle ABC$.



Definition

Similar Triangles

If the measurements of the three angles in two different triangles are the same, we say the two triangles are **similar triangles.**

Similar triangles have exactly the same shape, but their sizes may differ. In this case, they are "scale" versions of each other.

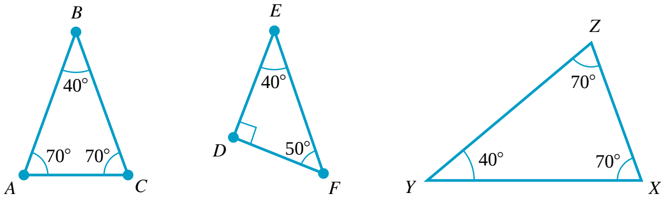
Example 4 Identifying Similar Triangles

< Objective 4 >

NOTE

This can be written $\triangle ABC \sim \triangle XYZ$

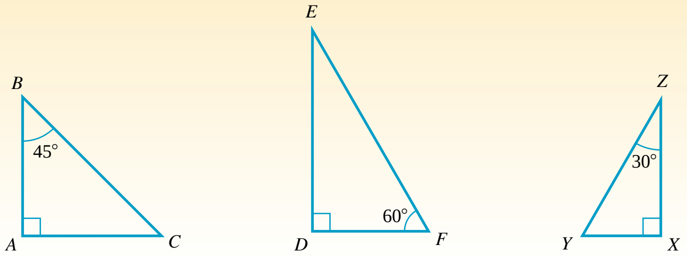
Which two triangles are similar?



Although they are of different size, $\triangle ABC$ and $\triangle XYZ$ are similar because they have the same angle measurements.

Check Yourself 4

Find the two triangles that are similar.



Finally, we return to an idea that we first saw in Chapter 5.

Property

Similar Triangles If two triangles are similar, their corresponding sides have the same ratio.

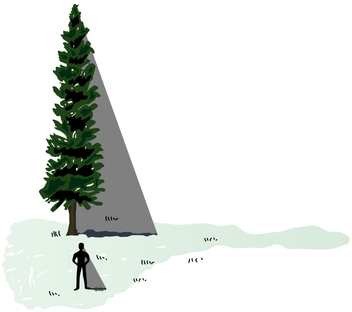
This property of similar triangles is used to find the height of a tall object as illustrated in Example 5.

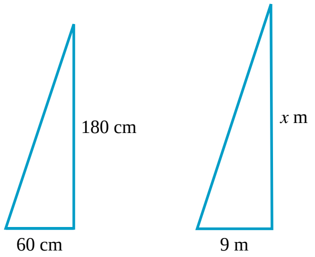
Example 5 Finding the Height of a Tree

< Objective 5 >

If a man who is 180 cm tall casts a shadow that is 60 cm long, how tall is a tree that casts a shadow that is 9 m long?

Because of the angle of the sun, the man and his shadow form a similar triangle to the tree and its shadow. Because of this, we can use the common ratio to find the height of the tree.





$$\begin{aligned}\frac{180\text{ cm}}{60\text{ cm}} &= \frac{x\text{ m}}{9\text{ m}}\\ x \cdot 60 &= 180 \cdot 9\\ x &= \frac{180 \cdot 9}{60}\\ x &= 27\end{aligned}$$

The tree is 27 m tall.

Check Yourself 5

If a man who is 160 cm tall casts a shadow that is 120 cm long, how tall is a building that casts a shadow that is 60 m long?

Check Yourself ANSWERS

1. $\triangle XYZ$ **2.** **(a)** Scalene; **(b)** equilateral; **(c)** isosceles **3.** 75°
4. $\triangle DEF$ and $\triangle XZY$ **5.** 80 m

Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 8.4

(a) A triangle is called an ________________ triangle if all three angles have the same measure.

(b) A triangle is called an ________________ triangle if two angles have the same measure.

(c) If the measurements of the three angles in two different triangles are the same, we say that the two triangles are ________________ triangles.

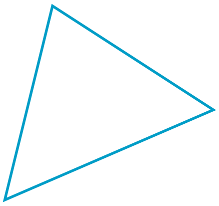
(d) If two triangles are similar, their ________________ sides have the same ratio.

8.4 exercises

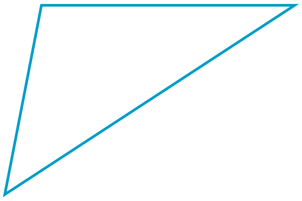
< **Objective 1** >

Label each triangle as acute or obtuse.

1.



2.



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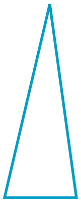

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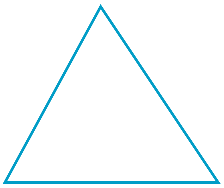
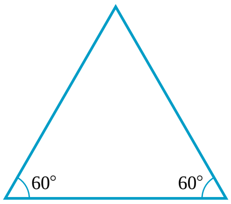
4.



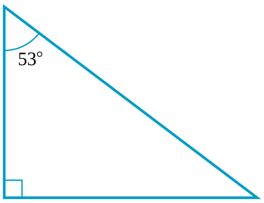
< **Objective 2** >

Label each triangle as equilateral, isosceles, or scalene.

5.



6.



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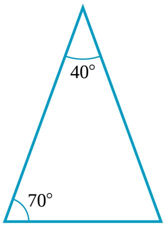

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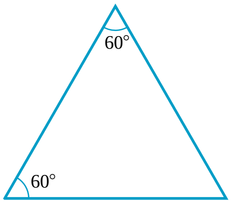
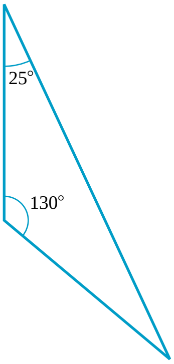


9.



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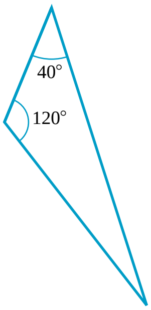
10.



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Name ____________________

Section __________ Date __________

Answers

1. ____________________

2. ____________________

3. ____________________

4. ____________________

5. ____________________

6. ____________________

7. ____________________

8. ____________________

9. ____________________

10. ____________________

Answers

11. ______________________

12. ______________________

13. ______________________

14. ______________________

15. ______________________

16. ______________________

17. ______________________

18. ______________________

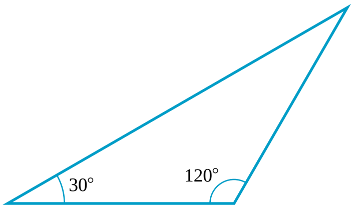
19. ______________________

20. ______________________

< Objective 3 >

Find the missing angle and then label the triangle as equilateral, isosceles, or scalene.

11.



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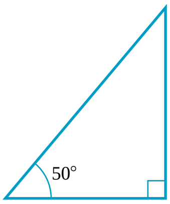

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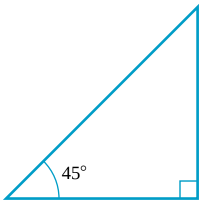

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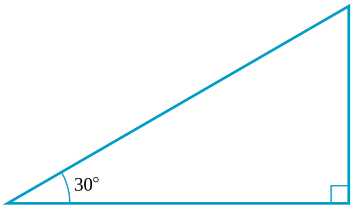
14.



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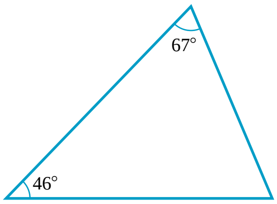

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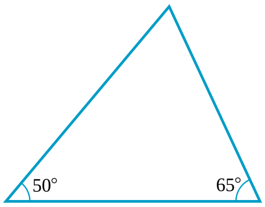
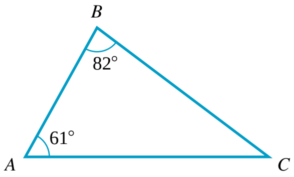
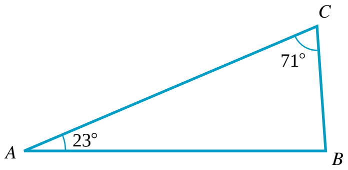


For each triangle shown, find the indicated angle.

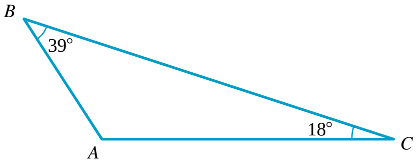
17. Find $m\angle C$.



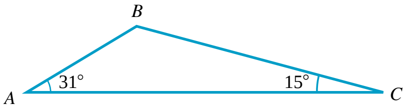
18. Find $m\angle B$.



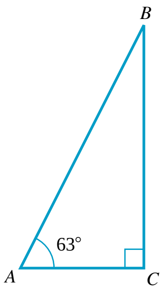
19. Find $m\angle A$.



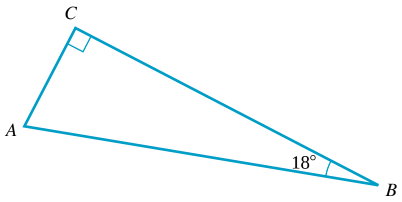
20. Find $m\angle B$.



21. Find $m\angle B$.

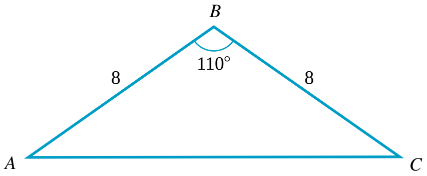


22. Find $m\angle A$.

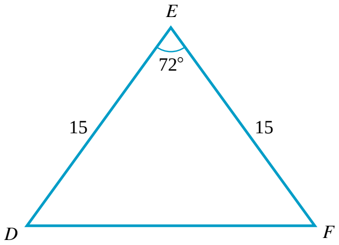


Assume that the given triangle is isosceles.

23. Find $m\angle A$ and $m\angle C$.

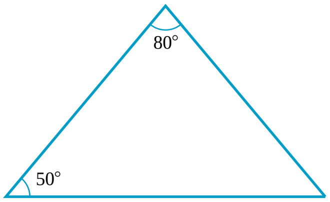


24. Find $m\angle D$ and $m\angle F$.

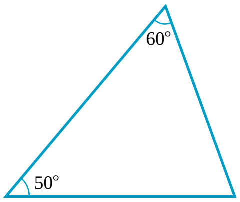


< **Objective 4** >

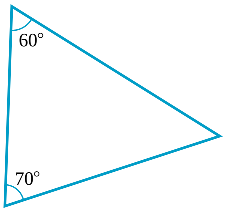
25. Which two triangles are similar?



(*a*)

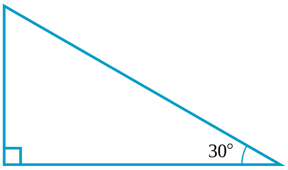


(*b*)



(*c*)

26. Which two triangles are similar?



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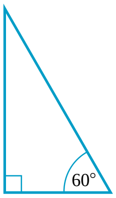

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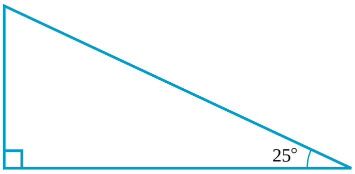


(*c*)

Answers

21. ____________________

22. ____________________

23. ____________________

24. ____________________

25. ____________________

26. ____________________

Answers

27. ______________________

28. ______________________

29. ______________________

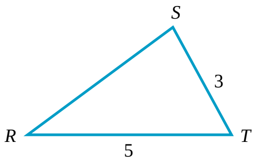
30. ______________________

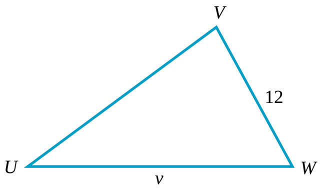
31. ______________________

< Objective 5 >

The two triangles shown are similar. Find the indicated side.

27. Find *v*.





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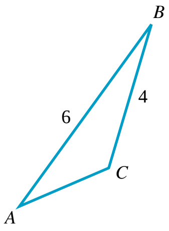
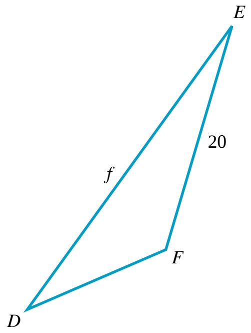
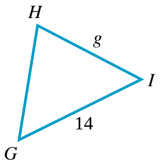
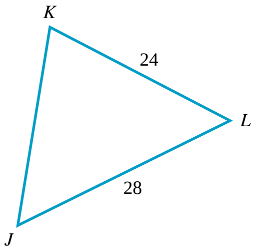
28. Find *f*.



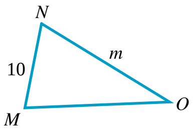


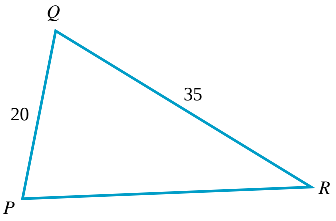
29. Find *g*.



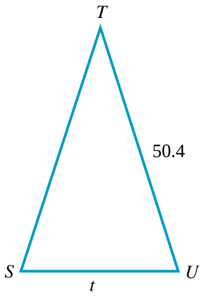


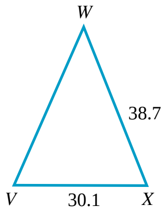
30. Find *m*.



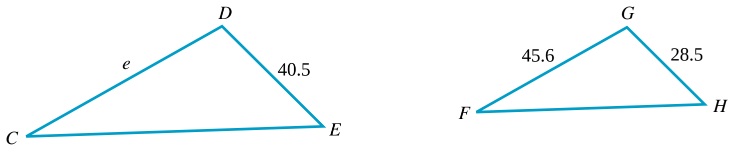


31. Find *t*.





32. Find *e*.


D
e
40.5
C
E
G
45.6
28.5
F
H

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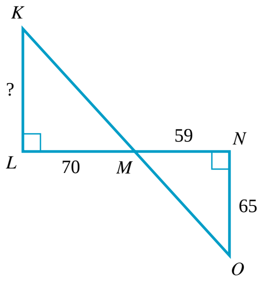
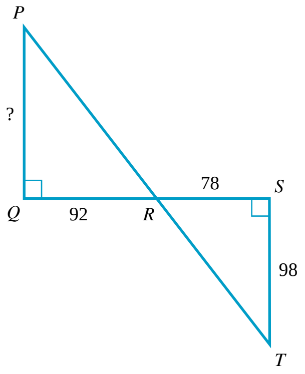
First indicate which two triangles are similar. Then find the indicated side. Round your answer to the nearest hundredth.

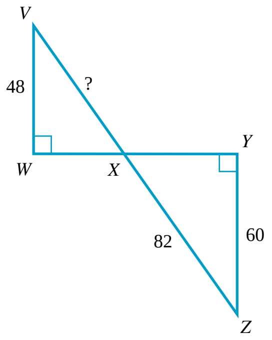
33. Find $\overline{KL}$.


K
?
L
70
M
59
N
65
O

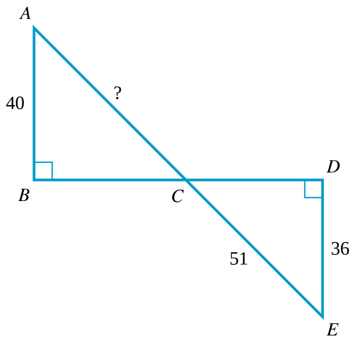
34. Find $\overline{PQ}$.


P
?
Q
92
R
78
S
98
T

35. Find $\overline{VX}$.

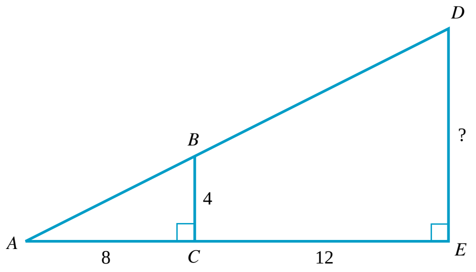

V
48
?
W
X
Y
82
60
Z

36. Find $\overline{AC}$.


A
40
?
B
C
D
51
36
E

Find the indicated side. If necessary, round to the nearest tenth of a unit.

37. Find $\overline{DE}$.



Answers

32. ____________________

33. ____________________

34. ____________________

35. ____________________

36. ____________________

37. ____________________

Answers

38. ____________________

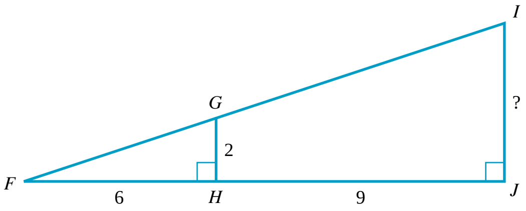
39. ____________________

40. ____________________

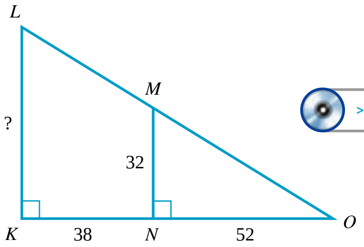
41. ____________________

42. ____________________

38. Find $\overline{IJ}$.

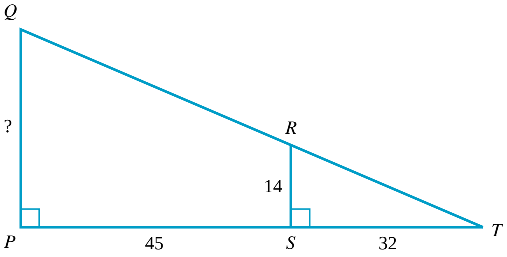


39. Find $\overline{KL}$.

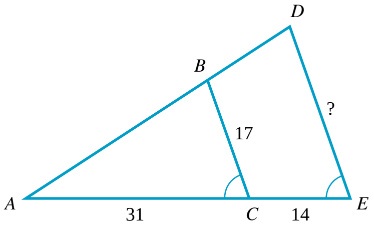

L
M
?
32
K
38
N
52
O

> Videos

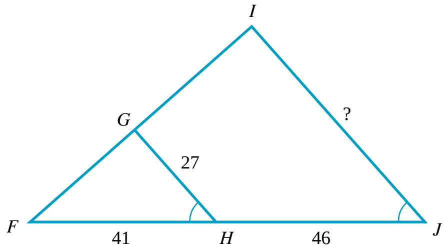
40. Find $\overline{PQ}$.


Q
?
R
14
P
45
S
32
T

41. Given: $m\angle BCA = m\angle DEA$. Find $\overline{DE}$.


D
B
?
17
A
31
C
14
E

42. Given: $m\angle GHF = m\angle IJF$. Find $\overline{IJ}$.


I
G
?
27
F
41
H
46
J

Answers

43. ____________________

44. ____________________

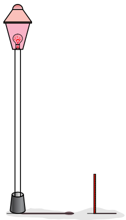
45. ____________________

46. ____________________

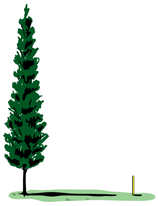
47. ____________________

48. ____________________

43. A light pole casts a shadow that measures 4 ft. At the same time, a yardstick casts a shadow that is 9 in. long. How tall is the pole?



44. A tree casts a shadow that measures 5 m. At the same time, a meter stick casts a shadow that is 0.4 m long. How tall is the tree?



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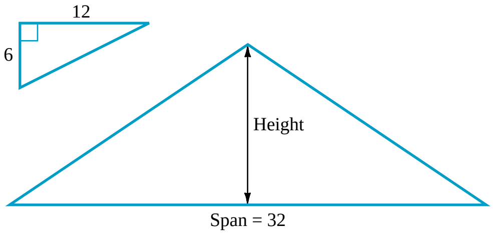
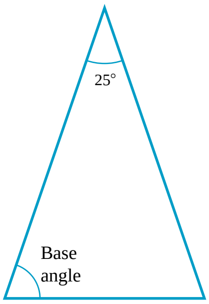
45. **MANUFACTURING TECHNOLOGY** In a common truss, the slope triangle tells you how high the truss is compared to the run. Find the height of this common truss. (*Hint:* The run is only half the span.)

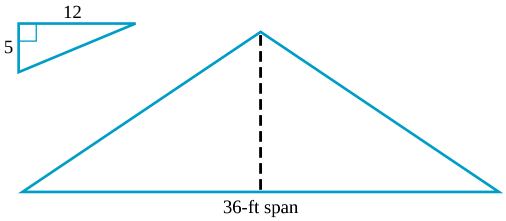


46. **MANUFACTURING TECHNOLOGY** A chipping hammer is shaped like a wedge with a tip angle of 25°.

Find the angle at the base of the isosceles triangle.



47. **MANUFACTURING TECHNOLOGY** The slope triangle on a truss tells you how high the truss is compared to the run.



What is the height of this truss, given the $\frac{5}{12}$-slope triangle? Remember that the run is only half the span.

Basic Skills | Challenge Yourself | Calculator/Computer | Career Applications | Above and Beyond

48. Use the ideas of similar triangles to determine the height of a pole or tree on your campus. Work with one or two partners.

Answers

49. __________

50. __________

51. __________

52. __________

53. __________

54. __________

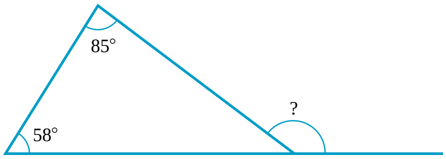
55. __________

56. __________

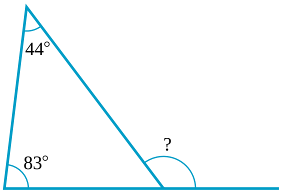
57. __________

In exercises 49 to 51, one side of the triangle has been extended, forming what is called an exterior angle. *In each case, find the measure of the indicated exterior angle.*

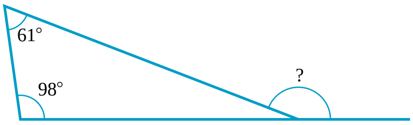
49.



50.



51.



52. What do you observe from exercises 49 to 51? Write a general conjecture about an exterior angle of a triangle.

53. Write an argument to show that an equilateral triangle cannot have a right angle.

54. Argue that, given an equilateral triangle, the measure of each angle must be 60°.

55. Argue that a triangle cannot have more than one obtuse angle.

56. Is it possible to have an isosceles right triangle? If such a triangle exists, what can be said about the angles? Defend your statements.

57. Create an argument to support the statement:

If $\triangle ABC$ is a right triangle, with $m\angle C = 90°$, then $\angle A$ and $\angle B$ must be acute and complementary.

Answers

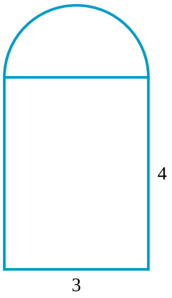
1. Acute **3.** Acute **5.** Equilateral **7.** Isosceles **9.** Isosceles
11. 30°; isosceles **13.** 45°; isosceles **15.** 67°; isosceles **17.** 37°
19. 123° **21.** 27° **23.** $m\angle A = 35°$; $m\angle C = 35°$ **25.** (*b*) and (*c*)
27. 20 **29.** 12 **31.** 39.2 **33.** 77.12 **35.** 65.60 **37.** 10
39. 55.4 **41.** 24.7 **43.** 16 ft **45.** 8 ft **47.** 7.5 ft **49.** 143°
51. 159° **53.** Above and Beyond **55.** Above and Beyond
57. Above and Beyond

chapter
8
> Make the Connection

Activity 22 ::
Composite Geometric Figures

When first introduced to geometry in your math class, you worked with fairly straightforward figures such as squares, circles, and triangles. Most real-world objects cannot be described by such simple geometric figures. Consider the chair that you are using right now. The chair is probably made up of several shapes put together.

Composite geometric figures are figures formed by combining two or more simple geometric figures. One example of a composite geometric figure is the Norman window. Norman windows are windows constructed by combining a rectangle with a half-circle (see the figure). Given such a figure, there are several questions we could ask. We might ask an area question to find the amount of glass used, a perimeter question to find the amount of frame, or a combination question to determine the amount of wall space necessary to accommodate such a window.



Assume all measurements are in feet and round answers to two decimal places.

1. Find the area of the rectangular piece of glass in the Norman window pictured.
2. Find the area of the half-circle piece of glass.
3. Find the total area of the glass.
4. If the glass costs \$3 per square foot for a rectangular piece and \$4.75 per square foot for the circular piece, find the total cost of the glass.
5. Find the outer perimeter of the figure.
6. Find the length of framework needed for the Norman window (do not forget to include the strip of frame separating the rectangle and half-circle).
7. Find the cost of the framework if it costs \$1.25 per foot for straight pieces and \$3.25 per foot for curved pieces.
8. Use your answers to exercises 4 and 7 to determine the total cost of the Norman window pictured.
9. Find the dimensions of the smallest rectangle that would completely contain the Norman window.
10. Find the area of the "leftover" rectangle created by cutting the Norman window from the rectangle found in exercise 9.

8.5 Square Roots and the Pythagorean Theorem

< 8.5 Objectives >

1 > Find the square root of a perfect square
2 > Identify the hypotenuse of a right triangle
3 > Identify a perfect triple
4 > Use the Pythagorean theorem
5 > Approximate the square root of a number

Some numbers can be written as the product of two identical factors, for example,

$9 = 3 \times 3$

The factor is called a **square root** of the number. The symbol $\sqrt{\ }$ (called a **radical sign**) is used to indicate a square root. Thus, $\sqrt{9} = 3$ because $3 \times 3 = 9$.

Example 1 Finding the Square Root

< Objective 1 >

NOTE

To use the $\boxed{\sqrt{\ }}$ key with a scientific calculator, first enter the 49 and then press the key. With a graphing calculator, press the radical key first and then enter the 49 and a closing parenthesis.

Find the square root of 49 and of 16.

(a) $\sqrt{49} = 7$ Because $7 \times 7 = 49$

(b) $\sqrt{16} = 4$ Because $4 \times 4 = 16$

Check Yourself 1

Find the indicated square root.

(a) $\sqrt{121}$ **(b)** $\sqrt{36}$

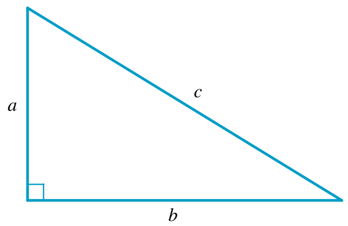
The most frequently used theorem in geometry is undoubtedly the Pythagorean theorem. In this section you will use that theorem. You will also learn a little about the history of the theorem. It is a theorem that applies only to right triangles.

The side opposite the right angle of a right triangle is called the **hypotenuse.** The other two sides are called **legs.** Note that the legs are perpendicular to each other.

Example 2 Identifying the Hypotenuse

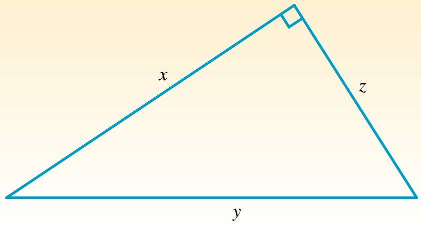
< Objective 2 >

In the right triangle, the side labeled *c* is the hypotenuse.



Check Yourself 2

Which side represents the hypotenuse of the given right triangle?



The numbers 3, 4, and 5 have a special relationship. Together they are called a **perfect triple,** which means that when you square all three numbers, the sum of the smaller squares equals the squared value of the largest number.

Example 3 Identifying Perfect Triples

< Objective 3 >

Show that each set of numbers is a perfect triple.

(a) 3, 4, and 5

$3^2 = 9 \qquad 4^2 = 16 \qquad 5^2 = 25$

and $9 + 16 = 25$, so we can say that $3^2 + 4^2 = 5^2$.

(b) 7, 24, and 25

$7^2 = 49 \qquad 24^2 = 576 \qquad 25^2 = 625$

and $49 + 576 = 625$, so we can say that $7^2 + 24^2 = 25^2$.

Check Yourself 3

Show that each set of numbers is a perfect triple.

(a) 5, 12, and 13 **(b)** 6, 8, and 10

All the triples that you have seen, and many more, were known by the Babylonians more than 4,000 years ago. Stone tablets that had dozens of perfect triples carved into them have been found. The basis of the Pythagorean theorem was understood long before the time of Pythagoras (ca. 540 B.C.). The Babylonians not only understood perfect triples but also knew how triples related to a right triangle.

Property

The Pythagorean Theorem (Version 1)

If the lengths of the three sides of a right triangle are all integers, they will form a perfect triple, with the hypotenuse as the longest side.

There are two other forms in which the Pythagorean theorem is regularly presented. It is important that you see the connection between the three forms.

Property

The Pythagorean Theorem (Version 2)

The square of the hypotenuse of a right triangle is equal to the sum of the squares of the other two sides.

Property

The Pythagorean Theorem (Version 3)

Given a right triangle with sides a and b and hypotenuse c, it is always true that

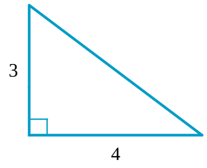
$c^2 = a^2 + b^2$

Example 4 Finding the Length of a Side of a Right Triangle

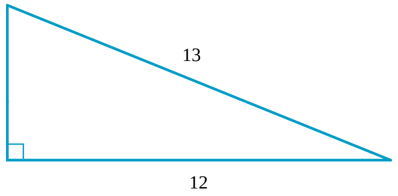
< Objective 4 >

Find the missing length for each right triangle.

(a)



(b)



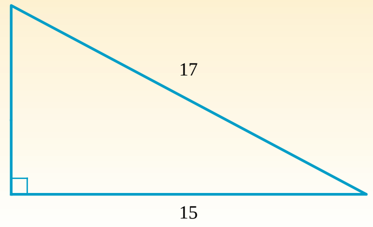
(a) A perfect triple is formed if the hypotenuse is 5 units long, creating the triple 3, 4, 5. Note that $3^2 + 4^2 = 9 + 16 = 25 = 5^2$.

(b) The triple must be 5, 12, 13, which makes the missing length 5 units. Here, $5^2 + 12^2 = 25 + 144 = 169 = 13^2$.

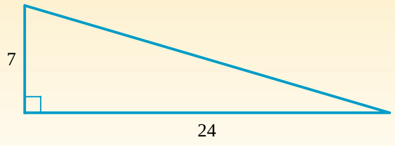
Check Yourself 4

Find the length of the unlabeled side for each right triangle.

(a)



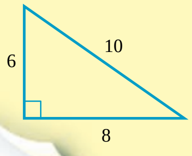
(b)



Example 5 Using the Pythagorean Theorem

NOTE

The triangle has sides 6, 8, and 10.



If the lengths of two sides of a right triangle are 6 and 8, find the length of the hypotenuse.

$c^2 = a^2 + b^2$ The value of the hypotenuse is found from the Pythagorean theorem with $a = 6$ and $b = 8$.

$c^2 = (6)^2 + (8)^2 = 36 + 64 = 100$

$c = \sqrt{100} = 10$ The length of the hypotenuse is 10 (because $10^2 = 100$).

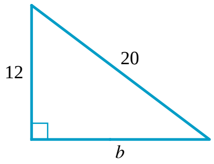
Check Yourself 5

Find the hypotenuse of a right triangle whose sides measure 9 and 12.

In some right triangles, the lengths of the hypotenuse and one side are given and we are asked to find the length of the missing side.

Example 6 Using the Pythagorean Theorem

Find the missing length.



$$a^2 + b^2 = c^2$$

Use the Pythagorean theorem with $a = 12$ and $c = 20$.

$$(12)^2 + b^2 = (20)^2$$

$$144 + b^2 = 400$$

$$b^2 = 400 - 144 = 256$$

$$b = \sqrt{256} = 16$$

The missing side is 16.

Check Yourself 6

Find the missing length for a right triangle with one leg measuring 8 centimeters (cm) and the hypotenuse measuring 10 cm.

Not every square root is a whole number. In fact, there are only 10 whole-number square roots for the numbers from 1 to 100. They are the square roots of 1, 4, 9, 16, 25, 36, 49, 64, 81, and 100. However, we can approximate square roots that are not whole numbers. For example, we know that the square root of 12 is not a whole number. We also know that its value must lie somewhere between the square root of 9 ($\sqrt{9} = 3$) and the square root of 16 ($\sqrt{16} = 4$). That is, $\sqrt{12}$ is between 3 and 4.

Example 7 Approximating Square Roots

< Objective 5 >

Approximate $\sqrt{29}$.

$\sqrt{25} = 5$ and $\sqrt{36} = 6$, so $\sqrt{29}$ must be between 5 and 6.

Check Yourself 7

$\sqrt{19}$ is between which pair of numbers?

(a) 4 and 5 **(b) 5 and 6** **(c) 6 and 7**

Check Yourself ANSWERS

1. **(a)** 11; **(b)** 6 **2.** Side y **3.** **(a)** $5^2 + 12^2 = 25 + 144 = 169$, $13^2 = 169$, so $5^2 + 12^2 = 13^2$; **(b)** $6^2 + 8^2 = 36 + 64 = 100$, $10^2 = 100$ so $6^2 + 8^2 = 10^2$ **4.** **(a)** 8; **(b)** 25 **5.** 15 **6.** 6 cm **7.** **(a)** 4 and 5

Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 8.5

(a) The symbol $\sqrt{}$ (called a ________________ sign) is used to indicate a square root.

(b) The side opposite the right angle of a right triangle is called the ________________.

(c) The ________________ theorem says that the square of the hypotenuse of a right triangle is equal to the sum of the squares of the other two sides.

(d) Not every square root is a ________________ number.

8.5 exercises

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Name ______________________

Section __________ Date __________

Answers

1. ______________________

2. ______________________

3. ______________________

4. ______________________

5. ______________________

6. ______________________

7. ______________________

8. ______________________

9. ______________________

10. ______________________

11. ______________________

12. ______________________

13. ______________________

14. ______________________

15. ______________________

< Objective 1 >

Find the square root.

1. $\sqrt{64}$ > Videos

2. $\sqrt{121}$

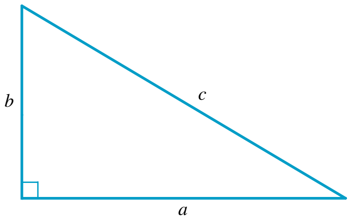
3. $\sqrt{169}$

4. $\sqrt{196}$

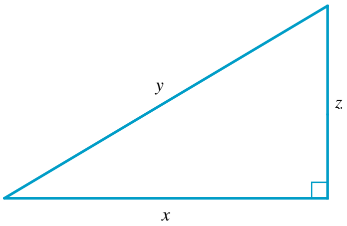
< Objective 2 >

Identify the hypotenuse of the given triangles by giving its letter.

5.



6.



< Objective 3 >

Identify which numbers are perfect triples.

7. 3, 4, 5 > Videos

8. 4, 5, 6

9. 7, 12, 13

10. 5, 12, 13

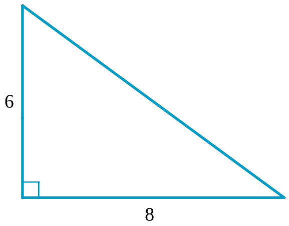
11. 8, 15, 17

12. 9, 12, 15

< Objective 4 >

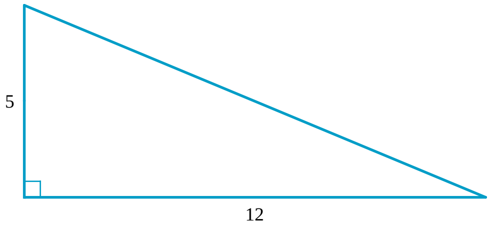
Find the missing length for each right triangle.

13.



> Videos

14.



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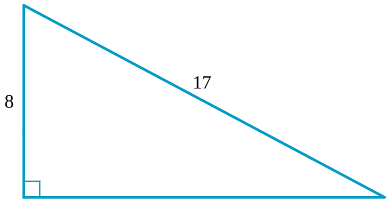
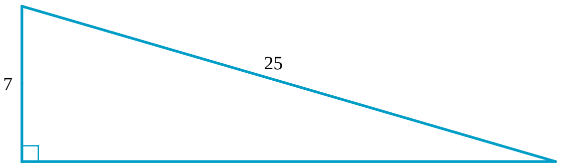


> Videos

16.



< **Objective 5** >

Select the correct approximation for each square root.

17. Is $\sqrt{23}$ between **(a)** 3 and 4, **(b)** 4 and 5, or **(c)** 5 and 6?

18. Is $\sqrt{15}$ between **(a)** 1 and 2, **(b)** 2 and 3, or **(c)** 3 and 4?

19. Is $\sqrt{44}$ between **(a)** 6 and 7, **(b)** 7 and 8, or **(c)** 8 and 9?

20. Is $\sqrt{31}$ between **(a)** 3 and 4, **(b)** 4 and 5, or **(c)** 5 and 6?

Basic Skills | **Challenge Yourself** | Calculator/Computer | Career Applications | Above and Beyond

Determine whether each statement is **true** *or* **false.**

21. For any triangle with side lengths a, b, and c, it is true that $a^2 + b^2 = c^2$.

22. If we know the lengths of two of the sides of a right triangle, we can find the length of the third side.

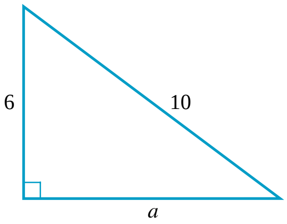
In each statement, fill in the blank with **always, sometimes,** *or* **never.**

23. The hypotenuse is __________ the longest side of a right triangle.

24. The square root of a whole number is ________ a whole number.

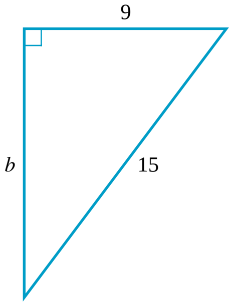
Find the perimeter of each triangle shown. (Hint: *First find the missing side.*)

25.

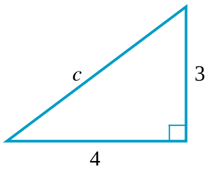


> Videos

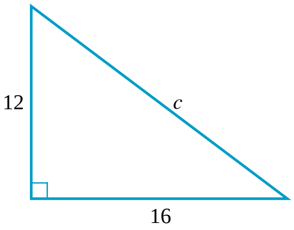
26.



27.



28.



Answers

16. ____________
17. ____________
18. ____________
19. ____________
20. ____________
21. ____________
22. ____________
23. ____________
24. ____________
25. ____________
26. ____________
27. ____________
28. ____________

Answers

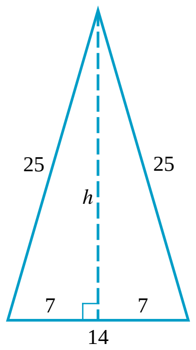
29. ______________________

30. ______________________

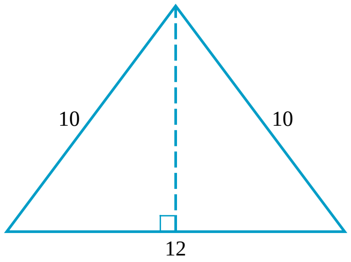
31. ______________________

32. ______________________

29. Find the altitude, h, of the isosceles triangle shown.

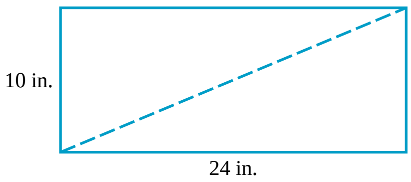


30. Find the altitude of the isosceles triangle shown. (*Hint:* The altitude shown bisects the base.)

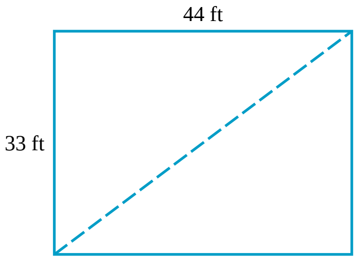


Find the length of the diagonal of each rectangle.

31.



32.



Basic Skills | Challenge Yourself | **Calculator/Computer** | Career Applications | Above and Beyond

To find a square root on your scientific calculator, you use the square root key. On some calculators, you simply enter the number and then press the square root key. With others, you must use the second function on the $\boxed{x^2}$ (or $\boxed{y^x}$) key and specify the root you wish to find.

For example, to find the square root of 256 with a scientific calculator, you must enter

256 $\boxed{\sqrt{\ }}$

or perhaps

256 $\boxed{\text{2nd}}$ $\boxed{\begin{matrix}\sqrt[x]{y}\\ y^x\end{matrix}}$ 2 $\boxed{=}$

The display should show 16.

It is very likely that the square root of a number will not be "nice." Your calculator can give you the *approximate* square root in such a case. To find, for example, the square root of 29, enter 29 $\boxed{\sqrt{\ }}$, and the display may show 5.385164807. This is an *approximation* of the square root. It is rounded to the nearest billionth. The calculator cannot display the exact answer because there is no end to the sequence of digits (and also no pattern). If you round to the nearest tenth, your approximation for the square root of 29 is 5.4.

Use your calculator to find the square root of each number.

33. 64 **34.** 144 **35.** 289

36. 1,024 **37.** 1,849 **38.** 784

39. 8,649 **40.** 5,329

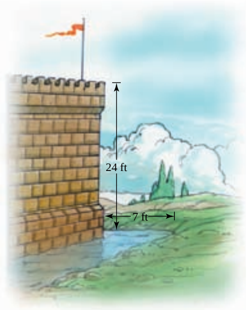
Use your calculator to approximate each square root. Round to the nearest tenth.

41. $\sqrt{23}$ **42.** $\sqrt{31}$

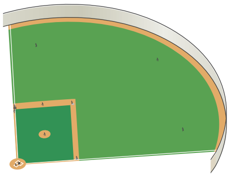
43. $\sqrt{51}$ **44.** $\sqrt{42}$

45. $\sqrt{134}$ **46.** $\sqrt{251}$

47. A castle wall, 24 feet high, is surrounded by a moat 7 feet across. Will a 26-foot ladder, placed at the edge of the moat, be long enough to reach the top of the wall?

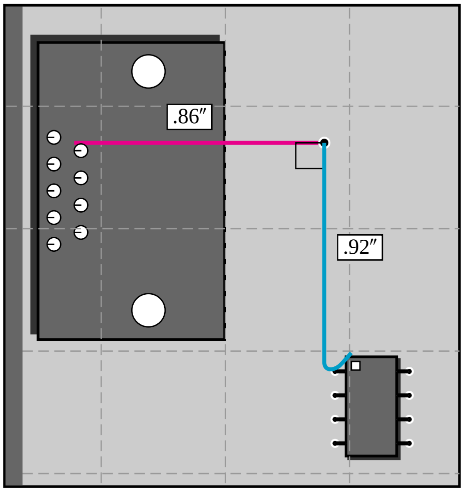


48. A baseball diamond is the shape of a square that has sides of length 90 feet. Find the distance from home plate to second base. Round to the nearest tenth.



Basic Skills | Challenge Yourself | Calculator/Computer | **Career Applications** | Above and Beyond

49. **ELECTRONICS** The image represents a small portion of a printed circuit board that is in the layout stage. The conductive trace being plotted between the solder pads is comprised of a vertical and a horizontal trace that meet at a right angle. The horizontal component is 0.86 in., and the vertical component is 0.92 in. If the trace could be run from point to point diagonally, its distance could be determined by finding $d = \sqrt{(0.86)^2 + (0.92)^2}$. How long would it be?



Answers

33. ____________

34. ____________

35. ____________

36. ____________

37. ____________

38. ____________

39. ____________

40. ____________

41. ____________

42. ____________

43. ____________

44. ____________

45. ____________

46. ____________

47. ____________

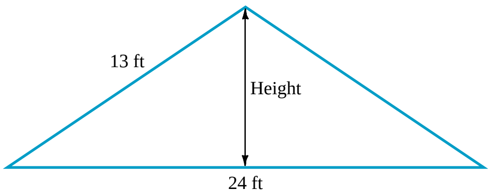
48. ____________

49. ____________

Answer

50. ______________________

50. **Manufacturing Technology** Find the height of this truss.



Answers

1. 8 **3.** 13 **5.** *c* **7.** Yes **9.** No **11.** Yes **13.** 10
15. 15 **17.** **(b)** **19.** **(a)** **21.** False **23.** always **25.** 24
27. 12 **29.** 24 **31.** 26 in. **33.** 8 **35.** 17 **37.** 43
39. 93 **41.** 4.8 **43.** 7.1 **45.** 11.6 **47.** Yes **49.** 1.26 in.

Activity 23 ::
The Pythagorean Theorem

In this activity, you and your group members will experiment with the Pythagorean theorem, one of the most famous results in all mathematics. This theorem is often used in construction to test that a manufactured (or built) right angle actually is a right angle.

For problems 1 and 2, we recommend that you work on graph paper to ensure that you have right angles.

1. Draw a right triangle of any size on your paper. Label the sides a, b, and c, where c is the hypotenuse. With a metric ruler, measure and record the lengths of a, b, and c to the nearest millimeter.

$a =$ $b =$ $c =$

Now compute: $a^2 =$ $b^2 =$ $c^2 =$

Compare the sum $a^2 + b^2$ with c^2. How close are they?

2. Repeat problem 1 with a completely new right triangle.

$a =$ $b =$ $c =$

Compute: $a^2 =$ $b^2 =$ $c^2 =$

How does the sum $a^2 + b^2$ compare to c^2?

3. Now draw a triangle that is *not* a right triangle. Again label the sides a, b, and c, where c is the longest side. Measure and record as before:

$a =$ $b =$ $c =$

Compute: $a^2 =$ $b^2 =$ $c^2 =$

How does the sum $a^2 + b^2$ compare to c^2?

4. Locate an example of a right triangle on your campus and with a measuring tape find the lengths of the sides. Record and compute the following to see if it really is a right triangle.

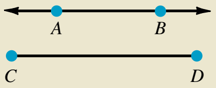
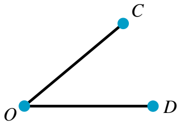
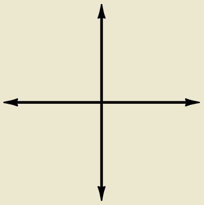
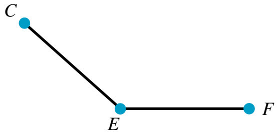
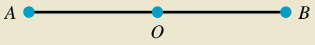
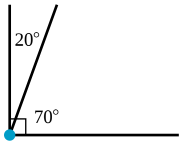
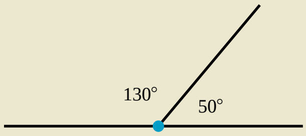
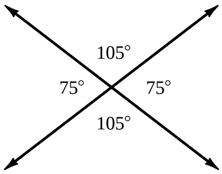
$a =$ $b =$ $c =$

Compute: $a^2 =$ $b^2 =$ $c^2 =$

How does the sum $a^2 + b^2$ compare to c^2?

Do you conclude that the angle is in fact a right angle?

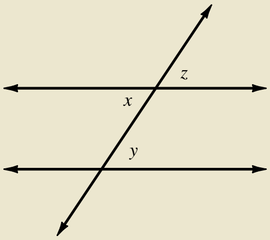
summary :: chapter 8

Definition/Procedure	Example	Reference
Lines and Angles		**Section 8.1**
Line A series of points that goes on forever. **Line segment** A piece of a line that has two endpoints.		*p.* 483
Angle A geometric figure consisting of two line segments that share a common endpoint.		*p.* 484
Perpendicular lines Lines are *perpendicular* if they intersect to form four equal angles. **Parallel lines** Lines are *parallel* if they never intersect.	These lines are perpendicular. 	*p.* 484
Acute angles have a measure less than 90°. **Obtuse angles** have a measure between 90° and 180°. **Right angles** have a measure of 90°.	∠*CEF* is obtuse. 	*pp.* 485
Straight angles have a measure of 180°.		*p.* 486
Complementary angles Two angles are complementary if the sum of their measures is 90°.		*p.* 488
Supplementary angles Two angles are supplementary if the sum of their measures is 180°.		*p.* 488
Vertical angles When two lines intersect, two pairs of vertical angles are formed. Vertical angles have equal measures.		*p.* 489

Definition/Procedure | **Example** | **Reference**

Parallel lines and a transversal When two parallel lines are intersected by a transversal,

1. Alternate interior angles have equal measures.
2. Corresponding angles have equal measures.



$m\angle x = m\angle y$

$m\angle y = m\angle z$

p. 490

Perimeter and Area

The **perimeter** of a figure is the distance around the figure. It can be found by taking the sum of its sides.

The **area** of a figure is the amount of space it fills.

Rectangle

$P = 2L + 2W$

$A = L \cdot W$

Square

$P = 4s$

$A = s^2$

Parallelogram

$P = 2b + 2s$

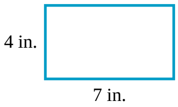
$A = b \cdot h$

Triangle

$A = \frac{1}{2}b \cdot h$

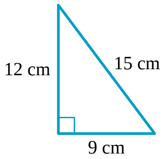
Trapezoid

$A = \frac{1}{2} \cdot h \cdot (b_1 + b_2)$



$$\begin{aligned} P &= 2L + 2W \\ &= 2(4 \text{ in.}) + 2(7 \text{ in.}) \\ &= 8 \text{ in.} + 14 \text{ in.} \\ &= 22 \text{ in.} \end{aligned}$$

$$\begin{aligned} A &= L \times W \\ &= (4 \text{ in.}) \times (7 \text{ in.}) \\ &= 28 \text{ in.}^2 \end{aligned}$$



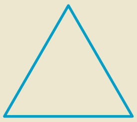
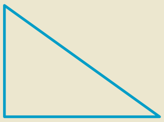
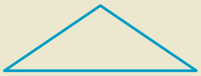
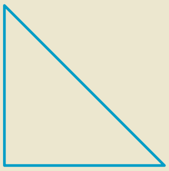
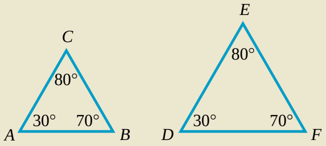
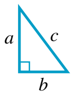
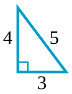
$$\begin{aligned} P &= (9 \text{ cm}) + (12 \text{ cm}) + (15 \text{ cm}) \\ &= 36 \text{ cm} \end{aligned}$$

$$\begin{aligned} A &= \frac{1}{2}b \cdot h \\ &= \frac{1}{2}\underbrace{(9 \text{ cm})}_{\text{base}} \cdot \underbrace{(12 \text{ cm})}_{\text{height}} \\ &= 54 \text{ cm}^2 \end{aligned}$$

Section 8.2

p. 500

Definition/Procedure	Example	Reference
To convert units of area, use the square of the one-dimensional conversion factor. *U.S. Customary System* $1\text{ ft}^2 = 144\text{ in.}^2$ $1\text{ yd}^2 = 9\text{ ft}^2$ $1\text{ mi}^2 = 640\text{ acres}$ $1\text{ acre} = 43{,}560\text{ ft}^2$ *Metric System* $1\text{ cm}^2 = 100\text{ mm}^2$ $1\text{ m}^2 = 10{,}000\text{ cm}^2$ $1\text{ hectare (ha)} = 10{,}000\text{ m}^2$ *U.S. Customary-Metric* $1\text{ in.}^2 = (2.54)^2\text{ cm}^2$ $= 6.45\text{ cm}^2$ $1\text{ cm}^2 = 0.15\text{ in.}^2$ $1\text{ mi}^2 = 2.59\text{ km}^2$ $1\text{ km}^2 = 0.38\text{ mi}^2$	$15\text{ ft}^2 = 15(144\text{ in.}^2)$ $= 2{,}160\text{ in.}^2$ $40\text{ cm}^2 = 40\,\cancel{\text{cm}^2} \times \frac{1\text{ m}^2}{10{,}000\,\cancel{\text{cm}^2}}$ $= \frac{40}{10{,}000}\text{ m}^2$ $= 0.004\text{ m}^2$	*p.* 506
Circles and Composite Figures The **circumference** of a circle is the distance around that circle. The **radius** is the distance from the center to a point on the circle. The **diameter** is twice the radius. The circumference is found using the formula $C = \pi d = 2\pi r$ where π is approximately 3.14. The area of a circle is found using the formula $A = \pi r^2$	If $r = 4.5$ cm, then $C = 2\pi r$ $\approx (2)(3.14)(4.5)$ $\approx 28.3\text{ cm}$ (rounded) If $r = 4.5$ cm, then $A = \pi r^2$ $\approx (3.14)(4.5)^2$ $\approx 63.6\text{ cm}^2$ (rounded)	**Section 8.3** *p.* 515

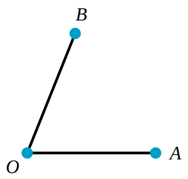
Definition/Procedure	Example	Reference
Triangles		Section 8.4
A triangle is **acute** if all three angles are less than 90°. A triangle is **right** if it has a right angle. A triangle is **obtuse** if it has an obtuse angle. A triangle is **equilateral** if all three sides have the same length. A triangle is **isosceles** if exactly two sides have the same length. A triangle is **scalene** if all three sides have different lengths.	 *Acute equilateral*  *Right scalene*  *Obtuse isosceles*  *Right isosceles*	*p.* 526
Similar triangles Two triangles are *similar* if the measures of the three angles in the two different triangles are the same. Corresponding sides of similar triangles are proportional.	 C 80° A 30° 70° B E 80° D 30° 70° F ΔACB and ΔDEF are similar triangles.	*p.* 529
Square Roots and the Pythagorean Theorem The square root of a number is a value that, when squared, gives us that number. The length of the three sides of a right triangle will form a perfect triple.  $a^2 + b^2 = c^2$	 $3^2 + 4^2 = 5^2$	Section 8.5 *p.* 540

summary exercises :: chapter 8

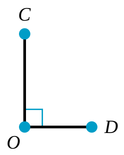
This summary exercise set is provided to give you practice with each of the objectives of this chapter. Each exercise is keyed to the appropriate chapter section. When you are finished, you can check your answers to the odd-numbered exercises against those presented in the back of the text. If you have difficulty with any of these questions, go back and reread the examples from that section. The answers to the even-numbered exercises appear in the *Instructor's Solutions Manual.* Your instructor will give you guidelines on how to best use these exercises in your instructional setting.

8.1 *Name the angle; label it as acute, obtuse, right, or straight; then estimate its measure with a protractor.*

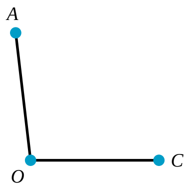
1.



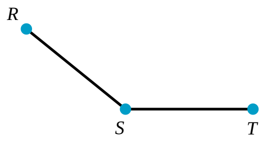
2.



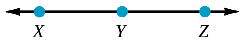
3.



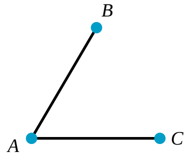
4.



5.



6.



Give the measure of each angle in degrees.

7. $\angle A$ represents $\frac{3}{8}$ of a complete circle.

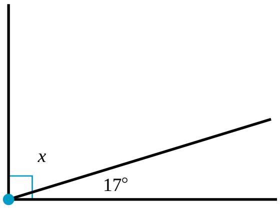
8. $\angle B$ represents $\frac{7}{10}$ of a complete circle.

9. If $m\angle x = 43°$, find the complement of $\angle x$.

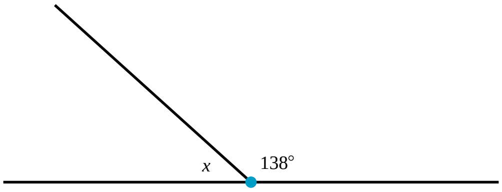
10. If $m\angle y = 82°$, find the supplement of $\angle y$.

Find $m\angle x$.

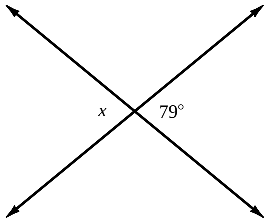
11.



12.



13.



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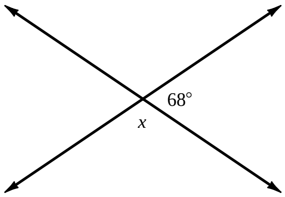

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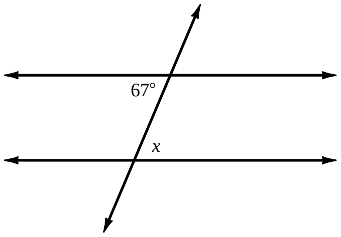

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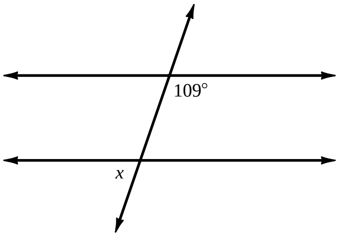
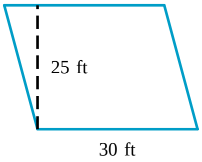
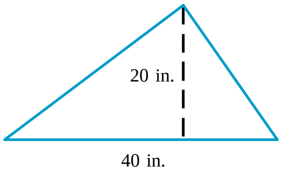


8.2 *Find the area of each figure.*

17.

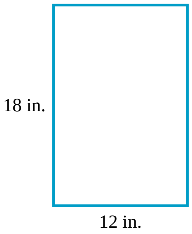


18.

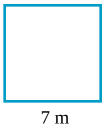


Find the perimeter and area of each figure.

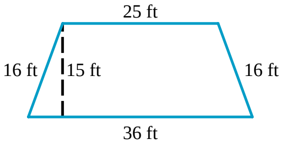
19.



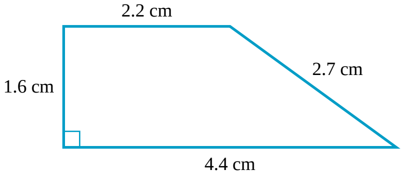
20.



21.


25 ft
16 ft
15 ft
16 ft
36 ft

22.


2.2 cm
1.6 cm
2.7 cm
4.4 cm

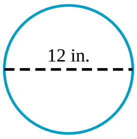
Solve each application.

23. **CRAFTS** How many square feet of vinyl floor covering will be needed to cover the floor of a room that is 10 ft by 18 ft? How many square yards (yd^2) will be needed? (*Hint:* How many square feet are in a square yard?)

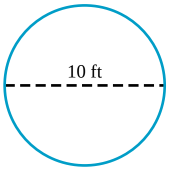
24. **CONSTRUCTION** A rectangular roof for a house addition measures 15 ft by 30 ft. A roofer will charge $175 per “square” (100 ft^2). Find the cost of the roofing for the addition.

8.3 *Find the circumference and area of each figure. Use 3.14 for π and round to the nearest tenth.*

25.

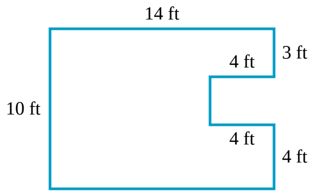


26.

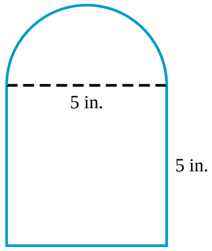


Find the perimeter and area of each figure. *Use 3.14 for π and round to the nearest tenth.*

27.

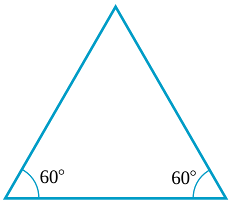

14 ft
4 ft
3 ft
10 ft
4 ft
4 ft

28.

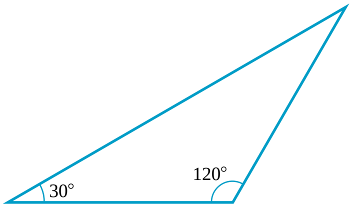


8.4 *Find the missing angle and then label the triangle as equilateral, isosceles, or scalene.*

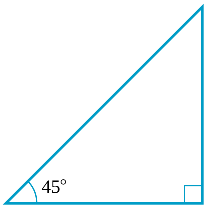
29.



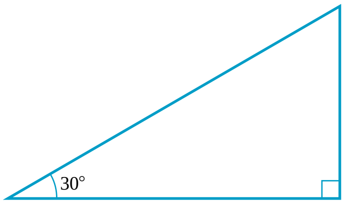
30.



31.

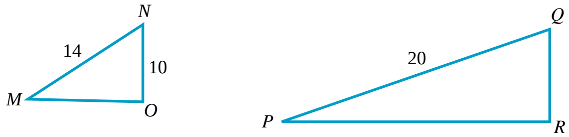


32.

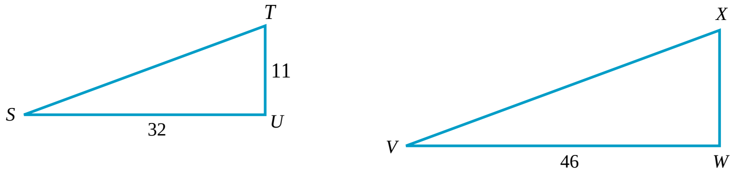


Find the indicated side. Round results to the nearest tenth.

33. Given: $\triangle MNO$ is similar to $\triangle PQR$. Find the length of $\overline{QR}$.


N
14
10
M
O
Q
20
P
R

34. Given: $\triangle SUT$ is similar to $\triangle VWX$. Find the length of $\overline{WX}$.


T
11
S
32
U
X
V
46
W

35. A tree casts a shadow that is 11.2 m long at the same time that a 4.0-m pole casts a shadow that is 1.4 m long. How tall is the tree?

8.5 *Find each square root. Where necessary, round to the nearest hundredth.*

36. $\sqrt{324}$

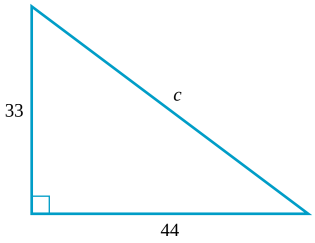
37. $\sqrt{784}$

38. $\sqrt{189}$

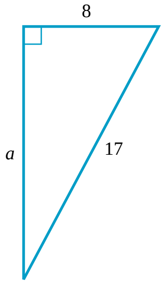
39. $\sqrt{91}$

Find the length of the unknown side.

40.



41.



self-test 8

CHAPTER 8

Name ____________________

Section ________ Date ________

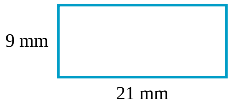
Answers

1. ____________________
2. ____________________
3. ____________________
4. ____________________
5. ____________________
6. ____________________
7. ____________________
8. ____________________
9. ____________________
10. ____________________
11. ____________________
12. ____________________

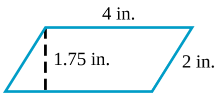
The purpose of this self-test is to help you assess your progress so that you can find concepts that you need to review before the next exam. Allow yourself about an hour to take this test. At the end of that hour, check your answers against those given in the back of this text. If you miss any, go back to the appropriate section to reread the examples until you have mastered that particular concept.

Find the perimeter or circumference of each figure. Use 3.14 for π and round to the nearest tenth when appropriate.

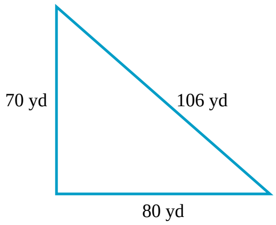
1.



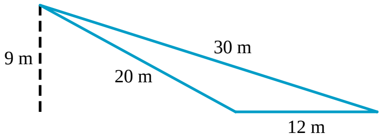
2.


4 in.
1.75 in.
2 in.

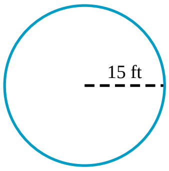
3.


70 yd
106 yd
80 yd

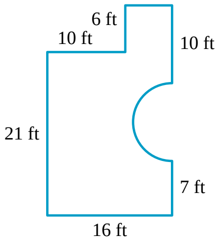
4.


9 m
30 m
20 m
12 m

5.

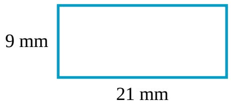


6.

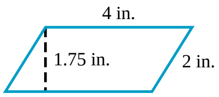

6 ft
10 ft
10 ft
21 ft
7 ft
16 ft

Find the area of each figure. Use 3.14 for π and round to the nearest tenth when appropriate.

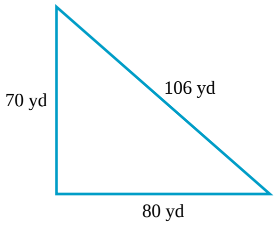
7.



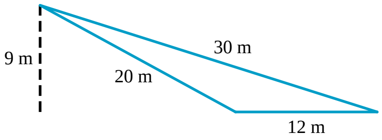
8.


4 in.
1.75 in.
2 in.

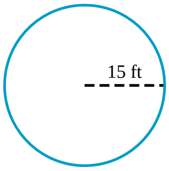
9.


70 yd
106 yd
80 yd

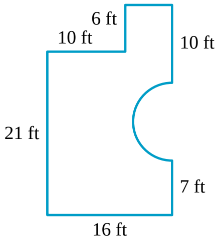
10.


9 m
30 m
20 m
12 m

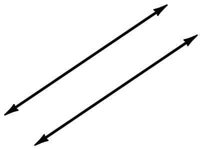
11.

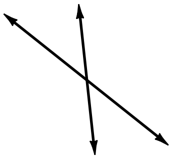


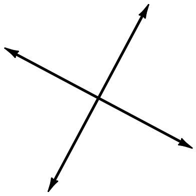
12.

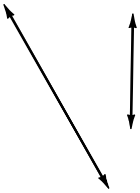

6 ft
10 ft
10 ft
21 ft
7 ft
16 ft

13. Label each pair of lines as parallel, perpendicular, or neither.

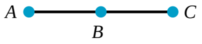

(*a*)


(*b*)

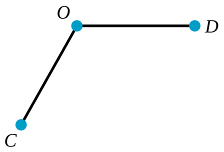

(*c*)


(*d*)

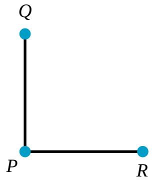
14. Label each of the angles as acute, obtuse, right, or straight.



(*a*)



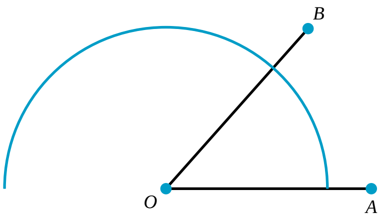
(*b*)



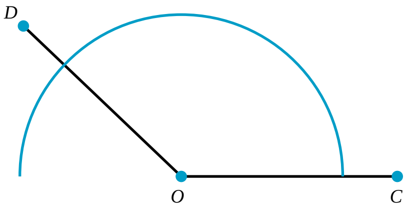
(*c*)

Use a protractor to estimate the measurement of the angles.

15.

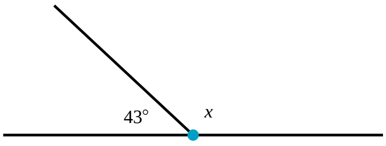


16.

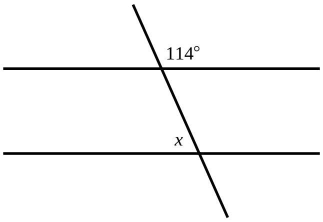


Find m∠x.

17.



18.



Answers

13. __________

14. __________

15. __________

16. __________

17. __________

18. __________

Answers

19. ____________________

20. ____________________

21. ____________________

22. ____________________

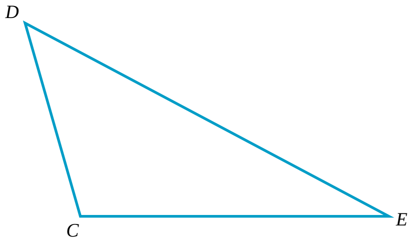
23. ____________________

24. ____________________

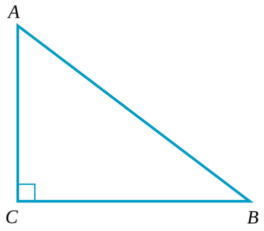
25. ____________________

Label the triangles as acute, obtuse, or right.

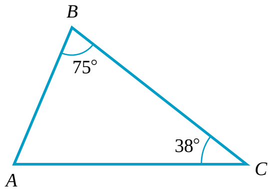
19.



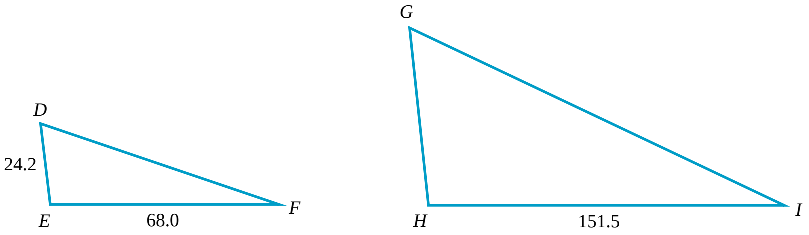
20.



21. Find $m\angle A$.



22. Given that $\triangle DEF$ is similar to $\triangle GHI$, find the length of $\overline{GH}$. Round to the nearest tenth.

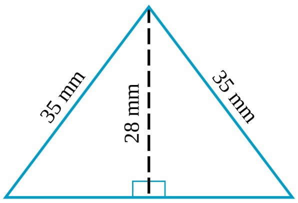

D
24.2
E
68.0
F
G
H
151.5
I

(*a*) (*b*)

23. Find the square root of 441.

24. The legs of a right triangle are 39 m and 52 m in length. Find the length of the hypotenuse.

25. Find the perimeter of the isosceles triangle shown.


35 mm
28 mm
35 mm

cumulative review chapters 1-8

Name ______________________

Section __________ Date __________

The following exercises are presented to help you review concepts from earlier chapters. This is meant as review material and not as a comprehensive exam. The answers are presented in the back of the text. Section references accompany the answers. If you have difficulty with any of these exercises, be certain to at least read through the summary related to those sections.

1. Give the place value of 6 in the number 4,865,201.

2. Evaluate: $82 - 2 \times 3^2$

3. Find the prime factorization for 630.

4. Find the greatest common factor (GCF) of 20, 24, and 32.

5. Find the least common multiple (LCM) of 20 and 24.

6. Multiply: $\frac{2}{3} \times 1\frac{4}{5} \times \frac{5}{8}$

7. Divide: $2\frac{5}{8} \div \frac{7}{12}$

8. Your living room measures $5\frac{2}{3}$ yd by $4\frac{1}{4}$ yd. If carpeting costs \$24 per square yard (yd^2), what will it cost to carpet the room?

9. If you drive 270 miles in $4\frac{1}{2}$ hours, what is your average speed?

10. Add: $\frac{3}{5} + \frac{1}{6} + \frac{2}{3}$

11. Subtract: $7\frac{3}{8} - 3\frac{5}{6}$

12. Adam's goal is to run 20 miles per week. So far he has run distances of $3\frac{1}{2}$ mi, $4\frac{2}{3}$ mi, and $5\frac{1}{4}$ mi. How much more must he do to reach his goal?

Answers

1. ______________________

2. ______________________

3. ______________________

4. ______________________

5. ______________________

6. ______________________

7. ______________________

8. ______________________

9. ______________________

10. ______________________

11. ______________________

12. ______________________

Answers

13. ____________________

14. ____________________

15. ____________________

16. ____________________

17. ____________________

18. ____________________

19. ____________________

20. ____________________

21. ____________________

22. ____________________

23. ____________________

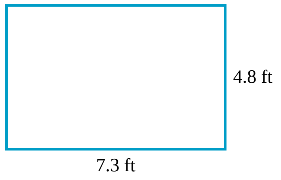
24. ____________________

Find the indicated place values.

13. 3 in 17.2396

14. 5 in 8.0915

15. Find the perimeter and area of the given figure.



16. Write 0.125 as a simplified fraction.

17. Write the decimal equivalent of $\frac{5}{16}$.

18. Express the rate in simplest form: $\frac{81{,}000 \text{ dollars}}{4 \text{ years}}$

19. Solve for the unknown: $\frac{35}{14} = \frac{10}{w}$

20. If one gallon of paint covers 250 square feet (ft^2), how many square feet will $4\frac{1}{2}$ gallons cover?

21. Write 8.5% as a decimal.

22. Write 37.5% as a fraction.

23. Write $\frac{27}{40}$ as a percent.

24. Find 16% of 320.

25. 35% of what number is 525?

26. The number of students at a certain high school dropped by 6% since last year. There are now 1,269 students. How many were there last year?

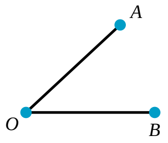
Complete each statement.

27. 3 mi = ______________ yd

28. 250 mg = ______________ g

29. 5.8 km = ______________ m

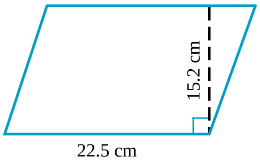
30. Use a protractor to find the measure of the given angle.



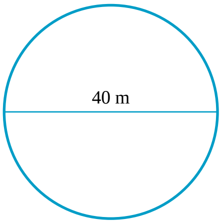
31. Find the circumference of a circle whose radius is 7.9 ft. Use 3.14 for π and round the result to the nearest tenth of a foot.

Find the area for each figure. Use 3.14 for π.

32.



33.



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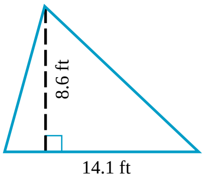


Answers

25. ______________

26. ______________

27. ______________

28. ______________

29. ______________

30. ______________

31. ______________

32. ______________

33. ______________

34. ______________

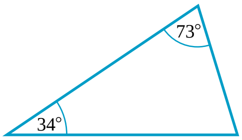
Answers

35. ____________________

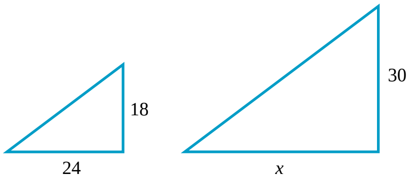
36. ____________________

37. ____________________

35. Find the missing angle and identify the triangle as equilateral, isosceles, or scalene.



36. The given triangles are similar. Find *x*.



37. If two legs of a right triangle have lengths 8 ft and 15 ft, find the length of the hypotenuse.

chapter 9 > Make the Connection

CHAPTER 9

Data Analysis and Statistics

INTRODUCTION

Alana is a statistical analyst for a small company in Woods Hole, Massachusetts. This particular company manufactures buoys that are used in the ocean to measure different properties of the water such as temperature and the amount of salt present. Alana tests the buoys for different properties such as strength and buoyancy. After the research is compiled, she meets with the biotechnology team to analyze and discuss ways to improve and streamline their models. Alana and the biotechnologists work together to create listings that are used to recognize certain patterns. The team also creates graphs to assess the quality of the data and performs statistical tests to enable them to make assumptions about the buoys.

When the research is analyzed and improvements are made, the buoys are ready to be sold to marine biology institutions. The marine scientists will place them in various locations in the ocean, where the buoys will collect data and transmit their findings to satellites. Marine scientists collect this information to research the oceans. You will get a sense of how this is accomplished when you complete the *Outliers in Scientific Data* Activity on page 628.

CHAPTER 9 OUTLINE

Chapter 9 :: Prerequisite Test 566
9.1 **Means, Medians, and Modes** 567
9.2 **Tables, Pictographs, and Bar Graphs** 583
9.3 **Line Graphs and Predictions** 596
9.4 **Creating Bar Graphs and Pie Charts** 604
9.5 **Describing and Summarizing Data Sets** 616
Chapter 9 :: Summary / Summary Exercises / Self-Test / Cumulative Review :: Chapters 1–9 629

9 prerequisite test

Name ______________________

Section __________ Date __________

Answers

1. ______________________

2. ______________________

3. ______________________

4. ______________________

5. ______________________

6. ______________________

7. ______________________

8. ______________________

9. ______________________

10. ______________________

11. ______________________

12. ______________________

13. ______________________

14. ______________________

This prerequisite test provides some exercises requiring skills that you will need to be successful in the coming chapter. The answers for these exercises can be found in the back of this text. This prerequisite test can help you identify topics that you will need to review before beginning the chapter.

Simplify each fraction.

1. $\frac{20}{25}$

2. $\frac{45{,}000}{60{,}000}$

Evaluate each expression.

3. $\frac{3 + 5 + 7 + 9}{4}$

4. $\frac{6 + 11}{2}$

5. $3(8 - 5)^2$

6. $2 + 3 \times 5^2$

7. $\frac{1}{4} \times 360$

8. $\frac{5}{6} \times 360$

Write each fraction as a percent.

9. $\frac{5}{8}$

10. $\frac{1}{6}$

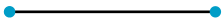
Write each list of numbers in ascending order (from smallest to largest).

11. 7, 1, 2, 11, 5, 0, 10, 13

12. 21, 50, 123, 81, 12, 7, 55, 56

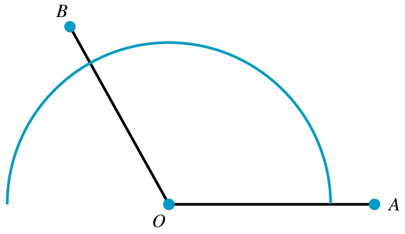
Use a ruler (with U.S. Customary units) to measure the line segment shown.

13.



Use a protractor to find the measure of the angle shown.

14.



9.1

Means, Medians, and Modes

< **9.1 Objectives** >

1 > Calculate the mean of a data set

2 > Find the median of a data set

3 > Compare the mean and median of a data set

4 > Find the mode of a data set

A very useful concept is the **average** of a group of numbers. An average is a number that is typical of a larger group of numbers. In mathematics we have several different kinds of averages that we can use to represent a larger group of numbers. The first of these is the **mean.**

Step by Step

Finding the Mean

To find the mean of a set of numbers:

Step 1 Add all the numbers in the set.

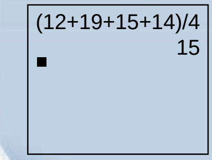
Step 2 Divide that sum by the number of items in the set.

Example 1 Finding the Mean

< **Objective 1** >

RECALL

On a calculator, you need to use parentheses to group the numerator.



```
(12+19+15+14)/4
               15
■
```

Find the mean of the set of numbers 12, 19, 15, and 14.

Step 1 Add all the numbers.

$12 + 19 + 15 + 14 = 60$

Step 2 Divide that sum by the number of items.

$60 \div 4 = 15$ There are four items in this group.

The mean of this set is 15.

You should see that we grouped the numbers together to form their sum before dividing.

$$\frac{12 + 19 + 15 + 14}{4} = \frac{60}{4}$$
$$= 15$$

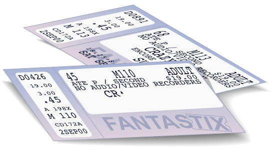
Check Yourself 1

Find the mean of the set of numbers 17, 24, 19, and 20.

Next, we apply the concept of mean to a word problem.

Example 2 Finding the Mean

The ticket prices (in dollars) for the nine concerts held at the Civic Arena this school year were



33, 31, 30, 59, 32, 35, 32, 36, 56

What was the mean price for these tickets?

NOTE

We round to the nearest hundredth because we are dealing with money.

Step 1 Add all the numbers.

$$33 + 31 + 30 + 59 + 32 + 35 + 32 + 36 + 56 = 344$$

Step 2 Divide by 9.

$344 \div 9 = 38.22$ Divide by 9 because there are 9 ticket prices.

The mean ticket price was $38.22.

Check Yourself 2

The costs (in dollars) of the six textbooks that Aaron needs for the fall quarter are

75, 69, 57, 87, 76, 80

Find the mean cost of these books.

The mean is the most common way of measuring the center or average of a set of numbers. However, if the set has an extreme value, then the mean is usually **not typical** of the data set.

Example 3 Computing a Mean with an Extreme Value

Recently, a university looked at the annual average salaries of students who graduated with a B.S. degree in geology 10 years earlier. They were able to make the claim that the average student earned over $7 million.

To verify this claim, we look at the graduates:

B.S. Geology Recipients

Occupation	Salary
Science teacher	$35,500
Graduate student	$12,000
Graduate student	$16,500
Oil prospector	$72,000
Pro basketball player	$35,000,000

Step 1 $35{,}500 + 12{,}000 + 16{,}500 + 72{,}000 + 35{,}000{,}000 = 35{,}136{,}000$

Step 2 $35{,}136{,}000 \div 5 = 7{,}027{,}200$

The mean annual salary was $7,027,200.

Check Yourself 3

A realtor has a list of six homes for sale. Their prices are $269,000, $249,900, $225,000, $290,000, $254,900, and $2,450,000.

Find the mean price of the houses listed.

NOTES

The median is often given when describing salaries, home prices, and people's ages.

In Example 3, the mean annual salary for the degree recipients is over $7 million. However, the typical student majoring in geology would not expect to earn $7 million per year after receiving a degree.

In situations such as these, it is better to use an alternate measure of the average or center of the set of numbers called the median.

The **median** of a set of numbers is the value in the middle when the numbers are sorted in (ascending) order. This way, half the numbers are above the median and half are below the median.

Step by Step

Finding a Median

To compute the median of a set of numbers, follow these steps.

Step 1 Sort the numbers in ascending order (lowest value to highest value).

Case 1 There are an odd number of data points.

- **Step 2** Select the middle data value; this is the median.

Case 2 There is an even number of data points.

- **Step 2** Select the two middle data values.
- **Step 3** Compute the mean of these two numbers; this is the median.

We demonstrate finding a median for both cases in Example 4.

Example 4 Finding the Median

< Objective 2 >

Find the median of each set of numbers.

(a) 35, 18, 27, 38, 19, 63, 22

Step 1 Rewrite the numbers in order from smallest to largest.

18, 19, 22, 27, 35, 38, 63

Step 2 There is an odd number of data points (7), so this is an example of the first case and we simply select the middle value.

$$18, 19, 22, \quad \underset{\substack{\uparrow \\ \text{Middle value}}}{27,} \quad 35, 38, 63$$

The median is 27.

NOTE

Three numbers are less than 27 and three are more than 27.

(b) 29, 88, 73, 81, 62, 37

Step 1 Rewrite the numbers in order from smallest to largest.

29, 37, 62, 73, 81, 88

Step 2 There is an even number of data points (6), so this is an example of the second case. We select the two middle values.

$$29, 37, \quad \underbrace{62, 73,}_{\text{Middle values}} \quad 81, 88$$

Step 3 Find the mean of the pair of middle values.

$$\frac{62 + 73}{2} = \frac{135}{2}$$

$$= 67\frac{1}{2}$$

The median is $67\frac{1}{2}$.

Check Yourself 4

Find the median of each set of numbers.

(a) 8, 6, 19, 4, 21, 5, 27 **(b)** 43, 29, 13, 38, 29, 53

In Example 5, we find the median of the data set that we first saw in Example 3. You should look at how the mean compares to the median for the salaries in question.

Example 5 Finding a Median

(a) Find the median salary for the B.S. recipients given above.

> CAUTION

You must sort the numbers before finding the median!

Step 1 Sort the data.

$12,000
$16,500
$35,500
$72,000
$35,000,000

Because there is an odd number of data points (5), this is an example of the first case.

Step 2 Select the middle value.

$$\$12{,}000,\ \$16{,}500,\ \underset{\substack{\uparrow \\ \text{Middle value}}}{\$35{,}500},\ \$72{,}000,\ \$35{,}000{,}000$$

The median salary for these degree recipients is $35,500. This is a much more reasonable expectation for someone who graduates with this degree.

(b) If there were a sixth graduate from the program (as shown below), find the median.

B.S. Geology Recipients

Occupation	Salary
Graduate student	$12,000
Graduate student	$16,500
Science teacher	$35,500
Researcher	$68,000
Oil prospector	$72,000
Pro basketball player	$35,000,000

Step 1 The data set has already been sorted.

There is now an even number of data points (6), so this is an example of the second case.

Step 2 Select the two middle data points.

$$\$12{,}000,\ \$16{,}500,\ \underbrace{\$35{,}500,\ \$68{,}000,}_{\text{Middle values}}\ \$72{,}000,\ \$35{,}000{,}000$$

$35,500 and $68,000 are the two points in the middle.

Step 3 Compute the mean of these two middle values.

$$\frac{35{,}500 + 68{,}000}{2} = \frac{103{,}500}{2} = 51{,}750$$

The median income for this set is $51,750.

Check Yourself 5

In each case, find the median of the data set given.

(a) The total wheat production (in millions of bushels) in Kansas between the years 1998 and 2002 is given.

Wheat Supply (Kansas)

	1998	1999	2000	2001	2002
Production	494.9	432.4	347.8	328.0	264.0

Source: Kansas Agricultural Statistics Service, Kansas Department of Agriculture (9/02).

(b) A realtor has a list of six homes for sale. Their prices are $269,000, $249,900, $225,000, $290,000, $254,900, and $2,450,000.

Note: You found the mean of this set in Check Yourself 3. How does the mean compare to the median? Which is a better measure of the average home price on the realtor's list?

Now let us look more explicitly at comparing means and medians.

Example 6 Comparing the Mean and the Median

< Objective 3 >

The following numbers represent the hourly wage of seven employees of a local chip manufacturing plant.

12, 11, 14, 16, 32, 13, 14

(a) Find the mean hourly wage.

Step 1 Add all the numbers in the set.

$12 + 11 + 14 + 16 + 32 + 13 + 14 = 112$

Step 2 Divide that sum by the number of items in the set.

$112 \div 7 = 16$

The mean wage is $16 an hour.

(b) Find the median wage for the seven workers.

Step 1 Rewrite the numbers in order from smallest to largest.

11, 12, 13, 14, 14, 16, 32

Step 2 There are 7 employees (odd), so we select the middle value.

$$11,\ 12,\ 13,\quad \underset{\substack{\uparrow \\ \text{Middle value}}}{14,}\quad 14,\ 16,\ 32$$

The median wage is $14 per hour.

(c) Compare the mean and median of this data set. Which better describes the typical hourly wage paid at this plant?

The mean hourly wage is $16 and the median is $14. The median is a better description of the typical wage because only one employee earns more than $16. The average worker should not expect to earn $16 or more at this plant because most of the employees earn closer to $14 per hour.

Check Yourself 6

The following are Jessica's phone bills over the last year.

26, 67, 31, 24, 15, 17, 41, 27, 17, 22, 26, 47

(a) Find the mean amount of her phone bills.
(b) Find the median amount of her phone bills.
(c) Compare the mean and median of this data set. Which better describes Jessica's typical monthly phone bill?

Mean and median are both useful ways of determining a typical member of many sets of numbers. In general, we prefer to use the mean because the techniques of inferential statistics are more effective when applied to the mean than with the median. However, as we stated earlier, if there are extreme values, the median may be more typical of a data set, and thus, the median may be a better measure of center.

Both the mean and median as measures of the typical member of a data set work with sets of numbers. When our data is not made up of numbers, then we cannot use either method to describe the *typical member*.

For instance, we can not add together a set of colors and divide by the size of the set. Nor can we get anything useful by sorting a set of phone numbers from smallest to largest.

As an example, if we surveyed respondents as to the color they prefer for new automobiles, we might get a data set as shown below.

red
blue
blue
purple
orange
blue
red
black
black
blue

We cannot compute a mean for this set (adding these ten colors, then dividing by 10 does not give anything meaningful). Similarly, there is no natural order in which to sort the data, so we are unable to find a middle value.

Another measure used as an average is the **mode.** The mode is always used when the data are not numbers.

Definition

Mode The *mode* of a set of data is the item or number that appears most frequently.

Example 7 Finding a Mode

< Objective 4 >

For the data set given above, the mode is blue (which appears four times).

Check Yourself 7

A researcher studying the amphibian population in Yellowstone National Park recorded the following sightings during the course of her visit. Determine the mode of her data set.

Columbia spotted frog
Columbia spotted frog
chorus frog
boreal toad
Columbia spotted frog
Columbia spotted frog
chorus frog
blotched tiger salamander
Columbia spotted frog
chorus frog
boreal toad
blotched tiger salamander
boreal toad
chorus frog
chorus frog
chorus frog
Columbia spotted frog
Columbia spotted frog
Columbia spotted frog
blotched tiger salamander

Adapted from information provided by the National Park Service; U.S. Department of the Interior

We can also find the mode of a set of numbers.

Example 8 Finding a Mode

Find the mode for the set of numbers given.

22, 24, 24, 24, 24, 27, 28, 32, 32

The mode, 24, is the number that appears most frequently.

Check Yourself 8

Find the mode for the set of numbers given.

7, 7, 7, 9, 11, 13, 13, 15, 15, 15, 15, 21

If a data set has two values that appear the most, we say it is *bimodal*. If there are three or more values that appear the most, the data set has no mode.

Example 9 Finding Modes

Find the mode of each set.

(a) 4, 6, 1, 5, 5, 4, 2

Both 4 and 5 appear twice. The other members of the set (1, 2, and 6) appear only once. Therefore, 4 and 5 are both modes and we say the set is bimodal.

(b) The computers in a lab, by manufacturer.

Apple, IBM, Apple, Compaq, Dell, Compaq, Dell, IBM

Each computer manufacturer appears twice. In this case, we say the data set does not have a mode.

Check Yourself 9

Find the mode of each set.

(a) A set of eye colors for a group of people.
blue, brown, brown, hazel, green, hazel, blue, brown, hazel

(b) 17, 43, 6, 22, 23, 19, 65, 51

To summarize our work so far, we have three methods for determining the typical member of a data set, or measuring its center. These are the mean, median, and mode.

Mean and median are used only for sets of numbers, whereas mode can be used for any set.

The mean is the more common, and desirable, method for determining an average, but median is more resistant to extreme values.

Example 10 A Health Sciences Application

A pregnant, adult female patient tested positive for gestational diabetes during the last 3 months of her pregnancy. Blood glucose levels (in milligrams per 100 milliliters) were gathered and recorded on a regular basis. The results are tabulated here.

78	104	103	101	78	120	103
97	75	128	90	98	80	128
119	106	83	99	101	108	127
118	96	125	78	123	124	92

Compute the mean.

To do this, we must first find the sum of the numbers. In this case, a calculator (or computer) proves very useful, since there are 28 numbers to add. The sum of these is 2,882. Dividing by 28 produces 102.928 . . . . To the nearest whole number, the mean glucose level is 103.

Check Yourself 10

For the data given in Example 10, find the median.

Check Yourself ANSWERS

1. 20 **2.** \$74 **3.** \$623,133.33 **4.** **(a)** 8; **(b)** $33\frac{1}{2}$
5. **(a)** 347.8 million bushels; **(b)** \$261,950 **6.** **(a)** \$30; **(b)** \$26; **(c)** The median is a better measure of Jessica's typical phone bill because 8 of 12 bills were below the mean. **7.** Columbia spotted frog **8.** 15
9. **(a)** Brown and hazel (bimodal); **(b)** no mode **10.** 102

Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 9.1

(a) To find the ______________ of a set of numbers, add the numbers in the set and divide by the number of items in the set.

(b) The ______________ is the middle value when an odd number of numbers are arranged in order.

(c) The ______________ of a set of data is the item or number that appears most frequently.

(d) A set with two different modes is called ______________.

9.1 exercises

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Name ____________________

Section __________ Date __________

< Objective 1 >

Find the mean of each set of numbers.

1. 6, 9, 10, 8, 12 > Videos

2. 13, 15, 17, 17, 18

3. 13, 15, 17, 19, 24, 25

4. 41, 43, 56, 67, 69, 72

5. 12, 14, 15, 16, 16, 16, 17, 22, 25, 27

6. 21, 25, 27, 32, 36, 37, 43, 43, 44, 51

7. 5, 8, 9, 11, 12

8. 7, 18, 11, 7, 12

9. 9, 8, 11, 14, 9

10. 21, 23, 25, 27, 22, 20

< Objective 2 >

Find the median of each set of numbers.

11. 2, 3, 5, 6, 10 > Videos

12. 12, 13, 15, 17, 18

13. 23, 24, 27, 31, 36, 38

14. 1, 4, 9, 15, 25, 36

15. 46, 13, 47, 25, 68, 51

16. 26, 71, 69, 33, 71, 25, 75

< Objective 3 >

17. Set: 45, 60, 70, 38, 54, 64, 70

(a) Calculate the mean of the set of numbers (to the nearest hundredth).
(b) Find the median of the set of numbers.
(c) Is the mean or median a better indicator of the average number in the set?

18. Set: 140, 125, 128, 150, 810, 112, 144, 153

(a) Calculate the mean of the set of numbers.
(b) Find the median of the set of numbers.
(c) Is the mean or median a better indicator of the average number in the set?

19. **BUSINESS AND FINANCE** A real estate agent lists eight houses for sale at the prices shown below.

$209,000	$224,900	$249,900	$215,000
$289,900	$265,000	$274,900	$749,900

(a) Calculate the mean home price of the agent's listings.
(b) Find the median home price of the agent's listings.
(c) Is the mean or median a better indicator of the typical home price listed?

20. A group of students takes an exam. Their test scores are shown below.

82, 76, 90, 94, 88, 64, 72, 92, 88, 76, 80

(a) Calculate the mean test score.
(b) Find the median test score.
(c) Is the mean or median a better indicator of the typical test score?

21. A physician sees 12 female patients in one day. As part of her examination, she measures the height of each patient. Their heights are listed below, in inches.

66, 60, 62, 64, 64, 58, 63, 68, 65, 64, 63, 65

(a) Calculate the mean height of her female patients.
(b) Find the median height of her female patients.
(c) Is the mean or median a better indicator of the typical height of her female patients?

Answers

1. __________ 2. __________
3. __________ 4. __________
5. __________ 6. __________
7. __________ 8. __________
9. __________ 10. __________
11. __________ 12. __________
13. __________ 14. __________
15. __________ 16. __________
17. ____________________
18. ____________________
19. ____________________
20. ____________________
21. ____________________

Answers

22. ____________________

23. ____________________

24. ____________________

25. ____________________

26. ____________________

27. ____________________

28. ____________________

29. ____________________

30. ____________________

31. ____________________

32. ____________________

33. ____________________

34. ____________________

35. ____________________

36. ____________________

22. An IT firm forms a group to solve server issues encountered by their customer base. The group is selected from throughout the firm's ranks. The annual salary of each group member is shown below.

$68,000	$46,000	$52,000
$85,400	$89,500	$105,000
$72,000	$68,000	$740,000

(a) Calculate the mean salary of the group members.

(b) Find the median salary of the group members.

(c) Is the mean or median a better indicator of the typical group member's salary?

< Objective 4 >

Find the mode of each set of numbers.

23. 17, 13, 16, 18, 17 > Videos

24. 41, 43, 56, 67, 69, 72

25. 21, 44, 25, 27, 32, 36, 37, 44

26. 9, 8, 10, 9, 9, 10, 8

27. 12, 13, 7, 14, 4, 11, 9

28. 8, 2, 3, 3, 4, 9, 9, 3

29. The following are eye colors from a class of eight students. Which color is the mode?

hazel, green, brown, brown, blue, green, hazel, green

30. The weather in Philadelphia over the last 7 days was as follows:

rain, sunny, cloudy, rain, sunny, rain, rain

What type of weather was the mode?

Solve each application.

31. **STATISTICS** High temperatures of 86°, 91°, 92°, 103°, and 98°F were recorded for the first 5 days of July. What was the mean high temperature? > Videos

chapter 9 > Make the Connection

32. **STATISTICS** A salesperson drove 238, 159, 87, 163, and 198 miles (mi) on a 5-day trip. What was the mean number of miles driven per day?

33. **STATISTICS** Highway mileage ratings for seven new diesel cars were 43, 29, 51, 36, 33, 42, and 32 miles per gallon (mi/gal). What was the mean rating?

34. **STATISTICS** The enrollments in the four elementary schools of a district are 278, 153, 215, and 198 students. What is the mean enrollment?

Basic Skills | **Challenge Yourself** | Calculator/Computer | Career Applications | Above and Beyond

Determine whether each statement is **true** *or* **false.**

35. The mode can only be found for sets of numbers.

36. The mean can be larger than most of the data.

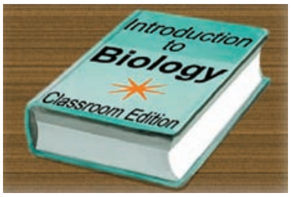
In each statement, fill in the blank with **always, sometimes,** *or* **never.**

37. The mean and the median are __________ the same number.

38. To find the median, you must ________ put the numbers in order.

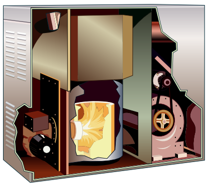
39. **Statistics** To get an A in history, you must have a mean of 90 on five tests. Your scores thus far are 83, 93, 88, and 91. How many points must you have on the final test to receive an A? (*Hint:* First find the total number of points you need to get an A.)

40. **Statistics** To pass biology, you must have a mean of 70 on six quizzes. So far your scores have been 65, 78, 72, 66, and 71. How many points must you have on the final quiz to pass biology?



41. **Statistics** Louis had scores of 87, 82, 93, 89, and 84 on five tests. Tamika had scores of 92, 83, 89, 94, and 87 on the same five tests. Who had the higher mean score? By how much?

42. **Statistics** The Wong family had heating bills of $105, $110, $90, and $67 in the first 4 months of 2003. The bills for the same months of 2004 were $110, $95, $75, and $76. In which year was the mean monthly bill higher? By how much?



Science and Medicine *Monthly energy use, in kilowatt-hours (kWh), by appliance type for four typical U.S. families is shown in the table.*

	Wong Family	McCarthy Family	Abramowitz Family	Gregg Family
Electric range	97	115	80	96
Electric heat	1,200	1,086	1,103	975
Water heater	407	386	368	423
Refrigerator	127	154	98	121
Lights	75	99	108	94
Air conditioner	123	117	96	120
Color TV	39	45	21	47

43. What is the mean number of kilowatt-hours used each month by the four families for heating their homes?

44. What is the mean number of kilowatt-hours used each month by the four families for hot water?

45. What is the mean number of kilowatt-hours used per appliance by the McCarthy family?

46. What is the mean number of kilowatt-hours used per appliance by the Gregg family?

Answers

37. ______________________
38. ______________________
39. ______________________
40. ______________________
41. ______________________
42. ______________________
43. ______________________
44. ______________________
45. ______________________
46. ______________________

Answers

47. ______

48. ______

49. ______

50. ______

51. ______

52. ______

53. ______

54. ______

55. ______

56. ______

57. ______

This explanation is followed by a set of exercises for which the calculator might be the preferred tool. As indicated by the placement of this explanation, you should refrain from using a calculator on the exercises that precede this.

Many calculators have built-in statistical functions that allow you to calculate the mean and median of a data set. Because these features vary widely between calculators, you will need to consult your instructor or the owner's manual for your calculator.

You can compute the mean of a data set without using built-in statistical functions as shown.

To compute the mean of the set: 2,253, 3,451, 2,157, 4,126, 967

Step 1 Press the *clear* key. $\boxed{\text{CLEAR}}$

Step 2 Press the *open parenthesis* key. $\boxed{(}$

Step 3 Enter the numbers, separated by *plus* signs. 2253 $\boxed{+}$
3451 $\boxed{+}$
2157 $\boxed{+}$
4126 $\boxed{+}$
967

Step 4 Press the *close parenthesis* key. $\boxed{)}$

Step 5 Enter *division* and the number of items. $\boxed{\div}$ 5

Step 6 Press *enter* or equals. $\boxed{\text{ENTER}}$ or $\boxed{=}$

Your display should read 2590.8.

Use your calculator to find the mean of each set of numbers.

47. 48, 50, 51, 52, 49, 50

48. 20, 18, 17, 24, 22, 19

49. 346, 351, 353, 347, 341, 382, 373, 363

50. 1,560, 1,540, 1,570, 1,555, 1,565, 1,545, 1,557

51. 16,430, 15,487, 17,982, 11,290, 21,908, 16,545

52. 311,431, 286,356, 356,090, 292,007, 301,857, 299,005

53. 18, 21, 20, 22

54. 356, 371, 366, 373, 359, 363

55. 1,898, 1,913, 1,875, 1,937

56. 15,865, 16,270, 16,090, 15,904

57. **BUSINESS AND FINANCE** The revenue for the leading apparel companies in the United States in 1997 is given in the table.

Company	Revenue (in millions)
Nike	$9,187
Vanity Fair	5,222
Reebok	3,637
Liz Claiborne	2,413
Fruit of the Loom	2,140
Nine West	1,865
Kellwood	1,521
Warmaio	1,437
Jones Apparel	1,387

What is the mean revenue taken in by these companies?

58. **BUSINESS AND FINANCE**

Unemployment in the United States (in thousands)

Year	Employed	Unemployed
1993	120,259	8,940
1994	123,060	7,996
1995	124,900	7,404
1996	126,708	7,236
1997	129,558	6,739
1998	131,463	6,210
1999	133,488	5,880
2000	136,891	5,692
2001	136,933	6,801
2002	136,485	6,378

Source: U.S. Department of Labor, Bureau of Labor Statistics.

Find the mean number of employed and unemployed people per year from 1993 to 2002. Round to the nearest thousand.

The work stoppages (strikes and lockouts) in the United States from 1995 to 2002 are given in the table.

Year	No. of Stoppages	Work Days Idle
1995	192	5,771
1996	273	4,889
1997	339	4,497
1998	387	5,116
1999	73	1,996
2000	394	20,419
2001	99	1,151
2002	46	6,596

Source: U.S. Department of Labor, Bureau of Labor Statistics.

59. Find the mean number of work stoppages per year from 1995 to 2002.

60. Find the mean number of work days idle from 1995 to 2002.

Basic Skills | Challenge Yourself | Calculator/Computer | Career Applications | Above and Beyond

61. **INFORMATION TECHNOLOGY** Response times in milliseconds (ms) from your computer to a local router using ping are given by the table.

2.2	2.5
2.3	2.4
1.9	2.2
2.0	2.4
2.5	2.5

Answers

58. ____________________

59. ____________________

60. ____________________

61. ____________________

Answers

62. __________

63. __________

64. __________

65. __________

(a) Compute the mean.
(b) Compute the median.
(c) Compute the mode.

62. CONSTRUCTION The thicknesses (in mm) of several parts are as follows:

30.9, 30.7, 29, 30.6, 29.3, 31.2, 29.3

(a) Calculate the mean. Round to the nearest hundredth.
(b) Find the median.
(c) Find the mode.

63. AUTOMOTIVE Early in 2005, twenty gas stations from around the United States were surveyed to determine the price of regular gasoline. The raw data are as follows:

2.14	1.93	2.16	2.19
1.99	2.36	2.25	2.02
2.48	2.31	2.04	2.15
2.33	2.55	2.22	2.24
2.18	2.19	2.38	2.33

(a) Calculate the mean gas price.
(b) Calculate the median gas price.
(c) Should you use the mean or median to describe the typical gas price?

64. WELDING Throughout the day, welds are randomly chosen and tested for strength. The results are shown in the following table. [Tensile strength can be expressed in pounds per square inch (lb/in.^{2}).]

2,314	2,318	2,307	2,291
2,289	2,301	2,320	2,318
2,322	2,297	2,314	2,296
2,309	2,311	2,304	2,321

(a) Calculate the mean tensile strength.
(b) Calculate the median tensile strength.
(c) Should you use the mean or median to describe the typical strength of a weld in this data set?

Basic Skills | Challenge Yourself | Calculator/Computer | Career Applications | **Above and Beyond** ▲

65. BUSINESS AND FINANCE Fred kept the following records of his utility bills for 12 months: \$53, \$51, \$43, \$37, \$32, \$29, \$34, \$41, \$58, \$55, \$49, and \$58.

(a) Find the mean of Fred's monthly utility bills.
(b) Find the median of Fred's monthly utility bills.
(c) Is the mean or median a more useful representative of Fred's monthly utility bills? Write a brief paragraph justifying your response.

66. **BUSINESS AND FINANCE** These scores were recorded on a 200-point final examination: 193, 185, 163, 186, 192, 135, 158, 174, 188, 172, 168, 183, 195, 165, 183.

(a) Find the mean final examination score.

(b) Find the median final examination score.

(c) Is the mean or median a more useful representative of the final examination scores? Write a brief paragraph justifying your response.

67. List the advantages and disadvantages of the mean, median, and mode.

68. In a certain math class, you take four tests and the final, which counts as two tests. Your grade is the average of the six tests. At the end of the course, you compute both the mean and the median.

(a) You want to convince the teacher to use the mean to compute your average. Write a note to your teacher explaining why this is a better choice. Choose numbers that make a convincing argument.

(b) You want to convince the teacher to use the median to compute your average. Write a note to your teacher explaining why this is a better choice. Choose numbers that make a convincing argument.

69. Create a set of five numbers such that the mean is equal to the median.

70. Create a set of five numbers such that the mean is greater than the median.

71. Create a set of five numbers such that the mean is less than the median.

72. Write a paragraph describing the conditions necessary for the mean of a data set to be greater than the median, less than the median, and equal to the median. How do you think the mode would compare to the mean and median in each of these situations?

Answers

1. 9 **3.** $18\frac{5}{6}$ **5.** 18 **7.** 9 **9.** 10.2 **11.** 5 **13.** 29
15. $46\frac{1}{2}$ **17.** **(a)** 57.29; **(b)** 60; **(c)** mean **19.** **(a)** $309,812.50;
(b) $257,450; **(c)** median **21.** **(a)** 63.5; **(b)** 64; **(c)** mean **23.** 17
25. 44 **27.** No mode **29.** Green **31.** 94°F **33.** 38 mi/gal
35. False **37.** sometimes **39.** 95 points **41.** Tamika, by two points
43. 1,091 kWh **45.** 286 kWh **47.** 50 **49.** 357 **51.** 16,607
53. 20.25 **55.** 1,905.75 **57.** $3,201,000,000 **59.** 225 stoppages
61. **(a)** 2.29 ms; **(b)** 2.35 ms; **(c)** 2.5 ms **63.** **(a)** $2.22; **(b)** $2.21; **(c)** mean
65. **(a)** $45; **(b)** $46; **(c)** Above and Beyond **67.** Above and Beyond
69. Answers will vary **71.** Answers will vary

Answers

66. ____________________

67. ____________________

68. ____________________

69. ____________________

70. ____________________

71. ____________________

72. ____________________

Activity 24 ::
Car Color Preferences

While we tend to use the mean and median to describe the center of a data set, people who work in marketing and manufacturing use the mode at least as often. In many such applications, the mode of a data set is the natural way to describe the center.

Out-of-Class Component

You should find a safe spot to observe cars. This could be an intersection, street, or even a parking lot. Record the color of the first 10 cars you see. Use broad color categories (such as *blue*), rather than more specific categories (such as *light blue* and *dark blue*). Make a second list of the next 25 cars you see.

In-Class Component

You should have two data sets: one list of 10 colors and one list of 25 colors.

1. Find the mode of each of the two data sets.

2. **(a)** Do the modes differ?

(b) If so, which do you feel is more accurate, and why?

3. Create a data set of 35 colors by combining your two lists. Find the mode of this data set.

4. The method you used to gather your data is called **convenience sampling.** Briefly describe why the method might take on that name.

5. **(a)** Describe two benefits and two weaknesses of convenience sampling as a method of gathering data.

(b) Statisticians generally avoid convenience sampling, believing its weaknesses outweigh its strengths. Briefly describe how you might create a sample of car colors that more accurately mirrors the preferences of the population as a whole.

9.2

Tables, Pictographs, and Bar Graphs

< 9.2 Objectives >

1 > Read a table

2 > Interpret a table

3 > Create a pictograph

4 > Read a bar graph

NOTE

Rows read left to right.
Columns read top to bottom.

A **table** is a display of information in rows or columns. Tables can be used anywhere that we need to summarize information.

The following is a table describing land area and world population. Each entry in the table is called a **cell.** This table will be used with Examples 1 and 2.

Continent or Region	Land Area ($1{,}000\ \text{mi}^2$)	% of Earth	Population 1900	Population 1950	Population 2000
North America	9,400	16.2	106,000,000	221,000,000	305,000,000
South America	6,900	11.9	38,000,000	111,000,000	515,000,000
Europe	3,800	6.6	400,000,000	392,000,000	510,000,000
Asia (including Russia)	17,400	30.1	932,000,000	1,591,000,000	4,028,000,000
Africa	11,700	20.2	118,000,000	229,000,000	889,000,000
Oceana (including Australia)	3,300	5.7	6,000,000	12,000,000	32,000,000
Antarctica	5,400	9.3	Uninhabited	—	—
World total	57,900		1,600,000,000	2,556,000,000	6,279,000,000

Source: Bureau of the Census, U.S. Dept. of Commerce.

Example 1 Reading a Table

< Objective 1 >

From the land area and world population table, find each of the following.

(a) What was the population of Africa in 1950?

Looking at the cell that is in the row labeled Africa and the column labeled 1950, we find a population of 229,000,000.

(b) What is the land area of Asia in square miles?

The cell in the row Asia and column labeled land area says 17,400. But note that the column is labeled "$1{,}000\ \text{mi}^2$." The land area is 17,400 thousand square miles, or $17{,}400{,}000\ \text{mi}^2$.

Check Yourself 1

Use the land area and world population table to answer each question.

(a) **What was the population of South America in 1900?**
(b) **What is the land area of Africa as a percent of Earth's land area?**

We can frequently use a table to find answers to questions that are not directly answered as part of the table.

Example 2 Interpreting a Table

< Objective 2 >

Use the world population and land area table to answer each of the following questions.

(a) To the nearest tenth of a percent, what percent of the world's population was in North America in the year 2000?

305,000,000 of the world's 6,279,000,000 people lived in North America.

$$\frac{305{,}000{,}000}{6{,}279{,}000{,}000} \approx 0.04857 \approx 4.9\%$$

Although North America has more than 16% of the Earth's land area, it has less than 5% of the world's population.

RECALL

Compute North America's portion of the Earth's total land area similarly.

$$\frac{9{,}400}{57{,}900} \approx 0.1623 \approx 16\%$$

(b) What percent of the Earth's habitable land is in Asia?

First, we must decide what is meant by "habitable land." We will assume anything outside of Antarctica is habitable. To find the amount of habitable land, we take the total of 57,900,000 and subtract Antarctica's 5,400,000. This leaves total habitable land of 52,500,000 mi^2.

$$\frac{17{,}400{,}000}{52{,}500{,}000} \approx 0.3314 \approx 33.1\%$$

(c) What was the mean population for the six populated regions in 1900?

Although we could add the six numbers, you should see that they have already been totaled. Using that total, we find the average.

$$\frac{1{,}600{,}000{,}000}{6} \approx 267{,}000{,}000$$

Check Yourself 2

Use the world population and land area table to answer each of the following questions.

(a) **To the nearest percent, what was the increase in the population of Africa between 1950 and 2000?**
(b) **Did world population increase by a greater percent between 1900 and 1950 or between 1950 and 2000?**

NOTE

Spreadsheets are a valuable tool for working with tables and their associated graphs.

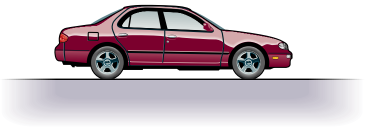
If a table has only one or two columns of numeric information, it is often easier to interpret it in picture form. A **graph** is a diagram that represents the connection between two or more things. A graph that uses pictures to represent quantities is called a **pictograph.**

Example 3 Creating a Pictograph

< Objective 3 >

Create a pictograph that displays the information in the table at the right.

First, we must decide what to use as our picture unit. A car is the obvious choice in this case.



Second, we must decide the "value" of each car. Given the units in the table, we will let each car represent 10,000,000 registrations.

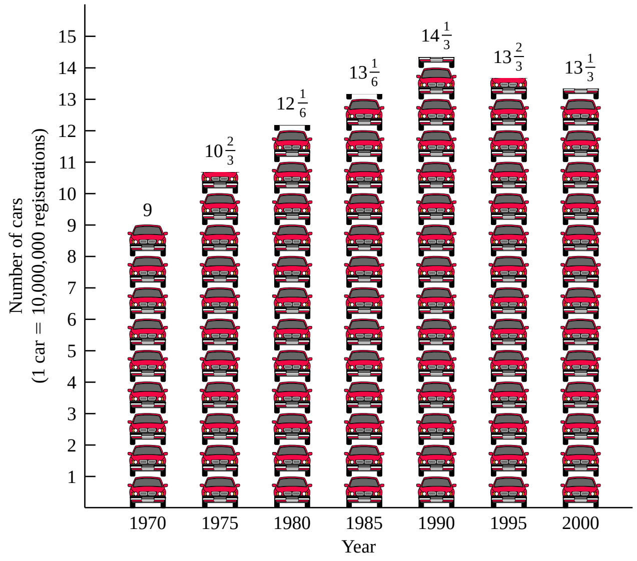
Cars Registered in the United States

Year	Cars Registered
1970	89,243,557
1975	106,705,934
1980	121,600,843
1985	131,664,029
1990	143,549,627
1995	136,066,045
2000	133,621,245

Source: Federal Highway Admin., U.S. Dept. of Transportation.

Federal Highway Admin., U.S. Dept. of Transportation

Year	Cars Pictured
1970	9
1975	$10\frac{2}{3}$
1980	$12\frac{1}{6}$
1985	$13\frac{1}{6}$
1990	$14\frac{1}{3}$
1995	$13\frac{2}{3}$
2000	$13\frac{1}{3}$



Check Yourself 3

Create a pictograph to represent the following table, which gives the percent of U.S. workers employed as farm workers.

Economic Research Service, U.S. Dept. of Agriculture

Year	Percent of Workers in Farm Occupations
1800	73.2
1840	68.6
1880	57.1
1920	27.0
1960	6.1
2000	2.3

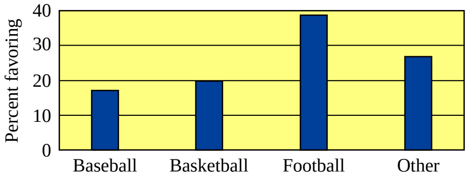
Other kinds of graphs also show the relationship of two sets of data. Perhaps the most common is the bar graph.

A bar graph is read in much the same manner as a pictograph.

Example 4 Reading a Bar Graph

< Objective 4 >

This bar graph represents the response to a Gallup poll that asked people what their favorite spectator sport was. In the graph, the information at the bottom describes the sport, and the information along the side describes the percentage of people surveyed. The height of the bar indicates the percentage of people who favor that particular sport.



(a) Find the percentage of people for whom football is their favorite spectator sport.

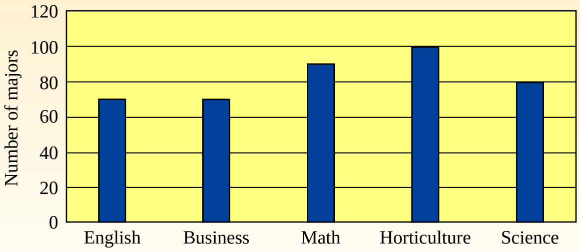
As is the case with pictographs, we frequently have to estimate our answer when reading a bar graph. In this case, 38% would be a good estimate.

(b) Find the percentage of people for whom baseball is their favorite spectator sport.

Again, we can only estimate our answer. It appears to be approximately 17% of the people responding who favor baseball.

Check Yourself 4

This bar graph represents the number of students who majored in each of five areas at Experimental Community College.

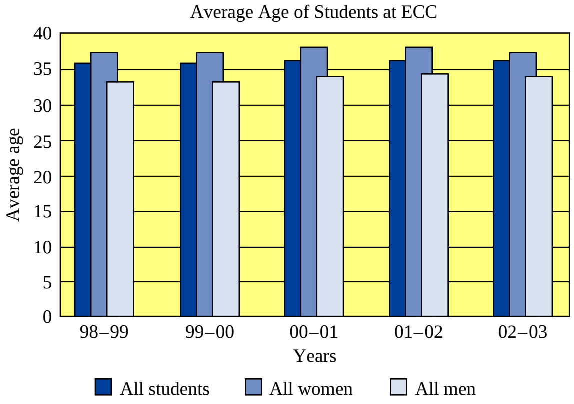


(a) How many mathematics majors were there?
(b) How many English majors were there?

Some bar graphs display additional information by using different colors or shading for different bars. With such graphs it is important to read the legend. The **legend** is the key that describes what each color or shade of the bar represents.

Example 5 Reading the Legend of a Graph

This bar graph represents the average student age at ECC.



(a) What was the average age of female students in 2002–2003?

The legend tells us that the ages of all women are represented as the medium blue color. Looking at the height of the medium blue column for the year 2002–2003, we see the average age was about 37.

(b) Who tends to be older, male students or female students?

The medium blue bar is higher than the light blue bar in every year. Female students tend to be older than male students at ECC.

Check Yourself 5

Use the graph in Example 5 to answer each question.

(a) Did the average age of female students increase or decrease between 2001–2002 and 2002–2003?
(b) What was the average age of male students in 2000–2001?

Check Yourself ANSWERS

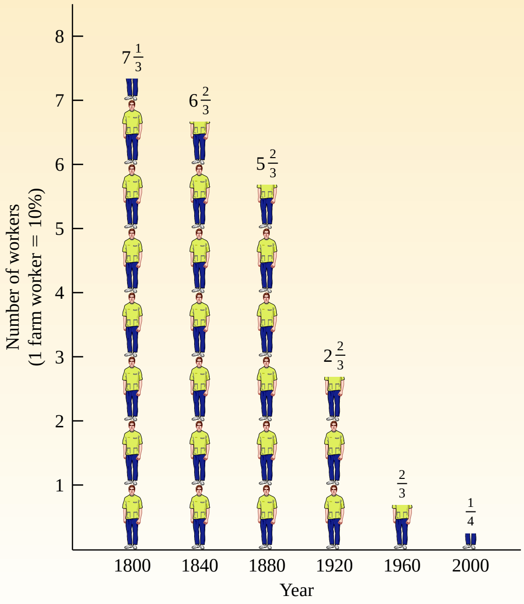
1. **(a)** 38,000,000; **(b)** 20.2% **2.** **(a)** 288%; **(b)** 1950–2000 (146% vs. 60%)

3.

1 Worker = 10%
(Economic Research Service, U.S. Dept. of Agriculture)

Year	Workers Pictured	Year	Workers Pictured
1800	$7\frac{1}{3}$	1920	$2\frac{2}{3}$
1840	$6\frac{2}{3}$	1960	$\frac{2}{3}$
1880	$5\frac{2}{3}$	2000	$\frac{1}{4}$

(*continued*)



4. **(a)** 90; **(b)** 70 **5.** **(a)** It decreased; **(b)** 34

Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 9.2

(a) A table is a display of information in ________________ or columns.

(b) Each entry in the table is called a ________________.

(c) A graph that uses pictures to represent quantities is called a ____________.

(d) The ________________ is the key that describes what each color or shade of a bar (in a bar graph) represents.

9.2 exercises

< Objectives 1–2 >

Use the world population and land area table reproduced here for exercises 1 to 10. Round answers to the nearest tenth or tenth of a percent.

Continent or Region	Land Area (1,000 mi^2)	% of Earth	Population 1900	Population 1950	Population 2000
North America	9,400	16.2	106,000,000	221,000,000	305,000,000
South America	6,900	11.9	38,000,000	111,000,000	515,000,000
Europe	3,800	6.6	400,000,000	392,000,000	510,000,000
Asia (including Russia)	17,400	30.1	932,000,000	1,591,000,000	4,028,000,000
Africa	11,700	20.2	118,000,000	229,000,000	889,000,000
Oceana (including Australia)	3,300	5.7	6,000,000	12,000,000	32,000,000
Antarctica	5,400	9.3	Uninhabited	—	—
World total	57,900		1,600,000,000	2,556,000,000	6,279,000,000

Source: Bureau of the Census, U.S. Dept. of Commerce.

1. **(a)** What was the population in North America in 1950? > Videos

(b) What is the total land area in North America as a percent of Earth?

2. **(a)** What is the population of Europe in 2000?

(b) What is the total area of Europe?

3. **(a)** What was the percent increase in population in Asia from 1900 to 1950?

(b) What was the percent increase in population in Asia from 1950 to 2000?

(c) What was the population per square mile in Asia in 1950?

(d) What was the population per square mile in Asia in 2000?

4. Compare the population per square mile in Asia to the population per square mile in North America for the year 2000.

5. What was the mean population of all inhabited areas except Asia in 1950?

6. What is the percent increase in the population for all six inhabited continents, excluding Asia, from 1950 to 2000? > Videos

7. **(a)** What percent of the Earth's inhabitable land is in North America?

(b) What percent of the world population in the year 2000 is in North America?

8. What was the percent increase in the population in South America from 1900 to 2000?

9. **(a)** What was the number of people per square mile for the entire world in 1950?

(b) What was the number of people per square mile for the entire world in 2000?

(c) What was the percent increase in the number of people per square mile for the entire world from 1950 to 2000?

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Name ____________________

Section ________ Date ________

Answers

1. ____________________

2. ____________________

3. ____________________

4. ____________________

5. ____________________

6. ____________________

7. ____________________

8. ____________________

9. ____________________

Answers

10. ______________

11. ______________

12. ______________

13. ______________

10. **(a)** What was the mean population of the six continents or land masses that were habitable in 2000?

(b) What was the mean population in 1950?

(c) What was the percent increase in the mean population from 1950 to 2000?

In exercises 11 to 13, use the given table.

Gasoline Retail Prices, U.S. City Average, 1978–2002

		(cents per gallon, including taxes)		
Average	**Leaded Regular**	**Unleaded Regular**	**Unleaded Premium**	**All Types**
1978	62.6	67.0	NA	65.2
1979	85.7	90.3	NA	88.2
1980	119.1	124.5	NA	122.1
1981	131.1	137.8	147.0	135.3
1982	122.2	129.6	141.5	128.1
1983	115.7	124.1	138.3	122.5
1984	112.9	121.2	136.6	119.8
1985	111.5	120.2	134.0	119.6
1986	85.7	92.7	108.5	93.1
1987	89.7	94.8	109.3	95.7
1988	89.9	94.6	110.7	96.3
1989	99.8	102.1	119.7	106.0
1990	114.9	116.4	134.9	121.7
1991	NA	114.0	132.1	119.6
1992	NA	112.7	131.6	119.0
1993	NA	110.8	130.2	117.3
1994	NA	111.2	130.5	117.4
1995	NA	114.7	133.6	120.5
1996	NA	123.1	141.3	128.8
1997	NA	123.4	141.6	129.1
1998	NA	105.9	125.0	111.5
1999	NA	116.5	135.7	122.1
2000	NA	151.0	169.3	156.3
2001	NA	146.1	165.7	153.1
2002	NA	135.5	157.8	144.1

Source: Energy Information Administration, U.S. Dept. of Energy, *Monthly Energy Review,* October 2003.

11. **(a)** What was the mean cost of unleaded regular gas in 1990?

(b) What was the mean cost of unleaded premium gas in 1997?

12. **(a)** What was the decrease in the price of unleaded regular gas from 1990 to 1998?

(b) What was the percent decrease in price of all types of gas from 1981 to 1998?

13. **(a)** What was the decrease in price of unleaded regular gas from 1982 to 1986?

(b) What was the decrease in price of unleaded premium gas from 1982 to 1986?

< Objective 3 >

14. Create a pictograph for the total world population in 1950, using the information given in the world population and land area table in the text.

1 Individual ≈ 200 Million

Continent/Region	Population Individuals
North America	1.1
South America	0.5
Europe	1.96
Asia	7.9
Africa	1.14
Oceana	0.06

15. Use the information in the table to create a pictograph.

U.S. Car Sales

Year	Car Sales
1970	8,403,000
1975	8,538,000
1980	8,979,000
1985	11,039,000
1990	9,484,000
1995	8,686,000
2000	8,847,000

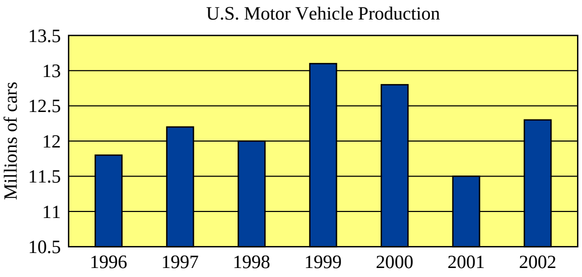
16. Use this table to create a pictograph.

World Motor Vehicle Production

Year	Vehicles
1970	29,419,000
1980	38,565,000
1985	44,909,000
1990	48,554,000
1995	49,983,000
2000	57,528,000

< Objective 4 >

Use the graph below, showing the total U.S. motor vehicle production for the years 1996 to 2002, to answer exercises 17 to 20.



Source: Automotive News Market Data Book.

Answers

14. ______

15. ______

16. ______

Answers

17.

18.

19.

20.

21.

22.

23.

24.

25.

26.

27.

28.

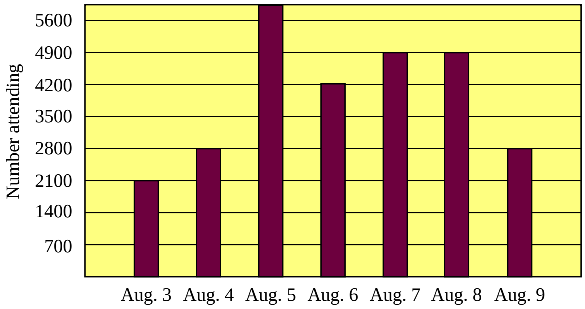
17. What was the production in 1998?

18. In what year did the greatest production occur?

19. Find the median number of cars produced in the 7 years.

20. In what year was the production decline the greatest compared to the previous year? > Videos

Use the bar graph, showing the attendance at a circus for 7 days in August, to solve exercises 21 to 24.



21. Find the attendance on August 4.

22. Which day had the greatest attendance?

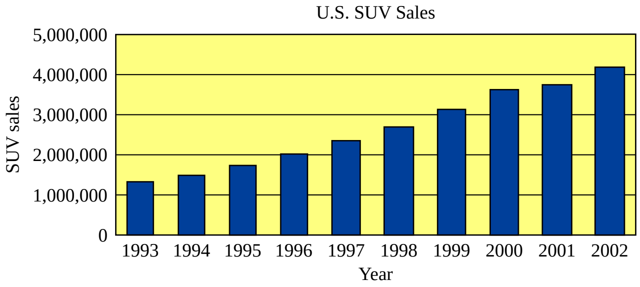
23. Which day had the lowest attendance? > Videos

24. Find the median attendance over the 7 days.

For exercises 25 to 28, use the bar graph below.

Sport Utility Vehicle Sales in the United States, 1993–2002

In 1993, 1,327,507 sport utility vehicles (SUVs) were sold in the United States, accounting for 26.3% of all sales of light vehicles (SUVs, minivans, vans, pickup trucks, and trucks under 14,000 lb). By 2002, sales of SUVs in the United States increased to 4,186,698, accounting for 48.3% of total light vehicle sales.



Source: Office of Transportation Technologies.

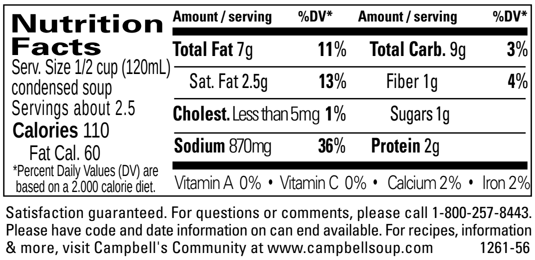
25. What were the sales of SUVs in 2000? > Videos

26. In what year did the greatest sales occur?

27. What was the percent increase in sales from 1993 to 2002?

28. In what year did the greatest increase in sales occur?

In exercises 29 to 32, use the nutritional facts given for Campbell's cream of mushroom soup. Assume you consumed 1 cup of soup.



Nutrition Facts
Serv. Size 1/2 cup (120mL) condensed soup
Servings about 2.5
Calories 110
Fat Cal. 60
*Percent Daily Values (DV) are based on a 2,000 calorie diet.

Amount / serving	%DV*	Amount / serving	%DV*
Total Fat 7g	**11%**	**Total Carb.** 9g	**3%**
Sat. Fat 2.5g	**13%**	Fiber 1g	**4%**
Cholest. Less than 5mg	**1%**	Sugars 1g	
Sodium 870mg	**36%**	**Protein** 2g	

Vitamin A 0% • Vitamin C 0% • Calcium 2% • Iron 2%

Satisfaction guaranteed. For questions or comments, please call 1-800-257-8443. Please have code and date information on can end available. For recipes, information & more, visit Campbell's Community at www.campbellsoup.com 1261-56

29. How many calories have you consumed?

30. What percent of the daily value of saturated fat have you consumed?

31. What percent of fiber did you get?

32. How many grams of sodium did you get?

In exercises 33 to 38, use the given table.

Soup	Calories	Fat	Total Protein	Sodium
Cream of mushroom	110	7 g	2 g	870 mg
Cream of chicken	130	8 g	11 g	890 mg
Split pea	180	3.5 g	10 g	860 mg
Tomato	100	0 g	2 g	760 mg

33. Which soup has the least fat?

34. Which soup has the most sodium?

35. Which soup has the least sodium?

36. Which soup has the fewest calories?

37. Find the mean number of calories in the soups.

38. Find the mean number of milligrams of sodium in the soups.

Basic Skills | Challenge Yourself | Calculator/Computer | Career Applications | Above and Beyond

39. **MECHANICAL ENGINEERING**

American Wire Gauge	Wire Diameter (in.)	American Wire Gauge	Wire Diameter (in.)
2	0.2576	12	0.0808
3	0.2294	14	0.0640
4	0.2043	16	0.0508
6	0.1620	18	0.0403
8	0.1285	20	0.0319
10	0.1019		

(a) What is the diameter of a 12-gauge wire?

(b) What is the difference in diameter between 14-gauge and 10-gauge wire?

Answers

29.

30.

31.

32.

33.

34.

35.

36.

37.

38.

39.

Answers

40. ____________________

41. ____________________

42. ____________________

43. ____________________

40. AUTOMOTIVE

Temperature Protection (°F)	Required Percent of Ethylene Glycol
15	22%
10	26%
5	29%
0	34%
−10	39%
−20	43%
−30	48%
−40	53%

(a) What percent ethylene glycol is required to provide protection down to −20°F?

(b) What is the temperature protection provided by a mixture of 29% ethylene glycol?

(c) If the percent of ethylene glycol is increased from 29% to 39%, how much does it change the temperature protection?

41. WELDING

Metal	Density (g/cm^3)	Melting Point (°C)
Iron	7.87	1,538
Aluminum	2.699	660.4
Copper	8.93	1,084.9
Tin	5.765	231.9
Titanium	4.507	1,668

(a) What is the density of titanium?

(b) What is the difference in melting point between copper and iron?

(c) Which metal has the highest melting point?

Basic Skills | Challenge Yourself | Calculator/Computer | Career Applications | **Above and Beyond**

42. Compare current gas prices in your area to those in the table given for exercises 11 to 13. Describe the differences.

43. Research national gas prices and compare them to the prices in the table given for exercises 11 to 13.

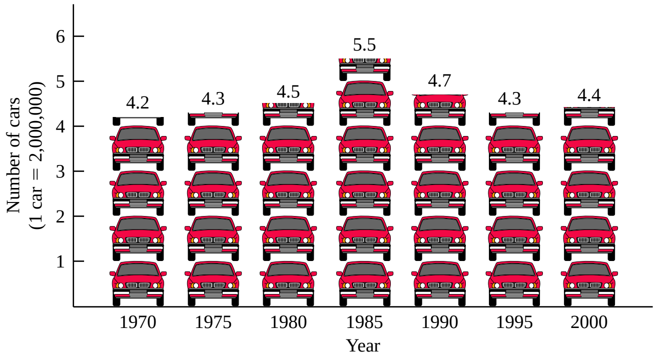
Answers

1. **(a)** 221,000,000; **(b)** 16.2% **3.** **(a)** 70.7%; **(b)** 153.2%; **(c)** 91.4 people; **(d)** 231.5 people **5.** 193,000,000 people **7.** **(a)** 17.9%; **(b)** 4.9% **9.** **(a)** 44.1; **(b)** 108.4; **(c)** 145.8% **11.** **(a)** 116.4 ¢/gal; **(b)** 141.6 ¢/gal

13. **(a)** 36.9 ¢/gal; **(b)** 33 ¢/gal

15.

Year	No. of Cars (in millions)
1970	4.2
1975	4.3
1980	4.5
1985	5.5
1990	4.7
1995	4.3
2000	4.4



17. 12,000,000 cars **19.** 12,200,000 cars **21.** 2,800 people
23. August 3 **25.** 3,600,000 SUVs **27.** 215.4% **29.** 220
31. 8% **33.** Tomato **35.** Tomato **37.** 130 cal
39. **(a)** 0.0808 in.; **(b)** 0.0379 in. **41.** **(a)** 4.507 g/cm^3; **(b)** 453.1°C;
(c) Titanium **43.** Above and Beyond

9.3 Line Graphs and Predictions

< **9.3 Objectives** >

1 > Read a line graph

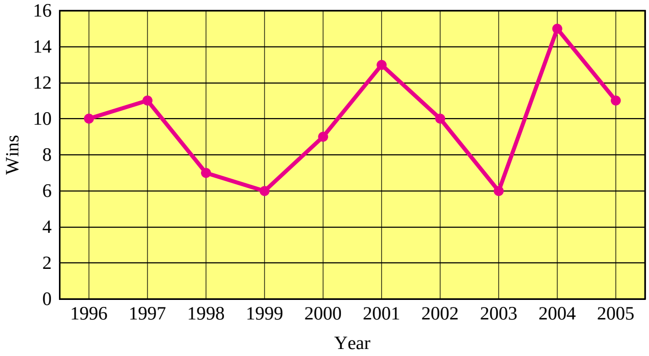
2 > Make a prediction from a line graph

We have seen that data can be represented graphically with a pictograph or a bar graph. Another useful type of graph is called a **line graph.** In a line graph, one of the types of information is almost always related to time (clock time, day, month, or year).

Example 1 Reading a Line Graph

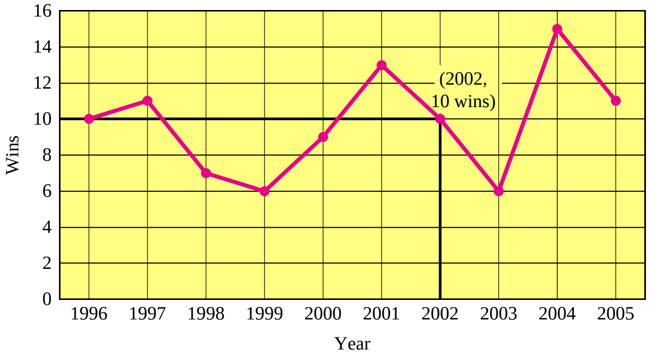
< Objective 1 >

The graph below shows the number of regular season games that the Pittsburgh Steelers won in each season over the 10-year period, 1996–2005. The years are displayed on the bottom (*horizontal axis*) and the number of victories is given along the side (*vertical axis*).



(a) How many games did the Steelers win in 2002?

We look across the bottom and find 2002. Then, we move straight up until we see the point indicated by the graph. Following to the left, we see that they won 10 games in 2002.



(b) Find the mean number of games won by the Steelers over this 10-year period.

For each dot on the line, we look to the left side to see the number of victories the dot represents. We then find the mean of these numbers.

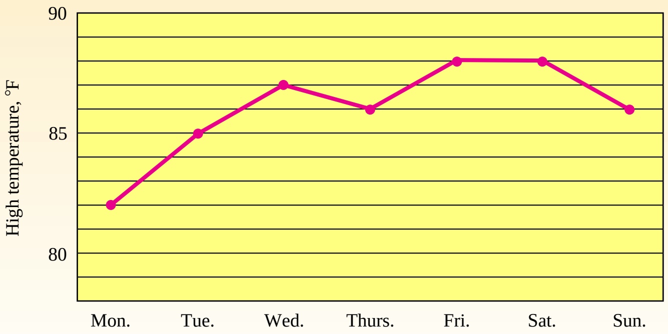
$$\text{Mean} = \frac{10 + 11 + 7 + 6 + 9 + 13 + 10 + 6 + 15 + 11}{10}$$

$$= \frac{98}{10} = 9.8$$

They averaged nearly 10 wins a season between 1996 and 2005.

Check Yourself 1

The following graph indicates the high temperatures in Baltimore, Maryland, for a week in September.



(a) What was the high temperature on Friday?
(b) Find the mean high temperature for that week.

It is often tempting, and sometimes useful, to use a line graph to predict a future value. Using an earlier trend to predict a future value is called **extrapolation.** This is something that statisticians warn us not to rely on, but it is done anyway. The key is not to predict very far from the data.

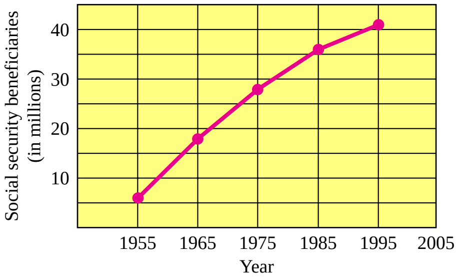
Example 2 Making a Prediction

< Objective 2 >

Use the line graph and table to predict the number of Social Security beneficiaries in the year 2005.

Social Security Admin.

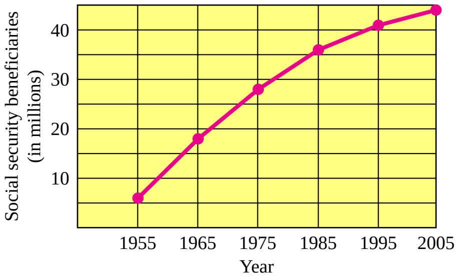
Year	Beneficiaries
1955	6,000,000
1965	18,000,000
1975	28,000,000
1985	36,000,000
1995	41,000,000
2005	?



From the shape of the line graph, it would be reasonable to guess that the next point on the graph would continue on the same “curve.”

Social Security Admin.

Year	Beneficiaries
2005	44,000,000



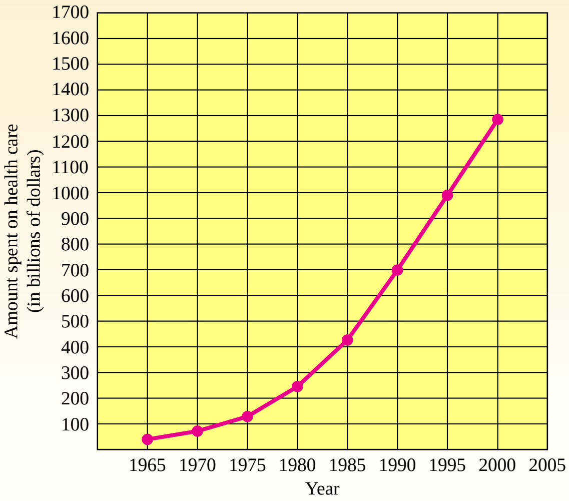
This point indicates that in 2005 we should expect about 44,000,000 Social Security beneficiaries. This number closely matches more sophisticated predictions for the number of beneficiaries in the year 2005.

Check Yourself 2

The graph and table show the amount spent on health care (in billions of dollars) in the United States every 5 years from 1965 to 2000. Use that information to predict the amount spent in the year 2005.

National Center for Health Stats.

Year	Health Care Expenditures (in billions of $)
1965	41
1970	73
1975	130
1980	247
1985	428
1990	700
1995	991
2000	1,285
2005	?

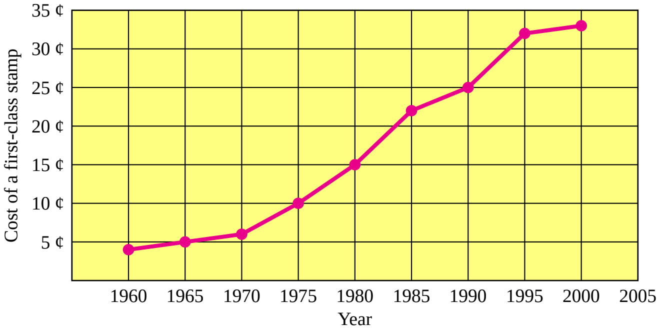


Here is an example yielding a result that is not quite as useful.

Example 3 Making a Prediction

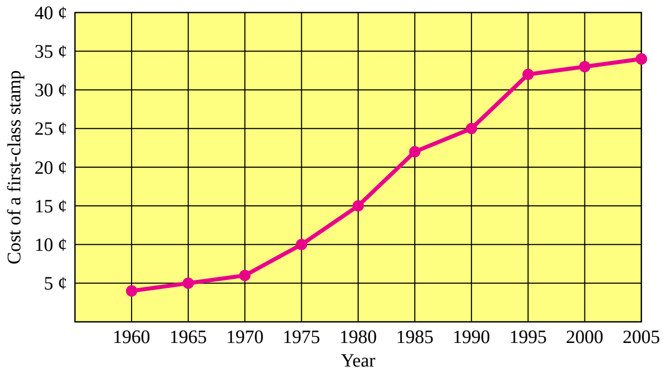
Use the line graph and table to "predict" the cost of a first-class stamp on January 1, 2005.

Year	Cost (¢)
1960	4
1965	5
1970	6
1975	10
1980	15
1985	22
1990	25
1995	32
2000	33
2005	?



Place a straightedge so that it comes close to going through all the points.

Year	Cost (¢)
1960	4
1965	5
1970	6
1975	10
1980	15
1985	22
1990	25
1995	32
2000	33
2005	34



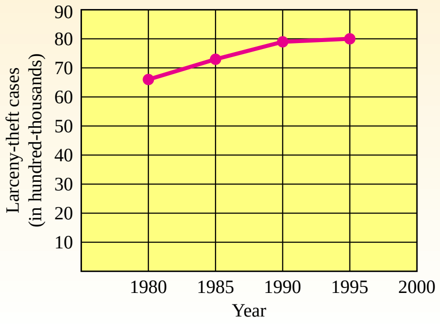
From the line graph, it would be reasonable to guess that in 2005 the cost would be 34 cents. In fact, on June 30, 2003, the cost was 37 cents. This is evidence of the danger of extrapolation, a danger we mentioned before Example 2.

Check Yourself 3

The graph represents the number of larceny-theft cases in the United States every 5 years from 1980 to 1995. Use the graph to predict the number of cases in 2000.

FBI Uniform Crime Report

Year	Larceny-Theft Cases (in hundred thousands)
1980	66
1985	73
1990	79
1995	80
2000	?



Check Yourself ANSWERS

1. **(a)** 88°F; **(b)** 86°F
2. A reasonable prediction from the data would be about 1,600 billion dollars.
3. A reasonable prediction from the data would be about 82 hundred thousand cases. There were actually 70 hundred thousand.

Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 9.3

(a) In a ______________ graph, one of the types of information is almost always related to time.

(b) It is often tempting to use a line graph to predict a ____________ value.

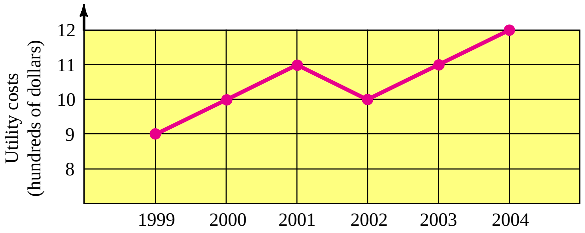
(c) Using an earlier trend to predict a future value is called ____________.

(d) The key, when using extrapolation, is not to ____________ very far from the data.

< Objective 1 >

Use the graph, showing the yearly utility costs of a family, for exercises 1 to 4.

> Videos



1. What was the cost in 2002?

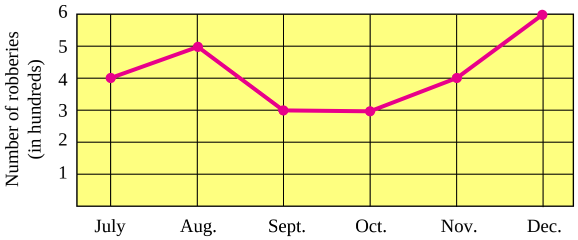
2. What was the mean cost of utilities for this family in the 6 years from 1999 to 2004?

3. What was the decrease in the cost of utilities from 2001 to 2002?

> Videos

4. In what year was the cost of utilities the smallest?

Use the graph, showing the number of robberies in a town during the last 6 months of a year, for exercises 5 to 8.



5. In which month did the greatest number of robberies occur?

> Videos

6. How many robberies occurred in November?

7. Find the decrease in the number of robberies between August and September.

> Videos

8. What was the mean number of robberies per month over the last 6 months?

< Objective 2 >

9. The graph and table show the income to the Hospital Insurance Trust Fund. Use this information to predict the income in the year 2005.

Year	Total Income (in millions of $)
1975	12,568
1980	25,415
1990	79,563
1995	114,847
2000	130,559

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Name ____________________

Section __________ Date __________

Answers

1. ____________________

2. ____________________

3. ____________________

4. ____________________

5. ____________________

6. ____________________

7. ____________________

8. ____________________

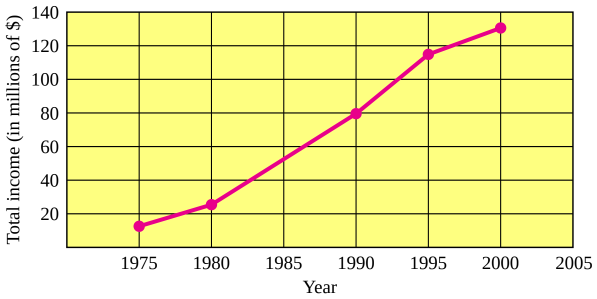
9. ____________________

Answers

10. ______

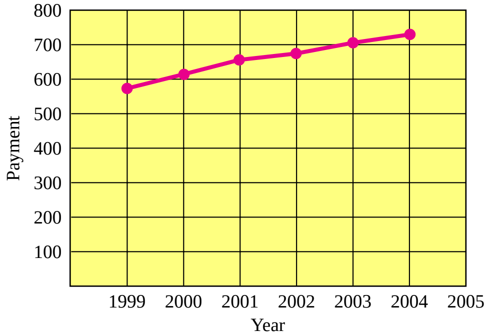
11. ______

12. ______



10. The graph and table give the monthly principal and interest payments for a mortgage from 1999 to 2004. Use this information to predict the payment for 2005.

Year	Payment
1999	$578
2000	613
2001	654
2002	675
2003	706
2004	730
2005	



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11. The table shows a relationship between years of formal education and typical income (in thousands of dollars) at age 30:

Years	8	10	12	14	16
Income (×1,000)	16	21	23	28	31

Use this information to make a line graph and then predict the yearly income associated with 18 years of formal education.

12. The table shows a relationship between the amount spent per week on advertising by a small fast-food shop and the total sales per week:

Amount spent	10	20	30	40
Sales	200	380	625	790

Use this information to make a line graph and then predict the weekly sales associated with the expenditure of $50 (per week) on advertising.

13. The table shows a relationship between the number of weeks on a special diet and the number of pounds lost during that time:

Number of weeks	2	4	6	8
Number of pounds	2	5	9	11

Use this information to make a line graph and then predict the number of pounds lost associated with 10 weeks on the special diet.

14. The table shows a relationship between the speed of a certain car over a 100-mi test trip and the gas mileage obtained:

Speed $\left(\text{in } \frac{\text{mi}}{\text{h}}\right)$	40	45	50	55	60
Gas mileage	28	25	21	18	16

Use this information to make a line graph and then predict the gas mileage associated with a speed of 65 mi/h.

Answers

1. $1,000 **3.** $100 **5.** December **7.** 200 **9.** $158,000,000,000
11. $35,000 **13.** 14 lb

Answers

13. ____________________

14. ____________________

9.4 Creating Bar Graphs and Pie Charts

< 9.4 Objectives >

1 > Use a table to create a bar graph
2 > Read a pie chart
3 > Create a pie chart from data

It is frequently easier to read information from a graph than it is from a table. In this section, we will look at two types of graphs that can be created from tables. We have already learned to read a bar graph. We create one in Example 1.

Example 1 Creating a Bar Graph

< Objective 1 >

NOTE

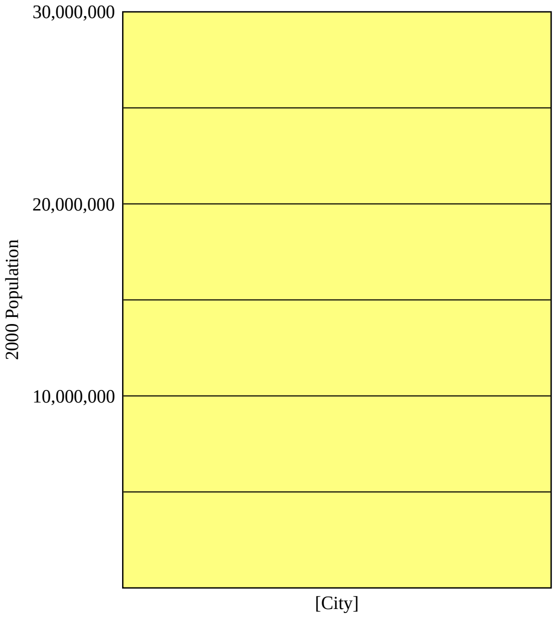
Mumbai was formerly listed as Bombay.

This table represents the 2000 population of the six most populated urban areas in the world. Each population represents the city and all of its suburbs. Create a bar graph from the information in the table.

Population of the World's Largest Urban Areas (U.N. Dept. for Economic and Social Info.)

City	2000 Population
Tokyo, Japan	26,400,000
Mexico City, Mexico	18,100,000
Mumbai, India	18,100,000
Sao Paulo, Brazil	17,800,000
New York City, USA	16,600,000
Lagos, Nigeria	13,400,000

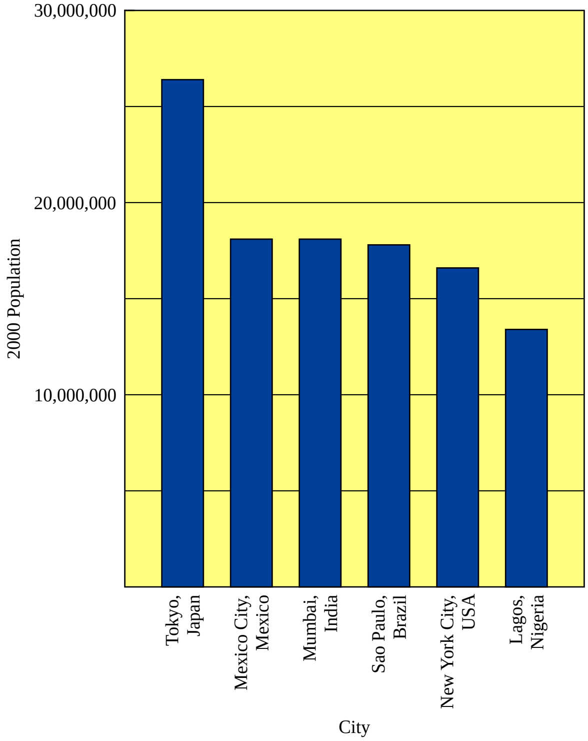
Source: United Nations Human Settlements Programme.



We will let the vertical axis, the vertical line to the left of the graph, represent population. We will place the six urban areas along the horizontal axis. To create a graph, we must decide on the scale for the vertical axis. The following steps will accomplish that.

1. Pick a number that is slightly larger than the biggest number we are to graph. 30,000,000 is slightly larger than 26,400,000.
2. Decide how long the axis will be. It is best if this length easily divides into the number of step 1. To accomplish this division, we choose 3 inches.
3. Scale the axis by dividing it with hashmarks. Label each hashmark with the appropriate number. In this graph, each inch will represent 10,000,000 people (the 30,000,000 divided by the 3 inches results in 10,000,000 people per inch).

Now, the height of each bar is determined by using the scale created for the axis. Remembering that we have 10,000,000 people per inch, we divide each population by 10,000,000. The result is the height of each bar. The height for Mexico City is 1.81 inches. Remember, all we can read from a bar graph is a rough approximation of the actual number.



Check Yourself 1

This table represents the 2000 population of the six most populated cities in the United States. Each population is the population within the city limits, which is why the New York population is so different from that in the table in Example 1. Create a bar graph from the information in the table.

Population of the Largest Cities in the United States (Bureau of the Census, U.S. Dept. of Commerce)

City	2000 Population
New York City, NY	8,008,000
Los Angeles, CA	3,695,000
Chicago, IL	2,896,000
Houston, TX	1,954,000
Philadelphia, PA	1,518,000
Phoenix, AZ	1,321,000

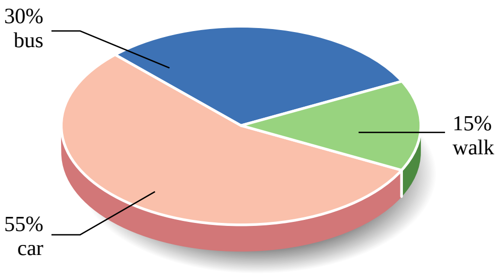
When a graph represents how some unit is divided, a *pie chart* is frequently used.

As you might expect, a **pie chart** is a circle. Wedges (or sectors) are drawn in the circle to show how much of the whole each part makes up.

Example 2 Reading a Pie Chart

< Objective 2 >

This pie chart represents the results of a survey that asked students how they get to school most often.



(a) What percent of the students walk to school?

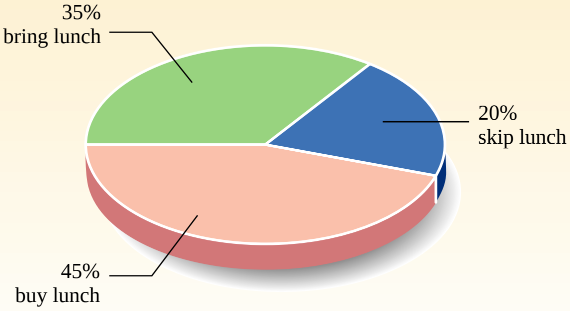
We see that 15% walk to school.

(b) What percent of the students do not arrive by car?

Because 55% arrive by car, 100% − 55%, or 45%, do not.

Check Yourself 2

This pie chart represents the results of a survey that asked students whether they bought lunch, brought it, or skipped lunch altogether.

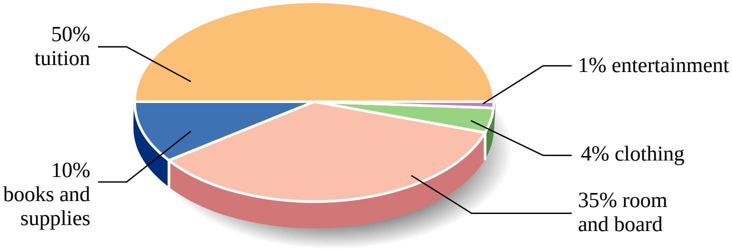


(a) **What percent of the students skipped lunch?**
(b) **What percent of the students did not buy lunch?**

If we know what the whole pie represents, we can also find out more about what each wedge represents. Example 3 illustrates this point.

Example 3 Interpreting a Pie Chart

This pie chart shows how Sarah spent her $12,000 college scholarship.



(a) How much did she spend on tuition?

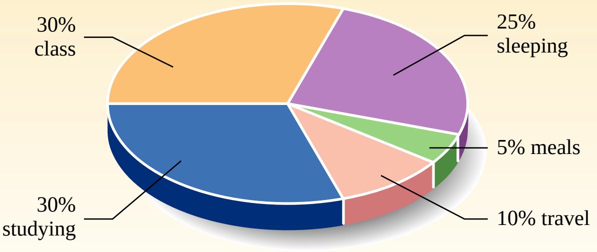
50% of her $12,000 scholarship, or $6,000.

(b) How much did she spend on clothing and entertainment?

Together, 5% of the money was spent on clothing and entertainment, and $0.05 \times 12{,}000 = 600$. Therefore, $600 was spent on clothing and entertainment.

Check Yourself 3

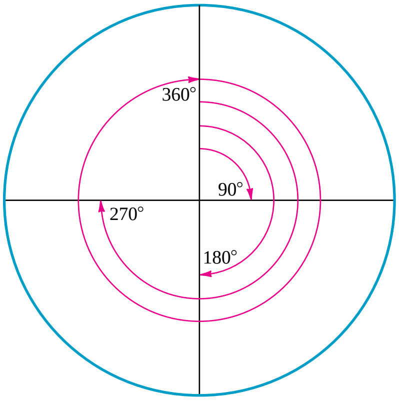
This pie chart shows how Rebecca spends an average 24-h school day.



(a) How many hours does she spend sleeping each day?
(b) How many hours does she spend altogether studying and in class?

If we are creating a pie chart, how do we know how much of the circle to use for each piece? To make this decision requires that a scale be used for the circle. A standard scale has been established for all circles. As we saw in Chapter 8, each circle has 360°. That means that $\frac{1}{4}$ of the circle has $\frac{1}{4}$ of 360°, which is 90°.

With a protractor, we can now create our own pie chart.



Example 4 Creating a Pie Chart

< Objective 3 >

This table gives the source of automobiles purchased in the United States in one year. Create a pie chart that represents the same data.

Source of Automobiles Purchased

Country of Origin	Number	% of Total
United States	6,500,000	80
Japan	800,000	10
Germany	400,000	5
All others	400,000	5

Source: American Automotive Manufacturers' Association.

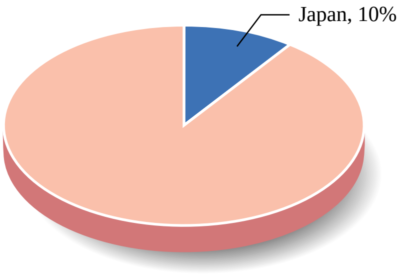
To find the size of the slice for each country, we take the given percent of 360°. We will create another table column to represent the degrees needed.

Source of Automobiles Purchased

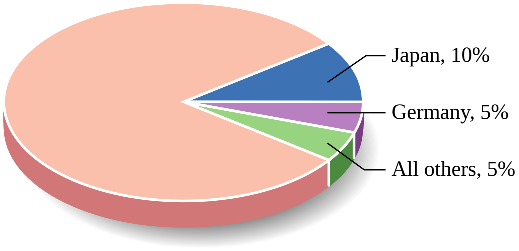
Country of Origin	Number	% of Total	Degrees
United States	6,500,000	80	288
Japan	800,000	10	36
Germany	400,000	5	18
All others	400,000	5	18

Source: American Automotive Manufacturers' Association.

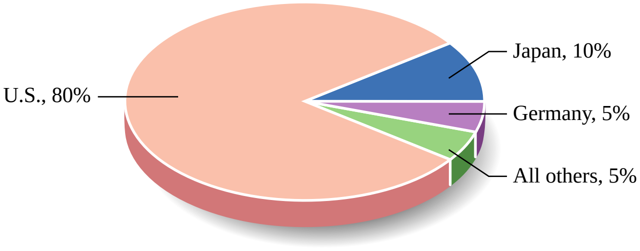
Using a protractor, we start with Japan and mark a section that is 36°.



Again, using the protractor, we mark the 18° section for Germany and the 18° section for the other countries.



There is no need to measure the remainder of the pie. What is left is the 288° section for U.S.-made cars. Note that we saved the largest section for last. It is much easier to mark the smaller sections and leave the largest for last.



Check Yourself 4

Create a pie chart for the table below, showing TV ownership for all U.S. homes.

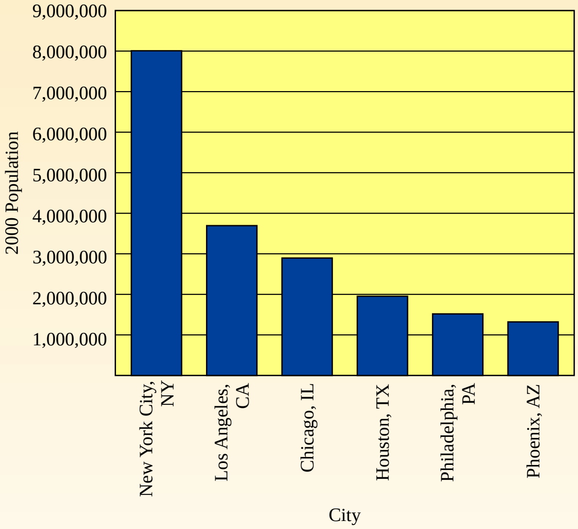
TV Ownership

Number of TVs	% of U.S. Homes
0	2
1	22
2	34
3 or more	42

Source: Nielsen Media Research.

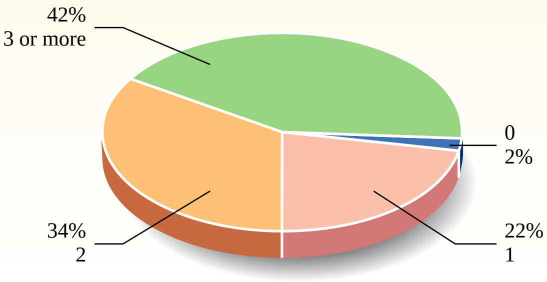
Check Yourself ANSWERS

1.



2. **(a)** 20%; **(b)** 55% **3.** **(a)** 6 h; **(b)** 14.4 h

4.



Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 9.4

(a) It is frequently easier to read information from a graph than it is from a ____________.

(b) All we can read from a bar graph is a rough ____________ of the actual number.

(c) In a ____________ chart, wedges are drawn in a circle to show how much of the whole each part makes up.

(d) Each ____________ has 360°.

9.4 exercises

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Name ____________________

Section __________ Date __________

Answers

1. ____________________

2. ____________________

3. ____________________

4. ____________________

5. ____________________

6. ____________________

7. ____________________

< Objective 1 >

1. This table gives the total U.S. population by age group, according to the 2000 census.

Age	Population
0–14	60,224,094
15–34	79,079,301
35–54	82,737,774
55–74	42,494,571
75+	16,603,839

Source: U.S. Census Bureau.

Construct a bar graph from this information.

2. This table gives the median earnings of women aged 25 and older who work full-time, year round, by educational attainment, according to the 2000 census.

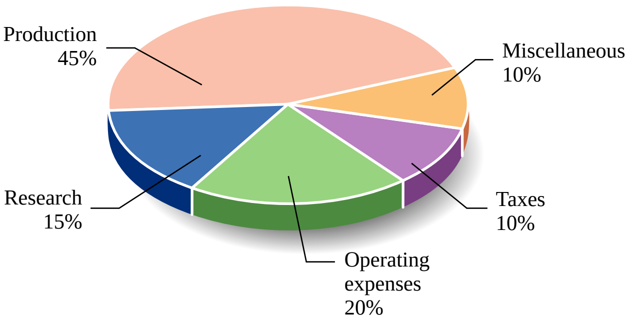
Education	Earnings
HS diploma or GED	$23,719
Associate's degree	30,178
Bachelor's degree	38,208
Master's degree	47,049
Doctorate's degree	55,620

Source: U.S. Census Bureau.

Create a bar graph from this information.

< Objective 2 >

This pie chart shows the budget for a local company. The total budget is $600,000. Find the amount budgeted in each of the following categories.



3. Production Videos

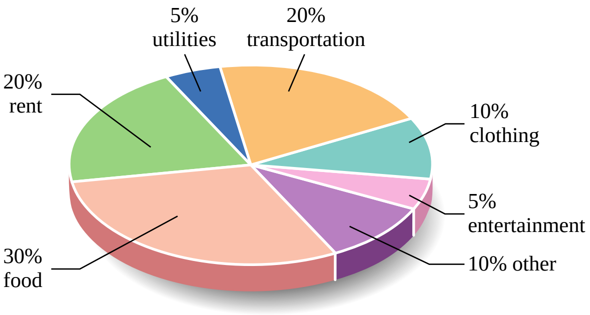
4. Taxes

5. Research Videos

6. Operating expenses

7. Miscellaneous

This pie chart shows the distribution of a person's total yearly income of $24,000. Find the amount budgeted for each category.



8. Food

9. Rent > Videos

10. Utilities

11. Transportation

12. Clothing

13. Entertainment > Videos

< **Objective 3** >

14. This table gives the number of Senate members with military service in the 106th U.S. Congress, by branch.

Branch	Count
Army	17
Navy	10
Air Force	4
Marines	6
Coast Guard	1
National Guard	2

Construct a pie chart from this information.

15. This table gives the number of foreign-born residents of the United States by region of birth in the year 2000.

Region	U.S. Population
Europe	4,400,000
Northern America	700,000
Latin America	14,500,000
Asia	7,200,000
Other areas	1,600,000

Source: U.S. Census Bureau.

Construct a pie chart from the information.

Answers

8. ____________________

9. ____________________

10. ____________________

11. ____________________

12. ____________________

13. ____________________

14. ____________________

15. ____________________

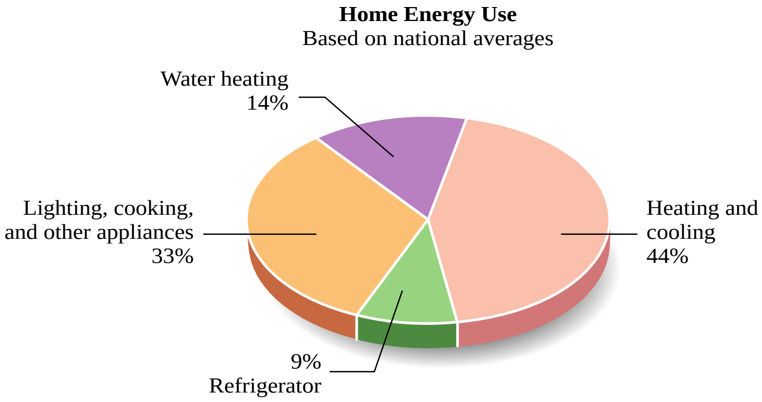
Answers

16. ______

17. ______

18. ______

16. MANUFACTURING



Source: "Small Electric Wind Systems: A U.S. Consumer's Guide," May 2001, revised October 2002. U.S. Department of Energy, Office of Energy Efficiency and Renewable Energy, Wind and Hydropower Technologies Program.

Assume that a household uses 10,600 kilowatt-hours (kWh) of electricity annually. Calculate the energy (in kWh) used annually for each category.

17. AUTOMOTIVE Create a circle graph for the manufacturing data.

Manufacturer	Sales (1,000s of Vehicles)
Chevrolet	43.4
Dodge	29.1
Ford	46.8
Honda	9.7
Toyota	28.1

18. HEALTH SCIENCES The table gives the number of live births, broken down by the race of the mother, in the United States for the year 2002.

Race	Number
White	3,174,760
Black	593,691
American Indian	42,368
Asian or Pacific Islander	210,907
Total	4,021,726

Source: National Vital Statistics Reports, Vol. 52, No. 10. December 17, 2003.

Construct a pie chart of this data.

19. **AUTOMOTIVE** Create a bar graph to display the data in the mileage table.

Model	Mileage
Taurus	27
Malibu	29
Stratus	26
Camry	32

20. **WELDING** Create a bar graph to display the melting points of the metals listed.

Metal	Density (g/cm^3)	Melting Point (°C)
Iron	7.87	1,538
Aluminum	2.699	660.4
Copper	8.93	1,084.9
Tin	5.765	231.9
Titanium	4.507	1,668

21. **AGRICULTURE** Construct a pie chart for the harvest total shown.

Crop	Harvest (1,000,000s of Tons)
Corn	127
Wheat	93.2
Barley	32.8
Soybeans	87.4

Answers

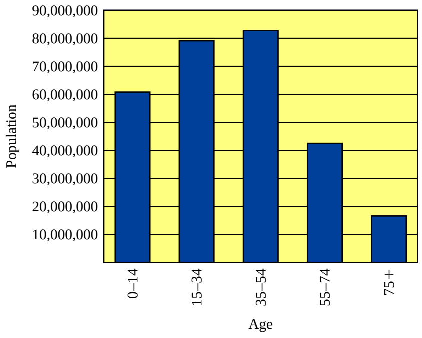
19. ______________________

20. ______________________

21. ______________________

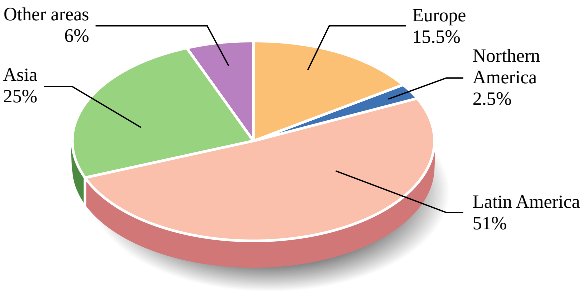
Answers

1.

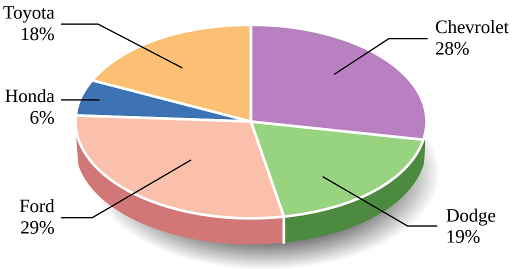


3. $270,000
5. $90,000
7. $60,000
9. $4,800
11. $4,800
13. $1,200

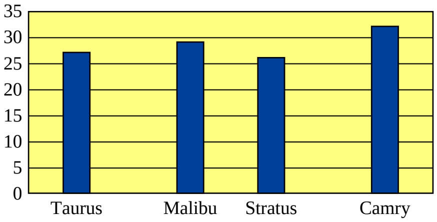
15.


Other areas
6%
Europe
15.5%
Northern
America
2.5%
Asia
25%
Latin America
51%

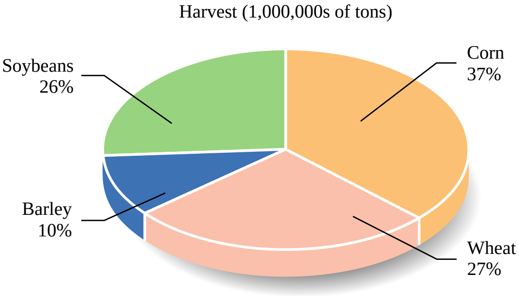
17.


Toyota
18%
Chevrolet
28%
Honda
6%
Ford
29%
Dodge
19%

19.


35
30
25
20
15
10
5
0
Taurus
Malibu
Stratus
Camry

21.


Harvest (1,000,000s of tons)
Soybeans
26%
Corn
37%
Barley
10%
Wheat
27%

Activity 25 ::
Graphing Car Color Data

Consider the data you gathered in Activity 24. If you worked in the automobile industry, information concerning customer color preferences would be important. To form conclusions, you need to present your data to other people (who are probably not statisticians). Bar graphs and pie charts are useful ways of presenting such data.

1. Create a bar graph using the set of 35 car colors compiled in Activity 24.
2. Create a pie chart using the set of 35 car colors compiled in Activity 24.
3. **(a)** Briefly describe the view of your data given by the two graphs.
   (b) In which graph is the mode more easily distinguished?
   (c) In which is it easier to get a sense of the "whole" data set?
4. Compare your graphs to those of a classmate. Briefly describe how yours differ from your classmate's graphs (consider the differences in data sets and in presentation).
5. Write a short letter to the manager of an auto body and paint shop, recommending levels of inventory for different color paints. Include either the bar graph or the pie chart in your letter.

9.5

Describing and Summarizing Data Sets

< **9.5 Objectives** >

1 > Compute the quartiles of a data set

2 > Give the five-number summary of a data set

3 > Construct a box-and-whisker plot to describe a data set

4 > Interpret a box-and-whisker plot

In Section 9.1, we learned to describe a data set by using various measures of center. That is, we described the "average" member of a set, using the mean, median, and mode.

Often, though, this is not sufficient to provide a clear picture of a data set.

Example 1 Describing Data with Measures of Center

< Objective 1 >

Find the mean and median of each set of numbers.

(a) 1, 2, 4, 3, 5, 1, 1, 3

Recall from Section 9.1 that the mean is computed by adding the numbers together and dividing by the number of elements in the set.

$$\text{Mean} = \frac{1 + 2 + 4 + 3 + 5 + 1 + 1 + 3}{8} = \frac{20}{8} = 2.5$$

To compute the median, we list the numbers in increasing order and find the midpoint.

1, 1, 1, **2, 3,** 3, 4, 5

Because we have two elements in the "middle," the median is the mean of the pair of middle numbers.

$$\text{Median} = \frac{2 + 3}{2} = \frac{5}{2} = 2.5$$

RECALL

There are always two values in the "middle" when there are an even number of terms in the list.

(b) 2, 2, 2, **2, 3,** 3, 3, 3

$$\text{Mean} = \frac{2 + 2 + 2 + 2 + 3 + 3 + 3 + 3}{8} = \frac{20}{8} = 2.5$$

$$\text{Median} = \frac{2 + 3}{2} = \frac{5}{2} = 2.5$$

Check Yourself 1

Find the median and mode of each set of numbers.

(a) 1, 2, 2, 7, 7, 7, 7, 10

(b) 24, 7, 7, 10, 6, 156, 7

In Example 1, both sets had the same mean and the same median, yet they are very different lists of numbers. Clearly, neither the mean nor the median alone is sufficient to distinguish a list of numbers.

NOTE

In fact, the median is the second quartile Q_2.

To assist us, we describe a data set using several numbers. One set of numbers we use is the **quartiles.** Just as the median divides a data set into halves, the quartiles divide the set into quarters (four equal parts). We use the notation Q_1, Q_2, and Q_3 to represent each of the three quartiles.

Step by Step

Finding the Quartiles Q_1 and Q_3

Step 1 Rewrite the given data set in ascending order.

Step 2 Find the median.

Step 3 Make a list of only those numbers (in order) that are to the left of the median.

Step 4 Find the median of the list created in step 3; this is the first quartile Q_1.

Step 5 Repeat steps 3 and 4 with those numbers that are to the right of the median; this gives the third quartile Q_3.

Example 2 Finding Quartiles

NOTE

We use a similar process to divide a set using deciles (tenths) and percentiles (hundredths).

Find the first and third quartiles, Q_1 and Q_3, of each data set.

(a) 1, 2, 4, 3, 5, 1, 1, 3

We sorted this list and found the median in Example 1.

1, 1, 1, 2, 3, 3, 4, 5

Median = 2.5

1, 1, 1, 2 are all to the left of the median. The median of this list is 1 (do you see why?), which is the first quartile.

$Q_1 = 1$

3, 3, 4, 5 are all to the right of the median. The median of this list is 3.5 (do you see why?), which is the third quartile.

$Q_3 = 3.5$

(b) 2, 2, 2, 2, 3, 3, 3, 3

We found the median to be 2.5 in Example 1.

2, 2, 2, 2 are all to the left of the median. The median of this set gives the first quartile

$Q_1 = 2$

Similarly, the third quartile is 3.

NOTE

We refer to the smallest and largest members of a set as the **min** and **max,** respectively.

Check Yourself 2

Find the first and third quartiles of each set of numbers.

(a) **1, 2, 2, 7, 7, 7, 7, 10** **(b)** **24, 7, 7, 10, 6, 156, 7**

The final pair of numbers we will use is the smallest (minimum) and largest (maximum) elements of the data set.

We are now ready to give a **five-number summary** of a data set.

Definition

Five-Number Summary

The five-number summary associated with a data set is given by

Min, Q_1, Median, Q_3, Max

The five-number summary serves to distinguish between sets of data where a single number is not sufficient. We now find some five-number summaries.

Example 3 Finding Five-Number Summaries

< Objective 2 >

Find the five-number summary of each data set.

(a) 1, 2, 4, 3, 5, 1, 1, 3

The smallest number is 1, so 1 is the min. The largest number is 5, so 5 is the max. Using our results from Examples 1 and 2, we have

1, 1, 2.5, 3.5, 5

(b) 2, 2, 2, 2, 3, 3, 3, 3

The five-number summary for this set is simply 2, 2, 2.5, 3, 3.

Check Yourself 3

Give the five-number summary for each set of numbers.

(a) 1, 2, 2, 7, 7, 7, 7, 10 **(b)** 24, 7, 7, 10, 6, 156, 7

We can now see that our two data sets from Example 1 are distinct. We can use a picture to display these differences based on the five-number summary. The graph we create is called a **box-and-whisker plot.**

Step by Step

Constructing Box-and-Whisker Plots

Step 1 Find the five-number summary of the given set of data.

Step 2 Construct a horizontal number line from the min to the max values in the summary.

Step 3 Mark off each of the numbers in the summary on the number line to scale (use small vertical lines).

Step 4 Draw a box from Q_1 to Q_3 (so the number line is in the middle of the box).

Example 4 Constructing Box-and-Whisker Plots

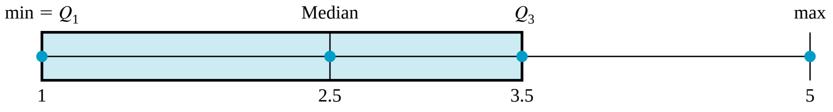
< Objective 3 >

NOTE

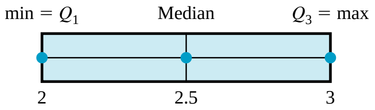
It is also possible to construct a vertical box-and-whisker plot.

Construct box-and-whisker plots for each data set.

(a) 1, 2, 4, 3, 5, 1, 1, 3



(b) 2, 2, 2, 2, 3, 3, 3, 3



NOTE

Many graphing calculators will compute five-number summaries and even produce box-and-whisker plots.

Check Yourself 4

Construct a box-and-whisker plot for each set of numbers.

(a) 1, 2, 2, 7, 7, 7, 7, 10 **(b)** 24, 7, 7, 10, 6, 56, 7

Our box-and-whisker plots for each set of data look different, which reflects differences between the sets.

A common application of box-and-whisker plots involves stock prices.

Example 5 Using Box-and-Whisker Plots

< Objective 4 >

Looking at investment options, we tracked the closing prices of two stocks (Microsoft—MSFT and Chevron-Texaco—CVX) on the New York Stock Exchange (NYSE) over a 2-week period. The following table shows closing prices for each stock.

Stock	Mon.	Tues.	Wed.	Thu.	Fri.	Mon.	Tues.	Wed.	Thu.	Fri.
MSFT	25.93	26.04	26.32	26.25	26.57	25.29	25.49	25.25	25.04	24.67
CVX	64.71	65.63	65.83	65.80	66.00	64.98	66.02	65.80	65.20	65.25

Source: Yahoo! Finance.

Construct box-and-whisker plots to compare the two stocks. Interpret your findings.

The five-number summaries are given for each stock.

MSFT: 24.67, 25.25, 25.71, 26.25, 26.57

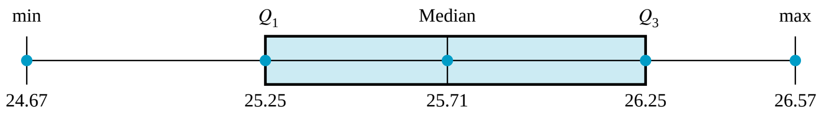
CVX: 64.71, 65.20, 65.72, 65.83, 66.02

This gives us the following pair of box-and-whisker plots.

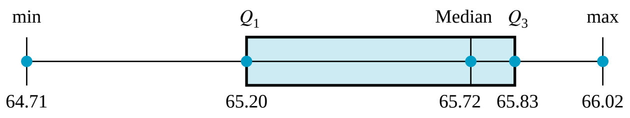
NOTE

We used the same scale for both plots.

MSFT



CVX



NOTE

See exercises 41 to 46 for more on the **range** of values.

While the closing prices of CVX stock are higher than the closing prices of MSFT, there are other (more) interesting characteristics that the box-and-whisker plots allow us to see.

The MSFT plot is wider than the CVX plot. This means that there is a wider range of values in the MSFT summary. The MSFT stock exhibits greater *volatility*.

Also notice that the median, third quartile value, and max are close together in the CVX plot. This indicates that the numbers in the lower half of the CVX data are more spread out (show more variation) than the numbers in the upper half.

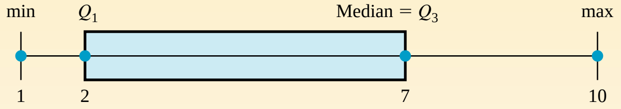
Check Yourself 5

A different 2-week period is shown in the following table. Construct box-and-whisker plots to compare the results. Interpret your findings.

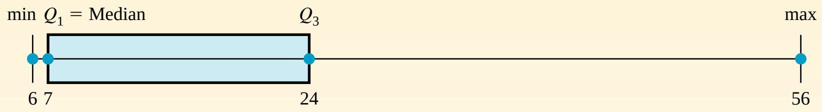
Stock	Mon.	Tues.	Wed.	Thu.	Fri.
MSFT	54.77	55.80	54.24	55.81	55.92
CVX	69.90	68.45	68.05	69.12	68.61
MSFT	56.39	56.97	56.27	55.35	51.46
CVX	68.18	68.50	68.15	68.32	68.11

Check Yourself Answers

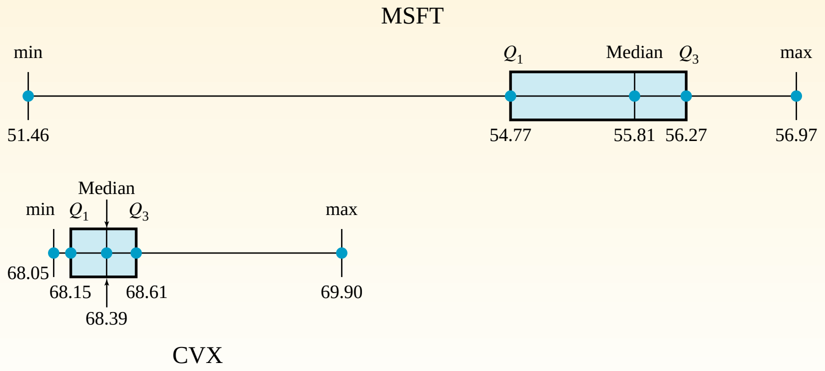
1. **(a)** Median 7, mode 7; **(b)** median 7, mode 7
2. **(a)** $Q_1 = 2$, $Q_3 = 7$; **(b)** $Q_1 = 7$, $Q_3 = 24$
3. **(a)** 1, 2, 7, 7, 10; **(b)** 6, 7, 7, 24, 156
4. **(a)**



(b)



5.



CVX still has higher prices than MSFT (though not nearly as much as in Example 5). MSFT exhibits greater volatility, but more of its values are at the high end.

Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 9.5

(a) The ______________ divide a data set into quarters (four equal parts).

(b) Just as a single number (the median) separates the data into two groups, it takes ______________ numbers to separate data into four groups.

(c) The five-number ______________ associated with a data set is given by min, Q_1, median, Q_3, max.

(d) A graph based on the five-number summary is called a ______________ plot.

9.5 exercises

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Name ____________________

Section __________ Date __________

< Objective 1 >

Find the median of each set of numbers.

1. 2, 8, 5, 6, 9, 7, 4, 4, 5, 4, 3 > Videos

2. 7, 7, 5, 4, 1, 9, 8, 8, 8, 5, 2

3. 11, 12, 16, 14, 14, 14, 8, 12, 10, 18

4. 26, 30, 38, 67, 59, 21, 17, 85, 22, 22

5. 326, 245, 123, 222, 245, 300, 350, 602, 256

6. 0.10, 0.25, 0.24, 0.24, 0.30, 0.20, 0.18, 0.21, 0.28, 0.26

Find the first and third quartiles, Q_1 and Q_3, of each set of numbers.

7. 2, 8, 5, 6, 9, 7, 4, 4, 5, 4, 3 > Videos

8. 7, 7, 5, 4, 1, 9, 8, 8, 8, 5, 2

9. 11, 12, 16, 14, 14, 14, 8, 12, 10, 18

10. 26, 30, 38, 67, 59, 21, 17, 85, 22, 22

11. 326, 245, 123, 222, 245, 300, 350, 602, 256

12. 0.10, 0.25, 0.24, 0.24, 0.30, 0.20, 0.18, 0.21, 0.28, 0.26

Find the min and max of each set of numbers.

13. 2, 8, 5, 6, 9, 7, 4, 4, 5, 4, 3 > Videos

14. 7, 7, 5, 4, 1, 9, 8, 8, 8, 5, 2

15. 11, 12, 16, 14, 14, 14, 8, 12, 10, 18

16. 26, 30, 38, 67, 59, 21, 17, 85, 22, 22

17. 326, 245, 123, 222, 245, 300, 350, 602, 256

18. 0.10, 0.25, 0.24, 0.24, 0.30, 0.20, 0.18, 0.21, 0.28, 0.26

< Objective 2 >

Give the five-number summary of each set of numbers.

19. 2, 8, 5, 6, 9, 7, 4, 4, 5, 4, 3 > Videos

20. 7, 7, 5, 4, 1, 9, 8, 8, 8, 5, 2

21. 11, 12, 16, 14, 14, 14, 8, 12, 10, 18

Answers

1. __________ **2.** __________

3. __________ **4.** __________

5. __________ **6.** __________

7. __________ **8.** __________

9. __________ **10.** __________

11. __________ **12.** __________

13. __________ **14.** __________

15. __________ **16.** __________

17. __________ **18.** __________

19. __________ **20.** __________

21. ____________________

Answers

22. ______________

23. ______________

24. ______________

25. ______________

26. ______________

27. ______________

28. ______________

29. ______________

30. ______________

31. ______________

32. ______________

33. ______________

34. ______________

22. 26, 30, 38, 67, 59, 21, 17, 85, 22, 22

23. 326, 245, 123, 222, 245, 300, 350, 602, 256

24. 0.10, 0.25, 0.24, 0.24, 0.30, 0.20, 0.18, 0.21, 0.28, 0.26

< Objective 3 >

Construct a box-and-whisker plot for each set of numbers.

25. 2, 8, 5, 6, 9, 7, 4, 4, 5, 4, 3

26. 7, 7, 5, 4, 1, 9, 8, 8, 8, 5, 2

27. 11, 12, 16, 14, 14, 14, 8, 12, 10, 18

28. 26, 30, 38, 67, 59, 21, 17, 85, 22, 22

29. 326, 245, 123, 222, 245, 300, 350, 602, 256

30. 0.10, 0.25, 0.24, 0.24, 0.30, 0.20, 0.18, 0.21, 0.28, 0.26

Basic Skills | **Challenge Yourself** | Calculator/Computer | Career Applications | Above and Beyond

Determine whether each statement is **true** *or* **false.**

31. The quartiles divide the data set into three equal parts.

32. The median is equal to the second quartile.

In each statement, fill in the blank with **always, sometimes,** *or* **never.**

33. Before finding the quartiles, we must ______________ sort the data.

34. The third quartile is ______________ smaller than the median.

< Objective 4 >

This table gives the maximum depth and total bottom time for 25 recreational scuba dives. Use this table for exercises 35 and 36.

Diving Data

Dive no.	Max Depth (ft)	Bottom Time (s)
1	24	55
2	24	10
3	26	22
4	30	35
5	45	31
6	58	45
7	109	25
8	40	35
9	42	30
10	42	26
11	48	29
12	64	31
13	50	32
14	72	24
15	42	35
16	55	33
17	64	24
18	71	32
19	63	27
20	45	30
21	43	30
22	59	29
23	59	26
24	51	20
25	78	23

chapter 9 > Make the Connection

35. SCIENCE AND MEDICINE

(a) Give the five-number summary of the depth data.

(b) Construct a box-and-whisker plot for the depth data.

chapter 9 > Make the Connection

(c) Describe the depth data based on the box-and-whisker plot.

36. SCIENCE AND MEDICINE

(a) Give the five-number summary of the bottom time data.

(b) Construct a box-and-whisker plot for the bottom time data.

chapter 9 > Make the Connection

(c) Describe the bottom time data based on the box-and-whisker plot.

Answers

35. __________

36. __________

Answers

37. ____________________

38. ____________________

39. ____________________

40. ____________________

37. **BUSINESS AND FINANCE** Closing prices for Adobe Systems stock are shown during a 2-week period in 2003. Construct and interpret a box-and-whisker plot for these data.

ADBE (Adobe Systems)

Date	2/3	2/4	2/5	2/6	2/7	2/10	2/11	2/12	2/13	2/14
Price	27.06	26.43	26.52	26.52	26.14	26.63	26.70	26.78	26.93	27.43

Source: Yahoo! Finance.

38. **BUSINESS AND FINANCE** The closing prices for Kellogg Company stock are shown during the same 2-week period in 2003 as the Adobe Systems data in exercise 37.

K (Kellogg Co.)

Date	2/3	2/4	2/5	2/6	2/7	2/10	2/11	2/12	2/13	2/14
Price	32.32	32.53	32.33	32.24	32.10	32.12	31.50	31.30	31.21	31.65

Source: Yahoo! Finance.

(a) Construct a box-and-whisker plot for the Kellogg Company closing stock price data.

(b) Describe any distinctive features shown by the plot.

(c) Compare your plot with the one constructed in exercise 37.

39. **STATISTICS** This table gives the mean temperature (in degrees Fahrenheit) for the month of July over a 20-year period in Roanoke, VA.

Year	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990
Temp.	76.1°	75.5°	77.0°	73.2°	76.5°	78.8°	79.2°	77.0°	75.9°	76.8°
Year	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000
Temp.	77.9°	76.9°	80.2°	77.5°	76.9°	74.4°	76.1°	77.5°	79.1°	73.3°

Source: NOAA; NCDC.

(a) Construct a box-and-whisker plot based on the data.

chapter 9 > Make the Connection

(b) Discuss any significant features of the plot.

40. **STATISTICS** This table gives the mean temperature (in degrees Fahrenheit) for the month of January over a 20-year period in Dickinson, ND.

Year	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990
Temp.	26.6°	−0.2°	27.3°	22.5°	13.1°	23.1°	24.7°	13.4°	18.5°	24.5°
Year	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000
Temp.	11.6°	27.7°	9.6°	6.1°	16.5°	6.2°	8.5°	17.1°	14.7°	19.3°

Source: NOAA; NCDC.

(a) Construct a box-and-whisker plot based on the data.

chapter 9 > Make the Connection

(b) Discuss any significant features of the plot.

(c) Compare the box-and-whisker plot constructed in exercise 39 with the one constructed in this exercise.

Basic Skills | Challenge Yourself | Calculator/Computer | Career Applications | **Above and Beyond**

One measure used to describe a data set is the **range.** The range of a data set is given by the difference between the max and the min of the set. The range describes the variability of the data (that is, how much do the numbers vary).

Range = max − min

Find the range of each set of numbers.

41. 2, 8, 5, 6, 9, 7, 4, 4, 5, 4, 3 > Videos

42. 7, 7, 5, 4, 1, 9, 8, 8, 8, 5, 2

43. 11, 12, 16, 14, 14, 14, 8, 12, 10, 18

44. 26, 30, 38, 67, 59, 21, 17, 85, 22, 22

45. 326, 245, 123, 222, 245, 300, 350, 602, 256

46. 0.10, 0.25, 0.24, 0.24, 0.30, 0.20, 0.18, 0.21, 0.28, 0.26

Another measure that we use is the **interquartile range** (IQR). The IQR is given by the difference between the third quartile and the first quartile. The IQR measures how large an interval is needed to contain the middle 50% of the data. It is used to measure variability and to assist in determining if there are any *outliers* in the data.

$\text{IQR} = Q_3 - Q_1$

chapter 9 > Make the Connection

Find the IQR of each set of numbers.

47. 2, 8, 5, 6, 9, 7, 4, 4, 5, 4, 3

48. 7, 7, 5, 4, 1, 9, 8, 8, 8, 5, 2

49. 11, 12, 16, 14, 14, 14, 8, 12, 10, 18

50. 26, 30, 38, 67, 59, 21, 17, 85, 22, 22

51. 326, 245, 123, 222, 245, 300, 350, 602, 256

52. 0.10, 0.25, 0.24, 0.24, 0.30, 0.20, 0.18, 0.21, 0.28, 0.26

One characteristic we look for when describing and analyzing a data set is the presence of **outliers.** An outlier of a data set is a number that is "far away" from most of the other numbers in the set. We use the IQR to determine whether a data set has any outliers.

Answers

41. ____________

42. ____________

43. ____________

44. ____________

45. ____________

46. ____________

47. ____________

48. ____________

49. ____________

50. ____________

51. ____________

52. ____________

A number from a set is an outlier if it is more than 1.5 IQRs from either the first quartile or the third quartile. That is, we compute the following “boundaries” for outliers.

Lower boundary: $Q_1 - 1.5 \times \text{IQR}$
Upper boundary: $Q_3 + 1.5 \times \text{IQR}$

chapter 9 > Make the Connection

Any number in the data set that is less than the lower boundary or greater than the upper boundary is considered an outlier.

Find any outliers in each set of numbers.

53. 2, 8, 5, 6, 9, 7, 4, 4, 5, 4, 3

54. 7, 7, 5, 4, 1, 9, 8, 8, 8, 5, 2

55. 11, 12, 16, 14, 14, 14, 8, 12, 10, 18

56. 26, 30, 38, 67, 59, 21, 17, 85, 22, 22

57. 326, 245, 123, 222, 245, 300, 350, 602, 256

58. 0.10, 0.25, 0.24, 0.24, 0.30, 0.20, 0.18, 0.21, 0.28, 0.26

Some outliers are so far from the rest of the data that we call them **extreme outliers.** An extreme outlier is more than 3 IQRs from one of the quartiles. Outliers that are not extreme outliers are called **mild outliers.**

To determine the extreme outliers, again we set boundaries.

Lower boundary: $Q_1 - 3 \times \text{IQR}$
Upper boundary: $Q_3 + 3 \times \text{IQR}$

Classify any outliers of each data set as extreme or mild.

59. 2, 8, 5, 6, 9, 7, 4, 4, 5, 4, 3

60. 7, 7, 5, 19, 4, 1, 9, 8, 8, 8, 5, 2

61. 11, 12, 16, 14, 14, 14, 8, 12, 10, 18

62. 26, 30, 38, 67, 59, 21, 17, 85, 22, 22

63. 326, 245, 61, 222, 245, 300, 350, 842, 256

64. 0.10, 0.25, 0.24, 0.24, 0.92, 0.20, 0.18, 0.21, 0.28, 0.26

65. Research Microsoft Corporation’s (MSFT) closing stock prices for the most recent 2-week period (see Example 5). Give the five-number summary of these data, construct a box-and-whisker plot for the data, and interpret your display.

66. Research Chevron-Texaco’s (CVX) closing stock prices for the most recent 2-week period (see Example 5). Give the five-number summary of these data, construct a box-and-whisker plot for the data, and interpret your display.

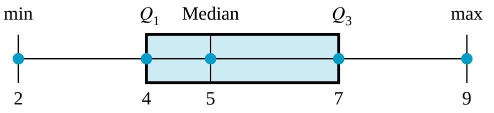
Answers

53. ____________________
54. ____________________
55. ____________________
56. ____________________
57. ____________________
58. ____________________
59. ____________________
60. ____________________
61. ____________________
62. ____________________
63. ____________________
64. ____________________
65. ____________________
66. ____________________

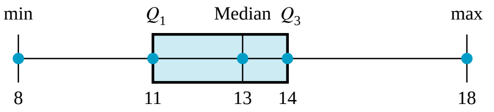
Answers

1. 5 **3.** 13 **5.** 256 **7.** 4; 7 **9.** 11; 14 **11.** 233.5; 338
13. 2; 9 **15.** 8; 18 **17.** 123; 602 **19.** 2, 4, 5, 7, 9
21. 8, 11, 13, 14, 18 **23.** 123, 233.5, 256, 338, 602

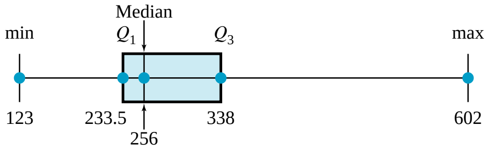
25.



27.

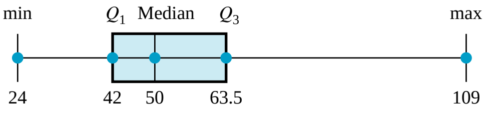


29.



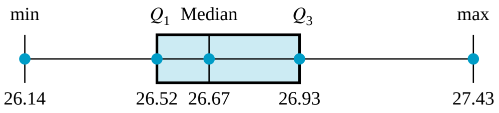
31. False **33.** always
35. **(a)** 24, 42, 50, 63.5, 109

(b)

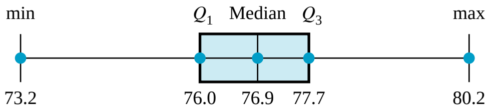


(c) answers will vary

37.



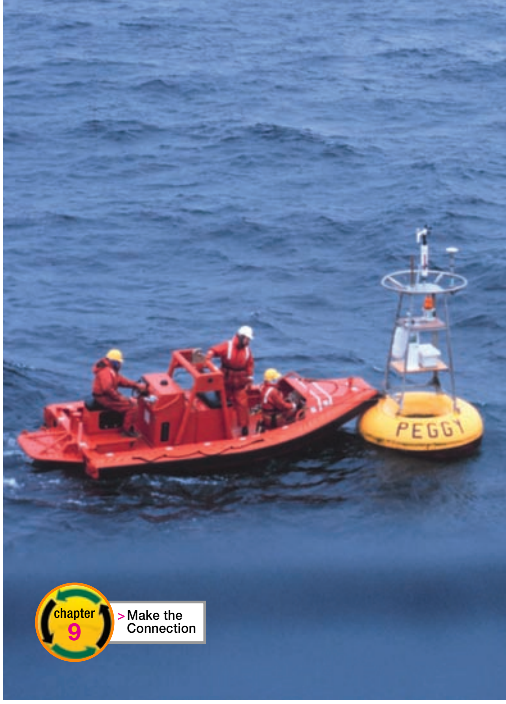
39. **(a)**



(b) answers will vary

41. 7 **43.** 10 **45.** 479 **47.** 3 **49.** 3 **51.** 104.5
53. None **55.** None **57.** 602 **59.** None **61.** None
63. Mild: 61; extreme: 842 **65.** Above and Beyond

Activity 26 ::
Outliers in Scientific Data



Marine scientists have placed numerous buoys in the oceans. These buoys are equipped to measure various properties of the water and to relay these measurements to satellites at regular intervals. The data are then collected and analyzed.

The table shows midday temperature data (in degrees Fahrenheit) collected off the coast of Cape Charles, VA, over a 12-day period in July 2003.

Date	7/1	7/2	7/3	7/4	7/5	7/6	7/7	7/8	7/9	7/10	7/11	7/12
Temp.	72.0°	73.2°	71.8°	72.6°	73.3°	74.1°	74.8°	76.0°	57.7°	77.5°	77.1°	78.2°

Source: NODC.

1. Find the mean temperature for the period shown.
2. Give the five-number summary for these data.
3. Compute the interquartile range of these data.
4. Identify any outliers in the data set.
5. Compute the mean of the data set without the outlier.
6. Give the five-number summary for the data set formed by removing the outlier from the table.

   Investigation showed that the buoy responsible for the given data was nonterminally damaged sometime between 7/8 and 7/10. It is believed that a small, personal watercraft struck the buoy about when it sent the July 9 data.

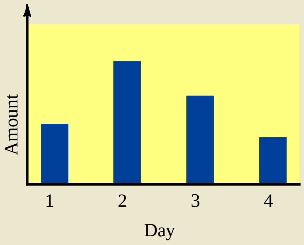
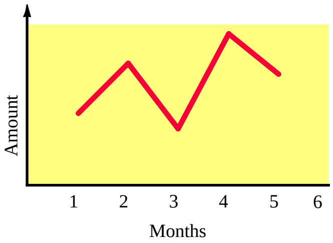
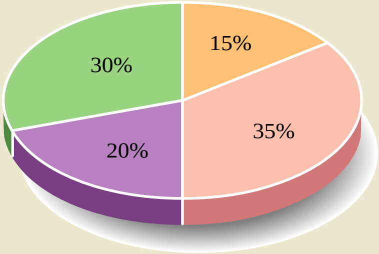
7. Describe how you might use this information to analyze salinity data measured by the same buoy.

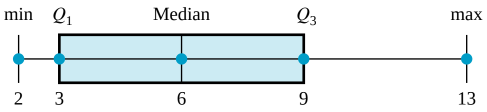
Definition/Procedure	Example	Reference
Means, Medians, and Modes		**Section 9.1**
Computing a Mean		
To find the *mean* for a set of numbers, follow these two steps: **Step 1** Add all the numbers in the set. **Step 2** Divide that sum by the number of items in the set.	Given the numbers 4, 8, 17, 23 $4 + 8 + 17 + 23 = 52$ $\text{Mean} = \frac{52}{4} = 13$	*p.* 567
Finding a Median		
Step 1 Sort the numbers in ascending order (lowest value to highest value). **Case 1** There are an odd number of data points. **Step 2** Select the middle data value; this is the median. **Case 2** There are an even number of data points. **Step 2** Select the two middle data values. **Step 3** Compute the mean of these two numbers; this is the median.	Given the 5 numbers 6, 4, 10, 7, 5 Rewrite the list in ascending order: $4, 5, \underbrace{6,}_{\text{Median}} 7, 10$ The middle value is the median: 6. Given the 6 numbers 9, 2, 5, 13, 7, 3 Rewrite the list in ascending order: $2, 3, \underbrace{5, 7,}_{\text{Middle values}} 9, 13$ Take the mean of the middle values: $\frac{5 + 7}{2} = \frac{12}{2} = 6$ The median is 6.	*p.* 569
Finding a Mode		
The *mode* is the number that occurs most frequently in a set of numbers.	Given the numbers 2, 3, 3, 3, 5, 5, 7, 7, 9, 11 3 is the mode.	*p.* 572
Tables, Pictographs, and Bar Graphs		**Section 9.2**
A **table** is a display of information in parallel columns or rows.		*p.* 583

Federal Highway Admin., U.S. Dept. of Transportation

Year	Cars Registered
1970	89,243,557
1975	106,705,934
1980	121,600,843
1985	131,664,029
1990	143,549,627
1995	136,066,045
2000	133,621,245

Continued

Definition/Procedure	Example	Reference
A **graph** is a diagram that relates two different pieces of information. One of the most common graphs is the **bar graph.** 		*p.* 584
Line Graphs and Predictions		**Section 9.3**
Line graph In line graphs, one of the axes is usually related to time. 		*p.* 596
Creating Bar Graphs and Pie Charts		**Section 9.4**
Pie chart Pie charts are graphs that show the component parts of a whole. 	Each percent is shown as the percent of a 360° circle. 30% of 360° = $0.30 \times 360^\circ = 108^\circ$ 20% of 360° = $0.20 \times 360^\circ = 72^\circ$ 15% of 360° = $0.15 \times 360^\circ = 54^\circ$ 35% of 360° = $0.35 \times 360^\circ = 126^\circ$	*p.* 605
Describing and Summarizing Data Sets		**Section 9.5**
Finding Quartiles		
To find the first quartile, find the median of the set of numbers to the left of the median. The third quartile is the median of the numbers to the right of the median.	Given 2, 3, 5, 7, 9, 13, we found the median to be 6. The list to the left of 6 is 2, 3, 5 and has a median of 3. Therefore, $Q_1 = 3$. Similarly, $Q_3 = 9$.	*p.* 617

Definition/Procedure	Example	Reference
Finding a Five-Number Summary The five-number summary is given by the list min, Q_1, median, Q_3, max	The five-number summary of the preceding list is 2, 3, 6, 9, 13	p. 617
Box-and-Whisker Plots Mark off the five-number summary on a number line from min to max and draw a rectangle between the quartiles.		p. 618

summary exercises :: chapter 9

This summary exercise set is provided to give you practice with each of the objectives of this chapter. Each exercise is keyed to the appropriate chapter section. When you are finished, you can check your answers to the odd-numbered exercises against those presented in the back of the text. If you have difficulty with any of these exercises, go back and reread the examples from that section. The answers to the even-numbered exercises appear in the *Instructor's Solutions Manual.* Your instructor will give you guidelines on how to best use these exercises in your instructional setting.

9.1 *Find the mean for each set of numbers.*

1. 8, 6, 7, 4, 5

2. 12, 14, 17, 19, 13

3. 117, 121, 122, 118, 115, 125, 123, 119

4. 134, 126, 128, 129, 133, 125, 122, 127

5. Elmer had test scores of 89, 71, 93, and 87 on his four math tests. What was his mean score?

6. The costs (in dollars) of the seven textbooks that Jacob needs for the spring semester are 77, 66, 55, 49, 85, 80, and 78. Find the mean cost of these books.

Find the median and the mode for each set of data.

7. 16, 20, 20, 19, 18

8. 8, 9, 9, 11, 11, 8, 7, 11, 12, 14, 10

9. 26, 31, 28, 35, 27, 28, 31, 30, 28, 30

10. 15, 18, 21, 23, 17, 19, 30, 35, 15, 32

11. Anita's first four test scores in her mathematics class were 88, 91, 86, and 93. What score must she get on her next test to have a mean of 90?

12. The weekly sales of a small company for 3 weeks were $2,400, $2,800, and $3,300. How much do sales need to be in the fourth week to achieve a mean of $3,000?

9.2 *Use the table for exercises 13 to 20.*

World Motor Vehicle Production, 1950–2000

			(in thousands)			
Year	**United States**	**Canada**	**Europe**	**Japan**	**Other**	**World Total**
2000	12,778	2,966	15,176	10,145	15,978	57,334
1999	13,025	3,057	15,395	9,895	14,388	55,760
1998	12,003	2,570	15,467	10,050	13,258	53,031
1995	11,985	2,408	17,045	10,196	8,349	49,983
1990	9,783	1,928	18,866	13,487	4,496	48,554
1985	11,653	1,933	16,113	12,271	2,939	44,909
1980	8,010	1,324	15,496	11,043	2,692	38,565
1970	8,284	1,160	13,049	5,289	1,637	29,419
1960	7,905	398	6,837	482	866	16,488
1950	8,006	388	1,991	32	160	10,577

Note: As far as can be determined, production refers to vehicles locally manufactured.
Source: American Automobile Manufacturers Assn.

13. What was the motor vehicle production in Japan in 1950? 2000?

14. What was the motor vehicle production in countries outside the United States in 1950? 2000?

15. What was the percent increase in motor vehicle production in the United States from 1950 to 2000?

16. What was the percent increase in motor vehicle production in countries outside the United States from 1950 to 2000?

17. What percent of world motor vehicle production occurred in Japan in 2000?

18. What percent of world motor vehicle production occurred in the United States in 2000?

19. What percent of world motor vehicle production occurred outside the United States and Japan in 2000?

20. Between 1950 and 2000, did the production of motor vehicles increase by a greater percent in Canada or Europe?

21. This table represents the 10 most expensive areas for home prices in the United States in 2000. Create a pictograph to represent the data.

City	Avg. Price
Palo Alto, CA	$974,237
Beverly Hills, CA	936,250
San Mateo, CA	873,250
La Jolla, CA	767,500
San Francisco, CA	759,250
Beverly Hills-South, CA	743,375
Greenwich, CT	706,500
Hollywood Hills, CA	663,375
Wellesley, MA	634,783

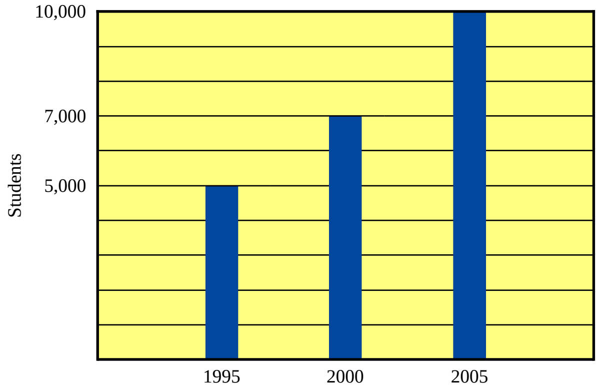
Source: Home Price Comparison Index.

22. This table represents the 10 most affordable areas in which to live in the United States in 2000. Create a pictograph to represent the data.

City	Avg. Price
Mt. Pleasant, MI	$103,640
Sioux City, IA	108,000
Eau Claire, WI	109,654
Hastings, NE	117,000
Minot, ND	124,400
Yankton, SD	124,750
Stroudsburg, PA	125,450
Helena, MT	126,475
Fort Wayne, IN	126,725
Tulsa, OK	129,950

Source: Home Price Comparison Index.

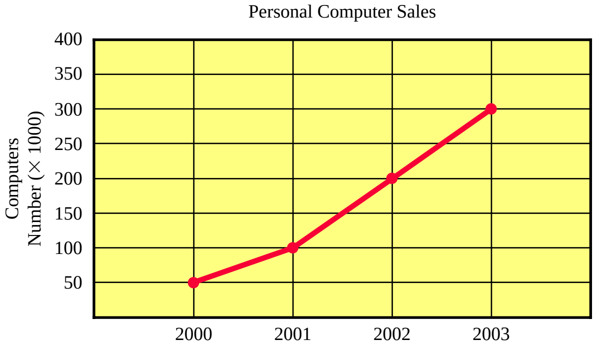
Use the bar graph for exercises 23 and 24.



23. How many more students were enrolled in 2005 than in 1995?

24. What was the percent increase from 1995 to 2000?

9.3 *Use the line graph for exercises 25 to 27.*



25. How many more personal computers were sold in 2003 than in 2000?

26. What was the percent increase in sales from 2000 to 2003?

27. Predict the sales of personal computers in the year 2004.

9.4 *This table shows the U.S. motor vehicle production in thousands, by source, in* 2002.

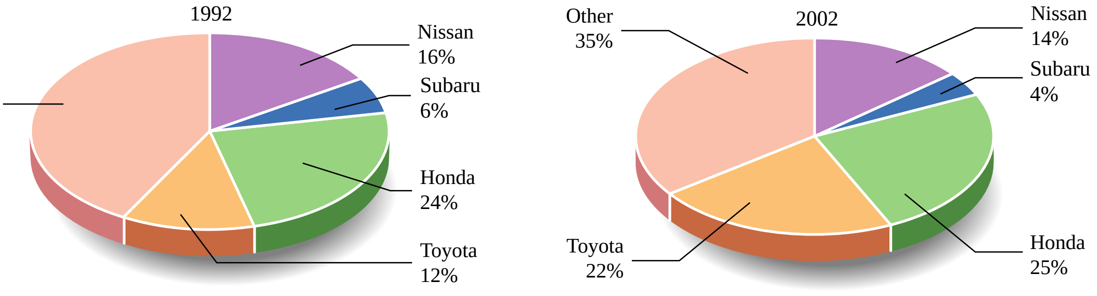
Source	Number (in thousands)	Percent
General Motors	4,093	33%
Ford	3,413	28%
Chrysler	1,837	15%
Foreign-based domestics	2,985	24%

Source: Automotive News Market Data Books.

28. Create a bar graph from the motor vehicle production table.

29. Create a pie chart from the motor vehicle production table.

The pie charts show the production of foreign-based domestic automobiles, by manufacturer, in the United States in 1992 *and* 2002.



Source: Automotive News Market Data Books.

30. What was the percent of Nissan and Subaru automobiles produced in 2002?

31. What was the percent of Honda and Toyota automobiles produced in 2002?

32. Which company's production increased the most between 1992 and 2002?

9.5

33. Give the five-number summary for the given set of numbers.

30, 32, 21, 35, 28, 28, 24, 23, 26, 30

34. Construct a box-and-whiskers plot of the given set of numbers.

30, 32, 21, 35, 28, 28, 24, 23, 26, 30

35. The scores on the first examination for an algebra class are as follows:

93, 79, 84, 62, 66, 94, 90, 87, 74, 76, 77, 72, 68, 62, 74, 85, 98, 69, 97, 78, 71

Construct a box-and-whisker plot for the examination grades and describe the results.

self-test 9

Name ______________________

Section __________ Date __________

The purpose of this self-test is to help you assess your progress so that you can find concepts that you need to review before the next exam. Allow yourself about an hour to take this test. At the end of that hour, check your answers against those given in the back of this text. If you miss any, go back to the appropriate section to reread the examples until you have mastered that particular concept.

1. Find the mean of the numbers 12, 19, 15, 20, 11, and 13.

2. Find the median of the numbers 8, 9, 15, 3, 1.

3. Find the median of the numbers 12, 18, 9, 10, 16, 6.

4. Find the mode of the numbers 6, 2, 3, 6, 2, 9, 2, 6, 6.

5. **CONSTRUCTION** A bus carried 234 passengers on the first day of a newly scheduled route. The next 4 days there were 197, 172, 203, and 214 passengers. What was the mean number of riders per day?

6. **STATISTICS** To earn an A in biology, you must have a mean of 90 on four tests. Your scores thus far are 87, 89, and 91. How many points must you have on the final test to earn the A?

7. These hair colors are from a class of seven students. What color is the mode?

brown, black, red, blonde, brown, brown, blue, gray

Answers

1. ______________________
2. ______________________
3. ______________________
4. ______________________
5. ______________________
6. ______________________
7. ______________________

Answers

8. ____________________

9. ____________________

10. ____________________

11. ____________________

12. ____________________

In exercises 8 to 11, use the table, which describes technology in the U.S. public schools from 1995 *to* 1998.

Technology in U.S. Public Schools, 1995–98

	Number of Schools			
Technology	**1995**	**1996**	**1997**	**1998**
Schools with modems[1]	**30,768**	**37,889**	**40,876**	**61,930**
Elementary	16,010	20,250	22,234	35,066
Junior high	5,652	6,929	7,417	10,996
Senior high[2]	8,790	10,277	10,781	14,540
Schools with networks[1]	**24,604**	**29,875**	**32,299**	**49,178**
Elementary	11,693	14,868	16,441	26,422
Junior high	4,599	5,590	6,035	9,003
Senior high[2]	8,159	9,166	9,565	12,853
Schools with CD-ROMs[1]	**34,480**	**43,499**	**46,388**	**64,200**
Elementary	18,343	24,353	26,377	37,908
Junior high	6,510	7,952	8,410	11,023
Senior high[2]	9,327	10,756	11,140	13,985
Schools with Internet access[1]	**NA**	**14,211**	**35,762**	**60,224**
Elementary	NA	7,608	21,026	34,195
Junior high	NA	2,707	5,752	10,888
Senior high[2]	NA	3,736	8,984	13,829

NA = Not applicable. (1) Includes schools for special and adult education, not shown separate with grade spans of K-3, K-5, K-6, K-8, and K-12. (2) Includes schools with grade spans of technical and alternative high schools and schools with grade spans of 7–12, 9–12, and 10.

Source: Quality Education Data, Inc., Denver, CO.

8. What is the increase in schools with modems from 1995 to 1998?

9. How many senior high schools had modems, networks, or CD-ROMs in 1998?

10. What is the percent increase in public schools with Internet access from 1996 to 1998?

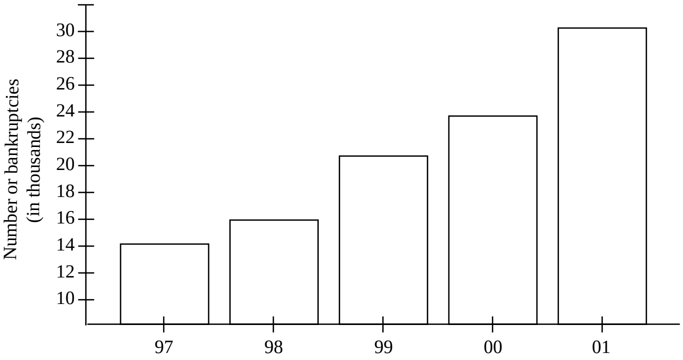
11. What is the percent increase in elementary schools that have modems, networks, or Internet access from 1996 to 1998?

12. Use the information in the table to create a pictograph.

Year	**Population of United States (in millions)**
1950	151
1960	179
1970	203
1980	227
1990	248
2000	281

Let each figure represent 30,000,000 people.

The bar graph represents the number of bankruptcy filings during a recent 5-year period.



13. How many people filed for bankruptcy in 1998?

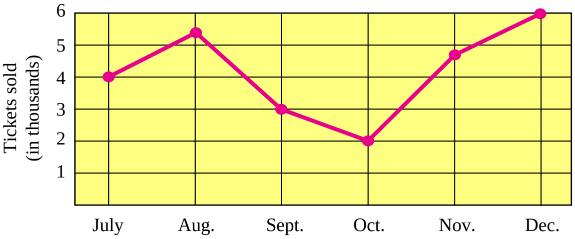
14. How many people filed for bankruptcy in 2001?

15. What was the increase in filings from 1999 to 2001?

16. What was the increase in filings from 1997 to 2001?

17. Which year had the greatest increase in filings?

The graph shows ticket sales for the last 6 months of the year.



18. What month had the greatest number of ticket sales?

19. Between what two months did the greatest decrease in ticket sales occur?

20. The information at the right shows a relationship between the number of workers absent from the assembly line and the number of defects coming off the line. Use this information to create a line graph and then predict the number of defects coming off the line if five workers are absent.

Number of Workers Absent	Number of Defects
0	9
1	10
2	12
3	16
4	18

21. The information at the right represents a relationship between age and college education. Create a bar graph from this information.

Age Group	Percent with 4 Years of College
25–34	24
35–44	27
45–54	21
55–64	15
65+	11

Answers

13. ____________________

14. ____________________

15. ____________________

16. ____________________

17. ____________________

18. ____________________

19. ____________________

20. ____________________

21. ____________________

Answers

22. ____________________

23. ____________________

24. ____________________

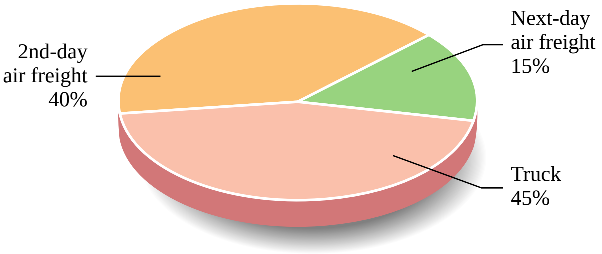
25. ____________________

22. The table gives the injuries suffered by students in classes involving participation.

Type of Injury	Number
Knee	24
Ankle	14
Elbow	4
Wrist	10
Others	8

Create a pie chart to show the distribution of the injury types.

The pie chart represents the way a new company ships its goods.



23. What percentage was shipped by truck?

24. What percentage was shipped by truck or second-day air freight?

25. November precipitation levels over a 20-year period in Amarillo, TX, are given by the table.

November Precipitation Levels; Amarillo, TX

Year	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990
Precipitation	1.50	0.76	0.36	1.10	0.42	1.83	0.44	0.30	0.00	0.52
Year	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000
Precipitation	0.68	0.81	0.52	0.60	0.06	1.32	1.17	0.34	0.00	0.96

Source: NOAA; NCDC.

Construct a box-and-whisker plot for the precipitation data.

cumulative review chapters 1-9

Name ______________________

Section __________ Date __________

The following exercises are presented to help you review concepts from earlier chapters. This is meant as review material and not as a comprehensive exam. The answers are presented in the back of the text. Beside each answer is a section reference for the concept. If you have difficulty with any of these exercises, be certain to at least read through the summary related to that section.

1. What is the place value of 6 in the numeral 126,489?

Perform the indicated operation.

2. $$\begin{array}{r} 5{,}306 \\ 389 \\ +\ 26{,}583 \\ \hline \end{array}$$

3. $$\begin{array}{r} 74{,}983 \\ -\ 35{,}695 \\ \hline \end{array}$$

4. 86×305

5. $27\overline{)8{,}322}$

6. $86{,}135 - 37{,}547$

7. 2.45×30.7

8. $\frac{4}{7} \times \frac{28}{24}$

9. $\frac{11}{15} \div \frac{121}{90}$

10. $3\frac{2}{3} + 5\frac{5}{6} - 2\frac{5}{12}$

Solve for the unknown.

11. $\frac{4}{7} = \frac{8}{x}$

12. $\frac{3}{5} = \frac{x}{15}$

13. Write 18% as a decimal and fraction.

14. Write $\frac{17}{40}$ as a decimal and percent.

Do the indicated operations.

15. $$\begin{array}{r} 7\text{ lb}\ \ 9\text{ oz} \\ +\ 3\text{ lb}\ 12\text{ oz} \\ \hline \end{array}$$

16. $$\begin{array}{r} 4\text{ min}\ 10\text{ s} \\ -\ 2\text{ min}\ 35\text{ s} \\ \hline \end{array}$$

Complete each statement.

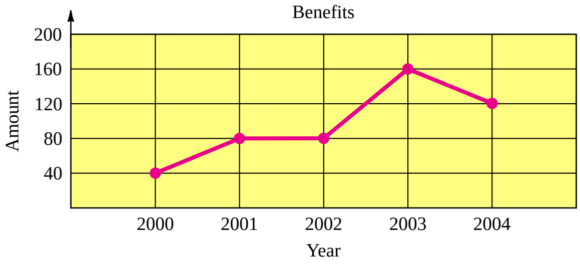
17. 8 km = __________ m

18. 3,000 mg = __________ g

19. 500 cm = __________ m

20. 25 cL = __________ mL

21. **STATISTICS** According to the line graph, between what two years was the increase in benefits the greatest?



Answers

1. ______________________
2. ______________________
3. ______________________
4. ______________________
5. ______________________
6. ______________________
7. ______________________
8. ______________________
9. ______________________
10. ______________________
11. ______________________
12. ______________________
13. ______________________
14. ______________________
15. __________ 16. __________
17. __________ 18. __________
19. __________ 20. __________
21. ______________________

Answers

22. ____________________

23. ____________________

24. ____________________

25. ____________________

26. ____________________

27. ____________________

28. ____________________

29. ____________________

30. ____________________

31. ____________________

32. ____________________

33. ____________________

34. ____________________

35. ____________________

22. **BUSINESS AND FINANCE** Construct a bar graph to represent the data.

Type of Stock	Number of Stocks
Industrial capital goods	110
Industrial consumer's goods	184
Public utilities	60
Railroads	15
Banks	25
Property liability insurance	16

23. Calculate the mean, median, and mode for the data.

11, 9, 3, 6, 7, 9, 8, 11, 12, 13, 11, 11, 4, 8, 12

24. If a boat uses 14 gal of gas to go 102 mi, how many gallons would be needed to go 510 mi?

Express each as a simplified rate.

25. $\frac{1{,}760 \text{ ft}}{20 \text{ s}}$

26. $\frac{133 \text{ pitches}}{7 \text{ innings}}$

27. **GEOMETRY** The floor of a room that is 12 ft by 18 ft is to be carpeted. If the price of the carpet is $17 per square yard, what will the carpet cost?

28. If you drive 152 miles in $3\frac{1}{6}$ hours, what is your average speed?

29. **GEOMETRY** A rectangle has length $8\frac{3}{5}$ cm and width $5\frac{7}{10}$ cm. Find its perimeter.

30. **GEOMETRY** The sides of a square each measure $13\frac{5}{6}$ ft. Find the perimeter of the square.

31. What is $9\frac{1}{2}\%$ of 1,400?

32. 15 is what percent of 7,500?

33. 111 is 60% of what number?

34. Find $\frac{2}{3}$ of $6\frac{1}{2}$.

35. **SOCIAL SCIENCE** The number of students attending a small college increased 6% since last year. This year there are 2,968 students. How many students attended last year?

chapter 10
> Make the Connection

CHAPTER 10

The Real Number System

INTRODUCTION

As he does the accounting for a medium-sized firm, Trinh reflects on how his knowledge of negative numbers helps him. Money comes in as revenue and goes out as costs. He also has lists of assets and debts to contend with.

Signed number arithmetic is used to model many situations that come up in business. Scientists, sports statisticians and fans, and even families considering their own personal finances find value in both positive and negative numbers.

In this chapter, we develop the skills necessary to perform arithmetic with real numbers. We also learn to model applications with real numbers; this leads into Chapter 11, which provides an introduction to algebra.

The real numbers are a tool that you will apply to several real-world applications in the activities, such as Activity 27 on p. 663, which requires you to gather data on the weather where you live.

CHAPTER 10 OUTLINE

Chapter 10 :: Prerequisite Test 644
10.1 **Real Numbers and Order** 645
10.2 **Adding Real Numbers** 654
10.3 **Subtracting Real Numbers** 664
10.4 **Multiplying Real Numbers** 672
10.5 **Dividing Real Numbers and the Order of Operations** 682
Chapter 10 :: Summary / Summary Exercises / Self-Test / Cumulative Review :: Chapters 1–10 694

10 prerequisite test

Name ____________

Section ________ Date ________

Answers

1. ____________

2. ____________

3. ____________

4. ____________

5. ____________

6. ____________

7. ____________

8. ____________

9. ____________

10. ____________

11. ____________

12. ____________

This prerequisite test provides some exercises requiring skills that you will need to be successful in the coming chapter. The answers for these exercises can be found in the back of this text. This prerequisite test can help you identify topics that you will need to review before beginning the chapter.

Evaluate each expression.

1. $12 + 9$

2. $147 - 68$

3. 23×16

4. $\frac{122}{8}$

5. $3 \times 8^2 - 5$

6. $56 - 3 \times 2^3$

Name the property being shown.

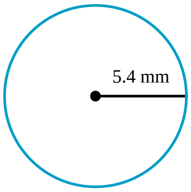
7. $12 + 5 = 5 + 12$

8. $3(3 + 1) = 3 \times 3 + 3 \times 1$

9. $(9 \times 7) \times 2 = 9 \times (7 \times 2)$

10. $\frac{14 + 23}{3} = \frac{14}{3} + \frac{23}{3}$

Use the circle shown to complete exercises 11–12. Use 3.14 for π and round your answers to two decimal places.



11. Find the circumference of the circle.

12. Find the area of the circle.

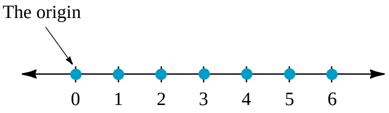
10.1

Real Numbers and Order

< **10.1 Objectives** >

1 > Represent an integer on a number line
2 > Order a set of real numbers
3 > Identify extreme values
4 > Simplify absolute value expressions

The numbers used to count things—1, 2, 3, 4, 5, and so on—are called the **natural** (or **counting**) **numbers.** The **whole numbers** consist of the natural numbers and zero—0, 1, 2, 3, 4, 5, and so on. They can be represented on a number line like the one shown. Zero (0) is considered the origin.

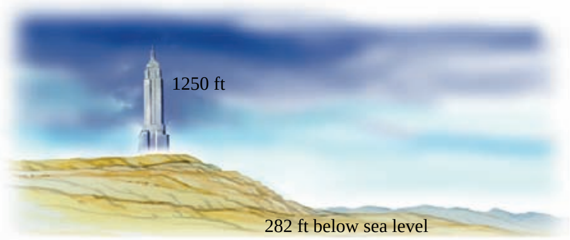


The number line continues indefinitely in both directions.

When numbers are used to represent physical quantities (such as altitude, temperature, and amount of money), it may be necessary to distinguish between *positive* and *negative* quantities. It is convenient to represent these quantities with plus (+) or minus (−) signs. For instance,

The Empire State building is 1,250 feet tall (+1,250).

The altitude at Badwater in Death Valley is 282 ft *below* sea level (−282).



The temperature in Chicago is 10° *below* zero (−10°).

An account could show a *gain* of \$100 (+100), or a *loss* of \$100 (−100).

These numbers suggest the need to extend the whole numbers to include both positive numbers (such as +100) and negative numbers (such as −282).

To represent the negative numbers, we extend the number line to the *left* of zero and name equally spaced points.

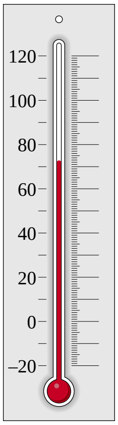
Numbers used to name points to the right of zero are positive numbers. They are written with a positive (+) sign or with no sign at all.

+6 and 9 are positive numbers

Numbers used to name points to the left of zero are negative numbers. They are always written with a negative (−) sign.

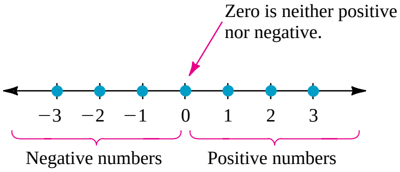
−3 and −20 are negative numbers

Read "negative 3."



Positive and negative numbers considered together (along with zero) are **real numbers.**

Here is the number line extended to include both positive and negative numbers.



The numbers used to name the points shown on the preceding number line are called the **integers.** The integers consist of the natural numbers, their negatives, and the number 0. We can write

$$\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots$$

A set of three dots is called an *ellipsis* and indicates that the pattern continues.

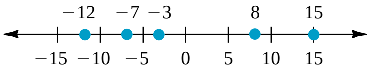
We mentioned that temperatures are real numbers. Note that the scale on the thermometer shown is similar to the scale on the number line.

Example 1 Representing Integers on the Number Line

< Objective 1 >

Represent the integers on the number line shown.

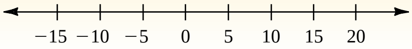
$-3, -12, 8, 15, -7$



Check Yourself 1

Represent the integers on a number line.

$-1, -9, 4, -11, 8, 20$



The set of numbers on the number line is *ordered.* The numbers get smaller as you move to the left on the number line and larger as you move to the right. When a set of numbers is written from smallest to largest, the numbers are said to be in *ascending order.*

Example 2 Ordering Real Numbers

< Objective 2 >

Place each set of numbers in ascending order.

(a) $9, -5, -8, 3, 7$

From smallest to largest, the numbers are

$-8, -5, 3, 7, 9$

Note that this is the order in which the numbers appear on a number line.

(b) $3, -2, 18, -20, -13$

From smallest to largest, the numbers are

$-20, -13, -2, 3, 18$

RECALL

In Section 9.5, we called these values the **min** and **max.**

Check Yourself 2

Place each set of numbers in ascending order.

(a) **$12, -13, 15, 2, -8, -3$** **(b)** **$3, 6, -9, -3, 8$**

The least and greatest numbers in a set are called the **extreme values.** The least element is called the **minimum,** and the greatest element is called the **maximum.**

Example 3 Labeling Extreme Values

< Objective 3 >

Determine the minimum and maximum values of each set of numbers.

(a) 9, -5, -8, 3, 7

From our previous ordering of these numbers, we see that -8, the least element, is the minimum, and 9, the greatest element, is the maximum.

(b) 3, -2, 18, -20, -13

-20 is the minimum and 18 is the maximum.

Check Yourself 3

Determine the minimum and maximum values of each set of numbers.

(a) **12, −13, 15, 2, −8, −3** **(b)** **3, 6, −9, −3, 8**

Integers are not the only kind of real numbers. Decimals and fractions are also real numbers.

Example 4 Identifying Real Numbers as Integers

Which real numbers are integers?

(a) 145 is an integer.

(b) -28 is an integer.

(c) 0.35 is not an integer.

(d) $-\frac{2}{3}$ is not an integer.

Check Yourself 4

Which real numbers are integers?

-23 1,054 -0.23 0 -500 $-\frac{4}{5}$

The set of real numbers corresponds to the points on the number line. That is, *every* point on the number line is associated with a real number and *every* real number can be represented by a point on the number line.

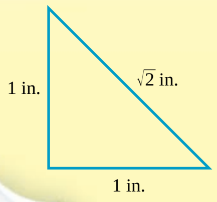
We divide the real numbers into two categories. You have worked primarily with one of the categories, the **rational numbers.** Every number that can be written as a fraction is a rational number. This includes all repeating or finite decimals, all integers, and all fractions.

RECALL

We used 3.14 to approximate π in many calculations in Section 8.3.

Numbers that are not rational are called **irrational numbers.** You have encountered some of the irrational numbers in your studies. For example, we worked with π (pi) in Section 8.1. Pi is given by the ratio of the circumference of a circle and its diameter. There is no nice fraction or decimal we can use to represent π. We can approximate π in calculations, but we do not have an exact decimal representation.

NOTE



There are many other irrational numbers, some of which are useful enough to "name," just as we named π. For instance, the square root of any whole number that is not a perfect square names an irrational number. We can use the Pythagorean theorem to find these numbers in nature.

There are even irrational numbers that we have not named. One example requires you to recognize the pattern that emerges in the following decimal.

0.10110111011110 . . .

You will learn much more about this rich set of numbers when you enroll in future math courses.

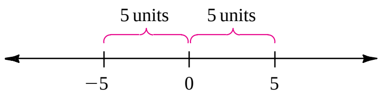
RECALL

We call zero (0) the origin.

NOTE

Distance and length are always given as positive numbers.

An important idea for our work in this chapter is the **absolute value** of a number. This represents the distance of the point named by the number from the origin on the number line.



The absolute value of 5 is 5. The absolute value of −5 is also 5.

In symbols we write

$|5| = 5$ and $|-5| = 5$

Read "the absolute value of 5."

Read "the absolute value of negative 5."

The absolute value of a number does *not* depend on whether the number is to the right or to the left of the origin, but on its *distance* from the origin.

Example 5 Simplifying Absolute Value Expressions

< Objective 4 >

(a) $|7| = 7$

(b) $|-7| = 7$

(c) $-|-7| = -7$

This is the *negative*, or opposite, of the absolute value of negative 7.

(d) $|-10| + |10| = 10 + 10 = 20$

(e) $|8 - 3| = |5| = 5$

Absolute value bars serve as another set of grouping symbols, so do the operation inside first.

(f) $|8| - |3| = 8 - 3 = 5$

Here, evaluate the absolute values and then subtract.

Check Yourself 5

Evaluate.

(a) $|8|$ **(b)** $|-8|$ **(c)** $-|-8|$

(d) $|-9| + |4|$ **(e)** $|9 - 4|$ **(f)** $|9| - |4|$

The language of real numbers can be applied to many situations. Example 6 gives you an idea of the variety of problems that are best modeled with real numbers.

Example 6 Describing Applications with Real Numbers

Represent each quantity with an integer.

(a) A sick infant's temperature drops by 4 degrees Fahrenheit (°F) after being given acetaminophen.

After taking the medication, the infant's temperature went down, so we need a negative number.

−4°F

RECALL

You should include units in your answer to an application.

(b) A project is $1,500 over budget.

The cost of the project went up, so we use a positive number.

+$1,500 (or simply $1,500)

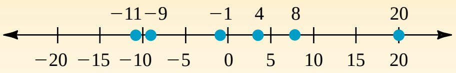
Check Yourself 6

Represent each quantity with an integer.

(a) After an hour, it is noticed that an intravenous solution (IV) is running 30 milliliters (mL) ahead of schedule.
(b) The bridge is on schedule to be completed 7 months early.

Check Yourself ANSWERS

1.



2. **(a)** $-13, -8, -3, 2, 12, 15$; **(b)** $-9, -3, 3, 6, 8$
3. **(a)** Minimum is -13; maximum is 15; **(b)** minimum is -9; maximum is 8
4. -23, 1,054, 0, and -500 **5.** **(a)** 8; **(b)** 8; **(c)** -8; **(d)** 13; **(e)** 5; **(f)** 5
6. **(a)** +30 mL; **(b)** –7 months

Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 10.1

(a) The whole numbers consist of the natural numbers and ______________.

(b) ______________ numbers are used to describe below-zero temperatures.

(c) When a set of numbers is written from smallest to largest, the numbers are said to be in ______________ order.

(d) The ______________ of a number is given by its distance from the origin on the number line.

10.1 exercises

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Name ______________________

Section __________ Date __________

Answers

1. __________ 2. __________

3. __________ 4. __________

5. ______________________

6. ______________________

7. ______________________

8. ______________________

9. ______________________

10. ______________________

11. ______________________

12. ______________________

13. ______________________

14. ______________________

15. ______________________

16. ______________________

17. ______________________

18. ______________________

19. __________ 20. __________

21. __________ 22. __________

23. __________ 24. __________

Represent each quantity with a real number.

1. An altitude of 400 feet (ft) above sea level

2. An altitude of 80 ft below sea level

3. A loss of $200

4. A profit of $400

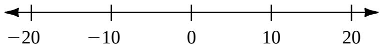
5. A decrease in population of 25,000

6. An increase in population of 12,500

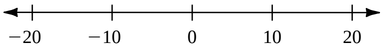
< Objective 1 >

Represent the integers on the number lines shown.

7. $5, -15, 18, -8, 3$



8. $-18, 4, -5, 13, 9$



Which numbers are integers?

9. $5, -\frac{2}{9}, 175, -234, -0.64$ > Videos

10. $-45, 0.35, \frac{3}{5}, 700, -26$

< Objective 2 >

Place each set in ascending order.

11. $3, -5, 2, 0, -7, -1, 8$ > Videos

12. $-2, 7, 1, -8, 6, -1, 0$

13. $9, -2, -11, 4, -6, 1, 5$

14. $23, -18, -5, -11, -15, 14, 20$

< Objective 3 >

For each set, determine the maximum and minimum values.

15. $5, -6, 0, 10, -3, 15, 1, 8$ > Videos

16. $9, -1, 3, 11, -4, 2, 5, -2$

17. $21, -15, 0, 7, -9, 16, -3, 11$

18. $-22, 0, 22, -31, 18, -5, 3$

< Objective 4 >

Evaluate.

19. $|17|$ > Videos

20. $|28|$

21. $|-10|$ > Videos

22. $|-7|$

23. $-|3|$

24. $-|5|$

25. $-|-8|$

26. $-|-13|$

27. $|-2| + |3|$

28. $|4| + |-3|$

29. $|-9| + |9|$ > Videos

30. $|11| + |-11|$

31. $|4| - |-4|$

32. $|5| - |-5|$

33. $|15| - |8|$

34. $|11| - |3|$

35. $|15 - 8|$

36. $|11 - 3|$

37. $|-9| + |2|$

38. $|-7| + |4|$

39. $|-8| - |-7|$

40. $|-9| - |-4|$

Represent each quantity with a real number.

41. Business and Finance The withdrawal of $50 from a checking account.

42. Business and Finance The deposit of $200 into a savings account.

43. Science and Medicine A temperature decrease of 10°F in 1 hour. chapter 10 > Make the Connection

44. Social Science An increase of 25,000 in a city's population.

45. Business and Finance An increase of 75 points in the Dow-Jones average.

46. Statistics An eight-game losing streak by the local baseball team.

47. Business and Finance A country exported $90,000,000 more than it imported, creating a positive trade balance.

48. Business and Finance A stock lost 8.5% of its value.

Basic Skills | **Challenge Yourself** | Calculator/Computer | Career Applications | Above and Beyond

Label each statement as **true** *or* **false.**

49. All whole numbers are integers.

50. All integers are real numbers.

51. All integers are whole numbers.

52. All real numbers are integers.

Fill in each blank with >, <, or = to make a true statement.

53. -9 ___ -6

54. $|-9|$ ___ $|-6|$

55. $|-9|$ ___ 6

56. 9 ___ $|-6|$

Answers

25. ________ 26. ________

27. ________ 28. ________

29. ________ 30. ________

31. ________ 32. ________

33. ________ 34. ________

35. ________ 36. ________

37. ________ 38. ________

39. ________ 40. ________

41. ________ 42. ________

43. ________________

44. ________________

45. ________________

46. ________________

47. ________________

48. ________ 49. ________

50. ________ 51. ________

52. ________ 53. ________

54. ________ 55. ________

56. ________________

Answers

57. ____

58. ____

59. ____

60. ____

61. ____

62. ____

63. ____

64. ____

65. ____

66. ____

67. ____

68. ____

69. ____

70. ____

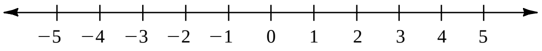
71. ____

72. ____

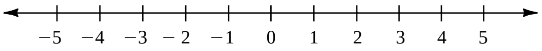
73. ____

Represent the real numbers on the number line shown.

57. $-\frac{3}{4}, 2\frac{1}{2}, -\frac{1}{2}, -3\frac{1}{4}, \frac{2}{3}$



58. 3.2, −1.4, 0.7, −0.9, −2.1



Place each set in ascending order.

59. $-\frac{1}{2}, \frac{3}{4}, -\frac{5}{6}, \frac{2}{3}, -\frac{1}{3}$

60. $\frac{3}{7}, -\frac{6}{7}, \frac{1}{7}, -\frac{1}{2}, \frac{2}{7}$

61. −6.1, −5.9, 6.1, 5.9, −6.0

62. 3.5, −5.3, −3.5, 5.3, 4

For each set, determine the maximum and minimum values.

63. $3, 0, \frac{1}{2}, -\frac{2}{3}, 5, \frac{3}{4}, -\frac{1}{6}$

64. $-3, 2, \frac{7}{12}, -\frac{3}{4}, \frac{5}{6}, -\frac{10}{3}, \frac{5}{2}$

65. $-3.3, 4\frac{1}{2}, -3, -2.8, 4.3, 4.8$

66. $-11, 4\frac{1}{2}, \frac{15}{4}, -10.9, -11.1, 0$

Place absolute value bars in the proper location on the left side of the equation in order to make it true.

67. $6 + (-2) = 4$

68. $8 + (-3) = 5$

69. $6 + (-2) = 8$

70. $8 + (-3) = 11$

Basic Skills | Challenge Yourself | Calculator/Computer | **Career Applications** | Above and Beyond

Represent each quantity with a real number.

71. **Agricultural Technology** The erosion of 5 cm of topsoil from an Iowa cornfield.

72. **Agricultural Technology** The formation of 2.5 cm of new topsoil on the African savanna.

73. **Construction Technology** The elevations, in inches, of several points on a jobsite are as follows:

−18, 27, −84, 37, 59, −13, 4, 92, 49, 66, −45

Arrange the elevations in ascending order.

74. **ELECTRONICS** Several 12-volt (V) batteries were tested using a voltmeter. The voltage values were entered into a table indicating their value in reference to 12 V. Determine the maximum and minimum voltage measurements taken.

Battery	Variance from 12 V (in V)
Cell 1	+1
Cell 2	0
Cell 3	−1
Cell 4	−3
Cell 5	+2

75. **ELECTRICAL ENGINEERING** Several resistors were tested using an ohmmeter. The resistance values were entered into a table indicating their value in reference to 10,000 ohms (10 kΩ). List the resistors in ascending order according to their measured resistance.

Battery	Variance from 10,000 Ω (in Ω)
Resistor 1	+175
Resistor 2	−60
Resistor 3	−188
Resistor 4	+10
Resistor 5	+218
Resistor 6	−65
Resistor 7	−302

76. **ELECTRICAL ENGINEERING** Which of the resistors in exercise 75 had a measured value furthest from 10,000 Ω?

Answers

74. ____________________

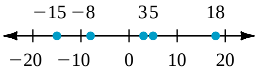
75. ____________________

76. ____________________

Answers

1. 400 ft or (+400 ft) **3.** −$200 **5.** −25,000 people

7.



9. 5, 175, −234 **11.** −7, −5, −1, 0, 2, 3, 8

13. −11, −6, −2, 1, 4, 5, 9 **15.** Min: −6; max: 15

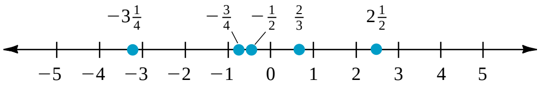
17. Min: −15; max: 21 **19.** 17 **21.** 10 **23.** −3 **25.** −8

27. 5 **29.** 18 **31.** 0 **33.** 7 **35.** 7 **37.** 11 **39.** 1

41. −$50 **43.** −10°F **45.** +75 points **47.** +$90,000,000

49. True **51.** False **53.** < **55.** >

57.



59. $-\frac{5}{6}, -\frac{1}{2}, -\frac{1}{3}, \frac{2}{3}, \frac{3}{4}$ **61.** −6.1, −6.0, −5.9, 5.9, 6.1

63. Min: $-\frac{2}{3}$; max: 5 **65.** Min: −3.3; max: 4.8

67. $6 + (-2) = 4$ or $|6 + (-2)| = 4$ **69.** $6 + |(-2)| = 8$ or $|6| + |(-2)| = 8$

71. −5 cm **73.** −84 in., −45 in., −18 in., −13 in., 4 in., 27 in., 37 in., 49 in., 59 in., 66 in., 92 in. **75.** 9,698 Ω, 9,812 Ω, 9,935 Ω, 9,940 Ω, 10,010 Ω, 10,175 Ω, 10,218 Ω

10.2

Adding Real Numbers

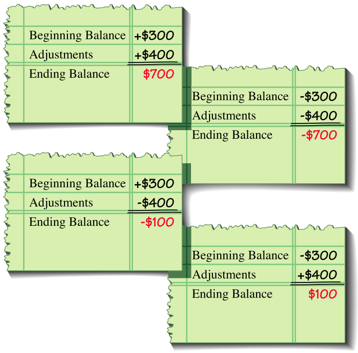
< 10.2 Objectives >

1 > Add two numbers with the same sign

2 > Add two numbers with opposite signs

In Section 10.1, we introduced the idea of real numbers. Now we will examine the four arithmetic operations (addition, subtraction, multiplication, and division) and see how those operations are performed when real numbers are involved. We start by considering addition.

An application may help. As before, we represent a gain of money as a positive number and a loss as a negative number.


Beginning Balance +$300
Adjustments +$400
Ending Balance $700
Beginning Balance -$300
Adjustments -$400
Ending Balance -$700
Beginning Balance +$300
Adjustments -$400
Ending Balance -$100
Beginning Balance -$300
Adjustments +$400
Ending Balance $100

If you gain \$300 and then gain \$400, the result is a gain of \$700:

$$300 + 400 = 700$$

If you lose \$300 and then lose \$400, the result is a loss of \$700:

$$-300 + (-400) = -700$$

If you gain \$300 and then lose \$400, the result is a loss of \$100:

$$300 + (-400) = -100$$

If you lose \$300 and then gain \$400, the result is a gain of \$100:

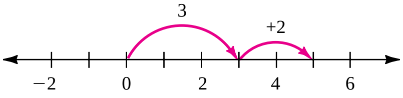
$$-300 + 400 = 100$$

The number line can be used to illustrate the addition of real numbers. Starting at the origin, we move to the *right* for positive numbers and to the *left* for negative numbers.

Example 1 Adding Real Numbers on the Number Line

< Objective 1 >

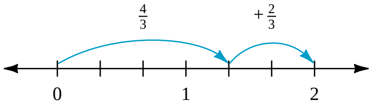
(a) Add 3 + 2.



Start at the origin and move 3 units to the right. Then, move 2 units more to the right to find the sum.

$$3 + 2 = 5$$

(b) Add $\frac{4}{3} + \frac{2}{3}$.


$\frac{4}{3}$
$+\frac{2}{3}$
0
1
2

Start at the origin and move $\frac{4}{3}$ units to the right. Then move $\frac{2}{3}$ more to the right to find the sum. So we have

$$\frac{4}{3} + \frac{2}{3} = \frac{6}{3} = 2$$

Check Yourself 1

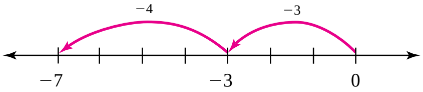
Add.

(a) 5 + 6 **(b)** $\frac{5}{4} + \frac{7}{4}$

The number line also helps you visualize the sum of two negative numbers. Remember to move left for negative numbers.

Example 2 Adding Real Numbers with the Same Sign

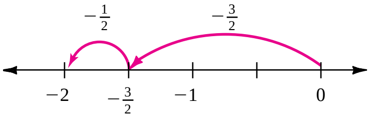
(a) Add $-3 + (-4)$.



Start at the origin and move 3 units to the left. Then move 4 more units to the left to find the sum. From the graph we see that the sum is

$$-3 + (-4) = -7$$

(b) Add $-\frac{3}{2} + \left(-\frac{1}{2}\right)$.



As before, we start at the origin. From that point move $\frac{3}{2}$ units left. Then move another $\frac{1}{2}$ unit left to find the sum. In this case

$$-\frac{3}{2} + \left(-\frac{1}{2}\right) = -2$$

Check Yourself 2

Add.

(a) $-4 + (-5)$ **(b)** $-3 + (-7)$

(c) $-5 + (-15)$ **(d)** $-\frac{5}{2} + \left(-\frac{3}{2}\right)$

You have probably noticed some helpful patterns in Examples 1 and 2. These patterns will allow you to do the work mentally without having to use the number line. Look at the following rule.

Property

Adding Real Numbers with the Same Sign

If two numbers have the same sign, add their absolute values. Give the sum the sign of the original numbers.

Example 3 Adding Real Numbers

NOTE

The sum of two positive numbers is positive; the sum of two negative numbers is negative.

(a) $-8 + (-5) = -13$

Add the absolute values ($8 + 5 = 13$) and give the sum the sign (−) of the original numbers.

(b) $[-3 + (-4)] + (-6)$

$= -7 + (-6) = -13$

Add inside the brackets as your first step.

Check Yourself 3

Add mentally.

(a) $7 + 9$ **(b)** $-7 + (-9)$

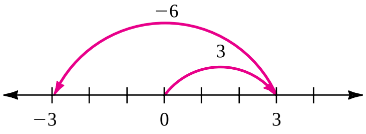
(c) $-5.8 + (-3.2)$ **(d)** $[-5 + (-2)] + (-3)$

We can also use the number line to illustrate the addition of real numbers with *different* signs.

Example 4 Adding Real Numbers with Opposite Signs

< Objective 2 >

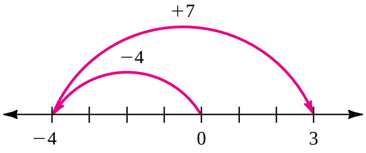
(a) Add $3 + (-6)$.



First move 3 units to the right of the origin. Then move 6 units to the left.

$3 + (-6) = -3$

(b) Add $-4 + 7$.



This time move 4 units to the left of the origin as the first step. Then move 7 units to the right.

$-4 + 7 = 3$

Check Yourself 4

Add.

(a) $7 + (-5)$ **(b)** $4 + (-8)$
(c) $-4 + 9$ **(d)** $-7 + 3$

You have no doubt noticed that, in adding a positive number and a negative number, sometimes the sum is positive and sometimes it is negative. The result depends on which of the numbers has the larger absolute value. This leads us to the second part of our addition rule.

Property

Adding Real Numbers with Different Signs

If two numbers have different signs, subtract their absolute values, the smaller from the larger. Give the result the sign of the number with the larger absolute value.

Example 5 Adding Real Numbers

Add.

(a) $7 + (-19) = -12$

Because the two numbers have different signs, subtract their absolute values ($19 - 7 = 12$). The sum has the sign ($-$) of the number with the larger absolute value, -19.

RECALL

Real numbers can be fractions and decimals as well as integers.

(b) $-13 + 7 = -6$

Subtract the absolute values ($13 - 7 = 6$). The sum has the sign ($-$) of the number with the larger absolute value, -13.

(c) $-4.5 + 8.2 = 3.7$

Subtract the absolute values ($8.2 - 4.5 = 3.7$). The sum has the sign ($+$) of the number with the larger absolute value, 8.2.

Check Yourself 5

Add mentally.

(a) $5 + (-14)$ **(b)** $-7 + (-8)$ **(c)** $-8 + 15$
(d) $7 + (-8)$ **(e)** $-\frac{2}{3} + \left(-\frac{7}{3}\right)$ **(f)** $5.3 + (-2.3)$

There are two properties of addition that we should mention before concluding this section. First, the sum of any number and 0 is always that number. In symbols,

Property

Additive Identity Property

For any number a,

$a + 0 = 0 + a = a$

Example 6 Adding Zero

NOTE

Numbers do not change their value after addition with 0. Zero is called the **additive identity.**

Add.

(a) $9 + 0 = 9$

(b) $0 + (-8) = -8$

(c) $-25 + 0 = -25$

Check Yourself 6

Add.

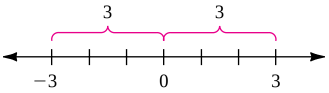
(a) $8 + 0$ **(b)** $0 + (-7)$ **(c)** $-36 + 0$

NOTES

The opposite of a number is also called its **additive inverse.**

3 and −3 are opposites.

We need one further definition to state our second property. Every number has an *opposite.* It corresponds to a point that is the same distance from the origin as the given number, but in the opposite direction.



The opposite of 9 is −9.

The opposite of −15 is 15.

Our second property states that the sum of any number and its opposite is 0.

Property

Additive Inverse Property

For any number a, there exists a number $-a$ such that

$a + (-a) = -a + a = 0$

In words: The sum of any number and its opposite, or additive inverse, is 0.

Example 7 Adding Inverses

Add.

(a) $9 + (-9) = 0$

(b) $-15 + 15 = 0$

(c) $-2.3 + 2.3 = 0$

(d) $\frac{4}{5} + \left(-\frac{4}{5}\right) = 0$

NOTE

All properties of addition from Section 1.2 apply when negative numbers are involved.

Check Yourself 7

Add.

(a) $-17 + 17$ **(b)** $12 + (-12)$

(c) $\frac{1}{3} + \left(-\frac{1}{3}\right)$ **(d)** $-1.6 + 1.6$

We can now use the associative and commutative properties of addition, first introduced in Section 1.2, to find the sum when more than two real numbers are involved. Example 8 illustrates these properties.

Example 8 Adding Real Numbers

NOTE

We use the commutative property to reverse the order of addition for -3 and 5. We then group -5 and 5. Do you see why?

$$\begin{aligned} -5 + (-3) + 5 &= -5 + 5 + (-3) \\ &= [-5 + 5] + (-3) \\ &= 0 + (-3) \\ &= -3 \end{aligned}$$

Check Yourself 8

Add.

(a) $-4 + 5 + (-3)$ **(b)** $-8 + 4 + 8$

Real numbers appear in many situations.

Example 9 An Application of Real Numbers

A vendor earned profits of −$86.75, $111.50, and $123 one weekend (Friday through Sunday). What was the vendor's total weekend profit?

We add the vendor's daily profits to get the weekend profit.

$$-86.75 + 111.50 + 123 = 147.75$$

So the vendor earned a weekend profit of $147.75.

Check Yourself 9

A softball team scored 3 runs in one inning, gave up 4 runs in another inning, scored 2 runs after that, and gave up 3 runs in the final inning. How far was the team ahead at the end of the game?

Check Yourself ANSWERS

1. **(a)** 11; **(b)** 3 **2.** **(a)** -9; **(b)** -10; **(c)** -20; **(d)** -4
3. **(a)** 16; **(b)** -16; **(c)** -9; **(d)** -10 **4.** **(a)** 2; **(b)** -4; **(c)** 5; **(d)** -4
5. **(a)** -9; **(b)** -15; **(c)** 7; **(d)** -1; **(e)** -3; **(f)** 3 **6.** **(a)** 8; **(b)** -7; **(c)** -36
7. **(a)** 0; **(b)** 0; **(c)** 0; **(d)** 0 **8.** **(a)** -2; **(b)** 4 **9.** -2 (they lost by 2)

Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 10.2

(a) The sum of two negative numbers is always ______________.

(b) Adding a ______________ number can be illustrated on a number line by moving to the left.

(c) When adding numbers with different signs, the result has the same sign as the number with the larger ______________ value.

(d) The sum of any number and its opposite, or additive inverse, is ______________.

10.2 exercises

< Objectives 1–2 >

Perform the indicated operation.

1. $3 + 6$ > Videos

2. $5 + 9$

3. $11 + 5$

4. $8 + 7$

5. $-2 + (-3)$

6. $-1 + (-9)$

7. $-13 + (-24)$ > Videos

8. $-1{,}234 + (-887)$

9. $9 + (-3)$

10. $10 + (-4)$

11. $8 + (-14)$

12. $7 + (-11)$

13. $-4 + 17$ > Videos

14. $-87 + 23$

15. $-732 + 1{,}104$

16. $2{,}417 + (-7{,}332)$

17. $\frac{3}{4} + \frac{5}{4}$

18. $\frac{1}{2} + \frac{4}{5}$

19. $-\frac{3}{5} + \left(-\frac{7}{5}\right)$ > Videos

20. $-\frac{1}{8} + \left(-\frac{3}{8}\right)$

21. $-\frac{2}{3} + \left(-\frac{1}{4}\right)$

22. $-\frac{3}{4} + \left(-\frac{5}{12}\right)$

23. $-2\frac{2}{3} + \left(-1\frac{1}{2}\right)$

24. $-6\frac{1}{2} + \left(-\frac{3}{5}\right)$

25. $\frac{3}{4} + \left(-\frac{1}{4}\right)$

26. $-\frac{7}{12} + \frac{1}{3}$

27. $-\frac{2}{5} + \frac{13}{20}$

28. $\frac{2}{3} + \left(-\frac{5}{6}\right)$

29. $5\frac{1}{3} + \left(-4\frac{4}{5}\right)$

30. $-17\frac{3}{4} + 21\frac{1}{3}$

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Name ______________________

Section __________ Date __________

Answers

1. __________ 2. __________
3. __________ 4. __________
5. __________ 6. __________
7. __________ 8. __________
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25. __________ 26. __________
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29. __________ 30. __________

Answers

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31. $-1.6 + (-2.3)$

32. $-3.5 + (-2.6)$

33. $-3.6 + 7.6$

34. $13.4 + (-11.4)$

35. $-9 + 0$

36. $0 + (-15)$

37. $14 + (-14)$

38. $-5 + 5$

39. **Statistics** Beach Channel High School's football team scored one field goal (3 points) and gave up a touchdown (7 points) in the first quarter. In the third quarter, the team scored another field goal and gave up a safety (2 points). The team gave up a final field goal in the fourth quarter. By how much did the team lose?

40. **Business and Finance** Jean deposited a check for $625, wrote two for $68.74 and $29.95, and used her debit card to pay for a purchase of $57.65. What is her new account balance?

41. **Science and Medicine** The temperature dropped by 23°F from a high of 8°F. What was the low temperature?

chapter 10 > Make the Connection

42. **Science and Medicine** The overnight low temperature was listed as −14°C. The temperature rose 19°C by noon. What was the noontime temperature?

chapter 10 > Make the Connection

Basic Skills | **Challenge Yourself** | Calculator/Computer | Career Applications | Above and Beyond

Label each statement as **true** *or* **false.**

43. $-10 + 6 = 6 + (-10)$

44. $5 + (-9) = -9 + 5$

45. $|-3| + |2| = |-3 + 2|$

46. $|-8| + |3| = |-8 + 3|$

Compute, as indicated.

47. $-9 + (-17) + 9$

48. $15 + (-3) + (-15)$

49. $2 + 5 + (-11) + 4$

50. $7 + (-9) + (-5) + 6$

51. $1 + (-2) + 3 + (-4)$

52. $(-9) + 0 + (-2) + 12$

53. $\frac{5}{3} + \left(-\frac{4}{3}\right) + \frac{5}{3}$

54. $-\frac{6}{5} + \left(-\frac{13}{5}\right) + \frac{4}{5}$

55. $-\frac{3}{2} + \left(-\frac{7}{4}\right) + \frac{3}{4}$

56. $\frac{2}{3} + \left(-\frac{5}{6}\right) + \left(-\frac{1}{2}\right)$

57. $2.8 + (-5.5) + (-2.9)$

58. $-5.4 + (-2.1) + (-3.5)$

59. $|3 + (-4)|$

60. $|-11 + 9|$

61. $|-17 + 8|$

62. $|-27 + 14|$

63. $|-5 + (-6)|$

64. $|-17 + (-14)|$

65. $|-3 + 2 + (-4)|$

66. $|-2 + 7 + (-5)|$

67. $|2 + (-3)| + |-3 + 2|$

68. $|8 + (-10)| + |-12 + 14|$

Evaluate and round each result to the nearest tenth.

69. $-4.1967 + 5.2943 + (-3.1698)$

70. $5.3297 + 4.1897 + (-3.2869)$

71. $-7.19863 + 4.8629 + 3.2689 + (-5.7936)$

72. $3.6829 + 4.5687 + 7.28967 + (-5.1623)$

Basic Skills | Challenge Yourself | Calculator/Computer | **Career Applications** | Above and Beyond

73. **INFORMATION TECHNOLOGY** Amir, a network administrator, has a budget of \$50,000 at the beginning of April. He has the following entries to enter in the budget for the month of April: \$1,000 for travel, \$9,550 for technology, \$542 for miscellaneous expenses, \$443 received from returns, \$123 for supplies, and \$150 for subscriptions. How much money does Amir have left in his budget? Make sure to write an integer expression that represents the change in the budget.

74. **INFORMATION TECHNOLOGY** Fred has been hired to redesign a database that is having performance issues. He finds that one table in the database called CUSTOMERS has field sizes of FIRST NAME and LAST NAME to be 100 bytes. He knows from experience that first names average around 30 bytes and last names average around 45 bytes. A character is a byte. Fred knows that wasting space on a very large database can cause performance issues. Write an integer expression that represents the change in the field sizes. By how much does Fred need to modify the field sizes?

75. **MECHANICAL ENGINEERING** A pneumatic actuator is operated by a pressurized air reservoir. At the beginning of the operator's shift, the pressure in the reservoir was 126 pounds per square inch (lb/in.^{2}). At the end of each hour, the operator records the change in pressure of the reservoir. The values (in lb/in.^{2}) recorded for this shift were a drop of 12, a drop of 7, a rise of 32, a drop of 17, a drop of 15, a rise of 31, a drop of 4, and a drop of 14. What is the pressure in the tank at the end of the shift?

76. **MECHANICAL ENGINEERING** A diesel engine for an industrial shredder has an 18-quart (qt) oil capacity. When the maintenance technician checked the oil, it was 7 qt low. Later that day, she added 4 qt to the engine. What was the oil level after the 4 qt were added?

Answers

61. ____________________

62. ____________________

63. ____________________

64. ____________________

65. ____________________

66. ____________________

67. ____________________

68. ____________________

69. ____________________

70. ____________________

71. ____________________

72. ____________________

73. ____________________

74. ____________________

75. ____________________

76. ____________________

Answers

77. ____________

78. ____________

79. ____________

80. ____________

81. ____________

82. ____________

ELECTRICAL ENGINEERING Dry cells or batteries have a positive and negative terminal. When correctly connected in series (positive to negative), the voltage of each cell can be added together. If a cell is connected and its terminals are reversed, the current will flow in the opposite direction.

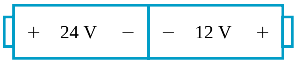
For example, if three 3-volt cells are connected in series and one cell is inserted backwards, the resulting voltage is 3 V.

$$3\text{ V} + 3\text{ V} + (-3)\text{ V} = 3\text{ V}$$

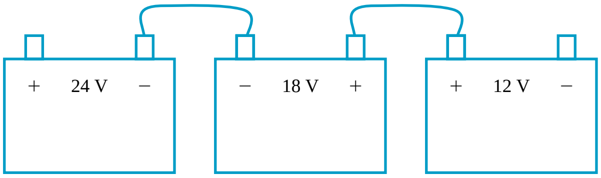
The voltages are added together because the cells are in series, but you must pay attention to the current flow.

Now complete exercises 77 and 78.

77. Assume you have a 24-V cell and a 12-V cell with their negative terminals connected. What would the resulting voltage be if measured from the positive terminals?



78. If a 24-V cell, an 18-V cell, and 12-V cell are supposed to be connected in series and the 18-V cell is accidentally reversed, what would the total voltage be?



Basic Skills | Challenge Yourself | Calculator/Computer | Career Applications | **Above and Beyond**

Place absolute value bars in the proper location on the left side of the equation in order to make the statement true.

79. $-3 + 7 = 10$

80. $-5 + 9 = 14$

81. $-6 + 7 + (-4) = 3$

82. $-10 + 15 + (-9) = 4$

Answers

1. 9 **3.** 16 **5.** -5 **7.** -37 **9.** 6 **11.** -6 **13.** 13
15. 372 **17.** 2 **19.** -2 **21.** $-\frac{11}{12}$ **23.** $-4\frac{1}{6}$ **25.** $\frac{1}{2}$
27. $\frac{1}{4}$ **29.** $\frac{8}{15}$ **31.** -3.9 **33.** 4 **35.** -9 **37.** 0
39. 6 points **41.** -15°F **43.** True **45.** False **47.** -17 **49.** 0
51. -2 **53.** 2 **55.** $-\frac{5}{2}$ **57.** -5.6 **59.** 1 **61.** 9 **63.** 11
65. 5 **67.** 2 **69.** -2.1 **71.** -4.9 **73.** \$39,078 **75.** 120 lb/in.^{2}
77. 12 V **79.** $|-3| + 7 = 10$ **81.** $|-6 + 7 + (-4)| = 3$

The Streeter/Hutchison Series in Mathematics Basic Mathematical Skills with Geometry

chapter 10 > Make the Connection

Activity 27 ::
Hometown Weather

The local weather provides us with many interesting applications of real numbers and data gathering.

1. Collect the daily high and low temperatures in your locale for the previous week.
2. Compute the mean high and mean low temperatures for the week.
3. List each day's high and low temperatures as their distance from the mean for the week. For example, if the weekly mean high temperature was 65°F, and Tuesday's high temperature was 62°F, then list it as −3°F.
4. Consider the differences from the weekly mean high temperature listed in exercise 3. What is the mean of this set of numbers?
5. Consider the differences from the weekly mean low temperature listed in exercise 3. What is the mean of this set of numbers?
6. Explain your answers to exercises 4 and 5.
7. Find the average annual high temperature for your locale.
8. Use real numbers to describe the difference between each of the high and low temperatures for the previous week and the annual averages.
9. Construct a line graph of the high temperatures for the previous week.
10. Add a second line graph to the graph constructed in exercise 9 for the low temperatures.
11. Add horizontal lines to your graph, one for the annual average high temperature and one for the annual average low temperature.
12. Describe the relation between the answers to exercise 8 and the graph.

10.3

Subtracting Real Numbers

< **10.3 Objective** >

1 > Find the difference of two real numbers

To begin our discussion of subtraction when real numbers are involved, we look back at a problem using natural numbers. Of course, we know that

$$8 - 5 = 3$$

From our work in adding real numbers in Section 10.2, we know that it is also true that

$$8 + (-5) = 3$$

Comparing these equations, we see that they have the same result. This leads us to an important pattern. Any subtraction problem can be written as an addition problem. Subtracting 5 is the same as adding the opposite of 5, or −5. We can write this fact as follows:

$$8 - 5 = 8 + (-5) = 3$$

This leads us to the following rule for subtracting real numbers.

Step by Step

Subtracting Real Numbers

Step 1 Rewrite the subtraction problem as an addition problem.

a. Change the subtraction symbol (−) to an addition symbol (+)

b. Replace the number being subtracted with its opposite

Step 2 Add the resulting real numbers as before.
In symbols,

$$a - b = a + (-b)$$

Example 1 illustrates the use of this process for subtracting.

Example 1 Subtracting Real Numbers

< **Objective 1** >

NOTE

Each subtraction is rewritten as an addition.

Change the subtraction symbol (−) to an addition symbol (+).

(a) $15 - 7 = 15 + (-7)$

$= 8$

Replace 7 with its opposite, −7.

(b) $9 - 12 = 9 + (-12) = -3$

(c) $-6 - 7 = -6 + (-7) = -13$

(d) $-\frac{3}{5} - \frac{7}{5} = -\frac{3}{5} + \left(-\frac{7}{5}\right) = -\frac{10}{5} = -2$

(e) $2.1 - 3.4 = 2.1 + (-3.4) = -1.3$

(f) Subtract 5 from −2.

We write the statement as $-2 - 5$ and proceed as before:

$$-2 - 5 = -2 + (-5) = -7$$

Check Yourself 1

Subtract.

(a) $18 - 7$ **(b)** $5 - 13$ **(c)** $-7 - 9$

(d) $-\frac{5}{6} - \frac{7}{6}$ **(e)** $-2 - 7$ **(f)** $5.6 - 7.8$

The subtraction rule is used in the same way when the number being subtracted is negative. Change the subtraction to addition. Replace the negative number being subtracted with its opposite, which is positive. Example 2 illustrates this principle.

Example 2 Subtracting Real Numbers

Subtract.

Change the subtraction to addition.

(a) $5 - (-2) = 5 + (+2) = 5 + 2 = 7$

Replace -2 with its opposite, $+2$ or 2.

(b) $7 - (-8) = 7 + (+8) = 7 + 8 = 15$

(c) $-9 - (-5) = -9 + 5 = -4$

(d) $-12.7 - (-3.7) = -12.7 + 3.7 = -9$

(e) $-\frac{3}{4} - \left(-\frac{7}{4}\right) = -\frac{3}{4} + \left(+\frac{7}{4}\right) = \frac{4}{4} = 1$

(f) Subtract -4 from -5. We write

$-5 - (-4) = -5 + 4 = -1$

Check Yourself 2

Subtract.

(a) $8 - (-2)$ **(b)** $3 - (-10)$ **(c)** $-7 - (-2)$

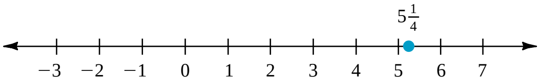
(d) $-9.8 - (-5.8)$ **(e)** $7 - (-7)$

We are now ready to describe negative mixed numbers, such as $-2\frac{1}{2}$.

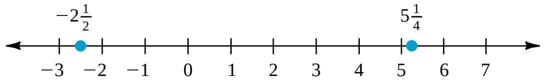
You should recall that we define a positive mixed number as the sum of the whole number part and the fraction part. For instance,

$$5\frac{1}{4} = 5 + \frac{1}{4}$$

This, of course, agrees with the number line approach, in which $5\frac{1}{4}$ is $\frac{1}{4}$ of a unit to the right of 5 on a number line.



Looking again at a number line, we can also locate $-2\frac{1}{2}$, which is $\frac{1}{2}$ of a unit to the left of -2 on the number line.



This suggests that $-2\frac{1}{2} = -2 - \frac{1}{2}$ or $-2 + \left(-\frac{1}{2}\right)$.

Example 3 An Application of Subtraction

From a seaside cliff 1,700 feet above sea level in the Cayman Islands, Nicole looks south to where the Cayman Trench is located. The Cayman Trench is the deepest part of the Caribbean, reaching a depth of 24,576 feet.

How far above the bottom of the Cayman Trench is Nicole standing?

To gauge the distance, we subtract, treating the depth of the trench as a negative number because the trench is below sea level.

$$\begin{aligned} 1{,}700 - (-24{,}576) &= 1{,}700 + (+24{,}576) \\ &= +26{,}276 \end{aligned}$$

Nicole is standing 26,276 feet above the bottom of the Cayman Trench.

Check Yourself 3

The high temperature one year for a midwestern town was 88°F. During the winter, the temperature dipped as low as −14°F. What was the temperature range experienced by the town that year?

Check Yourself ANSWERS

1. **(a)** 11; **(b)** −8; **(c)** −16; **(d)** −2; **(e)** −9; **(f)** −2.2
2. **(a)** 10; **(b)** 13; **(c)** −5; **(d)** −4; **(e)** 14 **3.** 102°F

Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 10.3

(a) Any subtraction problem can be written as an ____________ problem.

(b) We define the ______________ of real numbers by $a - b = a + (-b)$.

(c) To subtract real numbers, change the operation to addition and replace the second number with its ______________.

(d) The opposite of a negative number is a ____________ number.

10.3 exercises

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Name ______________________

Section __________ Date __________

< Objective 1 >

Perform the indicated operation.

1. $21 - 13$

2. $36 - 22$

3. $82 - 45$

4. $103 - 56$

5. $8 - 10$ > Videos

6. $14 - 19$

7. $24 - 45$

8. $136 - 352$

9. $-5 - 3$

10. $-15 - 8$

11. $-9 - 14$ > Videos

12. $-8 - 12$

13. $3 - (-4)$ > Videos

14. $6 - (-8)$

15. $5 - (-11)$

16. $7 - (-5)$

17. $7 - (-12)$

18. $3 - (-10)$

19. $-36 - (-24)$

20. $-28 - (-11)$

21. $-19 - (-27)$ > Videos

22. $-11 - (-16)$

23. $-11 - (-11)$

24. $-15 - (-15)$

25. $0 - (-8)$

26. $0 - (-11)$

27. **STATISTICS** On April 2, 2003, the high temperature in the United States was recorded as 94°F in Wink, Texas. The low temperature for the day was −23°F, recorded in both Deadhorse and Northway, Alaska (at −4°F, Presque Isle, Maine, recorded the lowest temperature in the contiguous states). What was the temperature range in the United States on that day? chapter 10 > Make the Connection

28. **STATISTICS** What was the temperature range in the contiguous 48 states on April 2, 2003 (from exercise 27)? chapter 10 > Make the Connection

29. **STATISTICS** The lowest temperature ever recorded in the state of Oregon was −54°F (in Seneca, on February 10, 1933). The state's record high temperature occurred in Pendleton on August 10, 1898, when it reached 119°F. What is the historical temperature range in the state of Oregon? chapter 10 > Make the Connection

30. **BUSINESS AND FINANCE** A government agency is operating despite a \$2.3 million budget deficit. Congress authorizes a \$3.5 million allocation in order for the agency to meet an additional \$1.9 million in payroll costs. How much is in the agency's budget after these transactions?

Answers

1. ________ 2. ________

3. ________ 4. ________

5. ________ 6. ________

7. ________ 8. ________

9. ________ 10. ________

11. ________ 12. ________

13. ________ 14. ________

15. ________ 16. ________

17. ________ 18. ________

19. ________ 20. ________

21. ________ 22. ________

23. ________ 24. ________

25. ________ 26. ________

27. ________ 28. ________

29. ____________________

30. ____________________

Answers

31. ________ 32. ________

33. ________ 34. ________

35. ________ 36. ________

37. ________ 38. ________

39. ________ 40. ________

41. ________ 42. ________

43. ________ 44. ________

45. ________ 46. ________

47. ________ 48. ________

49. ________ 50. ________

51. ________ 52. ________

53. ________ 54. ________

55. ________ 56. ________

57. ________ 58. ________

59. ________ 60. ________

61. ________ 62. ________

63. ________ 64. ________

65. ________ 66. ________

67. ________ 68. ________

69. ________ 70. ________

Basic Skills | **Challenge Yourself** | Calculator/Computer | Career Applications | Above and Beyond

Fill in each blank with **always, sometimes,** *or* **never.**

31. The difference between two negative numbers is ____________ negative.

32. A positive number subtracted from a negative number is ____________ negative.

33. The difference between two positive numbers is ____________ negative.

34. A negative number subtracted from a positive number is ____________ negative.

Compute, as indicated.

35. $\frac{15}{7} - \frac{8}{7}$

36. $\frac{17}{8} - \frac{9}{8}$

37. $\frac{7}{6} - \frac{19}{6}$

38. $\frac{5}{9} - \frac{32}{9}$

39. $-\frac{2}{5} - \frac{7}{10}$ > Videos

40. $-\frac{5}{9} - \frac{7}{18}$

41. $\frac{3}{4} - \left(-\frac{3}{2}\right)$

42. $\frac{5}{6} - \left(-\frac{7}{6}\right)$

43. $\frac{6}{7} - \left(-\frac{5}{14}\right)$

44. $\frac{11}{16} - \left(-\frac{7}{8}\right)$

45. $-\frac{3}{4} - \left(-\frac{11}{4}\right)$

46. $-\frac{1}{2} - \left(-\frac{5}{8}\right)$

47. $-\frac{2}{3} - \left(-\frac{3}{4}\right)$

48. $\frac{3}{10} - \frac{2}{3}$

49. $2\frac{3}{8} - 5\frac{3}{4}$

50. $11\frac{3}{4} - 6\frac{4}{5}$

51. $6\frac{1}{2} - \left(-5\frac{4}{5}\right)$

52. $3\frac{1}{6} - \left(-4\frac{2}{3}\right)$

53. $-2\frac{5}{6} - 3\frac{1}{2}$

54. $-4\frac{7}{10} - 3\frac{1}{3}$

55. $-8\frac{2}{3} - \left(-1\frac{1}{5}\right)$

56. $-3\frac{1}{2} - \left(-4\frac{1}{4}\right)$

57. $7.9 - 5.4$

58. $11.7 - 4.5$

59. $7.8 - 11.6$

60. $14.3 - 25.5$

61. $-3.4 - 4.7$

62. $-8.1 - 7.6$

63. $8.3 - (-5.7)$

64. $6.5 - (-4.3)$

65. $8.9 - (-11.7)$

66. $14.5 - (-24.6)$

67. $-12.7 - (-5.7)$

68. $-5.6 - (-2.6)$

69. $-6.9 - (-10.1)$

70. $-3.4 - (-7.6)$

Basic Skills | Challenge Yourself | **Calculator/Computer** | Career Applications | Above and Beyond

We now present a set of exercises for which the calculator might be the preferred tool. As indicated by the placement of this explanation, you should refrain from using a calculator on the exercises that precede this.

Your scientific or graphing calculator has a key that makes a number negative. This key is different from the *minus* key that we use for subtraction. The *negative* key is marked $\boxed{(-)}$ or $\boxed{+/-}$. With a scientific calculator, the negative key must be pressed *after* the number is entered. We assume you are using a graphing calculator.

To evaluate the expression: $132 + 547 + (-234) - 112 - (-327)$

Step 1 Press the *clear* key. $\boxed{\text{CLEAR}}$

Step 2 Enter the numbers as written. You do not need to use parentheses. Instead, use the negative key.

$132 \boxed{+}$
$547 \boxed{+}$
$\boxed{(-)}\ 234 \boxed{-}$
$112 \boxed{-}$
$\boxed{(-)}\ 327$

Step 3 Press *enter* or equals. $\boxed{\text{ENTER}}$ or $\boxed{=}$

Your display should read 660.

Use your calculator to evaluate each expression.

71. $8 + 4 - (-3) - 2$

72. $-27 - 43 - (-29) + 13$

73. $145 - (-547) + (-92) - 234$

74. $10{,}945 - (-2{,}347) + (-7{,}687) + 41$

Basic Skills | Challenge Yourself | Calculator/Computer | **Career Applications** | Above and Beyond

CONSTRUCTION *The elevation of the reference point used to set up a job is* 362 *inches (in.). Find the difference in elevation at the given points (use a negative sign for elevations below the reference point).*

75. 311 in.

76. 491 in.

MANUFACTURING TECHNOLOGY *At the beginning of the week, there were* 2,489 *pounds (lb) of steel in inventory. Report the weekly change in steel inventory for the given end-of-week inventories.*

77. 2,581 lb

78. 2,111 lb

79. **ELECTRICAL ENGINEERING** A certain electric motor spins at 5,400 rotations per minute (rpm) when unloaded. When a load is applied, the motor spins at 4,250 rpm. What is the change in rpm after loading?

80. **ELECTRICAL ENGINEERING** A cooling fan used to help dissipate heat from an electronic device has three modes of operation: off, low speed, and high speed. Low speed moves air at 34 cubic feet per minute (ft^3/min). High speed moves air at 52 ft^3/min. What is the difference in the volumes of air moved by the low and high speeds?

Answers

71.
72.
73.
74.
75.
76.
77.
78.
79.
80.

Answers

1. 8 **3.** 37 **5.** −2 **7.** −21 **9.** −8 **11.** −23 **13.** 7

15. 16 **17.** 19 **19.** −12 **21.** 8 **23.** 0 **25.** 8

27. 117°F **29.** 173°F **31.** sometimes **33.** sometimes **35.** 1

37. −2 **39.** $-\frac{11}{10}$ or $-1\frac{1}{10}$ **41.** $\frac{9}{4}$ or $2\frac{1}{4}$ **43.** $\frac{17}{14}$ or $1\frac{3}{14}$ **45.** 2

47. $\frac{1}{12}$ **49.** $-3\frac{3}{8}$ **51.** $12\frac{3}{10}$ **53.** $-6\frac{1}{3}$ **55.** $-7\frac{7}{15}$ **57.** 2.5

59. −3.8 **61.** −8.1 **63.** 14 **65.** 20.6 **67.** −7 **69.** 3.2

71. 13 **73.** 366 **75.** −51 in. **77.** +92 lb **79.** −1,150 rpm

Activity 28 ::
Plus/Minus Ratings in Hockey

The plus/minus statistic in professional hockey provides us with one application of the arithmetic of signed numbers. A player is awarded one point $(+1)$ if the player is on the ice when the player's team scores an even-strength or shorthanded goal (regardless of who actually scores the goal). A player is awarded -1 point if the opposing team scores such a goal while the player is on the ice.

Detroit Red Wing player Pavel Datsyuk finished the 2007–2008 season as the overall leader in the plus/minus category with $+41$.

1. Midway through the 2007–2008 season, Pavel Datsyuk entered a game against the San Jose Sharks with a plus/minus rating of $+28$. In the game, which Detroit won 6-3, Datsyuk was on the ice for three of his team's qualifying goals as well as one by the Sharks. What was Datsyuk's plus/minus rating for the game?
2. What was Datsyuk's plus/minus rating for the season, immediately after the game against San Jose?
3. In the same game, San Jose defenseman Douglas Murray was on the ice for two qualifying San Jose goals as well as for three such goals by Detroit. What was Murray's plus/minus rating for the game?
4. If Murray entered the game with a plus/minus rating of $+15$, what was his rating after the game?
5. Tampa Bay Lightning center Vincent Lecavalier finished the 2007–2008 season as the league's sixth leading scorer. However, his plus/minus rating for the season was -17.
   - **(a)** Lecavalier entered a pair of games against rival Florida Panthers with a -14 plus/minus rating. Florida won the first game 4-2. Lecavalier was on the ice for one qualifying Tampa Bay goal and three qualifying Florida goals. Tampa Bay won the second game 3-1. Lecavalier was on the ice for one qualifying goal by each team. What was his plus/minus rating for the two-game series?
   - **(b)** What was Lecavalier's season plus/minus rating after the two-game series?

Source: ESPN.

10.4

Multiplying Real Numbers

< 10.4 Objectives >

1 > Find the product of two or more real numbers

2 > Find the reciprocal of a real number

3 > Evaluate expressions involving real numbers

When you first considered multiplication in arithmetic, you thought of it as repeated addition. Our work with the addition of real numbers can tell us about multiplication when real numbers are involved. For example,

$$3 \cdot 4 = \underbrace{4 + 4 + 4}_{\text{We interpret multiplication as repeated addition to find the product, 12.}} = 12$$

NOTE

3(−4) is different from the subtraction problem 3 − 4.

Now, consider the product 3(−4):

$$3(-4) = (-4) + (-4) + (-4) = -12$$

Because multiplication is *commutative*, we know that the order of the factors does not matter. Therefore,

$$-4 \cdot 3 = 3 \cdot (-4) = -12$$

Looking at these products suggests the first portion of our rule for multiplying real numbers. The product of a positive number and a negative number is negative.

Property

Multiplying Real Numbers with Different Signs

The product of two numbers with different signs is negative.

To use this rule when multiplying two numbers with different signs, multiply their absolute values and attach a negative sign.

Example 1 **Multiplying Real Numbers**

< Objective 1 >

Multiply.

(a) $5(-6) = -30$

The product is negative.

(b) $-10(10) = -100$

(c) $8(-12) = -96$

(d) $-\frac{3}{4}\left(\frac{2}{5}\right) = -\frac{3}{10}$

NOTE

Multiply numerators together, and then multiply denominators together. Simplify the result.

Check Yourself 1

Multiply.

(a) $-7(5)$ **(b)** $-12(9)$ **(c)** $-15(8)$ **(d)** $\left(-\frac{5}{7}\right)\left(\frac{4}{5}\right)$

The product of two negative numbers is harder to visualize. The following pattern may help you see how we can determine the sign of the product.

This number is decreasing by 1.

$$\begin{aligned} 3(-2) &= -6 \\ 2(-2) &= -4 \\ 1(-2) &= -2 \\ 0(-2) &= 0 \\ -1(-2) &= 2 \end{aligned}$$

Do you see that the product is *increasing* by 2 each time?

NOTES

$-1(-2)$ is the opposite of -2.

We present a more detailed explanation at the end of the section.

What should the product $-2(-2)$ be? Continuing the pattern shown, we see that

$$-2(-2) = 4$$

This suggests that the product of two negative numbers is positive, which is the case.

Property

Multiplying Real Numbers with the Same Sign

The product of two numbers with the same sign is positive.

Example 2 Multiplying Real Numbers

Multiply.

(a) $9 \cdot 7 = 63$ The product of two positive numbers (same sign, +) is positive.

(b) $-8(-5) = 40$ The product of two negative numbers (same sign, −) is positive.

(c) $-\frac{1}{2}\left(-\frac{1}{3}\right) = \frac{1}{6}$

Check Yourself 2

Multiply.

(a) $10 \cdot 12$ **(b)** $-8(-9)$ **(c)** $-\frac{2}{3}\left(-\frac{6}{7}\right)$

Two numbers, 0 and 1, have special properties in multiplication.

Property

Multiplicative Identity Property

The product of 1 and any number is that number. The number 1 is called the **multiplicative identity.** In symbols,

$$a \cdot 1 = 1 \cdot a = a$$

Property

Multiplicative Property of Zero

The product of 0 and any number is 0. In symbols,

$$a \cdot 0 = 0 \cdot a = 0$$

Example 3 Multiplying Real Numbers

Find each product.

(a) $1(-7) = -7$

(b) $15(1) = 15$

(c) $-7(0) = 0$

(d) $0 \cdot 12 = 0$

(e) $-\frac{4}{5}(0) = 0$

Check Yourself 3

Multiply.

(a) $-10(1)$ **(b)** $0(-17)$ **(c)** $\frac{5}{7}(1)$ **(d)** $0\left(\frac{3}{4}\right)$

To complete our discussion of the properties of multiplication, we state the following.

Property

Multiplicative Inverse Property

For any nonzero number a, there is a number $\frac{1}{a}$ such that

$$a \cdot \frac{1}{a} = 1$$

$\frac{1}{a}$ is called the **multiplicative inverse,** or the **reciprocal,** of a. The product of any nonzero number and its reciprocal is 1.

Example 4 Finding a Reciprocal

< Objective 2 >

RECALL

The product of two negative numbers is a positive number.

(a) The reciprocal of 3 is $\frac{1}{3}$ because $3 \cdot \frac{1}{3} = 1$.

(b) The reciprocal of -5 is $\frac{1}{-5}$ or $-\frac{1}{5}$ because $-5\left(-\frac{1}{5}\right) = 1$.

(c) The reciprocal of $\frac{2}{3}$ is $\frac{1}{\frac{2}{3}}$ or $\frac{3}{2}$ because $\frac{2}{3} \cdot \frac{3}{2} = 1$.

Check Yourself 4

Find the multiplicative inverse (or reciprocal) of each number.

(a) 6 **(b)** -4 **(c)** $\frac{1}{4}$ **(d)** $-\frac{3}{5}$

In addition to the properties just mentioned, we can extend the commutative and associative properties for multiplication to real numbers. Example 5 is an application of the associative property of multiplication.

Example 5 Multiplying Real Numbers

< Objective 3 >

Find the product.

$-3(2)(-7)$

Applying the associative property, we can group the first two factors to write

$\overbrace{[(-3)(2)]}(-7)$ Evaluate first.

$= (-6)(-7)$

$= 42$

NOTE

This "grouping" can be done mentally.

Check Yourself 5

Find the product.

$-5(-8)(-2)$

We will take a closer look at how the order of operations comes into play when negative numbers are involved in Section 10.5. To do that, we need to learn some basic skills involving negative numbers.

The first such skill involves quantities with multiple negative signs. Our approach takes into account two ways of looking at positive and negative numbers.

First, a negative sign indicates the opposite of the number which follows. For instance, we have already said that the opposite of 5 is -5, whereas the opposite of -5 is 5. This last instance can be translated as $-(-5) = 5$.

Second, any number must correlate to some point on the number line. That is, any nonzero number is either positive or negative. No matter how many negative signs a quantity has, you can always simplify it so that it is represented by a negative or a positive number (one negative sign or none).

Example 6 Simplifying Real Numbers with Negative Signs

NOTE

You should see a pattern emerge. An even number of negative signs gives a positive number, whereas an odd number of negative signs produces a negative number.

Simplify the expression $-(-(-(-4)))$.

The opposite of -4 is 4, so $-(-4) = 4$.

The opposite of 4 is -4, so $-(-(-4)) = -4$. The opposite of this last number, -4, is 4, so

$-(-(-(-4))) = 4$

Check Yourself 6

Simplify the expression $-(-(-(-(-(-(-12))))))$.

We should also learn to evaluate expressions that contain both an exponent and a negative sign. Example 7 provides us with the opportunity to do just that.

Example 7 Evaluating Expressions

Evaluate each expression.

(a) $(-5)^2$

This means $(-5) \cdot (-5)$, which is 25.

> **CAUTION**
>
> Many students make careless errors when evaluating these types of expressions. Remember that $(-5)^2 \neq -5^2$.

(b) -5^2

This is not the same as the example in part **(a).** One way to look at this is to say that -5^2 is the *opposite* of $5^2 = 25$ so that

$$-5^2 = -25$$

Alternatively, we can look at -5 as shorthand notation for $-1 \cdot 5$. In which case, the order of operations requires that we compute the exponent prior to performing the multiplication. Therefore,

$$-5^2 = -(5^2) = -(25) = -25$$

(c) $(-5)^3$

This means $(-5) \cdot (-5) \cdot (-5)$.

From our earlier work in Example 5 of this section, we know we can use the associative property.

$$\begin{aligned}(-5)^3 &= (-5) \cdot (-5) \cdot (-5)\\ &= [(-5) \cdot (-5)] \cdot (-5)\\ &= 25 \cdot (-5)\\ &= -125\end{aligned}$$

> **RECALL**
>
> In Example 6, we saw that multiplying an odd number of negative signs yields a negative number.

Check Yourself 7

Evaluate each expression.

(a) -7^3 **(b)** $(-7)^3$ **(c)** $(-3)^4$

(d) -3^4 **(e)** $\left(-\frac{2}{3}\right)^2$ **(f)** $-\frac{2^2}{3}$

Of course, there are many applications that involve both positive and negative numbers.

Example 8 An Application of Real Numbers

The manager responsible for worker productivity at the TarCo manufacturing plant conducts a study on the amount of time workers do not engage in productive work. The manager finds that the average worker begins working 10 minutes after the shift begins, leaves for lunch 5 minutes early, and returns 10 minutes late. The manager also finds that the average employee works 15 minutes after the shift is over. Finally, the manager finds that employees spend an average of 22 minutes engaged in their 15-minute coffee break. If the plant employs 230 people, how much productivity does the plant lose each day?

We can compute the productivity lost in a day by the average worker as follows:

$$-10 + (-5) + (-10) + 15 + (15 - 22) = -17$$

Because the plant employs 230 people, the total lost productivity can be found by multiplication.

$$230(-17) = -3{,}910$$

The plant loses a total of 3,910 minutes of productivity per day (or about 65.2 hours).

Check Yourself 8

A math professor grades an exam as follows. Each correct answer is worth 4 points, each question left blank is worth 0 points, and each incorrect answer is worth −2 points. If a student answered 21 questions correctly, leaves 1 question blank, and answers 3 questions incorrectly, what score did the student earn on the exam?

Property

The Product of Two Negative Numbers

This is a detailed explanation of why the product of two negative numbers is positive.

From our earlier work, we know that the sum of a number and its opposite is 0:

$5 + (-5) = 0$

Multiply both sides of the equation by -3:

$(-3)[5 + (-5)] = (-3)(0)$

Because the product of 0 and any number is 0, on the right we have 0.

$(-3)[5 + (-5)] = 0$

We use the distributive property on the left.

$(-3)(5) + (-3)(-5) = 0$

We know that $(-3)(5) = -15$, so the equation becomes

$-15 + (-3)(-5) = 0$

We now have a statement of the form

$-15 + \square = 0$

in which $\square$ is the value of $(-3)(-5)$. We also know that $\square$ is the number that must be added to -15 to get 0, so $\square$ is the opposite of -15, or 15. This means that

$(-3)(-5) = 15$ The product is positive!

Regardless of which numbers we use in this argument the resulting product of two negative numbers will always be positive.

Check Yourself ANSWERS

1. **(a)** -35; **(b)** -108; **(c)** -120; **(d)** $-\frac{4}{7}$ **2.** **(a)** 120; **(b)** 72; **(c)** $\frac{4}{7}$

3. **(a)** -10; **(b)** 0; **(c)** $\frac{5}{7}$; **(d)** 0 **4.** **(a)** $\frac{1}{6}$; **(b)** $-\frac{1}{4}$; **(c)** 4; **(d)** $-\frac{5}{3}$ **5.** -80

6. -12 **7.** **(a)** -343; **(b)** -343; **(c)** 81; **(d)** -81; **(e)** $\frac{4}{9}$; **(f)** $-\frac{4}{3}$ **8.** 78

Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 10.4

(a) The product of two numbers with different signs is ______________.

(b) The product of two negative numbers is ______________.

(c) The number 1 is called the multiplicative ______________.

(d) $\frac{1}{a}$ is called the multiplicative inverse or ______________ of a.

10.4 exercises

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Name ______________________

Section __________ Date __________

Answers

1. ____________ 2. ____________

3. ____________ 4. ____________

5. ____________ 6. ____________

7. ____________ 8. ____________

9. ____________ 10. ____________

11. ____________ 12. ____________

13. ____________ 14. ____________

15. ____________ 16. ____________

17. ____________ 18. ____________

19. ____________ 20. ____________

21. ____________ 22. ____________

23. ____________ 24. ____________

25. ____________ 26. ____________

27. ____________ 28. ____________

29. ____________ 30. ____________

< Objective 1 >

Perform the indicated operation.

1. $4 \cdot 10$

2. $3 \cdot 14$

3. $5(-12)$ > Videos

4. $10(-2)$

5. $8(-10)$

6. $13(-7)$

7. $-8(9)$

8. $-12(3)$

9. $-11(12)$

10. $-17(5)$

11. $-8(-7)$ > Videos

12. $-9(-8)$

13. $-5(-12)$

14. $-7(-3)$

< Objective 2 >

15. $1(-18)$

16. $-3(1)$

17. $\frac{3}{4} \cdot \frac{4}{3}$

18. $-\frac{5}{3}\left(-\frac{3}{5}\right)$

19. $-5\left(-\frac{1}{5}\right)$ > Videos

20. $7 \cdot \frac{1}{7}$

21. $5\left(-\frac{1}{5}\right)$

22. $-7\left(\frac{1}{7}\right)$

23. $-5(0)$

24. $0\left(-\frac{2}{3}\right)$

< Objective 3 >

25. $-5(3)(-2)$

26. $-4(2)(-3)$

27. $8(-3)(7)$

28. $13(-2)(6)$

29. $-3(-5)(-2)$

30. $-6(4)(-3)$

31. $-5(-4)(2)$

32. $-5(2)(-6)$

33. $-9(-12)(0)$

34. $-13(0)(-7)$

35. $-(-3)$

36. $-(-9)$

37. $-(-(-1))$

38. $-(-(-11))$

39. $-(-(-(-(-123))))$

40. $-(-(-(-(-(-80)))))$

41. -6^3

42. $(-6)^3$

43. -6^2

44. $(-6)^2$

45. $(-8)^2$ > Videos

46. -8^2 > Videos

STATISTICS *A professor grades an exam by awarding 5 points for each correct answer and subtracting 2 points for each incorrect answer. Points are neither added nor subtracted for answers left blank. What is the exam score of each student?*

47. A student answers 14 questions correctly and 4 incorrectly while leaving 2 questions blank.

48. A student answers 16 questions correctly and 3 incorrectly while leaving 1 question blank.

49. **SOCIAL SCIENCE** A gambler lost \$45 per hour at a slot machine over a 4-hour period. How much money did the gambler lose?

50. **SOCIAL SCIENCE** A poker player loses \$325 per hour at a poker table over a 3-hour period. After a break, she wins \$145 per hour for two hours. How did she do, overall?

Basic Skills | **Challenge Yourself** | Calculator/Computer | Career Applications | Above and Beyond

Determine whether each statement is **true** *or* **false.**

51. The square of a negative number is negative.

52. The opposite of the square of a number is negative.

53. The opposite of the opposite of a negative number is negative.

54. The cube of a negative number is negative.

Compute, as indicated.

55. $4\left(-\frac{3}{2}\right)$

56. $9\left(-\frac{2}{3}\right)$

57. $-\frac{1}{4}(8)$

58. $-\frac{3}{2}(4)$

Answers

31. ________ 32. ________

33. ________ 34. ________

35. ________ 36. ________

37. ________ 38. ________

39. ________ 40. ________

41. ________ 42. ________

43. ________ 44. ________

45. ________

46. ________

47. ________

48. ________

49. ________

50. ________

51. ________

52. ________

53. ________

54. ________

55. ________

56. ________

57. ________

58. ________

59. $-9\left(-\frac{2}{3}\right)$

60. $-6\left(-\frac{3}{2}\right)$

61. $\frac{4}{5}\left(-\frac{3}{8}\right)$

62. $-\frac{2}{3}\left(-\frac{6}{7}\right)$

63. $-\frac{3}{4}\left(-\frac{10}{21}\right)$

64. $-\frac{1}{5}\left(\frac{3}{4}\right)$

65. $-\frac{1}{3}\left(\frac{6}{5}\right)(-10)$

66. $-\frac{1}{2}\left(\frac{4}{3}\right)(-6)$

67. $\left(-2\frac{5}{6}\right)\left(3\frac{1}{2}\right)$

68. $\left(-4\frac{7}{10}\right)\left(3\frac{1}{3}\right)$

69. $\left(-8\frac{2}{3}\right)\left(-1\frac{1}{5}\right)$

70. $\left(-3\frac{1}{2}\right)\left(-4\frac{1}{4}\right)$

71. $3.25(-4)$

72. $5.4(-5)$

73. $-1.25(-12)$

74. $-1.5(-20)$

Basic Skills | Challenge Yourself | **Calculator/Computer** | Career Applications | Above and Beyond

75. Use a calculator to complete the table.

$4 \cdot 3$	12
$4 \cdot 2$	8
$4 \cdot 1$	
$4 \cdot 0$	
$4(-1)$	
$4(-2)$	
$4(-3)$	
$4(-4)$	

76. Use a calculator to complete the table.

$-4 \cdot 3$	-12
$-4 \cdot 2$	-8
$-4 \cdot 1$	
$-4 \cdot 0$	
$-4(-1)$	
$-4(-2)$	
$-4(-3)$	
$-4(-4)$	

Answers

59. ____________________

60. ____________________

61. ____________________

62. ____________________

63. ____________________

64. ____________________

65. ____________________

66. ____________________

67. ____________________

68. ____________________

69. ____________________

70. ____________________

71. ____________________

72. ____________________

73. ____________________

74. ____________________

75. ____________________

76. ____________________

77. **MANUFACTURING TECHNOLOGY** Companies will occasionally sell products at a loss in order to draw in customers or as a reward to good customers. The theory is that customers will buy other products along with the discounted product and the net result will be a profit.

Beguhn Industries sells five different products. For each unit of product sold, the company makes or loses the following: product A, makes \$18; product B, loses \$4; product C, makes \$11; product D, makes \$38; and product E, loses \$15. During the previous month, Beguhn Industries sold 127 units of product A, 273 units of product B, 201 units of product C, 377 units of product D, and 43 units of product E.

Calculate the profit or loss for the month.

78. **MECHANICAL ENGINEERING** The bending moment created by a center support on a steel beam is approximated by the formula $-\frac{1}{4}PL^3$, in which P is the load on each side of the center support and L is the length of the beam on each side of the center support (assuming a symmetrical beam and load).

If the total length of the beam is 24 ft (12 ft on each side of the center) and the total load is 4,124 lb (2,062 lb on each side of the center), what is the bending moment (in ft-lb^3) at the center support?

Answers

1. 40 **3.** −60 **5.** −80 **7.** −72 **9.** −132 **11.** 56 **13.** 60 **15.** −18 **17.** 1 **19.** 1 **21.** −1 **23.** 0 **25.** 30 **27.** −168 **29.** −30 **31.** 40 **33.** 0 **35.** 3 **37.** −1 **39.** −123 **41.** −216 **43.** −36 **45.** 64 **47.** 62 **49.** \$180 **51.** False **53.** True **55.** −6 **57.** −2 **59.** 6 **61.** $-\frac{3}{10}$ **63.** $\frac{5}{14}$ **65.** 4 **67.** $-9\frac{11}{12}$ **69.** $10\frac{2}{5}$ **71.** −13 **73.** 15

75.

$4 \cdot 3$	12
$4 \cdot 2$	8
$4 \cdot 1$	4
$4 \cdot 0$	0
$4(-1)$	−4
$4(-2)$	−8
$4(-3)$	−12
$4(-4)$	−16

77. +\$17,086

Answers

77. ____________________

78. ____________________

10.5 Dividing Real Numbers and the Order of Operations

< 10.5 Objectives >

1 > Find the quotient of two real numbers

2 > Recognize that division by zero is undefined

3 > Use the order of operations to evaluate expressions involving real numbers

You know from your work in arithmetic that multiplication and division are related operations. We can use this fact, and our work from Section 10.4, to determine rules for the division of real numbers. Every division problem can be stated as an equivalent multiplication problem. For instance,

$$\frac{15}{5} = 3 \qquad \text{because} \qquad 15 = 5 \cdot 3$$

$$\frac{-24}{6} = -4 \qquad \text{because} \qquad -24 = (6)(-4)$$

$$\frac{-30}{-5} = 6 \qquad \text{because} \qquad -30 = (-5)(6)$$

These examples illustrate that because the two operations are related, the rule of signs that we stated in Section 10.4 for multiplication is also true for division.

Property

Dividing Real Numbers

The quotient of two numbers with different signs is negative.
The quotient of two numbers with the same sign is positive.

To divide two real numbers, divide their absolute values. Then attach the proper sign according to the preceding rule.

Example 1 **Dividing Real Numbers**

< Objective 1 >

Divide.

Positive
(a) $\dfrac{28}{7} = 28 \div 7 = 4 \longleftarrow$ Positive
Positive

Negative
(b) $\dfrac{-36}{-4} = -36 \div (-4) = 9 \longleftarrow$ Positive
Negative

Negative
(c) $\dfrac{-42}{7} = -42 \div 7 = -6 \longleftarrow$ Negative
Positive

Positive
(d) $\dfrac{75}{-3} = 75 \div (-3) = -25 \longleftarrow$ Negative
Negative

(e) $$\frac{\overset{\text{Positive}}{15.2}}{\underset{\text{Negative}}{-3.8}} = -4 \longleftarrow \text{Negative}$$

Check Yourself 1

Divide.

(a) $\frac{-55}{11}$ **(b)** $\frac{80}{20}$ **(c)** $\frac{-48}{-8}$ **(d)** $\frac{144}{-12}$ **(e)** $\frac{-13.5}{-2.7}$

You should be very careful when 0 is involved in a division problem. Remember that 0 divided by any nonzero number is just 0. Recall that

$$\frac{0}{-7} = 0 \qquad \text{because} \qquad 0 = (-7)(0)$$

However, if zero is the *divisor,* we have a special problem. Consider

$$\frac{9}{0} = ?$$

This means that $9 = 0 \cdot ?$.

Can 0 times a number ever be 9? No, so there is no solution.

Because $\frac{9}{0}$ cannot be replaced by any number, we agree that *division by* 0 *is not allowed.*

Property

Division by Zero Division by 0 is undefined.

Example 2 Division and Zero

< Objective 2 >

Divide, if possible.

(a) $\frac{7}{0}$ is undefined.

(b) $\frac{-9}{0}$ is undefined.

(c) $\frac{0}{5} = 0$

(d) $\frac{0}{-8} = 0$

NOTE

The expression $\frac{0}{0}$ is called an **indeterminate form.** You will learn more about this in later mathematics classes.

Check Yourself 2

Divide if possible.

(a) $\frac{0}{3}$ **(b)** $\frac{5}{0}$ **(c)** $\frac{-7}{0}$ **(d)** $\frac{0}{-9}$

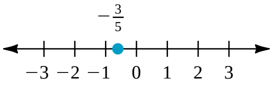
In Section 10.4, we began to develop the skills to evaluate more complex expressions involving negative numbers. We continue that work in Examples 3 to 7.

We begin with the reminder that any number must correlate to some point on the number line. That is, any nonzero number is either positive or negative.

With fractions, this fact is most easily seen as follows:

$$-\frac{3}{5} = \frac{-3}{5} = \frac{3}{-5}$$

All three quantities represent the same point on the number line



In this text, we generally choose to write negative fractions with the minus sign outside the fraction, such as $-\frac{3}{5}$.

In Section 10.4, we used these facts about multiple negative signs to help us evaluate integers. Here, we evaluate fractions with multiple negative signs.

Example 3 Simplifying Fractions with Negative Signs

Simplify each expression.

(a) $-\frac{-3}{4}$

This is the opposite of $\frac{-3}{4}$ which is $\frac{3}{4}$, a positive number.

(b) $-\frac{-3}{-4}$

The fraction part represents a negative number divided by another negative number, which is positive.

$$\frac{-3}{-4} = \frac{3}{4}$$

Therefore,

$$-\frac{-3}{-4} = -\left(\frac{-3}{-4}\right) = -\frac{3}{4}$$

Check Yourself 3

Simplify each expression.

(a) $-\frac{7}{-10}$ **(b)** $-\frac{-2}{-3}$

When symbols of grouping, or more than one operator, are involved in an expression, we must always remember to follow the rules for the order of operations.

Property

Order of Operations

We presented the order of operations in Section 1.7.

"**P**lease **E**xcuse **M**y **D**ear **A**unt **S**ally."

1. Perform all operations inside grouping symbols. Grouping symbols include parentheses, brackets, absolute value signs, fraction bars, and radical signs.
2. Apply all exponents.
3. Perform all multiplication and division operations, from left to right.
4. Perform all addition and subtraction operations, from left to right.

Example 4 Order of Operations

< Objective 3 >

Evaluate each expression.

(a) $7(-9 + 12)$
$= 7(3) = 21$

Evaluate inside the parentheses first.

(b) $-8(-7) - 40$
$= 56 - 40$
$= 16$

Multiply first, then subtract.

(c) $(-5)^2 - 3$
$= (-5)(-5) - 3$
$= 25 - 3$
$= 22$

Evaluate the power first.
Note that $(-5)^2 = (-5)(-5) = 25$

(d) $-5^2 - 3$
$= -25 - 3$
$= -28$

Note that $-5^2 = -25$. The power applies *only* to the 5.

Check Yourself 4

Evaluate each expression.

(a) $8(-9 + 7)$ **(b)** $-3(-5) + 7$
(c) $(-4)^2 - (-4)$ **(d)** $-4^2 - (-4)$

Because the fraction bar also serves as a grouping symbol, all operations in the numerator or denominator should each be done first, as illustrated in Example 5.

Example 5 Order of Operations

Evaluate each expression.

(a) $\dfrac{-6(-7)}{3} = \dfrac{42}{3} = 14$

Multiply in the numerator and then divide.

(b) $\dfrac{3 + (-12)}{3} = \dfrac{-9}{3} = -3$

Add in the numerator and then divide.

(c) $\dfrac{-4 + 2(-6)}{-6 - 2} = \dfrac{-4 + (-12)}{-6 - 2}$

Multiply in the numerator. Then add in the numerator and subtract in the denominator.

$= \dfrac{-16}{-8} = 2$

Divide as the last step.

Check Yourself 5

Evaluate each expression.

(a) $\dfrac{-4 + (-8)}{6}$ **(b)** $\dfrac{3 - 2(-6)}{-5}$ **(c)** $\dfrac{-2(-4) - (-6)(-5)}{(-4)(11)}$

Many students have difficulty applying the distributive property when negative numbers are involved. Just remember that the sign of a number “travels” with that number.

Example 6 Applying the Distributive Property with Negative Numbers

RECALL

We usually enclose negative numbers in parentheses in the middle of an expression to avoid careless errors.

Use the distributive property to evaluate each expression.

(a)
$$\begin{aligned} -7(3 + 6) &= -7 \cdot 3 + (-7) \cdot 6 && \text{Apply the distributive property.} \\ &= -21 + (-42) && \text{Multiply first and then add.} \\ &= -63 \end{aligned}$$

(b)
$$\begin{aligned} -3(5 - 6) &= -3[5 + (-6)] && \text{First, change the subtraction to addition.} \\ &= -3 \cdot 5 + (-3)(-6) && \text{Distribute the } -3. \\ &= -15 + 18 && \text{Multiply first and then add.} \\ &= 3 \end{aligned}$$

RECALL

We use brackets rather than nesting parentheses to avoid careless errors.

(c)
$$\begin{aligned} 5(-2 - 6) &= 5[-2 + (-6)] \\ &= 5 \cdot (-2) + 5 \cdot (-6) \\ &= -10 + -30 \\ &= -40 && \text{The sum of two negative numbers is negative.} \end{aligned}$$

Check Yourself 6

Use the distributive property to evaluate each expression.

(a) $-2(-3 + 5)$ **(b)** $4(-3 + 6)$ **(c)** $-7(-3 - 8)$

Combining the elements from Examples 3 to 6 requires that we carefully apply the order of operations. You must remain vigilant with any negative signs.

Example 7 Evaluating Expressions

Use the order of operations to evaluate each expression.

(a)
$$\begin{aligned} &4 + 2 \cdot (5 - 7)^2 \\ &= 4 + 2 \cdot (-2)^2 && \text{Evaluate inside parentheses first.} \\ &= 4 + 2 \cdot 4 && \text{Apply the exponent.} \\ &= 4 + 8 && \text{Multiply.} \\ &= 12 && \text{Add.} \end{aligned}$$

RECALL

Fraction bars are grouping symbols. Evaluate the numerator and denominator separately.

(b)
$$\begin{aligned} &\frac{3 - (-2)^3}{-7 + 3} \\ &= \frac{3 - (-8)}{-4} \\ &= \frac{3 + 8}{-4} \\ &= \frac{11}{-4} \\ &= -\frac{11}{4} \end{aligned}$$

NOTE

In future courses you will find that we rarely change an improper fraction to a mixed number.

Check Yourself 7

Use the order of operations to evaluate each expression.

(a) $-35 - (3 - 7)^3$ **(b)** $\dfrac{2 - 3 \cdot (1 - 5)^2}{(-3)^3 + (-2)^4}$

Check Yourself ANSWERS

1. (a) -5; **(b)** 4; **(c)** 6; **(d)** -12; **(e)** 5 **2. (a)** 0; **(b)** undefined; **(c)** undefined; **(d)** 0 **3. (a)** $\frac{7}{10}$; **(b)** $-\frac{2}{3}$ **4. (a)** -16; **(b)** 22; **(c)** 20; **(d)** -12 **5. (a)** -2; **(b)** -3; **(c)** $\frac{1}{2}$ **6. (a)** -4; **(b)** 12; **(c)** 77 **7. (a)** 29; **(b)** $\frac{46}{11}$

Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 10.5

(a) The quotient of two numbers with different signs is ______________.

(b) The quotient of two negative numbers is ______________.

(c) ______________ by 0 is undefined.

(d) Every nonzero number is either ______________ or negative.

10.5 exercises

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Name ______________________

Section __________ Date __________

Answers

1. __________ 2. __________

3. __________ 4. __________

5. __________ 6. __________

7. __________ 8. __________

9. __________ 10. __________

11. __________ 12. __________

13. __________ 14. __________

15. __________ 16. __________

17. __________ 18. __________

19. __________ 20. __________

21. __________ 22. __________

23. __________ 24. __________

25. __________ 26. __________

27. __________ 28. __________

29. __________ 30. __________

31. __________ 32. __________

< Objectives 1–3 >

Perform the indicated operations.

1. $\frac{70}{14}$

2. $\frac{48}{6}$

3. $\frac{-20}{-4}$ > Videos

4. $\frac{-75}{-3}$

5. $\frac{-24}{8}$

6. $\frac{56}{-7}$

7. $\frac{50}{-5}$

8. $\frac{-52}{4}$

9. $\frac{0}{-8}$ > Videos

10. $\frac{-9}{-1}$

11. $\frac{-17}{1}$

12. $\frac{18}{0}$ > Videos

13. $\frac{-27}{-1}$

14. $\frac{0}{8}$

15. $\frac{-10}{0}$

16. $\frac{-32}{1}$

17. $\frac{-8}{32}$

18. $\frac{-6}{-30}$

19. $\frac{24}{-16}$

20. $\frac{-25}{10}$

21. $\frac{-28}{-42}$

22. $\frac{-125}{-75}$

23. $-\frac{-13}{52}$

24. $-\frac{-52}{-13}$

25. $-\frac{-12}{-15}$

26. $-\frac{-91}{-7}$

27. $5(7 - 2)$

28. $7(8 - 5)$

29. $2(5 - 8)$

30. $6(14 - 16)$

31. $(-3)(9 - 7)$

32. $(-6)(12 - 9)$

33. $(-3)(-2 - 5)$

34. $(-2)(-7 - 3)$

35. $\dfrac{-6(-3)}{2}$

36. $\dfrac{-9(5)}{-3}$

37. $\dfrac{24}{-4 - 8}$ > Videos

38. $\dfrac{36}{-7 + 3}$

39. $\dfrac{55 - 19}{-12 - 6}$

40. $\dfrac{-11 - 7}{-14 + 8}$

41. $\dfrac{7 - 5}{2 - 2}$

42. $\dfrac{-11 - (-3)}{-4 - (-4)}$

43. $\dfrac{-9(-6) - 10}{18 - (-4)}$

44. $\dfrac{4 - 2(-6)}{-14 - (-6)}$

45. **BUSINESS AND FINANCE** Michelle deposits \$1,000 in her checking account each month. She writes a check for \$100 for car insurance and \$200 for her car payment. She also makes a \$55 payment on her student loan. How much money is left for her to use each week (assume there are 4 weeks in the month)?

46. **BUSINESS AND FINANCE** An advertising agency lost \$42,000 in revenue last year when a major client left for another agency. What was the agency's monthly loss in revenue?

47. **SCIENCE AND MEDICINE** At noon, the temperature was 70°F. It dropped at a constant rate until 5 P.M., when it was 58°F. What was the hourly change in temperature? chapter 10 > Make the Connection

48. **SCIENCE AND MEDICINE** A chemist has 84 ounces (oz) of a solution, which she pours into test tubes. If the chemist pours $\dfrac{2}{3}$ oz in each test tube, how many can she fill?

Basic Skills | **Challenge Yourself** | Calculator/Computer | Career Applications | Above and Beyond

Fill in each blank with **always, sometimes,** *or* **never.**

49. The sum of a positive number and the square of a negative number is ______________ negative.

50. The product of three negative numbers is ______________ negative.

51. The sum of a negative number and the product of negative numbers is ______________ negative.

52. A negative number subtracted from the square of a negative number is ______________ negative.

Compute, as indicated.

53. $(-2)(-7) + (2)(-3)$

54. $(-3)(-6) + (4)(-2)$

55. $(-7)(3) - (-2)(-8)$

56. $(-5)(2) - (3)(-4)$

Answers

33. ______________
34. ______________
35. ______________
36. ______________
37. ______________
38. ______________
39. ______________
40. ______________
41. ______________
42. ______________
43. ______________
44. ______________
45. ______________
46. ______________
47. ______________
48. ______________
49. ______________
50. ______________
51. ______________
52. ______________
53. ______________
54. ______________
55. ______________
56. ______________

Answers

57. ______ 58. ______

59. ______ 60. ______

61. ______ 62. ______

63. ______ 64. ______

65. ______ 66. ______

67. ______ 68. ______

69. ______ 70. ______

71. ______ 72. ______

73. ______ 74. ______

75. ______ 76. ______

77. ______ 78. ______

79. ______ 80. ______

57. $\dfrac{3(6) - (-4)(8)}{6 - (-4)}$

58. $\dfrac{5(-2) - (-4)(-5)}{-4 - 2}$

59. $\dfrac{2(-5) + 4(6 - 8)}{3(-4 + 2)}$

60. $\dfrac{-3(-5) - 3(5 - 8)}{4(-8 + 6)}$

61. $(-7)^2 - 17$

62. $(-6)^2 - 20$

63. $-7^2 - 17$

64. $-6^2 - 20$

65. $(-4)^2 - (-2)(-5)$

66. $-4^2 - (-2)(-5)$

67. $(-6)^2 - (-3)^2$

68. $-6^2 - 3^2$

69. $(-8)^2 - 8^2$

70. $-11^2 - (-11)^2$

71. $5 + 3(4 - 6)^2$

72. $8 - 2(3 - 6)^3$

73. $-20 \div 2 + 10 \cdot 2$

74. $-6 \cdot 3 \div 2 - 9$

75. $\dfrac{4 - (-3)^2}{-7(4) + 3}$

76. $\dfrac{4 - (-2)^3}{5^2 - (-2)^2}$

77. $-60 \div (-3)(4) - 2^3 + 4$

78. $\dfrac{16 \div (-2)(-4)}{2^3 + 4}$

79. $\dfrac{7 - (-1)}{3^2 - 9 \div (-3)}$

80. $\dfrac{4 - (-3) + 1}{2 - 2 \div (-2)}$

Basic Skills | Challenge Yourself | **Calculator/Computer** | Career Applications | Above and Beyond

We now present a set of exercises for which the calculator might be the preferred tool. As indicated by the placement of this explanation, you should refrain from using a calculator on the exercises that precede this.

Recall from Section 10.3 that your calculator has a *negative* key. Using this makes multiplication and division of real numbers a relatively straightforward process. Remember that you need to press the negative key *after* entering the number on a scientific calculator, but press it *before* entering the number on a graphing calculator.

To evaluate the expression: $457(-734)$

Step 1 Press the *clear* key. $\boxed{\text{CLEAR}}$

Step 2 Enter the numbers as written. $457\ \boxed{\times}$

Use the negative key, as necessary, and the proper operation. $\boxed{(-)}\ 734$

Step 3 Press *enter* or equals. $\boxed{\text{ENTER}}$ or $\boxed{=}$

Your display should read -335438.

If we replace $\boxed{\times}$ with $\boxed{\div}$ in step 2, we get $457 \div (-734) \approx -0.623$.

You can also use a calculator to raise real numbers to a power. You should be able to find a *power* key, either $\boxed{\wedge}$ (called a *caret*) or $\boxed{y^x}$. Use this key to separate the base from the power.

To evaluate the expression: $(-3)^6$

Step 1 Press the *clear* key. $\boxed{\text{CLEAR}}$

Step 2 Enter the *open parenthesis* key, the *negative* key, the base number 3, and then the *close parenthesis* key. $\boxed{(}\ \boxed{(-)}\ 3\ \boxed{)}$

Enter the *power* key and the exponent. $\boxed{\wedge}$ 6 or $\boxed{y^x}$ 6

Step 3 Press *enter* or equals. $\boxed{\text{ENTER}}$ or $\boxed{=}$

Your display should read 729.

Use your calculator to evaluate each expression. Round your answer to two decimal places.

81. 25(−21) **82.** 15(−45) **83.** −34(−28)

84. −71(−19) **85.** 345 ÷ (−25) **86.** 128 ÷ (−28)

87. −564 ÷ 36 **88.** −232 ÷ 52 **89.** −28 ÷ (−14)

90. −456 ÷ (−124) **91.** $(-4)^5$ **92.** $(-5)^4$

Basic Skills | Challenge Yourself | Calculator/Computer | Career Applications | Above and Beyond

93. **MANUFACTURING TECHNOLOGY** Peer's Pipe Fitters started the month of July with 1,789 gallons (gal) of liquified petrolium gas (LP) in their tank. After 21 working days, there were 676 gal left in the tank. If the same amount was used each day, how much LP was consumed each day?

94. **BUSINESS AND FINANCE** Three friends bought equal shares in an investment for a total of $21,000. They sold it later for $17,232. How much did each person profit?

Answers

1. 5 **3.** 5 **5.** −3 **7.** −10 **9.** 0 **11.** −17 **13.** 27

15. Undefined **17.** $-\frac{1}{4}$ **19.** $-\frac{3}{2}$ **21.** $\frac{2}{3}$ **23.** $\frac{1}{4}$ **25.** $-\frac{4}{5}$

27. 25 **29.** −6 **31.** −6 **33.** 21 **35.** 9 **37.** −2 **39.** −2

41. Undefined **43.** 2 **45.** $161.25 **47.** −2.4°F **49.** never

51. sometimes **53.** 8 **55.** −37 **57.** 5 **59.** 3 **61.** 32

63. −66 **65.** 6 **67.** 27 **69.** 0 **71.** 17 **73.** 10 **75.** $\frac{1}{5}$

77. 76 **79.** $\frac{2}{3}$ **81.** −525 **83.** 952 **85.** −13.8 **87.** −15.67

89. 2 **91.** −1,024 **93.** $53\ \frac{\text{gal}}{\text{day}}$

Answers

81. ____________

82. ____________

83. ____________

84. ____________

85. ____________

86. ____________

87. ____________

88. ____________

89. ____________

90. ____________

91. ____________

92. ____________

93. ____________

94. ____________

Activity 29 ::
Building Molecules

Every atom has an associated **valence number.** The valence of an atom is the number of electrons in its outermost electron-energy level.

From the valence number, we can determine the number of electrons that an atom must exchange in a *covalent bond* in order to form a stable molecule. We refer to this number as the **covalent bonding number.**

If an atom's valence is less than 4, then its covalent bonding number is the same as its valence number. In this case, the atom needs to give up electrons to another atom in order to form a covalent bond. The number of electrons that such an atom must give up is equal to its covalent bonding number.

Note that it is common, in chemistry, to include a "+" symbol before the covalent bonding number of an atom when it is a positive number.

If an atom's valence is greater than 4, then its covalent bonding number is equal to its valence minus 8 (this will always be zero or a negative number). These atoms gain electrons when forming covalent bonds. Specifically, they must gain the same number of electrons as the absolute value of their covalent bonding number in order to form a stable molecule.

An atom whose valence is exactly 4 can either gain or give up 4 electrons when forming a stable molecule. These atoms have a covalent bonding number of either $+4$ or -4, as the situation requires.

A stable molecule is formed when the sum of all the covalent bonding numbers of the atoms in the molecule equals 0.

The possible covalent bonding numbers for atoms are

$-4, -3, -2, -1, 0, 1, 2, 3, 4$

1. Find the covalent bonding number for each of the atoms given in the following table.

Atom	Valence Number	Covalent Bonding Number
Boron	3	
Calcium	2	
Carbon	4	
Chlorine	7	
Hydrogen	1	
Nitrogen	5	
Phosphorus	5	
Sulfur	6	

2. If the covalent bonding numbers of the atoms in a molecule add up to -2, and only one more atom is to be added, what covalent bonding number should it have? What valence number should it have?

3. A molecule contains 2 boron atoms. How many sulfur atoms would need to be added to make a stable molecule?

4. When combining hydrogen and chlorine, what is the fewest number of each atom that can be used to create a stable molecule?

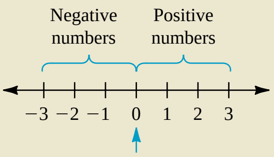
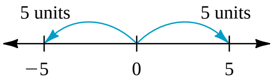
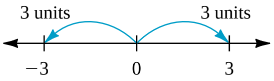
5. When combining hydrogen and nitrogen, what is the fewest number of each atom that can be used to create a stable molecule?

6. When combining phosphorus and calcium, what is the fewest number of each atom that can be used to create a stable molecule?

7. When combining carbon and boron, what is the fewest number of each atom that can be used to create a stable molecule?

8. When combining carbon and nitrogen, what is the fewest number of each atom that can be used to create a stable molecule?

summary :: chapter 10

Definition/Procedure	Example	Reference
Real Numbers and Order		**Section 10.1**
Positive numbers Numbers used to name points to the right of 0 on the number line. **Negative numbers** Numbers used to name points to the left of 0 on the number line. **Real numbers** The set containing all of the numbers corresponding to points on the number line.	 Zero is neither positive nor negative.	*p.* 645
Integers The set consisting of the natural numbers, their opposites, and 0.	The integers are $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$	
Absolute value The distance on the number line between the point named by a number and 0. The absolute value of a number is always positive or 0.	The absolute value of a number a is written $\lvert a \rvert$. $\lvert 7 \rvert = 7 \qquad \lvert -8 \rvert = 8$	*p.* 648
Adding Real Numbers		**Section 10.2**
To Add Real Numbers		
1. If two numbers have the same sign, add their absolute values. Give the sum the sign of the original numbers. **2.** If two numbers have different signs, subtract the smaller absolute value from the larger. Give the result the sign of the number with the larger absolute value.	$5 + 8 = 13$ $-3 + (-7) = -10$ $5 + (-3) = 2$ $7 + (-9) = -2$	*p.* 655
Opposites Two numbers are opposites if the points name the same distance from 0 on the number line, but in opposite directions. The opposite of a positive number is negative. The opposite of a negative number is positive. 0 is its own opposite.	 The opposite of 5 is −5.  The opposite of −3 is 3.	*p.* 657
Subtracting Real Numbers		**Section 10.3**
To Subtract Real Numbers		
To subtract real numbers, add the first number and the opposite of the number being subtracted.	$4 - (-2) = 4 + 2 = 6$ Replace −2 with its opposite, 2	*p.* 664

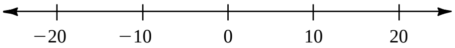
Definition/Procedure	Example	Reference
Multiplying Real Numbers *To Multiply Real Numbers*		**Section 10.4**
To multiply real numbers, multiply the absolute values of the numbers. Then attach a sign to the product according to the following rules: **1.** If the numbers have different signs, the product is negative. **2.** If the numbers have the same sign, the product is positive.	$5 \cdot 7 = 35$ $(-4)(-6) = 24$ $(8)(-7) = -56$	*p.* 672
Dividing Real Numbers and the Order of Operations *To Divide Real Numbers*		**Section 10.5**
To divide real numbers, divide the absolute values of the numbers. Then attach a sign to the quotient according to the following rules: **1.** If the numbers have the same sign, the quotient is positive. **2.** If the numbers have different signs, the quotient is negative.	$\frac{-8}{-2} = 4$ $27 \div (-3) = -9$ $\frac{-16}{8} = -2$	*p.* 682
Always follow the proper **order of operations** when evaluating an expression. **1.** Perform all operations inside grouping symbols. **2.** Apply any exponents. **3.** Perform all multiplication and division operations, from left to right. **4.** Perform all addition and subtraction operations, from left to right.	$4 + 2(5 - 7)^2$ $= 4 + 2(-2)^2$ **P**lease $= 4 + 2 \cdot (4)$ **E**xcuse $= 4 + 8$ **M**y **D**ear $= 12$ **A**unt **S**ally	*p.* 684

summary exercises :: chapter 10

This summary exercise set is provided to give you practice with each of the objectives of this chapter. Each exercise is keyed to the appropriate chapter section. When you are finished, you can check your answers to the odd-numbered exercises against those presented in the back of the text. If you have difficulty with any of these questions, go back and reread the examples from that section. The answers to the even-numbered exercises appear in the *Instructor's Solutions Manual.* Your instructor will give you guidelines on how to best use these exercises in your instructional setting.

10.1 *Represent the integers on the number line shown.*

1. $6, -18, -3, 2, 15, -9$



Place each set in ascending order.

2. $4, -3, 6, -7, 0, 1, -2$

3. $-\frac{1}{2}, -\frac{2}{3}, \frac{3}{5}, -\frac{4}{5}, \frac{5}{6}, \frac{7}{10}$

Determine the maximum and minimum values for each set of numbers.

4. $4, -2, 5, 1, -6, 3, -4$

5. $-4, 2, 5, -9, 8, 1, -6$

Evaluate.

6. $|9|$

7. $|-9|$

8. $-|9|$

9. $-|-9|$

10. $|12 - 8|$

11. $|8| - |12|$

12. $-|8 - 12|$

13. $|-8| - |-12|$

10.2 *Add.*

14. $-3 + (-8)$

15. $10 + (-4)$

16. $6 + (-6)$

17. $-16 + (-16)$

18. $-18 + 0$

19. $\frac{3}{8} + \left(-\frac{11}{8}\right)$

20. $5.7 + (-9.7)$

21. $-18 + 7 + (-3)$

10.3 *Subtract.*

22. $8 - 13$

23. $-7 - 10$

24. $10 - (-7)$

25. $-5 - (-1)$

26. $-9 - (-9)$

27. $0 - (-2)$

28. $-\frac{5}{4} - \left(-\frac{17}{4}\right)$

29. $7.9 - (-8.1)$

Perform the indicated operations.

30. $|4 - 8|$

31. $|4| - |8|$

32. $|-4 - 8|$

33. $|-4| - |-8|$

34. $-6 - (-2) + 3$

35. $-5 - (5 - 8)$

36. $7 - (3 - 7) + 4$

37. Subtract -7 from -8.

38. Subtract -9 from the sum of 6 and -2.

10.4 *Multiply.*

39. $10(-7)$

40. $-8(-5)$

41. $-3(-15)$

42. $1(-15)$

43. $0(-8)$

44. $\frac{2}{3}\left(-\frac{3}{2}\right)$

45. $-4\left(\frac{3}{8}\right)$

46. $-\frac{5}{4}(-1)$

47. $-8(-2)(5)$

48. $-4(-3)(2)$

49. $\frac{2}{5}(-10)\left(-\frac{5}{2}\right)$

50. $\frac{4}{3}(-6)\left(-\frac{3}{4}\right)$

Perform the indicated operations.

51. $2(-4 + 3)$

52. $2(-3) - (-5)(-3)$

53. $(2 - 8)(2 + 8)$

10.5 *Divide.*

54. $\frac{80}{16}$

55. $\frac{-63}{7}$

56. $\frac{-81}{-9}$

57. $\frac{0}{-5}$

58. $\frac{32}{-8}$

59. $\frac{-7}{0}$

Perform the indicated operations.

60. $\frac{-8 + 6}{-8 - (-10)}$

61. $\frac{2(-3) - 1}{5 - (-2)}$

62. $\frac{(-5)^2 - (-2)^2}{-5 - (-2)}$

self-test 10

Name ____________________

Section ________ Date ________

Answers

1. ____________________

2. ____________________

3. ____________________

4. ________ 5. ________

6. ________ 7. ________

8. ________ 9. ________

10. ________ 11. ________

12. ________ 13. ________

14. ________ 15. ________

16. ________ 17. ________

18. ________ 19. ________

20. ________ 21. ________

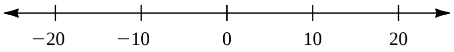
22. ________ 23. ________

24. ________ 25. ________

The purpose of this self-test is to help you assess your progress so that you can find concepts that you need to review before the next exam. Allow yourself about an hour to take this test. At the end of that hour, check your answers against those given in the back of this text. If you miss any, go back to the appropriate section to reread the examples until you have mastered that particular concept.

Represent the integers on the number line shown.

1. $5, -12, 4, -7, 18, -17$



2. Place the numbers in ascending order: $4, -3, -6, 5, 0, \frac{3}{4}, \frac{1}{2}, 2, -2$

3. Determine the maximum and minimum of the data set:

$3, 2, -5, 6, 1, -2$

Evaluate, as indicated.

4. $|7|$

5. $|-7|$

6. $-8 + (-5)$

7. $6 + (-9)$

8. $9 - 15$

9. $-9 - 15$

10. $(-8)(5)$

11. $(-9)(-7)$

12. $\frac{75}{-3}$

13. $\frac{-27}{-9}$

14. $|18 - 7|$

15. $|18| - |-7|$

16. $-9 + (-12)$

17. $-\frac{5}{3} + \frac{8}{3}$

18. $5 - (-4)$

19. $-7 - (-7)$

20. $(4.5)(-6)$

21. $(-2)(-3)(-4)$

22. $\frac{-45}{9}$

23. $\frac{9}{0}$

24. $-8 - (-3 + 7)^2$

25. $\frac{5 + (-9) - 6}{(-3)^2 + (-2)^3}$

cumulative review chapters 1-10

Name ____________

Section ________ Date ________

The following exercises are presented to help you review concepts from earlier chapters. This is meant as review material and not as a comprehensive exam. The answers are presented in the back of the text. Beside each answer is a section reference for the concept. If you have difficulty with any of these exercises, be certain to at least read through the summary related to that section.

1. What is the place value of 9 in the numeral 4,593,657?

Perform the indicated operation.

2. $\begin{array}{r} 7{,}623 \\ 3{,}006 \\ +\ 131{,}602 \\ \hline \end{array}$

3. $\begin{array}{r} 125{,}678 \\ -\ \ 96{,}105 \\ \hline \end{array}$

4. 105×509

5. $56\overline{)22{,}540}$

6. $103.456 - 89.769$

7. 30.45×60.34

8. $\dfrac{8}{11} \times \dfrac{33}{56}$

9. $\dfrac{7}{15} \div \dfrac{21}{45}$

10. $6\dfrac{1}{9} - 3\dfrac{4}{27} + 5\dfrac{7}{18}$

11. Solve for the unknown: $\dfrac{6}{3} = \dfrac{8}{x}$.

12. Write 58% as a decimal and fraction.

13. Write $\dfrac{12}{25}$ as a decimal and percent.

Simplify.

14. 7 ft 22 in.

15. 8 lb 20 oz

Do the indicated operations.

16. $\begin{array}{rr} & 5\text{ ft}\ \ 8\text{ in.} \\ + & 6\text{ ft } 10\text{ in.} \\ \hline \end{array}$

17. $\begin{array}{rr} & 5\text{ lb}\ \ 8\text{ oz} \\ - & 2\text{ lb } 10\text{ oz} \\ \hline \end{array}$

18. $3 \times (3\text{ h } 30\text{ min})$

19. $\dfrac{10\text{ min } 45\text{ s}}{5}$

Solve each application.

20. **CRAFTS** A plan for a bookcase requires three pieces of lumber 2 ft 8 in. long and two pieces 3 ft 4 in. long. What is the total length of material that is needed?

21. **BUSINESS AND FINANCE** You can buy three bottles of dishwashing liquid, each containing 1 pt 6 fl oz, on sale for $2.40. For the same price you can buy a large container holding 2 qt. Which is the better buy?

Answers

1. ____________

2. ____________

3. ____________

4. ____________

5. ____________

6. ____________

7. ____________

8. ____________

9. ____________

10. ____________

11. ____________

12. ____________

13. ____________

14. ____________

15. ____________

16. ____________

17. ____________

18. ____________

19. ____________

20. ____________

21. ____________

Answers

22. ____________

23. ____________

24. ____________

25. ____________

26. ____________

27. ____________

28. ____________

29. ____________

30. ____________

31. ____________

32. ____________

33. ____________

34. ____________

35. ____________

36. ____________

37. ____________

38. ____________

39. ____________

40. ____________

22. GEOMETRY A rectangle has length 8.6 cm and width 5.7 cm. Find the area of the rectangle.

23. GEOMETRY The sides of a square each measure $8\frac{1}{2}$ in. Find the area of the square.

24. GEOMETRY Find the circumference of a circle whose diameter is 8.2 ft. Use 3.14 for π, and round the result to the nearest tenth.

25. 116 is 145% of what number?

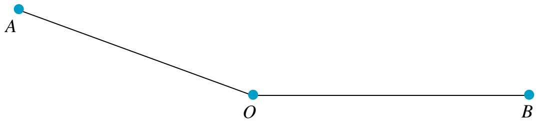
26. BUSINESS AND FINANCE The sales tax on an item costing $136 is $10.20. What is the sales tax rate?

Complete each statement.

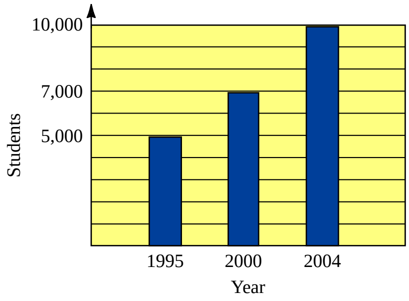
27. 17 g = __________ kg

28. 82 cm = __________ mm

29. Use a protractor to find the measure of the given angle.



30. SOCIAL SCIENCE According to the bar graph, how many more students were enrolled in the university in 2004 than in 1995?



31. Calculate the mean, median, and mode for the data.

15, 16, 18, 13, 17, 19, 17, 21

Evaluate each expression.

32. $-9 + 13$

33. $17 - (-3)$

34. $-9 - 23$

35. $|-8| - |-23|$

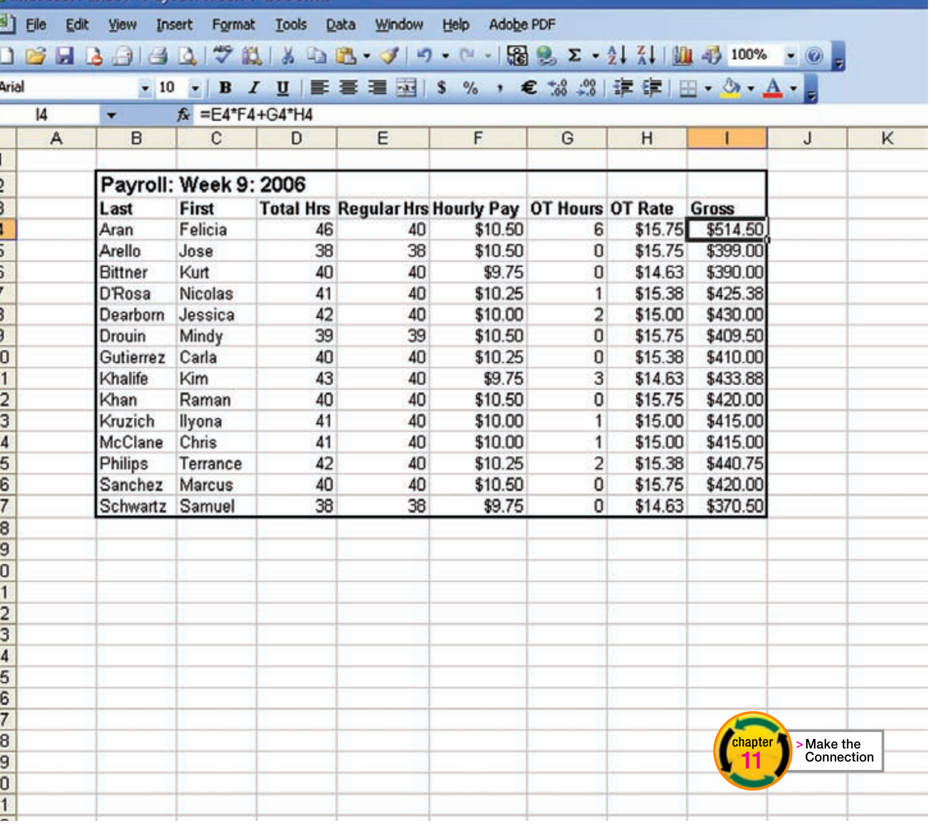
36. $|-8 - 23|$

37. $(-9)(-12)$

38. $\frac{-36}{-9}$

39. $(-7)^2 - (16 \div 2)$

40. $45 \div 5 \times 2^3$



I4 =E4*F4+G4*H4

Payroll: Week 9: 2006							
Last	First	Total Hrs	Regular Hrs	Hourly Pay	OT Hours	OT Rate	Gross
Aran	Felicia	46	40	$10.50	6	$15.75	$514.50
Arello	Jose	38	38	$10.50	0	$15.75	$399.00
Bittner	Kurt	40	40	$9.75	0	$14.63	$390.00
D'Rosa	Nicolas	41	40	$10.25	1	$15.38	$425.38
Dearborn	Jessica	42	40	$10.00	2	$15.00	$430.00
Drouin	Mindy	39	39	$10.50	0	$15.75	$409.50
Gutierrez	Carla	40	40	$10.25	0	$15.38	$410.00
Khalife	Kim	43	40	$9.75	3	$14.63	$433.88
Khan	Raman	40	40	$10.50	0	$15.75	$420.00
Kruzich	Ilyona	41	40	$10.00	1	$15.00	$415.00
McClane	Chris	41	40	$10.00	1	$15.00	$415.00
Philips	Terrance	42	40	$10.25	2	$15.38	$440.75
Sanchez	Marcus	40	40	$10.50	0	$15.75	$420.00
Schwartz	Samuel	38	38	$9.75	0	$14.63	$370.50

chapter 11 > Make the Connection

CHAPTER 11

An Introduction to Algebra

INTRODUCTION

Small businesses often use spreadsheet software to keep track of things. This is especially true of the many home-based businesses that have cropped up since computers became so common.

Spreadsheets, such as Microsoft Excel and OpenOffice Calc, became popular because they are easy to use. Most people can learn to use spreadsheet software by taking a single course at a local community college, or even by working through a tutorial on their own. While larger firms need more complex software, requiring extensive training, small businesses find that spreadsheets (and perhaps database software, such as Access and Base) can help them with most of their bookkeeping needs.

While many people have come to rely on spreadsheets, those who can realize their full power have a strong background in algebra. This is because spreadsheets can be thought of as multidimensional algebra machines. Consider a typical payroll spreadsheet.

While it may seem that each "cell" is simply typed in, the truth is that only the name, total hours, and hourly rate are manually entered on each line. The other fields, such as the gross pay, are determined by formulas. You can see such a formula, which refers to other cells in the spreadsheet, in the formula line. This allows someone to copy the formula to every employee, without having to retype it each time.

CHAPTER 11 OUTLINE

Chapter 11 :: Prerequisite Test 702
11.1 From Arithmetic to Algebra 703
11.2 Evaluating Algebraic Expressions 711
11.3 Simplifying Algebraic Expressions 721
11.4 Using the Addition Property to Solve an Equation 730
11.5 Using the Multiplication Property to Solve an Equation 743
11.6 Combining the Properties to Solve Equations 754
Chapter 11 :: Summary / Summary Exercises / Self-Test / Cumulative Review :: Chapters 1–11 764

11 prerequisite test

Name ______________

Section ________ Date ________

Answers

1. ______________
2. ______________
3. ______________
4. ______________
5. ______________
6. ______________
7. ______________
8. ______________
9. ______________
10. ______________
11. ______________
12. ______________

This prerequisite test provides some exercises requiring skills that you will need to be successful in the coming chapter. The answers for these exercises can be found in the back of this text. This prerequisite test can help you identify topics that you will need to review before beginning the chapter.

Write each phrase as an arithmetic expression and solve.

1. 8 less than 10

2. The sum of 3 and the product of 5 and 6

Find the reciprocal of each number.

3. -12

4. $4\frac{5}{8}$

Evaluate, as indicated.

5. $\left(-\frac{3}{2}\right) \times \left(-\frac{2}{3}\right)$

6. $(4)\left(\frac{1}{4}\right)$

7. $\frac{2}{2}$

8. $5 + 2 \times 3^2$

9. -8^2

10. $(-8)^2$

11. **Business and Finance** An $8\frac{1}{2}$-acre plot of land is on sale for \$120,000. What is the price per acre?

12. **Business and Finance** A grocery store adds a 30% markup to the wholesale price of goods to determine their retail price. What is the retail price of a box of cookies if its wholesale price is \$1.19?

11.1

From Arithmetic to Algebra

< **11.1 Objectives** >

1 > Use the symbols and language of algebra

2 > Identify algebraic expressions

In arithmetic, you learned how to do calculations with numbers by using the basic operations of addition, subtraction, multiplication, and division.

In algebra, we still use numbers and the same four operations. However, we also use letters to represent numbers. Letters such as x, y, L, and W are called **variables** when they represent numerical values.

You are familiar with the four symbols ($+$, $-$, $\times$, $\div$) used to indicate the fundamental operations of arithmetic.

To see how these operations are indicated in algebra, we begin with addition.

Definition

Addition $x + y$ means the *sum* of x and y or x *plus* y.

Example 1 Writing Expressions That Indicate Addition

< Objective 1 >

(a) The *sum* of a and 3 is written as $a + 3$.

(b) L *plus* W is written as $L + W$.

(c) 5 *more than* m is written as $m + 5$.

(d) x *increased by* 7 is written as $x + 7$.

Check Yourself 1

Write, using symbols.

(a) The sum of y and 4

(b) a plus b

(c) 3 more than x

(d) n increased by 6

Next, we look at how subtraction is indicated in algebra.

Definition

Subtraction $x - y$ means the *difference* of x and y or x *minus* y. Subtracting y is the same as adding its opposite, so $x - y = x + (-y)$.

Example 2 Writing Expressions That Indicate Subtraction

(a) r *minus* s is written as $r - s$.

(b) The *difference* of m and 5 is written as $m - 5$.

(c) x *decreased by* 8 is written as $x - 8$.

(d) 4 *less than* a is written as $a - 4$.

Check Yourself 2

Write, using symbols.

(a) w minus z

(b) The difference of a and 7

(c) y decreased by 3

(d) 5 less than b

You have seen that the operations of addition and subtraction are written exactly the same way in algebra as in arithmetic. This is not true in multiplication because the sign × looks like the letter x, so we use other symbols to show multiplication to avoid any confusion. Here are some ways to write multiplication.

Definition

Multiplication

A centered dot	$x \cdot y$	These all indicate the *product* of x and y or x times y. x and y are called the **factors** of the product xy.
Parentheses	$(x)(y)$	
Writing the letters next to each other	xy	

Example 3 Writing Expressions That Indicate Multiplication

NOTE

You can place letters next to each other or numbers and letters next to each other to show multiplication. But you *cannot* place numbers side by side to show multiplication: 37 means the number "thirty-seven," not 3 times 7.

(a) The product of 5 and a is written as $5 \cdot a$, $(5)(a)$, or $5a$. The last expression, $5a$, is the shortest and the most common way of writing the product.

(b) 3 times 7 can be written as $3 \cdot 7$ or $(3)(7)$.

(c) Twice z is written as $2z$.

(d) The product of 2, s, and t is written as $2st$.

(e) 4 more than the product of 6 and x is written as $6x + 4$.

Check Yourself 3

Write, using symbols.

(a) m times n

(b) The product of h and b

(c) The product of 8 and 9

(d) The product of 5, w, and y

(e) 3 more than the product of 8 and a

Before moving on to division, look at how we combine the symbols learned so far.

Definition

Expression

An **expression** is a meaningful collection of numbers, variables, and signs of operation.

Example 4 Identifying Expressions

< Objective 2 >

NOTE

Not every collection of symbols is an expression.

(a) $2m + 3$ is an expression. It means that we multiply 2 and m, and then add 3.

(b) $x + \cdot + 3$ is not an expression. The three operations in a row have no meaning.

(c) $y = 2x - 1$ is not an expression. The equal sign is not an operation sign.

(d) $3a + 5b - 4c$ is an expression. Its meaning is clear.

Check Yourself 4

Identify which are expressions and which are not.

(a) $7 - \cdot x$

(b) $6 + y = 9$

(c) $a + b - c$

(d) $3x - 5yz$

To write more complicated products in algebra, we need some "punctuation marks." Parentheses ( ) mean that an expression is to be thought of as a single quantity. Brackets [ ] are used in exactly the same way as parentheses in algebra. Example 5 shows the use of these signs of grouping.

Example 5 Expressions with More Than One Operation

NOTES

This can be read as "3 times the quantity *a* plus *b*."

No parentheses are needed in (b) because the 3 multiplies *only* the *a*.

(a) 3 times the sum of a and b is written as

$$3(\underbrace{a + b})$$

The sum of *a* and *b* is a single quantity, so it is enclosed in parentheses.

(b) The sum of 3 times a and b is written as $3a + b$.

(c) 2 times the difference of m and n is written as $2(m - n)$.

(d) The product of s plus t and s minus t is written as $(s + t)(s - t)$.

(e) The product of b and 3 less than b is written as $b(b - 3)$.

Check Yourself 5

Write, using symbols.

(a) Twice the sum of *p* and *q*
(b) The sum of twice *p* and *q*
(c) The product of *a* and the quantity *b* − *c*
(d) The product of *x* plus 2 and *x* minus 2
(e) The product of *x* and 4 more than *x*

NOTE

In algebra the fraction form is usually used.

Now look at the operation of division. In arithmetic, you use the division sign ÷, the long division symbol $\overline{)\;\;}$, and fraction notation. For example, to indicate the quotient when 9 is divided by 3, you could write

$$9 \div 3 \qquad \text{or} \qquad 3\overline{)9} \qquad \text{or} \qquad \frac{9}{3}$$

Definition

Division $\frac{x}{y}$ means *x divided by y* or the *quotient* of *x* and *y*.

Example 6 Writing Expressions That Indicate Division

RECALL

The fraction bar is a grouping symbol.

Write, using symbols.

(a) m divided by 3 is written as $\frac{m}{3}$.

(b) The quotient of a plus b and 5 is written as $\frac{a + b}{5}$.

(c) The sum of p and q divided by the difference of p and q is written as $\frac{p + q}{p - q}$.

Check Yourself 6

Write, using symbols.

(a) *r* divided by *s*
(b) The quotient when *x* minus *y* is divided by 7
(c) The difference of *a* and 2 divided by the sum of *a* and 2

Of everything we have studied so far, algebra lends itself to the greatest variety of applications. In Example 7, we model one such application.

When choosing a letter to use as a variable, it is often a good idea to choose one that reminds us of what it represents.

Example 7 Modeling Applications with Algebra

NOTE

We are being asked to describe her pay given that her hours may vary.

Carla earns $10.25 per hour in her job. Write an expression that describes her weekly gross pay in terms of the number of hours she works.

We represent the number of hours she works in a week by the variable h. Carla's pay is figured by taking the product of her hourly wage and the number of hours she works.

So, the expression

$10.25h$

describes Carla's weekly gross pay.

Check Yourself 7

NOTE

The word *twice* indicates multiplication by 2.

The specifications for an engine cylinder call for the stroke length to be two more than twice the diameter of the cylinder. Write an expression for the stroke length of a cylinder based on its diameter.

We close this section by listing many of the common words used to indicate arithmetic operations.

Words Indicating Operations

The operations listed are usually indicated by the words shown.

Addition (+)	Plus, and, more than, increased by, sum
Subtraction (−)	Minus, from, less than, decreased by, difference
Multiplication (·)	Times, of, by, product
Division (÷)	Divided, into, per, quotient

Check Yourself ANSWERS

1. **(a)** $y + 4$; **(b)** $a + b$; **(c)** $x + 3$; **(d)** $n + 6$
2. **(a)** $w - z$; **(b)** $a - 7$; **(c)** $y - 3$; **(d)** $b - 5$
3. **(a)** mn; **(b)** hb; **(c)** $8 \cdot 9$ or $(8)(9)$; **(d)** $5wy$; **(e)** $8a + 3$
4. **(a)** Not an expression; **(b)** not an expression; **(c)** an expression; **(d)** an expression
5. **(a)** $2(p + q)$; **(b)** $2p + q$; **(c)** $a(b - c)$; **(d)** $(x + 2)(x - 2)$; **(e)** $x(x + 4)$
6. **(a)** $\frac{r}{s}$; **(b)** $\frac{x - y}{7}$; **(c)** $\frac{a - 2}{a + 2}$ **7.** $2d + 2$

Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 11.1

(a) In algebra, we use letters, called ________________, to represent unknown quantities.

(b) $x + y$ gives the ________________ of x and y.

(c) $x \cdot y$, $(x)(y)$, and xy are all ways of indicating ________________ in algebra.

(d) An ________________ is a meaningful collection of numbers, variables, and symbols of operation.

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11.1 exercises

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Name ____________________

Section ________ Date ________

< Objective 1 >

Write each phrase, using symbols.

1. The sum of c and d > Videos

2. a plus 7

3. w plus z

4. The sum of m and n

5. x increased by 2

6. 3 more than b

7. 10 more than y

8. m increased by 4

9. a minus b

10. 5 less than s

11. b decreased by 7

12. r minus 3

13. 6 less than r > Videos

14. x decreased by 3

15. w times z

16. The product of 3 and c

17. The product of 5 and t > Videos

18. 8 times a

19. The product of 8, m, and n

20. The product of 7, r, and s

21. The product of 3 and the quantity p plus q

22. The product of 5 and the sum of a and b

23. Twice the sum of x and y > Videos

24. 3 times the sum of m and n

25. The sum of twice x and y

26. The sum of 3 times m and n

27. Twice the difference of x and y

28. 3 times the difference of c and d

29. The quantity a plus b times the quantity a minus b

30. The product of x plus y and x minus y

Answers

1. ____________ **2.** ____________

3. ____________ **4.** ____________

5. ____________ **6.** ____________

7. ____________ **8.** ____________

9. ____________ **10.** ____________

11. ____________ **12.** ____________

13. ____________ **14.** ____________

15. ____________ **16.** ____________

17. ____________ **18.** ____________

19. ____________ **20.** ____________

21. ____________ **22.** ____________

23. ____________ **24.** ____________

25. ____________ **26.** ____________

27. ____________ **28.** ____________

29. ________________________

30. ________________________

Answers

31. ______ 32. ______

33. ______ 34. ______

35. ______ 36. ______

37. ______ 38. ______

39. ______ 40. ______

41. ______

42. ______

43. ______

44. ______

45. ______

46. ______

47. ______

48. ______

49. ______ 50. ______

51. ______ 52. ______

53. ______ 54. ______

55. ______ 56. ______

31. The product of m and 3 less than m

32. The product of a and 7 more than a

33. x divided by 5 > Videos

34. The quotient when b is divided by 8

35. The quotient of a plus b, and 7

36. The difference x minus y, divided by 9

37. The difference of p and q, divided by 4

38. The sum of a and 5, divided by 9

39. The sum of a and 3, divided by the difference of a and 3

40. The difference of m and n, divided by the sum of m and n

< Objective 2 >

Identify which are expressions and which are not.

41. $2(x + 5)$

42. $4 + (x - 3)$

43. $4 + \div m$

44. $6 + a = 7$

45. $2b = 6$

46. $x(y + 3)$

47. $2a + 5b$

48. $4x + \cdot 7$

49. **Social Science** The Earth's population has doubled in the last 40 years. If we let x represent the Earth's population 40 years ago, what is the population today, in terms of x?

50. **Science and Medicine** It is estimated that the Earth is losing 4,000 species of plants and animals each year to extinction. Let S represent the number of species living last year. Represent the number of species alive this year, in terms of S.

51. **Business and Finance** The simple interest earned when a principal P is invested at a rate r for a time period t is given by the product of the principal, the rate, and the time. Write an expression for the simple interest earned.

52. **Science and Medicine** The kinetic energy of a particle of mass m is found by taking one-half of the product of the mass and the square of the velocity v. Write an expression for the kinetic energy of a particle.

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Match each phrase with the proper expression.

53. 8 decreased by x

54. 8 less than x

55. The difference between 8 and x

56. 8 from x

(a) $x - 8$

(b) $8 - x$

Write each phrase using symbols. Use the variable x to represent the number in each case.

57. 5 more than a number

58. A number increased by 8

59. 7 less than a number

60. A number decreased by 10

61. 9 times a number

62. Twice a number

63. 6 more than 3 times a number

64. 5 times a number, decreased by 10

65. Twice the sum of a number and 5

66. 3 times the difference of a number and 4

67. The product of 2 more than a number and 2 less than that same number

68. The product of 5 less than a number and 5 more than that same number

69. The quotient of a number and 7

70. A number divided by 3

71. The sum of a number and 5, divided by 8

72. The quotient when 7 less than a number is divided by 3

73. 6 more than a number divided by 6 less than that same number

74. The quotient when 3 less than a number is divided by 3 more than that same number

Basic Skills | Challenge Yourself | Calculator/Computer | Career Applications | Above and Beyond

75. **Allied Health** The standard dosage given to a patient is equal to the product of the desired dose D and the available quantity Q divided by the available dose H. Write the standard dosage calculation formula.

76. **Information Technology** Mindy is the manager of the help desk at a large cable company. She notices that, on average, her staff can handle 50 calls per hour. Last week, during a thunderstorm, the call volume increased from 65 to 150 calls per hour. To figure out the average number of customers in the system, she needs to take the quotient of the average rate of customer arrivals (the call volume) a and the average rate at which customers are served h minus the average rate of customer arrivals a. Write a formula for the average number of customers in the system.

Answers

57. ____

58. ____

59. ____

60. ____

61. ____

62. ____

63. ____

64. ____

65. ____

66. ____

67. ____

68. ____

69. ____

70. ____

71. ____

72. ____

73. ____

74. ____

75. ____

76. ____

Answers

77. ____________________

78. ____________________

77. **Construction Technology** K Jones Manufacturing produces hex bolts and carriage bolts. It sold 284 more hex bolts than carriage bolts last month. Write a formula that describes the number of carriage bolts it sold last month. Let H be the number of hex bolts sold last month.

78. **Electrical Engineering** Electrical power P is the product of voltage V and current I. Express this relationship algebraically.

Answers

1. $c + d$ **3.** $w + z$ **5.** $x + 2$ **7.** $y + 10$ **9.** $a - b$
11. $b - 7$ **13.** $r - 6$ **15.** wz **17.** $5t$ **19.** $8mn$
21. $3(p + q)$ **23.** $2(x + y)$ **25.** $2x + y$ **27.** $2(x - y)$
29. $(a + b)(a - b)$ **31.** $m(m - 3)$ **33.** $\frac{x}{5}$ **35.** $\frac{a + b}{7}$
37. $\frac{p - q}{4}$ **39.** $\frac{a + 3}{a - 3}$ **41.** Expression **43.** Not an expression
45. Not an expression **47.** Expression **49.** $2x$ **51.** Prt
53. (b) **55.** (b) **57.** $x + 5$ **59.** $x - 7$ **61.** $9x$ **63.** $3x + 6$
65. $2(x + 5)$ **67.** $(x + 2)(x - 2)$ **69.** $\frac{x}{7}$ **71.** $\frac{x + 5}{8}$
73. $\frac{x + 6}{x - 6}$ **75.** $\frac{DQ}{H}$ **77.** $H - 284$

11.2

Evaluating Algebraic Expressions

< **11.2 Objectives** >

1 > Substitute real numbers for the variables in an expression

2 > Use the order of operations to evaluate an expression

When using algebra to solve problems, we often want to find the value of an algebraic expression, given particular values for the variables. Finding the value of an expression is called *evaluating the expression* and uses the following steps.

Step by Step

To Evaluate an Algebraic Expression

Step 1 Replace each variable with its given number value.

Step 2 Do the necessary arithmetic operations, following the rules for the order of operations.

Example 1 Evaluating Algebraic Expressions

< **Objective 1** >

NOTE

We use parentheses when we make the initial substitution. This helps us to avoid careless errors.

Suppose that $a = 5$ and $b = 7$.

(a) To evaluate $a + b$, we replace a with 5 and b with 7.

$$a + b = (5) + (7) = 12$$

(b) To evaluate $3ab$, we again replace a with 5 and b with 7.

$$3ab = 3(5)(7) = 105$$

Check Yourself 1

If $x = 6$ and $y = 7$, evaluate.

(a) $y - x$ **(b)** $5xy$

Some algebraic expressions require us to follow the rules for the order of operations.

Example 2 Evaluating Algebraic Expressions

< **Objective 2** >

Evaluate each expression if $a = 2$, $b = 3$, $c = 4$, and $d = 5$.

(a) $5a + 7b = 5(2) + 7(3)$ Multiply first.

$= 10 + 21 = 31$ Then add.

(b) $3c^2 = 3(4)^2$ Apply the exponent.

$= 3 \cdot 16 = 48$ Then multiply.

(c) $7(c + d) = 7[(4) + (5)]$ Add inside the parentheses.

$= 7 \cdot 9 = 63$

(d) $5a^4 - 2d^2 = 5(2)^4 - 2(5)^2$ Apply the exponents.

$= 5 \cdot 16 - 2 \cdot 25$ Multiply.

$= 80 - 50 = 30$ Subtract.

> **CAUTION**

The expression in part **(b)** is different from

$(3c)^2 = [3(4)]^2$

$= 12^2 = 144$

Check Yourself 2

If $x = 3$, $y = 2$, $z = 4$, and $w = 5$, evaluate each expression.

(a) $4x^2 + 2$ **(b)** $5(z + w)$ **(c)** $7(z^2 - y^2)$

To evaluate algebraic expressions when a fraction bar is used, start by doing all the work in the numerator, and then do the work in the denominator. Divide the numerator by the denominator as the last step.

Example 3 Evaluating Algebraic Expressions

RECALL

The fraction bar, like parentheses, is a grouping symbol. Work first in the numerator and then in the denominator.

If $p = 2$, $q = 3$, and $r = 4$, evaluate:

(a) $\dfrac{8p}{r}$

Replace p with 2 and r with 4.

$$\frac{8p}{r} = \frac{8(2)}{(4)} = \frac{16}{4} = 4$$ Divide as the last step.

(b) $\dfrac{7q + r}{p + q} = \dfrac{7(3) + (4)}{(2) + (3)}$ Now evaluate the top and bottom separately.

$$= \frac{21 + 4}{2 + 3} = \frac{25}{5} = 5$$

Check Yourself 3

Evaluate if $c = 5$, $d = 8$, and $e = 3$.

(a) $\dfrac{6c}{e}$ **(b)** $\dfrac{4d + e}{c}$ **(c)** $\dfrac{10d - e}{d + e}$

When the algebraic expression contains parentheses or other grouping symbols, remember to follow the proper order of operations.

Example 4 Evaluating Expressions

NOTE

We can interchange parentheses and brackets as convenient.

Evaluate each expression if $x = 3$, $y = -4$, and $z = 1$.

(a) $3x(y + 2z)$

Replace x with 3, y with -4, and z with 1 and evaluate properly.

$$\begin{aligned} 3x(y + 2z) &= 3(3)[(-4) + 2(1)] \\ &= 3(3)(-4 + 2) \\ &= 3(3)(-2) \\ &= -18 \end{aligned}$$

(b) $(1 - 2x)(z - y)$

$$\begin{aligned} (1 - 2x)(z - y) &= [1 - 2(3)][(1) - (-4)] \\ &= (-5)(5) \\ &= -25 \end{aligned}$$

Check Yourself 4

Evaluate each expression if $a = 3$, $b = 0$, and $c = -6$.

(a) $3c(c - a)$ **(b)** $\dfrac{ab}{c - a}$

Example 5 Evaluating Expressions

RECALL

The rules for the order of operations require that we multiply first and then add.

Evaluate $5a + 4b$ if $a = -2$ and $b = \frac{3}{4}$.

Replace a with -2 and b with $\frac{3}{4}$.

$$
\begin{aligned}
5a + 4b &= 5(-2) + 4\left(\frac{3}{4}\right) \\
&= -10 + 3 \\
&= -7
\end{aligned}
$$

Check Yourself 5

Evaluate $3x + 5y$ if $x = -2$ and $y = -\frac{4}{5}$.

We follow the same rules no matter how many variables are in the expression.

Example 6 Evaluating Expressions

Evaluate each expression if $a = -4$, $b = 2$, $c = -5$, and $d = 6$.

This becomes $-(-20)$, or $+20$.

(a) $$\begin{aligned} 7a - 4c &= 7(-4) \overbrace{- 4(-5)} \\ &= -28 + 20 \\ &= -8 \end{aligned}$$

Apply the exponent first, and then multiply by 7.

(b) $$\begin{aligned} 7c^2 &= 7(-5)^2 = 7 \cdot 25 \\ &= 175 \end{aligned}$$

(c) $$\begin{aligned} b^2 - 4ac &= (2)^2 - 4(-4)(-5) \\ &= 4 - 4(-4)(-5) \\ &= 4 - 80 \\ &= -76 \end{aligned}$$

Add inside the parentheses first.

(d) $$\begin{aligned} b(a + d) &= (2)[(-4) + (6)] \\ &= 2(2) \\ &= 4 \end{aligned}$$

> **CAUTION**

When a squared variable is replaced by a negative number, square the negative.

$(-5)^2 = (-5)(-5) = 25$

The exponent applies to -5!

$-5^2 = -(5 \cdot 5) = -25$

The exponent applies only to 5!

Check Yourself 6

Evaluate if $p = -4$, $q = 3$, and $r = -2$.

(a) $5p - 3r$
(b) $2p^2 + q$
(c) $p(q + r)$
(d) $-q^2$
(e) $(-q)^2$

If an expression involves a fraction, remember that the fraction bar is a grouping symbol. This means that you should do the required operations first in the numerator and then in the denominator separately. Divide as the last step.

Example 7 Evaluating Expressions

Evaluate each expression if $x = 4$, $y = -5$, $z = 2$, and $w = -3$.

(a) $$\frac{z - 2y}{x} = \frac{(2) - 2(-5)}{(4)} = \frac{2 + 10}{4}$$

$$= \frac{12}{4} = 3$$

(b) $$\frac{3x - w}{2x + w} = \frac{3(4) - (-3)}{2(4) + (-3)} = \frac{12 + 3}{8 + (-3)}$$

$$= \frac{15}{5} = 3$$

Check Yourself 7

Evaluate if $m = -6$, $n = 4$, and $p = -3$.

(a) $\frac{m + 3n}{p}$ **(b)** $\frac{4m + n}{m + 4n}$

When a calculator is used to evaluate an expression, the same order of operations that we introduced in Section 1.7 is followed.

	Algebraic Notation	**Calculator Notation**
Addition	$6 + 2$	6 $\boxed{+}$ 2
Subtraction	$4 - 8$	4 $\boxed{-}$ 8
Multiplication	$(3)(-5)$	3 $\boxed{\times}$ ($\boxed{(-)}$ 5) or 3 $\boxed{\times}$ 5 $\boxed{+/-}$
Division	$\frac{8}{6}$	8 $\boxed{\div}$ 6
Exponential	3^4	3 $\boxed{\wedge}$ 4 or 3 $\boxed{y^x}$ 4
	$(-3)^4$	$\boxed{(}$ $\boxed{(-)}$ 3 $\boxed{)}$ $\boxed{\wedge}$ 4 or $\boxed{(}$ 3 $\boxed{+/-}$ $\boxed{)}$ $\boxed{y^x}$ 4

The use of this notation is illustrated in Example 8.

Example 8 Evaluating Expressions

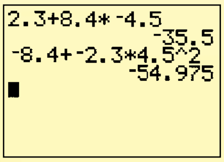
Evaluate each expression if $A = 2.3$, $B = 8.4$, and $C = 4.5$. Round your answer to the nearest tenth.

(a) $A + B(-C)$

Letting A, B, and C take on the given values, we have

2.3 $\boxed{+}$ 8.4 $\boxed{\times}$ $\boxed{(-)}$ 4.5 $\boxed{\text{Enter}}$ -35.5

NOTE



```
2.3+8.4*-4.5
          -35.5
-8.4+-2.3*4.5^2
         -54.975
▮
```

(b) $-B + (-A)C^2$

Substituting the given values, we have

$\boxed{(-)}\ 8.4\ \boxed{+}\ \boxed{(-)}\ 2.3\ \boxed{\times}\ 4.5\boxed{\wedge}2\ \boxed{\text{Enter}}\quad -54.975$

Rounding to the nearest tenth gives us -55.0.

Check Yourself 8

Evaluate each expression when $A = -2$, $B = 3$, and $C = 5$.

(a) $A + B(-C)$ **(b)** $C + BA^3$ **(c)** $4(B - C)/(2A)$

Many applications require us to evaluate algebraic expressions.

Example 9 Applying Algebra

The lens formula (in the study of *optics*) states that the *focal length* (the distance between a *lens* and the *focal point*) of a lens is given by the formula

$$\frac{d_od_i}{d_o + d_i}$$

in which d_o is the distance of an object from a thin lens and d_i is the distance of the object's image from the lens. Find the focal length of a lens if an object 24 inches (in.) from a lens produces an image 1 in. from the lens.

We substitute as before.

$$\frac{d_od_i}{d_o + d_i} = \frac{(24)(1)}{(24) + (1)}$$

$$= \frac{24}{25}$$

So the focal length is $\frac{24}{25}$ in. (or 0.96 in.).

Check Yourself 9

In an electric circuit with electromotive force of E volts and resistance R ohms, the rate of change in the current with respect to resistance is given by

$$-\frac{E}{R^2} \qquad \text{amperes per ohm}$$

Find the rate of change in the current with respect to resistance if $E = 100$ volts and $R = 12$ ohms.

Check Yourself ANSWERS

1. **(a)** 1; **(b)** 210 **2.** **(a)** 38; **(b)** 45; **(c)** 84 **3.** **(a)** 10; **(b)** 7; **(c)** 7
4. **(a)** 162; **(b)** 0 **5.** -10 **6.** **(a)** -14; **(b)** 35; **(c)** -4; **(d)** -9; **(e)** 9
7. **(a)** -2; **(b)** -2 **8.** **(a)** -17; **(b)** -19; **(c)** 2
9. $-\frac{25}{36}$ amperes per ohm (approximately -0.69)

Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 11.2

(a) Finding the value of an expression is called ________________ the expression.

(b) To evaluate an algebraic expression, first replace each instance of a ________________ with its given number value.

(c) To evaluate an algebraic expression, you must follow the rules for the order of ________________.

(d) When a squared variable is replaced by a negative number, the result is ________________.

11.2 exercises

< Objectives 1–2 >

Evaluate each expression if $a = -2$, $b = 5$, $c = -4$, and $d = 6$.

1. $3c - 2b$ > Videos

2. $4c - 2b$

3. $8b + 2c$

4. $7a - 2c$

5. $-b^2 + b$

6. $(-b)^2 + b$

7. $3a^2$

8. $6c^2$

9. $c^2 - 2d$

10. $3a^2 + 4c$

11. $2a^2 + 3b^2$ > Videos

12. $4b^2 - 2c^2$

13. $2(a + b)$

14. $5(b - c)$

15. $4(2a - d)$ > Videos

16. $6(3c - d)$

17. $a(b + 3c)$

18. $c(3a - d)$

19. $\frac{6d}{c}$

20. $\frac{8b}{5c}$

21. $\frac{3d + 2c}{b}$ > Videos

22. $\frac{2b + 3d}{2a}$

23. $\frac{2b - 3a}{c + 2d}$ > Videos

24. $\frac{3d - 2b}{5a + d}$

25. $d^2 - b^2$

26. $c^2 - a^2$

27. $(d - b)^2$

28. $(c - a)^2$

29. $(d - b)(d + b)$

30. $(c - a)(c + a)$

31. $d^3 - b^3$

32. $c^3 + a^3$

33. $(d - b)^3$

34. $(c + a)^3$

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Name ____________________

Section __________ Date __________

Answers

1. ______
2. ______
3. ______
4. ______
5. ______
6. ______
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34. ______

Answers

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42. __________

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48. __________

49. __________

50. __________

51. __________

52. __________

53. __________

54. __________

55. __________

56. __________

35. $(d - b)(d^2 + db + b^2)$

36. $(c + a)(c^2 - ac + a^2)$

37. $b^2 + a^2$

38. $d^2 - a^2$

39. $(b + a)^2$

40. $(d - a)^2$

41. $a^2 + 2ad + d^2$

42. $b^2 - 2bc + c^2$

43. **GEOMETRY** The formula for the area of a triangle is given by $A = \frac{1}{2}bh$. Find the area of a triangle if $b = 4$ cm and $h = 8$ cm.

44. **GEOMETRY** The perimeter of a rectangle with length L and width W is given by the formula $P = 2L + 2W$. Find the perimeter of a rectangle if its length is 10 in. and its width is 5 in.

45. **BUSINESS AND FINANCE** The simple interest I on a principal P dollars at interest rate r for time t is given by $I = Prt$. Find the simple interest earned on a principal of \$6,000 at 3% for 2 years. *Hint:* 3% = 0.03.

46. **BUSINESS AND FINANCE** Use the simple interest formula in exercise 45 to find the interest earned on a principal of \$12,500 at 4.5% for 3 years.

47. **SCIENCE AND MEDICINE** A formula that relates Celsius and Fahrenheit temperatures is $F = \frac{9}{5}C + 32$. If the low temperature is −10°C one day, what was the Fahrenheit equivalent?

48. **GEOMETRY** The area of a circle with radius r is $A = \pi r^2$. Use $\pi \approx 3.14$ to approximate the area of a circle if the radius is 3 ft.

Basic Skills | **Challenge Yourself** | Calculator/Computer | Career Applications | Above and Beyond

Evaluate each expression if $x = -3$, $y = 5$, *and* $z = \frac{2}{3}$.

49. $x^2 - y$

50. $\frac{y - x}{z}$

51. $z - y^2$

52. $z - \frac{z + x}{y - x}$

In each problem, decide if the given values for the variables make the statement **true** *or* **false.**

53. $x - 7 = 2y + 5$; $x = 22$, $y = 5$

54. $3(x - y) = 6$; $x = 5$, $y = -3$

55. $2(x + y) = 2x + y$; $x = -4$, $y = -2$

56. $x^2 - y^2 = x - y$; $x = 4$, $y = -3$

Basic Skills | Challenge Yourself | **Calculator/Computer** | Career Applications | Above and Beyond

Use your calculator to evaluate each expression if $x = -2.34$, $y = -3.14$, *and* $z = 4.12$. *Round your results to the nearest tenth.*

57. $x + yz$

58. $y - 2z$

59. $x^2 - z^2$

60. $x^2 + y^2$

61. $\frac{xy}{z - x}$

62. $\frac{y^2}{zy}$

63. $\frac{2x + y}{2x + z}$

64. $\frac{x^2y^2}{xz}$

Basic Skills | Challenge Yourself | Calculator/Computer | **Career Applications** | Above and Beyond

65. **Allied Health** The concentration, in micrograms per milliliter (μg/mL), of an antihistamine in a patient's bloodstream can be approximated using the formula $-2t^2 + 13t + 1$, in which t is the number of hours since the drug was administered. Approximate the concentration of the antihistamine 1 hour after it has been administered.

66. **Allied Health** Use the formula given in exercise 65 to approximate the concentration of the antihistamine 3 hours after it has been administered.

67. **Electrical Engineering** Evaluate $\frac{rT}{5{,}252}$ for $r = 1{,}180$ and $T = 3$ (round to the nearest thousandth).

68. **Mechanical Engineering** The kinetic energy (in joules) of a particle is given by $\frac{1}{2}mv^2$. Find the kinetic energy of a particle if its mass is 60 kg and its velocity is 6 m/s.

Basic Skills | Challenge Yourself | Calculator/Computer | Career Applications | **Above and Beyond**

69. Write an English interpretation for each algebraic expression.

(a) $(2x^2 - y)^3$

(b) $3n - \frac{n - 1}{2}$

(c) $(2n + 3)(n - 4)$

70. Is $a^n + b^n = (a + b)^n$? Try a few numbers and decide if you think this is true for all numbers, true for some numbers, or never true. Write an explanation of your findings and give examples.

Answers

1. –22 **3.** 32 **5.** –20 **7.** 12 **9.** 4 **11.** 83 **13.** 6
15. –40 **17.** 14 **19.** –9 **21.** 2 **23.** 2 **25.** 11 **27.** 1
29. 11 **31.** 91 **33.** 1 **35.** 91 **37.** 29 **39.** 9 **41.** 16
43. 16 cm^2 **45.** \$360 **47.** 14°F **49.** 4 **51.** $-\frac{73}{3}$ **53.** True
55. False **57.** –15.3 **59.** –11.5 **61.** 1.1 **63.** 14 **65.** 12 μg/mL
67. 0.674 **69.** Above and Beyond

Answers

57. ____________

58. ____________

59. ____________

60. ____________

61. ____________

62. ____________

63. ____________

64. ____________

65. ____________

66. ____________

67. ____________

68. ____________

69. ____________

70. ____________

Activity 30 ::
Evaluating Net Pay

chapter
11
> Make the Connection

Many people are paid based on the number of hours they work. However, while a person may earn a fixed number of dollars per hour, the person's actual paycheck differs from this straightforward multiplication.

The **gross pay** of an hourly employee is determined by multiplying the number of hours worked by the amount paid per hour. More generally, gross pay is the amount earned before any money is deducted. A person's **net pay** is the amount the person actually receives, after all deductions.

1. Ilyona earns \$12.50 per hour working at her local library. Find her gross pay if she works a 35-hour week.

2. The federal government deducts 6% of her gross pay for taxes and an additional 7% for FICA. The state also deducts 5% of her gross for state taxes. How much do the federal and state governments deduct from her pay?

3. Ilyona contributes \$25 each week to her benefits package, and \$8 each week is paid as city employees' union dues. Find Ilyona's net weekly pay.

4. Find her yearly gross and net earnings. Assume she is paid for 52 weeks.

Obviously, it is not efficient for a large company to compute steps 1 to 4 manually, one at a time, for each employee. By creating and using formulas, the process can be made more efficient.

5. Create an expression using r for hourly pay and t for the number of hours worked that describes a person's gross pay.

6. Create an expression that describes a person's net pay. Assume the deductions stated in part 2 apply.

7. Ilyona's supervisor is paid \$15.75 per hour and works 40 hours per week. Use the expression found in exercise 6 to find the supervisor's net pay.

11.3

Simplifying Algebraic Expressions

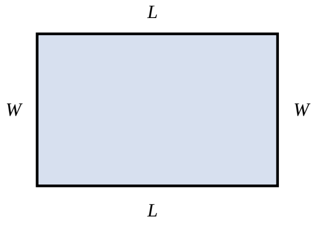
< **11.3 Objectives** >

1 > Identify terms and coefficients
2 > Identify like terms
3 > Combine like terms

To find the perimeter of a rectangle, we add 2 times the length and 2 times the width. In the language of algebra, this can be written as

RECALL

The perimeter of a figure is the distance around that figure.



$$\text{Perimeter} = 2L + 2W$$

We call $2L + 2W$ an **algebraic expression,** or more simply an **expression.** Recall from Section 11.1 that an expression is a mathematical idea written in symbols. It can be thought of as a meaningful collection of letters, numbers, and operation signs.

Some expressions are

1. $5x^2$
2. $3a + 2b$
3. $4x^3 - 2y + 1$
4. $3(x^2 + y^2)$

In algebraic expressions, the addition and subtraction signs break the expressions into smaller parts called *terms*.

Definition

Term	A **term** is an expression that can be written as a number, or the product of a number and one or more variables, raised to a power.

In an expression, each sign (+ or −) is a part of the term that follows the sign.

Example 1 Identifying Terms

< **Objective 1** >

RECALL

Each term "owns" the sign that precedes it.

(a) $5x^2$ has one term.

(b) $\underbrace{3a}_{\text{Term}} \underbrace{+\ 2b}_{\text{Term}}$ has two terms: $3a$ and $2b$.

(c) $\underbrace{4x^3}_{\text{Term}} \underbrace{-\ 2y}_{\text{Term}} \underbrace{+\ 1}_{\text{Term}}$ has three terms: $4x^3$, $-2y$, and 1.

Check Yourself 1

List the terms of each expression.

(a) $2b^4$ **(b)** $5m + 3n$ **(c)** $2s^2 - 3t - 6$

A term in an expression may have any number of factors. For instance, $5xy$ is a term. It has factors of 5, x, and y. The number factor of a term is called the **numerical coefficient.** So for the term $5xy$, the numerical coefficient is 5.

Example 2 Identifying the Numerical Coefficient

(a) $4a$ has the numerical coefficient 4.

(b) $6a^3b^4c^2$ has the numerical coefficient 6.

(c) $-7m^2n^3$ has the numerical coefficient -7.

(d) Because $1 \cdot x = x$, the numerical coefficient of x is understood to be 1.

Check Yourself 2

Give the numerical coefficient of each term.

(a) $8a^2b$ **(b)** $-5m^3n^4$ **(c)** y

If terms contain exactly the *same letters* (or variables) raised to the *same powers*, they are called **like terms.**

Example 3 Identifying Like Terms

< Objective 2 >

(a) The following are like terms.

$6a$ and $7a$

$5b^2$ and b^2

$10x^2y^3z$ and $-6x^2y^3z$

$-3m^2$ and m^2

Each pair of terms has the same letters, with each letter raised to the same power—the numerical coefficients do not need to be the same.

(b) The following are *not* like terms.

Different letters

$6a$ and $7b$

Different exponents

$5b^2$ and b^3

Different exponents

$3\overbrace{x^2y}$ and $4\overbrace{xy^2}$

Check Yourself 3

Circle the like terms.

$5a^2b$ ab^2 a^2b $-3a^2$ $4ab$ $3b^2$ $-7a^2b$

Like terms of an expression can always be combined into a single term. Consider the following:

$$\overbrace{x + x}^{2x} + \overbrace{x + x + x + x + x}^{5x} = \overbrace{x + x + x + x + x + x + x}^{7x}$$

Rather than having to write out all those x's, try

$2x + 5x = (2 + 5)x = 7x$

In the same way,

$9b + 6b = (9 + 6)b = 15b$

RECALL

Here we use the distributive property from Section 1.5.

and

$10a - 4a = (10 - 4)a = 6a$

This leads us to the following rule.

Step by Step

Combining Like Terms

To combine like terms, use the following steps.

Step 1 Add or subtract the numerical coefficients.
Step 2 Attach the common variables.

Example 4 Combining Like Terms

< **Objective 3** >

RECALL

When any factor is multiplied by 0, the product is 0.

Combine like terms.

(a) $8m + 5m = (8 + 5)m = 13m$

(b) $5pq^3 - 4pq^3 = 1pq^3 = pq^3$ Multiplication by 1 is understood.

(c) $7a^3b^2 - 7a^3b^2 = 0a^3b^2 = 0$

Check Yourself 4

Combine like terms.

(a) $6b + 8b$ **(b)** $12x^2 - 3x^2$
(c) $8xy^3 - 7xy^3$ **(d)** $9a^2b^4 - 9a^2b^4$

The idea is the same for expressions involving more than two terms.

Example 5 Combining Like Terms

NOTE

The distributive property can be used over any number of like terms.

Combine like terms.

(a) $4xy - xy + 2xy$
$= (4 - 1 + 2)xy$
$= 5xy$

Only like terms can be combined.

(b) $\overbrace{8x - 2x} + 5y$
$= 6x + 5y$

NOTE

With practice you will do these steps mentally instead of writing them out.

Like terms Like terms

(c) $5m + 8n + 4m - 3n$ We use the associative and commutative properties.
$= (5m + 4m) + (8n - 3n)$
$= 9m + 5n$

(d) $4x^2 + 2x - 3x^2 + x$
$= (4x^2 - 3x^2) + (2x + x)$
$= x^2 + 3x$

As these examples illustrate, combining like terms often means changing the grouping and the order in which the terms are written. Again, all this is possible because of the properties of addition that we introduced in Section 1.2.

Check Yourself 5

Combine like terms.

(a) $4m^2 - 3m^2 + 8m^2$ **(b)** $9ab + 3a - 5ab$
(c) $4p + 7q + 5p - 3q$

You may not realize it, but adding and subtracting algebraic expressions occurs all the time in the world. You have probably combined like terms successfully many times before ever taking this course.

Example 6 An Application of Algebra

In anticipation of a holiday rush, a produce market receives 6 cases of apples and 4 cases of oranges from their supplier. The market already had 2 cases of apples and 2 cases of oranges in stock. How many cases of each does the market have after the delivery?

We add

6 apples + 4 oranges + 2 apples + 2 oranges

and combine like terms using the *commutative* property.

(6 apples + 2 apples) + (4 oranges + 2 oranges) = 8 apples + 6 oranges

Therefore, the market begins the day with 8 cases of apples and 6 cases of oranges.

If we let a represent the number of cases of apples and r represent the number of cases of oranges, then this calculation is performed as follows.

$$\begin{aligned} 6a + 4r + 2a + 2r &= (6a + 2a) + (4r + 2r) \\ &= 8a + 6r \end{aligned}$$

NOTE

We cannot add apples and oranges.

Check Yourself 6

An electronics store has 8 two-packs and 20 ten-packs of 3.5-in. floppy disks in stock. They receive a shipment of 48 two-packs and 24 ten-packs. Algebraically represent the number and type of packages of disks that the store has after the shipment arrives.

Check Yourself ANSWERS

1. **(a)** $2b^4$; **(b)** $5m, 3n$; **(c)** $2s^2, -3t, -6$ **2.** **(a)** 8; **(b)** -5; **(c)** 1
3. The like terms are $5a^2b$, a^2b, and $-7a^2b$ **4.** **(a)** $14b$; **(b)** $9x^2$; **(c)** xy^3; **(d)** 0
5. **(a)** $9m^2$; **(b)** $4ab + 3a$; **(c)** $9p + 4q$ **6.** $56x + 44y$

Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 11.3

(a) The product of a number and a variable is called a ______________.

(b) The ______________ factor of a term is called the numerical coefficient.

(c) Terms that contain exactly the same variables raised to the same powers are called ______________ terms.

(d) The ______________ property enables us to combine like terms into a single term.

11.3 exercises

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Name

Section Date

< Objective 1 >

List the terms of each expression.

1. $5a + 2$ > Videos

2. $7a - 4b$

3. $4x^3$

4. $3x^2$

5. $3x^2 + 3x - 7$

6. $2a^3 - a^2 + a$

< Objective 2 >

Circle the like terms in each group of terms.

7. $5ab, 3b, 3a, 4ab$ > Videos

8. $9m^2, 8mn, 5m^2, 7m$

9. $4xy^2, 2x^2y, 5x^2, -3x^2y, 5y, 6x^2y$

10. $8a^2b, 4a^2, 3ab^2, -5a^2b, 3ab, 5a^2b$

< Objective 3 >

Combine the like terms.

11. $3m + 7m$ > Videos

12. $6a^2 + 8a^2$

13. $7b^3 + 10b^3$

14. $7rs + 13rs$

15. $21xyz + 7xyz$

16. $4mn^2 + 15mn^2$

17. $9z^2 - 3z^2$

18. $7m - 6m$

19. $5a^3 - 5a^3$

20. $13xy - 9xy$

21. $19n^2 - 18n^2$

22. $7cd - 7cd$

23. $21p^2q - 6p^2q$

24. $17r^3s^2 - 8r^3s^2$

25. $10x^2 - 7x^2 + 3x^2$

26. $13uv + 5uv - 12uv$

27. $9a - 7a + 4b$ > Videos

28. $5m^2 - 3m + 6m^2$

29. $7x + 5y - 4x - 4y$

30. $6a^2 + 11a + 7a^2 - 9a$

31. $4a + 7b + 3 - 2a + 3b - 2$

32. $5p^2 + 2p + 8 + 4p^2 + 5p - 6$

33. $\frac{2}{3}m + 3 + \frac{4}{3}m$

34. $\frac{1}{5}a - 2 + \frac{4}{5}a$

Answers

1. ____ **2.** ____

3. ____ **4.** ____

5. ____ **6.** ____

7. ____ **8.** ____

9. ____

10. ____

11. ____ **12.** ____

13. ____ **14.** ____

15. ____ **16.** ____

17. ____ **18.** ____

19. ____ **20.** ____

21. ____ **22.** ____

23. ____ **24.** ____

25. ____ **26.** ____

27. ____ **28.** ____

29. ____ **30.** ____

31. ____

32. ____

33. ____ **34.** ____

Answers

35. ________ 36. ________

37. ________ 38. ________

39. ________ 40. ________

41. ________ 42. ________

43. ________ 44. ________

45. ________ 46. ________

47. ________ 48. ________

49. ________________

50. ________________

51. ________ 52. ________

53. ________ 54. ________

55. ________ 56. ________

57. ________ 58. ________

59. ________________

60. ________________

61. ________________

62. ________________

63. ________________

64. ________________

65. ________________

66. ________________

35. $\frac{13}{5}x + 2 - \frac{3}{5}x + 5$ **36.** $\frac{17}{12}y + 7 + \frac{7}{12}y - 3$

37. $2.3a + 7 + 4.7a + 3$ **38.** $5.8m + 4 - 2.8m + 11$

Perform the indicated operations.

39. Find the sum of $5a^4$ and $8a^4$. **40.** What is the sum of $9p^2$ and $12p^2$?

41. Subtract $12a^3$ from $15a^3$. **42.** Subtract $5m^3$ from $18m^3$.

43. Subtract $4x$ from the sum of $8x$ and $3x$.

> Videos

44. Subtract $8ab$ from the sum of $7ab$ and $5ab$.

45. Subtract $3mn^2$ from the sum of $9mn^2$ and $5mn^2$.

46. Subtract $4x^2y$ from the sum of $6x^2y$ and $12x^2y$.

47. **GEOMETRY** A rectangle has sides that measure $8x + 9$ and $6x - 7$. Find the simplified expression that represents its perimeter.

48. **GEOMETRY** A triangle has sides measuring $3x + 7$, $4x - 9$, and $5x + 6$. Find the simplified expression that represents its perimeter.

Basic Skills | **Challenge Yourself** | Calculator/Computer | Career Applications | Above and Beyond

49. **BUSINESS AND FINANCE** The cost of producing x units of an item is $150 + 25x$. The revenue from selling x units is $90x - x^2$. The profit is given by the revenue minus the cost. Find the simplified expression that represents the profit.

chapter 11 > Make the Connection

50. **BUSINESS AND FINANCE** The revenue from selling y units is $3y^2 - 2y + 5$ and the cost of producing y units is $y^2 + y - 3$. Find the simplified expression that represents the profit.

chapter 11 > Make the Connection

Use the distributive property to remove the parentheses in each expression. Then, simplify the expression by combining like terms.

51. $2(3x + 2) + 4$ **52.** $3(4z + 5) - 9$ **53.** $5(6a - 2) + 12a$

54. $7(4w - 3) - 25w$ **55.** $4s + 2(s + 4) + 4$ **56.** $5p + 4(p + 3) - 8$

Evaluate each expression if $a = 2$, $b = 3$, *and* $c = 5$. *Be sure to combine like terms, when possible, as the first step.*

57. $7a^2 + 3a$ **58.** $11b^2 - 9b$ **59.** $3c^2 + 5c^2$ **60.** $9b^3 - 5b^3$

61. $5b + 3a - 2b$ **62.** $7c - 2b + 3c$ **63.** $5ac^2 - 2ac^2$ **64.** $5a^3b - 2a^3b$

Basic Skills | Challenge Yourself | **Calculator/Computer** | Career Applications | Above and Beyond

Use your calculator to evaluate each expression for the given values of the variables. Round your results to the nearest tenth.

65. $7x^2 - 5y^3$; $x = 7.1695$, $y = 3.128$

66. $2x^2 + 3y + 5x$; $x = 3.61$, $y = 7.91$

67. $(4x^2y)(2xy^2) - 5x^3y; x = 1.29, y = 2.56$

68. $3x^3y - 4xy + 2x^2y^2; x = 3.26, y = 1.68$

Basic Skills | Challenge Yourself | Calculator/Computer | **Career Applications** | Above and Beyond

69. **ALLIED HEALTH** A person's body mass index (BMI) can be calculated using their height h, in inches, and their weight w, in pounds, with the formula

$$\frac{703w}{h^2}$$

Compute the BMI of a 69-inch, 190-pound man (to the nearest tenth).

70. **ALLIED HEALTH** A person's body mass index (BMI) can be calculated using their height h, in centimeters, and their weight w, in kilograms, with the formula

$$\frac{10{,}000w}{h^2}$$

Compute the BMI of a 160-cm, 70-kg woman (to the nearest tenth).

71. **MECHANICAL ENGINEERING** A primary beam can support a load of $54p$. A second beam is added that can support a load of $32p$. What is the total load that the two beams can support?

72. **MECHANICAL ENGINEERING** Two objects are spinning on the same axis. The moment of inertia of the first object is $\frac{6^3}{12}b$. The moment of inertia of the second object is given by $\frac{30^3}{36}b$. The total moment of inertia is given by the sum of the moments of inertia of the two objects. Write a simplified expression for the total moment of inertia for the two objects described.

Basic Skills | Challenge Yourself | Calculator/Computer | Career Applications | **Above and Beyond**

73. A toy store begins the day with four Frisbees and eight basketballs in stock. During the morning shift, two Frisbees and one basketball are sold. In the afternoon, a shipment containing six Frisbees arrived. The afternoon shift sells three Frisbees and two basketballs.

Algebraically represent the number of Frisbees and of basketballs that are left at the end of the day (use f to represent the number of Frisbees and b to represent the number of basketballs).

74. Determine the number of pounds of each type of coffee that a retailer has at the end of the day, given the following information.

A retailer begins the day with 24 pounds (lb) of Kona coffee, 17 lb of Italian roast, and 12 lb of Sumatran roast.

The retailer sells 8 lb of the Kona variety, 11 lb of the Italian, and 7 lb of the Sumatran. A delivery brings 4 lb of Kona and 16 lb of Sumatran coffees.

Express your answer algebraically, using K, I, and S to represent the number of pounds of Kona, Italian, and Sumatran coffees, respectively.

75. Write a paragraph explaining the difference between n^2 and $2n$.

76. Complete the explanation: "x^3 and $3x$ are not the same because . . . ."

Answers

67. ____________

68. ____________

69. ____________

70. ____________

71. ____________

72. ____________

73. ____________

74. ____________

75. ____________

76. ____________

Answers

77. ____________________

78. ____________________

79. ____________________

80. ____________________

77. Complete the statement: "$x + 2$ and $2x$ are different because . . . ."

78. Write an English phrase for each algebraic expression.

(a) $2x^3 + 5x$ **(b)** $(2x + 5)^3$ **(c)** $6(n + 4)^2$

79. Work with another student to complete this exercise. Place >, <, or = in the blank in these statements.

1^2 ______ 2^1

2^3 ______ 3^2

3^4 ______ 4^3

4^5 ______ 5^4

What happens as the table of numbers is extended? Try more examples.

What sign seems to occur the most in your table: >, <, or =?

Write an algebraic statement for the pattern of signs in this table. Do you think this is a pattern that continues? Add more lines to the table and extend the pattern to the general case by writing the pattern in algebraic notation. Write a short paragraph stating your conjecture.

80. Work with other students on this exercise.

Part 1: Evaluate the three expressions $\dfrac{n^2 - 1}{2}$, n, and $\dfrac{n^2 + 1}{2}$, using odd values of n: 1, 3, 5, 7, etc. Make a chart like the following one and complete it.

n	$a = \dfrac{n^2 - 1}{2}$	$b = n$	$c = \dfrac{n^2 + 1}{2}$	a^2	b^2	c^2
1						
3						
5						
7						
9						
11						
13						
15						

Part 2: The numbers, a, b, and c that you get in each row have a surprising relationship to each other. Complete the last three columns and work together to discover this relationship. You may want to find out more about the history of this famous number pattern.

Answers

1. $5a, 2$ **3.** $4x^3$ **5.** $3x^2, 3x, -7$ **7.** $5ab, 4ab$
9. $2x^2y, -3x^2y, 6x^2y$ **11.** $10m$ **13.** $17b^3$ **15.** $28xyz$ **17.** $6z^2$
19. 0 **21.** n^2 **23.** $15p^2q$ **25.** $6x^2$ **27.** $2a + 4b$ **29.** $3x + y$
31. $2a + 10b + 1$ **33.** $2m + 3$ **35.** $2x + 7$ **37.** $7a + 10$
39. $13a^4$ **41.** $3a^3$ **43.** $7x$ **45.** $11mn^2$ **47.** $28x + 4$
49. $P = -x^2 + 65x - 150$ **51.** $6x + 8$ **53.** $42a - 10$ **55.** $6s + 12$
57. 34 **59.** 200 **61.** 15 **63.** 150 **65.** 206.8 **67.** 260.6
69. 28.1 **71.** $86p$ **73.** $5f + 5b$ **75.** Above and Beyond
77. Above and Beyond **79.** Above and Beyond

Activity 31 ::
Writing Equations

In Section 11.1, you learned to translate phrases to algebraic expressions. In most applications, you need more than an expression; you need an equation.

1. Write an algebraic equation for the statement “Three more than a number is 9.”
2. Write an algebraic equation describing “an employee’s gross pay is the hourly pay times the number of hours worked.”
3. Use the equation in exercise 2 to determine the gross pay of someone who works 40 hours, earning $9.75 per hour.
4. Create an equation that determines the net pay if the employee in exercise 3 pays a total of 16% of the gross pay to the federal and state governments.
5. Determine the net pay for the employee in exercise 3.
6. Write a paragraph describing some reasons why, or situations in which, forming an equation to compute net pay might be useful.
7. **(a)** Describe another situation in which constructing an equation would be useful.
   (b) Construct an equation for the situation described in part (a).

11.4

Using the Addition Property to Solve an Equation

< **11.4 Objectives** >

1 > Determine whether a given number is a solution for an equation

2 > Use the addition property to solve an equation

In this section we begin working with one of the most important tools of mathematics, the equation. The ability to recognize and solve various types of equations is probably the most useful algebraic skill you will learn. To begin with, we define the word *equation*.

Definition

Equation An **equation** is a mathematical statement that two expressions are equal.

Some examples are $3 + 4 = 7$, $x + 3 = 5$, $P = 2L + 2W$.

As you can see, an equal sign (=) separates the two equal expressions. We call these expressions the *left side* and the *right side* of the equation.

$$\underbrace{x + 3}_{\text{Left side}} \underset{\text{Equals}}{=} \underbrace{5}_{\text{Right side}}$$

NOTE

An equation such as

$x + 3 = 5$

is called a **conditional equation** because it can be either true or false depending on the value given to the variable.

An equation may be either true or false. For instance, $3 + 4 = 7$ is true because both sides name the same number. What about an equation such as $x + 3 = 5$ that has a letter or variable on one side? Any number can replace x in the equation. However, only one number will make this equation a true statement.

$$x + 3 = 5$$

If $x = 1$: $(1) + 3 = 5$ is false

If $x = 2$: $(2) + 3 = 5$ is true

If $x = 3$: $(3) + 3 = 5$ is false

The number 2 is called a **solution** (or *root*) of the equation $x + 3 = 5$ because substituting 2 for x gives a true statement.

Definition

Solution A **solution** for an equation is any value for the variable that makes the equation a true statement.

Example 1 Verifying a Solution

< **Objective 1** >

(a) Is 3 a solution for the equation $2x + 4 = 10$?

To find out, replace x with 3 and evaluate $2x + 4$ on the left.

Left Side		Right Side
$2(3) + 4$	$\stackrel{?}{=}$	10
$6 + 4$	$\stackrel{?}{=}$	10
10	=	10

Because $10 = 10$ is a true statement, 3 is a solution of the equation.

(b) Is 5 a solution for the equation $3x - 2 = 2x + 1$?

To find out, replace x with 5 and evaluate each side separately.

Left Side		Right Side
$3(5) - 2$	$\stackrel{?}{=}$	$2(5) + 1$
$15 - 2$	$\stackrel{?}{=}$	$10 + 1$
13	$\stackrel{?}{=}$	11

RECALL

The rules for the order of operation require that we multiply first; then add or subtract.

Because the two sides do not name the same number, we do not have a true statement, and 5 is not a solution.

Check Yourself 1

For the equation

$2x - 1 = x + 5$

(a) Is 4 a solution? **(b)** Is 6 a solution?

You may be wondering whether an equation can have more than one solution. It certainly can. For instance,

$$x^2 = 9$$

has two solutions. They are 3 and -3 because

$$3^2 = 9 \qquad \text{and} \qquad (-3)^2 = 9$$

In this chapter, we work with *linear equations in one variable.* These are equations that can be put into the form

$$ax + b = 0$$

in which x is the variable, a and b are numbers, with a not equal to 0. In a linear equation, the variable can appear only to the first power. No other power (x^2, x^3, etc.) can appear. Linear equations are also called **first-degree equations.** The degree of an equation in one variable is the highest power to which the variable appears.

Property

Linear Equations

Linear equations in one variable that can be written in the form

$$ax + b = 0 \qquad a \neq 0$$

have exactly one solution.

Example 2 Identifying Expressions and Equations

Label each as an expression, a linear equation, or an equation that is not linear.

(a) $4x + 5$ is an expression.

(b) $2x + 8 = 0$ is a linear equation.

(c) $3x^2 - 9 = 0$ is an equation that is not linear.

(d) $5x = 15$ is a linear equation.

Check Yourself 2

Label each as an expression, a linear equation, or an equation that is nonlinear.

(a) $2x^2 = 8$ **(b)** $2x - 3 = 0$

(c) $5x - 10$ **(d)** $2x + 1 = 7$

It is not difficult to find the solution for an equation such as $x + 3 = 8$ by guessing the answer to the question, What plus 3 is 8? Here the answer to the question is 5, which is also the solution for the equation. But for more complicated equations you are going to need something more than guesswork. A better method is to transform the given equation to an *equivalent equation* whose solution can be found by inspection. Here is a definition.

Definition

Equivalent Equations

Equations that have exactly the same solution(s) are called **equivalent equations.**

NOTE

In some cases we write the equation in the form

$\square = x$

The number will be our solution when the equation has the variable isolated on either side.

These are all equivalent equations:

$2x + 3 = 5 \qquad 2x = 2 \qquad$ and $\qquad x = 1$

They all have the same solution, 1. We say that a linear equation is *solved* when it is transformed to an equivalent equation of the form

$x = \square$

The variable is alone on the left side.

The right side is some number, the solution.

The addition property of equality is the first property you need in order to transform an equation to an equivalent form.

Property

The Addition Property of Equality

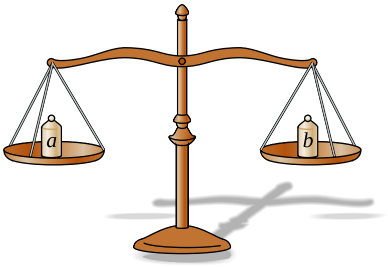
If $\qquad a = b$

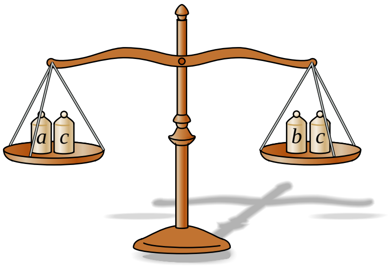
then $\qquad a + c = b + c$

In words, adding the same quantity to both sides of an equation gives an equivalent equation.

NOTE

An equation is a statement that the two sides are equal. Adding the same quantity to both sides does not change the equality or "balance."





Here is an example of applying this property to solve an equation.

Example 3 Using the Addition Property to Solve an Equation

< Objective 2 >

Solve

$x - 3 = 9$

Remember that our goal is to isolate x on one side of the equation. Because 3 is being subtracted from x, we can add 3 to remove it. We must use the addition property to add 3 to both sides of the equation.

$$\begin{array}{rcr} x - 3 & = & 9 \\ +\,3 & & +3 \\ \hline x \quad & = & 12 \end{array}$$

Adding 3 leaves x alone on the left.

NOTE

To check, replace x with 12 in the original equation:

$$\begin{aligned} x - 3 &= 9 \\ (12) - 3 &\stackrel{?}{=} 9 \\ 9 &= 9 \end{aligned}$$

Because we have a true statement, 12 is the solution.

Because 12 is the solution for the equivalent equation $x = 12$, it is the solution for our original equation.

Check Yourself 3

Solve and check.

$x - 5 = 4$

The addition property also allows us to add a negative number to both sides of an equation. This is really the same as subtracting the same quantity from both sides.

Example 4 Using the Addition Property to Solve an Equation

RECALL

Earlier we stated that we could write an equation in either form $x = \square$ or $\square = x$, in which $\square$ represents some number. Suppose we have an equation like

$12 = x + 7$

Subtracting 7 isolates x *on the right*

$$\begin{array}{rcl} 12 & = & x + 7 \\ -7 & & -7 \\ \hline 5 & = & x \end{array}$$

The solution is 5.

Solve

$x + 5 = 9$

In this case, 5 is *added* to x on the left. We can use the addition property to subtract 5 from both sides. This leaves the variable x alone on one side of the equation.

$$\begin{array}{lcr} x + 5 & = & 9 \\ -\ 5 & & -5 \\ \hline x & = & 4 \end{array}$$

The solution is 4. To check, replace x with 4 in the original equation.

$(4) + 5 = 9$ (True)

Check Yourself 4

Solve and check.

$x + 6 = 13$

What if the equation has a variable term on both sides? Then we use the addition property to add a term involving the variable to get the desired result.

Example 5 Using the Addition Property to Solve an Equation

Solve

$5x = 4x + 7$

We start by subtracting $4x$ from both sides of the equation. Do you see why? Remember that an equation is solved when we have an equivalent equation of the form $x = \square$.

$$\begin{array}{rcr} 5x & = & 4x + 7 \\ -4x & & -4x \\ \hline x & = & 7 \end{array}$$

Subtracting $4x$ from both sides *removes* $4x$ from the right.

To check: Because 7 is a solution for the equivalent equation $x = 7$, it should be a solution for the original equation. To find out, replace x with 7:

$$\begin{aligned} 5(7) &\stackrel{?}{=} 4(7) + 7 \\ 35 &\stackrel{?}{=} 28 + 7 \\ 35 &= 35 \end{aligned}$$

(True)

Check Yourself 5

Solve and check.

$7x = 6x + 3$

You may have to apply the addition property more than once to solve an equation. Look at Example 6.

Example 6 Using the Addition Property to Solve an Equation

NOTE

This equation could also be written as

$7x + (-8) = 6x$

Solve

$$7x - 8 = 6x$$

We want all variables on *one* side of the equation. If we choose the left, we subtract $6x$ from both sides of the equation. This removes $6x$ from the right:

$$\begin{array}{rr} 7x - 8 = & 6x \\ -6x \qquad & -6x \\ \hline x - 8 = & 0 \end{array}$$

We want the variable alone, so we add 8 to both sides. This isolates x on the left.

$$\begin{array}{rr} x - 8 = & 0 \\ +8 \quad & +8 \\ \hline x \qquad = & 8 \end{array}$$

The solution is 8. We leave it to you to check this result.

Check Yourself 6

Solve and check.

$9x + 3 = 8x$

Often an equation has more than one variable term *and* more than one number. You have to apply the addition property twice when solving these equations.

Example 7 Using the Addition Property to Solve an Equation

Solve

$$5x - 7 = 4x + 3$$

NOTE

You could just as easily have added 7 to both sides and *then* subtracted $4x$. The result is the same. With practice, you will learn to combine the two steps.

We would like the variable terms on the left, so we start by subtracting $4x$ from both sides of the equation to remove that term from the right side of the equation:

$$\begin{array}{rr} 5x - 7 = & 4x + 3 \\ -4x \qquad & -4x \qquad \\ \hline x - 7 = & 3 \end{array}$$

Now, to isolate the variable, we add 7 to both sides.

$$\begin{array}{rr} x - 7 = & 3 \\ +7 \quad & +7 \\ \hline x \qquad = & 10 \end{array}$$

The solution is 10. To check, replace x with 10 in the original equation:

$$\begin{array}{r} 5(10) - 7 \stackrel{?}{=} 4(10) + 3 \\ 43 = 43 \qquad \end{array}$$

(True)

RECALL

By *simplify* we mean to combine all like terms.

Check Yourself 7

Solve and check.

(a) $4x - 5 = 3x + 2$ **(b)** $6x + 2 = 5x - 4$

In solving an equation, you should always simplify each side as much as possible before using the addition property.

Example 8 Combining Like Terms to Solve an Equation

Solve

Like terms Like terms

$$5 + 8x - 2 = 2x - 3 + 5x$$

Because like terms appear on each side of the equation, we start by combining the numbers on the left (5 and -2). Then we combine the like terms ($2x$ and $5x$) on the right. We have

$$3 + 8x = 7x - 3$$

Now we apply the addition property, as before:

$$\begin{array}{rrl}
3 + 8x = & 7x - 3 & \\
-\ 7x = & -7x & \text{Subtract } 7x. \\
\hline
3 + \ \ x = & -\ 3 & \text{Subtract 3.} \\
-3 \qquad\ \ & -\ 3 & \\
\hline
x = & -\ 6 & \text{Isolate } x.
\end{array}$$

The solution is -6. To check, always return to the original equation. That catches any possible errors in simplifying. Replacing x with -6 gives

$$5 + 8(-6) - 2 \overset{?}{=} 2(-6) - 3 + 5(-6)$$

$$5 - 48 - 2 \overset{?}{=} -12 - 3 - 30$$

$$-45 = -45 \qquad \text{(True)}$$

Check Yourself 8

Solve and check.

(a) $3 + 6x + 4 = 8x - 3 - 3x$ **(b)** $5x + 21 + 3x = 20 + 7x - 2$

We may need to apply some of the properties discussed in Chapter 1 in solving equations. Example 9 illustrates the use of the distributive property to clear an equation of parentheses.

Example 9 Using the Distributive Property and Solving Equations

RECALL

$2(3x + 4)$
$= 2(3x) + 2(4)$
$= 6x + 8$

Solve

$$2(3x + 4) = 5x - 6$$

Applying the distributive property on the left gives

$$6x + 8 = 5x - 6$$

We can then proceed as before:

$$\begin{array}{rcrl} 6x + 8 &=& 5x - 6 & \\ -5x && -5x & \quad \text{Subtract } 5x. \\ \hline x + 8 &=& -6 & \\ -8 && -8 & \quad \text{Subtract 8.} \\ \hline x &=& -14 & \end{array}$$

The solution is -14. We leave it to you to check this result.

Remember: Always return to the original equation to check.

Check Yourself 9

Solve and check each equation.

(a) $4(5x + 2) = 19x + 20$ **(b)** $3(5x + 1) = 2(7x + 3) - 16$

Of course, there are many applications that require us to use the addition property to solve an equation. Consider the consumer application in Example 10.

Example 10 A Consumer Application

An appliance store is having a sale on washers and dryers. It charges \$999 for a washer and dryer combination. If the washer sells for \$649, how much is someone paying for the dryer as part of the combination?

Let d be the cost of the dryer and solve the equation $d + 649 = 999$ to answer the question.

RECALL

You should always answer an application problem with a full sentence.

$$\begin{array}{rcrl} d + 649 &=& 999 & \\ -649 && -649 & \quad \text{Subtract 649 from both sides.} \\ \hline d &=& 350 & \end{array}$$

The dryer adds \$350 to the price.

Check Yourself 10

Of 18,540 votes cast in the school board election, 11,320 went to Carla. How many votes did her opponent Marco receive? Who won the election?

Let m be the number of votes Marco received and solve the equation $11{,}320 + m = 18{,}540$ in order to answer the questions.

Check Yourself ANSWERS

1. **(a)** 4 is not a solution; **(b)** 6 is a solution
2. **(a)** Nonlinear equation; **(b)** linear equation; **(c)** expression; **(d)** linear equation
3. $x = 9$ **4.** $x = 7$ **5.** $x = 3$ **6.** $x = -3$ **7.** **(a)** $x = 7$; **(b)** $x = -6$
8. **(a)** $x = -10$; **(b)** $x = -3$ **9.** **(a)** $x = 12$; **(b)** $x = -13$
10. Marco received 7,220 votes; Carla won the election.

Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 11.4

(a) An ______________ is a mathematical statement that two expressions are equal.

(b) A solution for an equation is a value for the variable that makes the equation a ______________ statement.

(c) Equivalent equations have the same ______________.

(d) The answer to an application should always be given using a full ______________.

11.4 exercises

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Section __________ Date __________

Answers

1. __________ 2. __________
3. __________ 4. __________
5. __________ 6. __________
7. __________ 8. __________
9. __________ 10. __________
11. __________ 12. __________
13. __________ 14. __________
15. __________ 16. __________
17. __________ 18. __________
19. __________ 20. __________
21. __________ 22. __________
23. ____________________
24. ____________________
25. ____________________
26. ____________________
27. ____________________
28. ____________________
29. ____________________
30. ____________________

< Objective 1 >

Is the number shown in parentheses a solution for the given equation?

1. $x + 4 = 9$ (5)

2. $x + 2 = 11$ (8)

3. $x - 15 = 6$ (-21)

4. $x - 11 = 5$ (16)

5. $5 - x = 2$ (4)

6. $10 - x = 7$ (3)

7. $4 - x = 6$ (-2)

8. $5 - x = 6$ (-3)

9. $3x + 4 = 13$ (8) > Videos

10. $5x + 6 = 31$ (5)

11. $4x - 5 = 7$ (2)

12. $2x - 5 = 1$ (3)

13. $5 - 2x = 7$ (-1)

14. $4 - 5x = 9$ (-2)

15. $4x - 5 = 2x + 3$ (4)

16. $5x + 4 = 2x + 10$ (4)

17. $x + 3 + 2x = 5 + x + 8$ (5)

18. $5x - 3 + 2x = 3 + x - 12$ (-2)

19. $\frac{3}{4}x = 18$ (20)

20. $\frac{3}{5}x = 24$ (40)

21. $\frac{3}{5}x + 5 = 11$ (10)

22. $\frac{2}{3}x + 8 = -12$ (-6)

Label each as an expression or a linear equation.

23. $2x + 1 = 9$ > Videos

24. $7x + 14$

25. $2x - 8$ > Videos

26. $5x - 3 = 12$

27. $7x + 2x + 8 - 3$

28. $x + 5 = 13$

29. $2x - 8 = 3$

30. $12x - 5x + 2 + 5$

< Objective 2 >

Solve and check each equation.

31. $x + 9 = 11$ > Videos

32. $x - 4 = 6$

33. $x - 8 = 3$

34. $x + 11 = 15$

35. $x - 8 = -10$

36. $x + 5 = 2$

37. $x + 4 = -3$

38. $x - 5 = -4$

39. $11 = x + 5$

40. $x + 7 = 0$

41. $4x = 3x + 4$

42. $7x = 6x - 8$

43. $11x = 10x - 10$

44. $9x = 8x + 5$

45. $6x + 3 = 5x$

46. $12x - 6 = 11x$

47. $8x - 4 = 7x$

48. $9x - 7 = 8x$

49. $2x + 3 = x + 5$ > Videos

50. $3x - 2 = 2x + 1$

51. $5x - 7 = 4x - 3$

52. $8x + 5 = 7x - 2$

53. $7x - 2 = 6x + 4$

54. $10x - 3 = 9x - 6$

55. $3 + 6x + 2 = 3x + 11 + 2x$

56. $6x - 3 + 2x = 7x + 8$

57. $4x + 7 + 3x = 5x + 13 + x$

58. $5x + 9 + 4x = 9 + 8x - 7$

59. $3x - 5 + 2x - 7 + x = 5x + 2$

60. $5x + 8 + 3x - x + 5 = 6x - 3$

61. **CRAFTS** Jeremiah had found 50 bones for a Halloween costume. In order to complete his 62-bone costume, how many more does he need?

Let b be the number of bones he needs, and use the equation $b + 50 = 62$ to solve the problem.

62. **BUSINESS AND FINANCE** Four hundred tickets to the opening of an art exhibit were sold. General admission tickets cost \$5.50, whereas students were only required to pay \$4.50 for tickets. If total ticket sales were \$1,950, how many of each type of ticket were sold?

Let x be the number of general admission tickets sold and $400 - x$ be the number of student tickets sold. Use the equation $5.5x + 4.5(400 - x) = 1,950$ to solve the problem.

Answers

31. ________ 32. ________
33. ________ 34. ________
35. ________ 36. ________
37. ________ 38. ________
39. ________ 40. ________
41. ________ 42. ________
43. ________ 44. ________
45. ________ 46. ________
47. ________ 48. ________
49. ________
50. ________
51. ________
52. ________
53. ________
54. ________
55. ________
56. ________
57. ________
58. ________
59. ________
60. ________
61. ________
62. ________

Answers

63.

64.

65.

66.

67.

68.

69.

70.

71.

72.

73.

74.

75.

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77.

78.

63. **BUSINESS AND FINANCE** A shop pays $2.25 for each copy of a magazine and sells the magazines for $3.25 each. If the fixed costs associated with the sale of these magazines are $50 per month, how many must the shop sell in order to realize $175 in profit from the magazines?

Let m be the number of magazines the shop must sell, and use the equation $3.25m - 2.25m - 50 = 175$ to solve the problem.

chapter 11 > Make the Connection

64. **NUMBER PROBLEM** The sum of a number and 15 is 22. Find the number.

Let x be the number and solve the equation $x + 15 = 22$ to find the number.

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65. Which equation is equivalent to $8x + 5 = 9x - 4$?

(a) $17x = -9$ **(b)** $x = -9$

(c) $8x + 9 = 9x$ **(d)** $9 = 17x$

66. Which equation is equivalent to $5x - 7 = 4x - 12$?

(a) $9x = 19$ **(b)** $9x - 7 = -12$

(c) $x = -18$ **(d)** $x - 7 = -12$

67. Which equation is equivalent to $12x - 6 = 8x + 14$?

(a) $4x - 6 = 14$ **(b)** $x = 20$

(c) $20x = 20$ **(d)** $4x = 8$

68. Which equation is equivalent to $7x + 5 = 12x - 10$?

(a) $5x = -15$ **(b)** $7x - 5 = 12x$

(c) $-5 = 5x$ **(d)** $7x + 15 = 12x$

True *or* **false?**

69. Every linear equation with one variable has exactly one solution.

70. Isolating the variable on the right side of the equation results in a negative solution.

Solve and check each equation.

71. $4(3x + 4) = 11x - 2$

72. $2(5x - 3) = 9x + 7$

73. $3(7x + 2) = 5(4x + 1) + 17$

74. $5(5x + 3) = 3(8x - 2) + 4$

75. $\frac{5}{4}x - 1 = \frac{1}{4}x + 7$

76. $\frac{7}{5}x + 3 = \frac{2}{5}x - 8$

77. $\frac{9}{2}x - \frac{3}{4} = \frac{7}{2}x + \frac{5}{4}$

78. $\frac{11}{3}x + \frac{1}{6} = \frac{8}{3}x + \frac{19}{6}$

79. **CONSTRUCTION TECHNOLOGY** K-Jones Manufacturing produces hex bolts and carriage bolts.

(a) They sold 284 more cases of hex bolts than carriage bolts last month. Write an equation for the number of cases of carriage bolts sold last month given the number of cases of hex bolts sold h.

(b) If they sold 2,680 carriage bolts, how many cases of hex bolts did they sell?

80. **ENGINEERING TECHNOLOGY** The specifications for an engine cylinder of a particular ship calls for the stroke length to be two more than twice the diameter of the cylinder.

(a) Write an equation for the required stroke length given a cylinder's diameter d.

(b) Find the stroke length specified for a cylinder with a 52 in. diameter.

81. **ELECTRONICS TECHNOLOGY** Berndt Electronics earns a marginal profit of $560 on the sale of each server. If other costs amount to $4,500, will it earn a profit of at least $5,000 on the sale of 15 servers?

82. **INFORMATION TECHNOLOGY** A student has saved $2,350 for a computer setup costing $3,675. How much more must the student save?

Answers

79. ____________________

80. ____________________

81. ____________________

82. ____________________

Answers

1. Yes **3.** No **5.** No **7.** Yes **9.** No **11.** No **13.** Yes
15. Yes **17.** Yes **19.** No **21.** Yes **23.** Linear equation
25. Expression **27.** Expression **29.** Linear equation **31.** 2
33. $x = 11$ **35.** $x = -2$ **37.** $x = -7$ **39.** $x = 6$ **41.** $x = 4$
43. $x = -10$ **45.** $x = -3$ **47.** $x = 4$ **49.** $x = 2$ **51.** $x = 4$
53. $x = 6$ **55.** $x = 6$ **57.** $x = 6$ **59.** $x = 14$ **61.** 12
63. 225 **65.** (c) **67.** (a) **69.** True **71.** $x = -18$
73. $x = 16$ **75.** $x = 8$ **77.** $x = 2$ **79.** **(a)** $c = h - 284$;
(b) 2,964 cases **81.** No

Activity 32 ::
Graphing Solutions

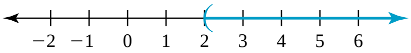
You have now solved many equations using algebra. Often, it is convenient to present a picture of the solutions for an equation instead of giving a set of numbers. One method to present a picture uses the familiar number line.

1. Plot the points $\{-2, 0.5, 3\}$ on a number line.

2. Solve the equation $x + 5 = 8$.

3. Plot the solution to the equation in exercise 2 on a number line.

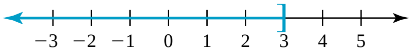
We often use a number line to present sets of numbers. For example, the set of numbers greater than 2 is written algebraically as $\{x \mid x > 2\}$, and is shown on a number line as



NOTE

We read $\{x \mid x > 2\}$ as "the set of every value of x for which x is greater than 2."

Whereas the set of numbers less than or equal to 3 is written as $\{x \mid x \le 3\}$ and shown on a number line as



4. Graph the set of numbers $\{x \mid x < 3\}$ on a number line.

5. Graph the set of numbers $\{x \mid x \ge -1\}$ on a number line.

6. Solve the inequality $x + 2 > 0$.

7. Graph every solution to exercise 6 on a number line.

11.5 Using the Multiplication Property to Solve an Equation

< 11.5 Objectives >

1 > Use the multiplication property to solve an equation

2 > Combine like terms before solving an equation

3 > Use algebra to solve percent problems

NOTE

Subtracting 6 from both sides yields $6x - 6 = 12$

In this section, we look at a different type of equation. What if we wanted to solve the following equation?

$$6x = 18$$

The addition property that you just learned does not help us in this situation. We need a second property for solving such an equation.

Property

The Multiplication Property of Equality

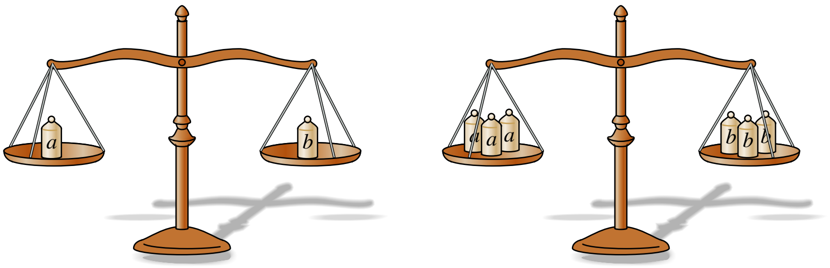
If $a = b$ then $ac = bc$ when $c \neq 0$

In words, multiplying both sides of an equation by the same nonzero number gives an equivalent equation.

NOTES

Do you see why the number cannot be 0? Multiplying by 0 gives $0 = 0$. We have lost the variable!

Again, as long as you do the *same* thing to *both* sides of an equation, the "balance" is maintained.



We work through some examples using this rule.

Example 1 Solving Equations by Using the Multiplication Property

< Objective 1 >

RECALL

Multiplying both sides by $\frac{1}{6}$ is equivalent to dividing both sides by 6.

Solve

$$6x = 18$$

Here the variable x is multiplied by 6. So we apply the multiplication property and multiply both sides by $\frac{1}{6}$. Keep in mind that we want an equation of the form

$$x = \square$$

$$6x = 18$$

$$\frac{1}{6}(6x) = \frac{1}{6}(18)$$

$$\frac{1}{6} \cdot \frac{6x}{1} = \frac{1}{6} \cdot \frac{18}{1}$$

$$\frac{\cancel{6}x}{\cancel{6}} = \frac{\cancel{18}^{3}}{\cancel{6}_{1}} \qquad 18 \div 6 = 3$$

$$x = 3$$

NOTE

$\frac{1}{6}(6x) = \frac{\cancel{6}x}{\cancel{6}} = x$

We then have x alone on the left, which is what we want.

The solution is 3. To check, replace x with 3:

$$6(3) \overset{?}{=} 18$$

$$18 = 18 \qquad \text{(True)}$$

Check Yourself 1

Solve and check.

$8x = 32$

Example 2 Solving Equations by Using the Multiplication Property

Solve

$$-9x = 54$$

$$\frac{-9x}{-9} = \frac{54}{-9}$$

$$x = -6$$

NOTES

Because division is defined in terms of multiplication, we can also divide both sides of an equation by the same nonzero number.

In this case, x is multiplied by -9, so we divide both sides by -9 to isolate x on the left.

The solution is -6. To check:

$$(-9)(-6) \overset{?}{=} 54$$

$$54 = 54 \qquad \text{(True)}$$

Check Yourself 2

Solve and check.

$-10x = -60$

Example 3 illustrates the use of the multiplication property when fractions appear in an equation.

Example 3 Solving Equations by Using the Multiplication Property

(a) Solve

$$\frac{x}{3} = 6$$

Here x is *divided* by 3. We use multiplication to isolate x.

$$3\left(\frac{x}{3}\right) = 3 \cdot 6$$

$$x = 18$$

This leaves x alone on the left because

$3\left(\frac{x}{3}\right) = \frac{3}{1} \cdot \frac{x}{3} = \frac{x}{1} = x$

NOTE

$\frac{x}{3} = \frac{1}{3}x$

To check:

$$\frac{(18)}{3} \overset{?}{=} 6$$

$$6 = 6 \qquad \text{(True)}$$

NOTE

$\frac{x}{5} = \frac{1}{5}x$

(b) Solve

$$\frac{x}{5} = -9$$

$$5\left(\frac{x}{5}\right) = 5(-9)$$

Because *x* is divided by 5, multiply both sides by 5.

$$x = -45$$

The solution is −45. To check, we replace *x* with −45:

$$\frac{(-45)}{5} \overset{?}{=} -9$$

$$-9 = -9 \quad \text{(True)}$$

The solution is verified.

Check Yourself 3

Solve and check.

(a) $\frac{x}{7} = 3$ **(b)** $\frac{x}{4} = -8$

When the variable is multiplied by a fraction with a numerator other than 1, there are two approaches to finding the solution.

Example 4 Solving Equations by Using Reciprocals

Solve

$$\frac{3}{5}x = 9$$

One approach is to multiply by 5 as the first step.

$$5\left(\frac{3}{5}x\right) = 5 \cdot 9$$

$$3x = 45$$

Now we divide by 3.

$$\frac{3x}{3} = \frac{45}{3}$$

$$x = 15$$

To check:

$$\frac{3}{5}(15) \overset{?}{=} 9$$

$$9 = 9 \quad \text{(True)}$$

A second approach combines the multiplication and division steps and is generally more efficient. We multiply by $\frac{5}{3}$.

$$\frac{5}{3}\left(\frac{3}{5}x\right) = \frac{5}{3} \cdot 9$$

$$x = \frac{5}{\underset{1}{\cancel{3}}} \cdot \frac{\overset{3}{\cancel{9}}}{1} = 15$$

So $x = 15$, as before.

NOTE

Recall that $\frac{5}{3}$ is the *reciprocal* of $\frac{3}{5}$, and the product of a number and its reciprocal is just 1! So

$\left(\frac{5}{3}\right)\left(\frac{3}{5}\right) = 1$

Check Yourself 4

Solve and check.

$$\frac{2}{3}x = 18$$

You may sometimes need to simplify an equation before applying the methods of this section. Example 5 illustrates this situation.

Example 5 Combining Like Terms and Solving Equations

< Objective 2 >

Solve and check

$$3x + 5x = 40$$

Using the distributive property, we can combine the like terms on the left to write

$$8x = 40$$

We can now proceed as before.

$$\frac{8x}{8} = \frac{40}{8} \qquad \text{Divide by 8.}$$

$$x = 5$$

The solution is 5. To check, we return to the original equation. Substituting 5 for x yields

$$3(5) + 5(5) \stackrel{?}{=} 40$$
$$15 + 25 \stackrel{?}{=} 40$$
$$40 = 40 \qquad \text{(True)}$$

Check Yourself 5

Solve and check.

$$7x + 4x = -66$$

RECALL

In percent problems, A is the amount, B is the base, and R is the rate.

RECALL

In Section 6.3, you learned to identify the base, rate, and amount in a percent problem.

As with the addition property, many applications require the multiplication property. One of the most useful set of applications involves percent problems.

In Section 6.4, you learned to use the percent relationship

$$\frac{A}{B} = R$$

to solve percent problems. We did this by writing the percent relationship as a proportion in which $R = \dfrac{r}{100}$.

$$\frac{A}{B} = \frac{r}{100}$$

We then used the proportion rule to rewrite the equation.

$$100A = rB$$

So, for instance, if the question asked us to find 45% of 80, we would identify $r = 45$ and $B = 80$.

$$100A = (45)(80)$$
$$100A = 3{,}600$$

The next step was to divide both sides by the coefficient of the variable, just as we have been doing throughout this section.

$$\frac{100A}{100} = \frac{3{,}600}{100}$$
$$A = 36$$

So, 45% of 80 is 36.

We can simplify this process by using algebra. First, we can rewrite the percent relationship by multiplying both sides by the base.

NOTE

This says that the amount is equal to the product of the base and the rate.

$$\frac{A}{B} = R$$
$$\cancel{B}\left(\frac{A}{\cancel{B}}\right) = R \cdot B$$
$$A = RB$$

This form is especially useful if we write the rate as a decimal rather than a fraction.

Example 6 Using Algebra to Solve a Percent Problem

< Objective 3 >

(a) What is 75% of 360?

The rate is given as 75%, which we write as a decimal, $R = 0.75$. Translating the question, we have

$$A = RB$$
$$A = (0.75)(360)$$
$$A = 270$$

75% of 360 is 270.

Alternatively, we can translate the question directly into an algebraic equation.

$$\begin{array}{ccccc} \text{What} & \text{is} & 75\% & \text{of} & 360? \\ \uparrow & \uparrow & \uparrow & \uparrow & \uparrow \\ A & = & 0.75 & \text{Multiplication} & 360 \end{array}$$

Which gives,

$$A = 0.75 \cdot 360$$
$$= 270$$

(b) What percent of 246 is 342?

This time, we begin by translating the question directly into an algebraic equation.

NOTE

The base is 246 and the amount is 342.

$$\begin{array}{ccccc} \text{What percent} & \text{of} & 246 & \text{is} & 342? \\ \uparrow & \uparrow & \uparrow & \uparrow & \uparrow \\ R & \text{Multiplication} & 246 & = & 342 \end{array}$$

This gives us

$$R \cdot 246 = 342$$
$$\text{or} \quad 246R = 342$$

RECALL

Because the amount is larger than the base, the rate is greater than 100%.

We can solve this equation using the methods of this section.

$$\frac{\cancel{246}R}{\cancel{246}} = \frac{342}{246}$$

$$R = \frac{342}{246}$$

$$= \frac{57}{41} \qquad \text{The GCF of 246 and 342 is 6.}$$

$$\approx 1.39$$

$$= 139\% \qquad \text{Move the decimal two places to the right and attach the percent symbol.}$$

Therefore, 342 is about 139% of 246.

Check Yourself 6

Use algebra to solve each percent application.

(a) 240 is what percent of 400?
(b) 57 is 30% of what number?

Of course, we can use algebra to solve applications that involve percents, as well.

Example 7 Solving a Percent Application with Algebra

A saleswoman earns a 5% commission on her sales. If she wants to earn $1,800 in commissions in one month, how much does she need to sell?

The question we are being asked is, “$1,800 is 5% of what number?” We can translate the question into an algebraic equation by writing the rate in decimal form, 5% = 0.05.

$$1{,}800 = 0.05x$$

$$\frac{1{,}800}{0.05} = \frac{0.05x}{0.05}$$

$$36{,}000 = x$$

She must sell $36,000 in order to earn $1,800 in commissions.

Check Yourself 7

Patrick pays $525 interest for a 1-year loan at 10.5%. What was the amount of his loan?

We can solve many types of problems with algebra besides percent problems.

Example 8 An Application Involving the Multiplication Property

On his first day on the job in a photography lab, Samuel processed all the film given to him. The following day, his boss gave him four times as much film to process. Over the two days, he processed 60 rolls of film. How many rolls did he process on the first day?

Let x be the number of rolls Samuel processed on his first day and solve the equation $x + 4x = 60$ to answer the question.

$$
\begin{aligned}
x + 4x &= 60 \\
5x &= 60 && \text{Combine like terms first.} \\
\frac{1}{5}(5x) &= \frac{1}{5}(60) && \text{Multiply by } \frac{1}{5}\text{, to isolate the variable.} \\
x &= 12
\end{aligned}
$$

Samuel processed 12 rolls of film on his first day.

RECALL

You should always use a sentence to give the answer to an application.

NOTE

The yen (¥) is the monetary unit of Japan.

Check Yourself 8

On a recent trip to Japan, Marilyn exchanged $1,200 and received 139,812 yen. What exchange rate did she receive?

Let x be the exchange rate and solve the equation $1{,}200x = 139{,}812$ to answer the question (to the nearest hundredth).

Check Yourself ANSWERS

1. $x = 4$ **2.** $x = 6$ **3.** **(a)** $x = 21$; **(b)** $x = -32$ **4.** $x = 27$
5. $x = -6$ **6.** **(a)** 60%; **(b)** 190 **7.** $5,000
8. Marilyn received 116.51 yen for each dollar.

Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 11.5

(a) Multiplying both sides of an equation by the same nonzero number yields an ______________ equation.

(b) Always return to the ______________ equation to check a solution.

(c) Multiplying an equation by 5 is the same as ______________ by $\frac{1}{5}$.

(d) The product of a nonzero number and its ______________ is 1.

11.5 exercises

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Section __________ Date __________

Answers

1. __________ 2. __________
3. __________ 4. __________
5. __________ 6. __________
7. __________ 8. __________
9. __________ 10. __________
11. __________ 12. __________
13. __________ 14. __________
15. __________ 16. __________
17. __________ 18. __________
19. __________ 20. __________
21. __________ 22. __________
23. __________ 24. __________
25. __________ 26. __________
27. __________ 28. __________
29. __________ 30. __________
31. __________ 32. __________
33. __________ 34. __________

< Objective 1 >

Solve and check.

1. $5x = 20$ > Videos

2. $6x = 30$

3. $9x = 54$

4. $6x = -42$

5. $63 = 9x$

6. $66 = 6x$

7. $4x = -16$

8. $-3x = 27$

9. $-9x = 72$

10. $10x = -100$

11. $6x = -54$

12. $-7x = 49$

13. $-4x = -12$ > Videos

14. $52 = -4x$

15. $-42 = 6x$

16. $-7x = -35$

17. $-6x = -54$

18. $-4x = -24$

19. $\frac{x}{2} = 4$ > Videos

20. $\frac{x}{3} = 2$

21. $\frac{x}{5} = 3$

22. $\frac{x}{8} = 5$

23. $6 = \frac{x}{7}$

24. $6 = \frac{x}{3}$

25. $\frac{x}{5} = -4$

26. $\frac{x}{7} = -5$

27. $-\frac{x}{3} = 8$

28. $-\frac{x}{8} = -3$

29. $\frac{2}{3}x = 6$ > Videos

30. $\frac{4}{5}x = 8$

31. $\frac{3}{4}x = -15$

32. $\frac{7}{8}x = -21$

33. $-\frac{2}{5}x = 10$ > Videos

34. $-\frac{5}{6}x = -15$

< **Objective 2** >

35. $5x + 4x = 36$

36. $8x - 3x = -50$

37. $16x - 9x = -42$

38. $5x + 7x = 60$

39. $4x - 2x + 7x = 36$

40. $6x + 7x - 5x = -48$

< **Objective 3** >

Solve each percent problem.

41. What is 65% of 300?

42. 15% of 140 is what number?

43. Find 80% of 80.

44. What is 6% of 550?

45. What percent of 220 is 66?

46. 104 is what percent of 260?

47. 102 is what percent of 85?

48. What percent of 130 is 299?

49. 15 is 4% of what number?

50. 16% of what number is 24?

51. Find the base if 240% of the base is 36.

52. Find the base if 375% of the base is 600.

Solve each application.

53. **BUSINESS AND FINANCE** Roberto has 26% of his pay withheld for deductions. If he earns $550 per week, what amount is withheld?

chapter 11 > Make the Connection

54. **BUSINESS AND FINANCE** A real estate agent's commission rate is 6%. What is the amount of commission on the sale of a $185,000 home?

chapter 11 > Make the Connection

55. **SOCIAL SCIENCE** Of the 60 people who started a training program, 45 were successful. What is the dropout rate?

56. **BUSINESS AND FINANCE** In a shipment of 250 parts, 40 are found to be defective. What percent of the shipment is in good working order?

57. **SCIENCE AND MEDICINE** There are 117 mL of acid in 900 mL of a solution (acid and water). What percent of the solution is water?

58. **STATISTICS** Marla needs to answer 70% of the questions correctly on her final exam in order to receive a C for the course. If the exam has 120 questions, how many can she miss?

Answers

35. ________ 36. ________

37. ________ 38. ________

39. ________ 40. ________

41. ________ 42. ________

43. ________

44. ________

45. ________

46. ________

47. ________

48. ________

49. ________

50. ________

51. ________

52. ________

53. ________

54. ________

55. ________

56. ________

57. ________

58. ________

Answers

59. __________

60. __________

61. __________

62. __________

63. __________

64. __________

65. __________

66. __________

67. __________

68. __________

69. __________

70. __________

71. __________

72. __________

73. __________

74. __________

75. __________

76. __________

59. CRAFTS Returning from Mexico City, Sung-A exchanged her remaining 450 pesos for \$41.70. What exchange rate did she receive?

Use the equation $450x = 41.70$ to solve this problem (round to the nearest thousandth).

60. BUSINESS AND FINANCE Upon arrival in Portugal, Nicolas exchanged \$500 and received 417.35 euros (€). What exchange rate did he receive?

Use the equation $500x = 417.35$ to solve this problem (round to the nearest hundredth).

61. SCIENCE AND TECHNOLOGY On Tuesday, there were twice as many patients in the clinic as on Monday. Over the two-day period, 48 patients were treated. How many patients were treated on Monday?

Let p be the number of patients that came in on Monday and use the equation $p + 2p = 48$ to answer the question.

62. NUMBER PROBLEM Two-thirds of a number is 46. Find the number.

Use the equation $\frac{2}{3}x = 46$ to solve the problem.

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Certain equations involving decimals can be solved by the methods of this section. For instance, to solve $2.3x = 6.9$, we use the multiplication property to divide both sides of the equation by 2.3. This isolates x on the left, as desired. Use this idea to solve each equation.

63. $3.2x = 12.8$

64. $5.1x = -15.3$

65. $-4.5x = 13.5$

66. $-8.2x = -32.8$

67. $1.3x + 2.8x = 12.3$

68. $2.7x + 5.4x = -16.2$

69. $9.3x - 6.2x = 12.4$

70. $12.5x - 7.2x = -21.2$

Basic Skills | Challenge Yourself | **Calculator/Computer** | Career Applications | Above and Beyond

Use your calculator to solve each equation. Round your answers to the nearest hundredth.

71. $230x = 157$

72. $31x = -15$

73. $-29x = 432$

74. $-141x = -3{,}467$

75. $23.12x = 94.6$

76. $46.1x = -1$

Basic Skills | Challenge Yourself | Calculator/Computer | **Career Applications** | Above and Beyond

77. **INFORMATION TECHNOLOGY** A 50-Gbyte-capacity hard drive contains 30 Gbytes of used space. What percent of the hard drive is full?

78. **INFORMATION TECHNOLOGY** A compression program reduces the size of files and folders by 36%. If a folder contains 17.5 Mbytes, how large will it be after it is compressed?

79. **AUTOMOTIVE TECHNOLOGY** It is estimated that 8% of rebuilt alternators do not last through the 90-day warranty period. If a parts store had six bad alternators returned during the year, how many did it sell?

chapter 11 > Make the Connection

80. **AGRICULTURAL TECHNOLOGY** A farmer sold 2,200 bushels of barley on the futures market. Because of a poor harvest, he was only able to make 94% of his bid. How many bushels did he actually harvest?

chapter 11 > Make the Connection

Basic Skills | Challenge Yourself | Calculator/Computer | Career Applications | **Above and Beyond**

81. Describe the difference between the multiplication property and the addition property for solving equations. Give examples of when to use each property.

82. Describe when you should add a quantity to or subtract a quantity from both sides of an equation as opposed to when you should multiply or divide both sides by the same quantity.

Motors, Windings, and More! sells every motor, regardless of type, for \$2.50. *This vendor also has a deal in which customers can choose whether to receive a markdown or free shipping. Shipping costs are* \$1.00 *per item. If you do not choose the free shipping option, you can deduct* 17.5% *from your total order (but not the cost of shipping).*

83. If you buy six motors, calculate the total cost for each of the two options. Which option is cheaper?

84. Is one option *always* cheaper than the other? Justify your result.

Answers

1. $x = 4$ **3.** $x = 6$ **5.** $x = 7$ **7.** $x = -4$ **9.** $x = -8$
11. $x = -9$ **13.** $x = 3$ **15.** $x = -7$ **17.** $x = 9$ **19.** $x = 8$
21. $x = 15$ **23.** $x = 42$ **25.** $x = -20$ **27.** $x = -24$ **29.** $x = 9$
31. $x = -20$ **33.** $x = -25$ **35.** $x = 4$ **37.** $x = -6$ **39.** $x = 4$
41. 195 **43.** 64 **45.** 30% **47.** 120% **49.** 375 **51.** 15
53. \$143 **55.** 25% **57.** 87% **59.** 0.093 dollars per peso
61. 16 patients **63.** $x = 4$ **65.** $x = -3$ **67.** $x = 3$ **69.** $x = 4$
71. 0.68 **73.** −14.9 **75.** 4.09 **77.** 60% **79.** 75 alternatives
81. Above and Beyond **83.** Above and Beyond

Answers

77. ____________________

78. ____________________

79. ____________________

80. ____________________

81. ____________________

82. ____________________

83. ____________________

84. ____________________

11.6 Combining the Properties to Solve Equations

< **11.6 Objectives** >

1 > Solve an equation using both the addition and multiplication properties

2 > Combine like terms first, and then solve an equation using both properties

In our examples in Sections 11.4 and 11.5, either the addition property or the multiplication property was used in solving an equation. Often, finding a solution requires the use of both properties.

Example 1 Solving Equations

< Objective 1 >

> **CAUTION**

Use the addition property before applying the multiplication property. That is, do not divide by 4 until after you have added 5!

(a) Solve

$$4x - 5 = 7$$

Here x is *multiplied* by 4. The result, $4x$, then has 5 subtracted from it on the left side of the equation. These two operations mean that both properties must be applied in solving the equation.

Because there is only one variable term, we start by adding 5 to both sides:

$$4x - 5 + 5 = 7 + 5 \qquad \text{or} \qquad 4x = 12$$

The first step is to *isolate* the variable term on one side of the equation.

We now divide both sides by 4:

$$\frac{4x}{4} = \frac{12}{4}$$

$$x = 3$$

Next, *isolate* the variable.

The solution is 3. To check, replace x with 3 in the original equation. Be careful to follow the rules for the order of operations.

$$4(3) - 5 \stackrel{?}{=} 7$$

$$12 - 5 \stackrel{?}{=} 7$$

$$7 = 7$$

(True)

NOTES

Isolate the variable term.

Isolate the variable.

(b) Solve

$$3x + 8 = -4$$

$$3x + 8 - 8 = -4 - 8$$

Subtract 8 from both sides.

$$3x = -12$$

Now divide both sides by 3 to isolate x on the left.

$$\frac{3x}{3} = \frac{-12}{3}$$

$$x = -4$$

The solution is -4. We leave it to you to check this result.

Check Yourself 1

Solve and check.

(a) $6x + 9 = -15$

(b) $5x - 8 = 7$

The variable may appear in any position in an equation. Just apply the rules carefully as you try to write an equivalent equation, and you will find the solution. Example 2 illustrates this property.

Example 2 Solving Equations

Solve

$$\begin{aligned} 3 - 2x &= 9 \\ 3 - 3 - 2x &= 9 - 3 \\ -2x &= 6 \end{aligned}$$

First subtract 3 from both sides.

Now divide both sides by -2. This leaves x alone on the left.

$$\frac{-2x}{-2} = \frac{6}{-2}$$

$$x = -3$$

The solution is -3. We leave it to you to check this result.

NOTE

$\frac{-2}{-2} = 1$, so we divide by -2 to isolate x.

Check Yourself 2

Solve and check.

$10 - 3x = 1$

You may also have to combine multiplication with addition or subtraction to solve an equation. Consider Example 3.

Example 3 Solving Equations

(a) Solve

$$\frac{x}{5} - 3 = 4$$

To get the x term alone, we first add 3 to both sides.

$$\begin{aligned} \frac{x}{5} - 3 + 3 &= 4 + 3 \\ \frac{x}{5} &= 7 \end{aligned}$$

To undo the division, multiply both sides of the equation by 5.

$$5\left(\frac{x}{5}\right) = 5 \cdot 7$$

$$x = 35$$

The solution is 35. Just return to the original equation to check the result.

$$\begin{aligned} \frac{(35)}{5} - 3 &\overset{?}{=} 4 \\ 7 - 3 &\overset{?}{=} 4 \\ 4 &= 4 \end{aligned}$$

(True)

(b) Solve

$$\frac{2}{3}x + 5 = 13$$

$$\frac{2}{3}x + 5 - 5 = 13 - 5$$ First subtract 5 from both sides.

$$\frac{2}{3}x = 8$$

Now multiply both sides by $\frac{3}{2}$, the reciprocal of $\frac{2}{3}$.

$$\left(\frac{3}{2}\right)\left(\frac{2}{3}x\right) = \left(\frac{3}{2}\right)8$$

or

$$x = 12$$

The solution is 12. We leave it to you to check this result.

Check Yourself 3

Solve and check.

(a) $\frac{x}{6} + 5 = 3$ **(b)** $\frac{3}{4}x - 8 = 10$

In Section 11.4, you learned how to solve certain equations when the variable appeared on both sides. Example 4 shows you how to extend that work by using the multiplication property of equality.

Example 4 Combining Properties to Solve an Equation

Solve

$$6x - 4 = 3x - 2$$

We begin by bringing all the variable terms to one side. To do this, we subtract $3x$ from both sides. This eliminates the variable term from the right side.

$$\begin{aligned} 6x - 4 &= 3x - 2 \\ 6x - 4 - 3x &= 3x - 2 - 3x \\ 3x - 4 &= -2 \end{aligned}$$

We now isolate the variable term by adding 4 to both sides.

$$\begin{aligned} 3x - 4 &= -2 \\ 3x - 4 + 4 &= -2 + 4 \\ 3x &= 2 \end{aligned}$$

Finally, divide by 3.

$$\frac{3x}{3} = \frac{2}{3}$$

$$x = \frac{2}{3}$$

Check:

$$\begin{aligned} 6\left(\frac{2}{3}\right) - 4 &\overset{?}{=} 3\left(\frac{2}{3}\right) - 2 \\ 4 - 4 &\overset{?}{=} 2 - 2 \\ 0 &= 0 \qquad \text{(True)} \end{aligned}$$

The basic idea is to use our two properties to form an equivalent equation with the x isolated. Here we subtracted $3x$ and then added 4. You can do these steps in either order. Try it for yourself the other way. In either case, the multiplication property is then used as the *last step* in finding the solution.

Check Yourself 4

Solve and check.

$7x - 5 = 3x + 5$

Here are two approaches to solving equations in which the coefficient on the right side is greater than the coefficient on the left side.

Example 5 Combining Properties to Solve an Equation (Two Methods)

Solve $4x - 8 = 7x + 7$.

Method 1

$$\begin{aligned} 4x - 8 - 7x &= 7x + 7 - 7x && \text{Bring the variable terms to the same (left) side.} \\ -3x - 8 &= 7 \\ -3x - 8 + 8 &= 7 + 8 && \text{Isolate the variable term.} \\ -3x &= 15 \\ \frac{-3x}{-3} &= \frac{15}{-3} && \text{Isolate the variable.} \\ x &= -5 \end{aligned}$$

We let you check this result.

To avoid a negative coefficient (-3, in this example), some students prefer a different approach.

This time we work toward having the number on the *left* and the x term on the *right*, or $\square = x$.

Method 2

$$\begin{aligned} 4x - 8 &= 7x + 7 \\ 4x - 8 - 4x &= 7x + 7 - 4x && \text{Bring the variable terms to the same (right) side.} \\ -8 &= 3x + 7 \\ -8 - 7 &= 3x + 7 - 7 && \text{Isolate the variable term.} \\ -15 &= 3x \\ \frac{-15}{3} &= \frac{3x}{3} && \text{Isolate the variable.} \\ -5 &= x \end{aligned}$$

NOTE

It is usually easier to isolate the variable term on the side that results in a positive coefficient.

Because $-5 = x$ and $x = -5$ are equivalent equations, it really makes no difference; the solution is still -5! You may use whichever approach you prefer.

Check Yourself 5

Solve $5x + 3 = 9x - 21$ by finding equivalent equations of the form $x = \square$ and $\square = x$ to compare the two methods of finding the solution.

When possible, we start by combining like terms on each side of the equation.

Example 6 Combining Terms to Solve an Equation

< Objective 2 >

Solve.

$$\begin{aligned} 7x - 3 + 5x + 4 &= 6x + 25 && \text{Start by combining like terms.} \\ 12x + 1 &= 6x + 25 \\ 12x + 1 - 6x &= 6x + 25 - 6x && \text{Bring the variables to one side.} \\ 6x + 1 &= 25 \\ 6x + 1 - 1 &= 25 - 1 && \text{Isolate the variable term.} \\ 6x &= 24 \\ \frac{6x}{6} &= \frac{24}{6} && \text{Isolate the variable.} \\ x &= 4 \end{aligned}$$

The solution is 4. We leave the check to you.

Check Yourself 6

Solve and check.

$9x - 6 - 3x + 1 = 2x + 15$

It may also be necessary to remove grouping symbols to solve an equation. Example 7 illustrates this property.

Example 7 Solving Equations That Contain Parentheses

NOTE

$5(x - 3)$
$= 5[x + (-3)]$
$= 5x + 5(-3)$
$= 5x + (-15)$
$= 5x - 15$

Solve and check.

$$\begin{aligned} 5(x - 3) - 2x &= x + 7 && \text{Apply the distributive property.} \\ 5x - 15 - 2x &= x + 7 && \text{Combine like terms.} \\ 3x - 15 &= x + 7 \end{aligned}$$

We now have an equation that we can solve by the usual methods. First, bring the variable terms to one side, then isolate the variable term, and finally, isolate the variable.

$$\begin{aligned} 3x - 15 - x &= x + 7 - x && \text{Subtract } x \text{ to bring the variable terms to the same side.} \\ 2x - 15 &= 7 \\ 2x - 15 + 15 &= 7 + 15 && \text{Add 15 to isolate the variable term.} \\ 2x &= 22 \\ \frac{2x}{2} &= \frac{22}{2} && \text{Divide by 2 to isolate the variable.} \\ x &= 11 \end{aligned}$$

The solution is 11. To check, substitute 11 for x in the original equation. Again note the use of our rules for the order of operations.

$5[(11) - 3] - 2(11) \overset{?}{=} (11) + 7$ Simplify terms in parentheses.

$5 \cdot 8 - 2 \cdot 11 \overset{?}{=} 11 + 7$ Multiply.

$40 - 22 \overset{?}{=} 11 + 7$ Add and subtract.

$18 = 18$ A true statement.

Check Yourself 7

Solve and check.

$7(x + 5) - 3x = x - 7$

We say that an equation is “solved” when we have an equivalent equation of the form

$x = \square$ or $\square = x$ in which $\square$ is some number

The steps of solving a linear equation are as follows:

Step by Step

To Solve a Linear Equation

Step 1 Use the distributive property to remove any grouping symbols.

Step 2 Combine like terms on each side of the equation.

Step 3 Add or subtract variable terms to bring the variable term to one side of the equation.

Step 4 Add or subtract numbers to isolate the variable term.

Step 5 Multiply by the reciprocal of the coefficient to isolate the variable.

Step 6 Check your result.

There are a host of applications involving linear equations.

Example 8 Applying Algebra

In an election, the winning candidate had 160 more votes than the loser did. If the total number of votes cast was 3,260, how many votes did each candidate receive?

We first set up the problem. Let x represent the number of votes received by the loser. Then the winner received $x + 160$ votes.

We can set up an equation by adding the number of votes the candidates received. This must total 3,260.

$x + (x + 160) = 3{,}260$ Remove the parentheses and combine like terms.

$2x + 160 = 3{,}260$ Subtract 160 from both sides.

$2x = 3{,}100$ Divide both sides by 2.

$x = 1{,}550$

The loser received 1,550 votes. Therefore, the winner received $x + 160 = 1{,}550 + 160 = 1{,}710$ votes.

Check Yourself 8

The Randolphs used 12 more gallons (gal) of fuel oil in October than in September and twice as much oil in November as in September. If they used 132 gal for the 3 months, how much was used each month?

Check Yourself ANSWERS

1. **(a)** $x = -4$; **(b)** $x = 3$ **2.** $x = 3$ **3.** **(a)** $x = -12$; **(b)** $x = 24$

4. $x = \frac{5}{2}$ **5.** $x = 6$ **6.** $x = 5$ **7.** $x = -14$

8. 30 gal in September, 42 gal in October, 60 gal in November

Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 11.6

(a) The first goal for solving an equation is to ________________ the variable term on one side of the equation.

(b) Apply the ________________ property before applying the multiplication property.

(c) Always return to the ________________ equation to check your result.

(d) An equation in the form $x = \square$ or $\square = x$ has been ________________.

11.6 exercises

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Name ____________________

Section __________ Date __________

< Objective 1 >

Solve and check.

1. $2x + 1 = 9$ > Videos

2. $3x - 1 = 17$

3. $3x - 2 = 7$

4. $5x + 3 = 23$

5. $4x + 7 = 35$

6. $7x - 8 = 13$

7. $2x + 9 = 5$

8. $6x + 25 = -5$

9. $4 - 7x = 18$

10. $8 - 5x = -7$

11. $3 - 4x = -9$ > Videos

12. $5 - 4x = 25$

13. $\frac{x}{2} + 1 = 5$ > Videos

14. $\frac{x}{3} - 2 = 3$

15. $\frac{x}{4} - 5 = 3$

16. $\frac{x}{5} + 3 = 8$

17. $\frac{2}{3}x + 5 = 17$

18. $\frac{3}{4}x - 5 = 4$

19. $\frac{4}{5}x - 3 = 13$

20. $\frac{5}{7}x + 4 = 14$

21. $5x = 2x + 9$ > Videos

22. $7x = 18 - 2x$

23. $3x = 10 - 2x$

24. $11x = 7x + 20$

25. $9x + 2 = 3x + 38$

26. $8x - 3 = 4x + 17$

27. $4x - 8 = x - 14$

28. $6x - 5 = 3x - 29$

29. $5x + 7 = 2x - 3$ > Videos

30. $9x + 7 = 5x - 3$

31. $7x - 3 = 9x + 5$

32. $5x - 2 = 8x - 11$

33. $5x + 4 = 7x - 8$

34. $2x + 23 = 6x - 5$

Answers

1. __________ 2. __________

3. __________ 4. __________

5. __________ 6. __________

7. __________ 8. __________

9. __________ 10. __________

11. __________ 12. __________

13. __________ 14. __________

15. __________ 16. __________

17. __________ 18. __________

19. __________ 20. __________

21. __________ 22. __________

23. __________ 24. __________

25. __________ 26. __________

27. __________ 28. __________

29. __________ 30. __________

31. __________ 32. __________

33. __________ 34. __________

Answers

35. 36.

37. 38.

39. 40.

41.

42.

43.

44.

45.

46.

47.

48.

49.

50.

51.

52.

53.

54.

55.

56.

57.

< Objective 2 >

35. $2x - 3 + 5x = 7 + 4x + 2$

36. $8x - 7 - 2x = 2 + 4x - 5$

37. $6x + 7 - 4x = 8 + 7x - 26$

38. $7x - 2 - 3x = 5 + 8x + 13$

39. $9x - 2 + 7x + 13 = 10x - 13$

40. $5x + 3 + 6x - 11 = 8x + 25$

41. $8x - 7 + 5x - 10 = 10x - 12$

42. $10x - 9 + 2x - 3 = 8x - 18$

43. **SOCIAL SCIENCE** There were 55 more yes votes than no votes on an election measure. If 735 votes were cast in all, how many yes votes were there?

44. **BUSINESS AND FINANCE** Juan worked twice as many hours as Jerry. Marcia worked 3 more hours than Jerry. If they worked a total of 31 hours, how many hours did each employee work?

45. **BUSINESS AND FINANCE** Francine earns $120 per month more than Rob. If they earn a total of $2,680 per month, how much does Francine earn each month?

46. **SCIENCE AND MEDICINE** To determine the upper limit for a person's heart rate during aerobic training, subtract the person's age from 220, and then multiply the result by $\frac{9}{10}$. Determine the age of a person if the person's upper limit heart rate is 153.

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Solve each equation.

47. $7(2x - 1) - 5x = x + 25$

48. $9(3x + 2) - 10x = 12x - 7$

49. $3x + 2(4x - 3) = 6x - 9$

50. $7x + 3(2x + 5) = 10x + 17$

51. $\frac{8}{3}x - 3 = \frac{2}{3}x + 15$

52. $\frac{12}{5}x + 7 = 31 - \frac{3}{5}x$

53. $\frac{2x}{5} - 5 = \frac{12x}{5} + 8$

54. $\frac{3x}{7} - 5 = \frac{24x}{7} + 7$

55. $5.3x - 7 = 2.3x + 5$

56. $9.8x + 2 = 3.8x + 20$

Basic Skills | Challenge Yourself | Calculator/Computer | **Career Applications** | Above and Beyond

57. **AGRICULTURAL TECHNOLOGY** The estimated yield Y of a field of corn (in bushels per acre) can be found by multiplying the rainfall r, in inches, during the growing season by 16 and then subtracting 15. This relationship can be modeled by the formula

$$Y = 16r - 15$$

If a farmer wants a yield of 159 bushels per acre, then we can write the equation shown to determine the amount of rainfall required.

$$159 = 16r - 15$$

How much rainfall is necessary to achieve a yield of 159 bushels of corn per acre?

58. **CONSTRUCTION TECHNOLOGY** The number of studs s required to build a wall (with studs spaced 16 inches on center) is equal to the one more than $\frac{3}{4}$ times the length of the wall w, in feet. We model this with the formula

$$s = \frac{3}{4}w + 1$$

If a contractor uses 22 studs to build a wall, how long is the wall?

59. **ALLIED HEALTH** The internal diameter D [in millimeters (mm)] of an endotracheal tube for a child is calculated using the formula

$$D = \frac{t + 16}{4}$$

in which t is the child's age (in years).

How old is a child who requires an endotracheal tube with an internal diameter of 7 mm?

60. **MECHANICAL ENGINEERING** The number of BTUs required to heat a house is $2\frac{3}{4}$ times the volume of the air in the house (in cubic feet). What is the maximum air volume that can be heated with a 90,000-BTU furnace?

Basic Skills | Challenge Yourself | Calculator/Computer | Career Applications | **Above and Beyond**

61. Create an equation of the form $ax + b = c$ that has 2 as a solution.

62. Create an equation of the form $ax + b = c$ that has -6 as a solution.

63. The equation $3x = 3x + 5$ has no solution, whereas the equation $7x + 8 = 8$ has zero as a solution. Explain the difference between an equation that has zero as a solution and an equation that has no solution.

64. Construct an equation for which every real number is a solution.

Answers

1. $x = 4$ **3.** $x = 3$ **5.** $x = 7$ **7.** $x = -2$ **9.** $x = -2$ **11.** $x = 3$
13. $x = 8$ **15.** $x = 32$ **17.** $x = 18$ **19.** $x = 20$ **21.** $x = 3$
23. $x = 2$ **25.** $x = 6$ **27.** $x = -2$ **29.** $x = -\frac{10}{3}$ **31.** $x = -4$
33. $x = 6$ **35.** $x = 4$ **37.** $x = 5$ **39.** $x = -4$ **41.** $x = \frac{5}{3}$
43. 395 votes **45.** \$1,400 **47.** $x = 4$ **49.** $x = -\frac{3}{5}$ **51.** $x = 9$
53. $x = -\frac{13}{2}$ **55.** $x = 4$ **57.** $10\frac{7}{8}$ in. **59.** 12 yr
61. Above and Beyond **63.** Above and Beyond

Answers

58. ____________________

59. ____________________

60. ____________________

61. ____________________

62. ____________________

63. ____________________

64. ____________________

summary :: chapter 11

Definition/Procedure	Example	Reference
From Arithmetic to Algebra		**Section 11.1**
Addition $x + y$ means the **sum** of x and y, or x **plus** y. Some other words indicating addition are *more than* and *increased by*. **Subtraction** $x - y$ means the **difference** of x and y, or x **minus** y. Some other words indicating subtraction are *less than* and *decreased by*. **Multiplication** $\left.\begin{array}{l} x \cdot y \\ (x)(y) \\ xy \end{array}\right\}$ These all mean the **product** of x and y, or x **times** y. **Division** $\frac{x}{y}$ means x **divided by** y, or the **quotient** when x is divided by y.	The sum of x and 5 is $x + 5$. 7 more than a is $a + 7$. b increased by 3 is $b + 3$. The difference of x and 3 is $x - 3$. 5 less than p is $p - 5$. a decreased by 4 is $a - 4$. The product of m and n is mn. The product of 2 and the sum of a and b is $2(a + b)$. n divided by 5 is $\frac{n}{5}$. The sum of a and b, divided by 3, is $\frac{a + b}{3}$.	p. 703
Evaluating Algebraic Expressions		**Section 11.2**
Step 1 Replace each variable with the given number value. **Step 2** Do the necessary arithmetic operations, following the rules for the order of operations.	Evaluate $\frac{4a - b}{2c}$ if $a = -6$, $b = 8$, and $c = -4$. $\frac{4a - b}{2c} = \frac{4(-6) - (8)}{2(-4)}$ $= \frac{-24 - 8}{-8}$ $= \frac{-32}{-8} = 4$	p. 711
Simplifying Algebraic Expressions		**Section 11.3**
Term A number, or the product of a number and one or more variables, raised to a power. **Like terms** Terms that contain exactly the same variables raised to the same powers.	$4a^2$ and $3a^2$ are like terms. $5x^2$ and $2xy^2$ are not like terms.	p. 721
Combining Like Terms		
Step 1 Add or subtract the numerical coefficients. **Step 2** Attach the common variables.	$5a + 3a = 8a$ $7xy - 3xy = 4xy$	p. 723

Definition/Procedure	Example	Reference
Using the Addition Property to Solve an Equation		**Section 11.4**
Equation A statement that two expressions are *equal*. **Solution** Any value for the variable that makes an equation a true statement.	$3x - 5 = 7$ is an equation. 4 is a solution to the equation because $3(4) - 5 \overset{?}{=} 7$ $12 - 5 \overset{?}{=} 7$ $7 = 7$ (True)	p. 730
Equivalent equations Equations that have exactly the same set of solutions.	$3x - 5 = 7$ and $x = 4$ are equivalent equations.	p. 732
The addition property If $a = b$, then $a + c = b + c$. Adding (or subtracting) the same quantity to both sides of an equation yields an equivalent equation.	$x - 5 = 7$ $\underline{\quad + 5 \quad +5}$ $x = 12$	p. 732
Using the Multiplication Property to Solve an Equation		**Section 11.5**
The multiplication property If $a = b$ and $c \neq 0$, then $ac = bc$. Multiplying (or dividing) both sides of an equation by the same nonzero number yields an equivalent equation.	$5x = 20$ $\frac{5x}{5} = \frac{20}{5}$ $x = 4$	p. 743
To solve a percent problem algebraically, translate the problem into algebra (writing the rate as a decimal) and use the multiplication rule to solve.	30% of what number is 45? $0.3x = 45$ $\frac{0.3x}{0.3} = \frac{45}{0.3}$ $x = 150$	p. 747
Combining the Properties to Solve Equations		**Section 11.6**
Solving linear equations We say that an equation is solved when we have an equivalent equation of the form $x =$ ☐ or ☐ $= x$ in which ☐ is some number. The steps for solving a linear equation follow. **Step 1** Use the distributive property to remove any grouping symbols. **Step 2** Combine like terms on each side of the equation. **Step 3** Add or subtract variable terms to bring the variable term to one side of the equation. **Step 4** Add or subtract numbers to isolate the variable term. **Step 5** Multiply by the reciprocal of the coefficient to isolate the variable. **Step 6** Check your result.	Solve: $3x - 6 + 4x = 3x + 14$ $7x - 6 = 3x + 14$ $7x - 6 - 3x = 3x + 14 - 3x$ $4x - 6 = 14$ $4x - 6 + 6 = 14 + 6$ $4x = 20$ $\frac{4x}{4} = \frac{20}{4}$ $x = 5$ Check: $3(5) - 6 + 4(5) \overset{?}{=} 3(5) + 14$ $29 = 29$ True	p. 759

summary exercises :: chapter 11

This summary exercise set is provided to give you practice with each of the objectives of this chapter. Each exercise is keyed to the appropriate chapter section. When you are finished, you can check your answers to the odd-numbered exercises against those presented in the back of the text. If you have difficulty with any of these questions, go back and reread the examples from that section. The answers to the even-numbered exercises appear in the *Instructor's Solutions Manual*. Your instructor will give you guidelines on how to best use these exercises in your instructional setting.

11.1 *Write, using symbols.*

1. 5 more than y

2. c decreased by 10

3. The product of 8 and a

4. The quotient when y is divided by 3

5. 5 times the product of m and n

6. The product of a and 5 less than a

7. 3 more than the product of 17 and x

8. The quotient when a plus 2 is divided by a minus 2

11.2 *Evaluate each expression if* $x = -3, y = 6, z = -4$, *and* $w = 2$.

9. $3x + w$

10. $5y - 4z$

11. $x + y - 3z$

12. $5z^2$

13. $3x^2 - 2w^2$

14. $3x^3$

15. $5(x^2 - w^2)$

16. $\frac{6z}{2w}$

17. $\frac{2x - 4z}{y + (-z)}$

18. $\frac{3x - y}{w - x}$

19. $\frac{x(y^2 - z^2)}{(y + z)(y - z)}$

20. $\frac{y(x - w)^2}{x^2 - 2xw + w^2}$

11.3 *List the terms of each expression.*

21. $4a^3 - 3a^2$

22. $5x^2 - 7x + 3$

Circle like terms.

23. $5m^2, -3m, -4m^2, 5m^3, m^2$

24. $4ab^2, 3b^2, -5a, ab^2, 7a^2, -3ab^2, 4a^2b$

Combine like terms.

25. $5c + 7c$

26. $2x + 5x$

27. $4a - 2a$

28. $6c - 3c$

29. $9xy - 6xy$

30. $5ab^2 + 2ab^2$

31. $7a + 3b + 12a - 2b$

32. $6x - 2x + 5y - 3x$

33. $5x^3 + 17x^2 - 2x^3 - 8x^2$

34. $3a^3 + 5a^2 + 4a - 2a^3 - 3a^2 - a$

35. Subtract $4a^3$ from the sum of $2a^3$ and $12a^3$.

36. Subtract the sum of $3x^2$ and $5x^2$ from $15x^2$.

11.4 *Tell whether the number shown in parentheses is a solution for the given equation.*

37. $7x + 2 = 16 \quad (2)$

38. $5x - 8 = 3x + 2 \quad (4)$

39. $7x - 2 = 2x + 8 \quad (2)$

40. $4x + 3 = 2x - 11 \quad (-7)$

41. $x + 5 + 3x = 2 + x + 23 \quad (6)$

42. $\frac{2}{3}x - 2 = 10 \quad (21)$

Solve each equation and check your result.

43. $x + 5 = 7$

44. $x - 9 = 3$

45. $5x = 4x - 5$

46. $3x - 9 = 2x$

47. $5x - 3 = 4x + 2$

48. $9x + 2 = 8x - 7$

49. $7x - 5 = 6x + (-4)$

50. $3 + 4x - 1 = x - 7 + 2x$

51. $4(2x + 3) = 7x + 5$

52. $5(5x - 3) = 6(4x + 1)$

11.5–11.6

53. $5x = 35$

54. $7x = -28$

55. $-6x = 24$

56. $-9x = -63$

57. $\frac{x}{4} = 8$

58. $-\frac{x}{5} = -3$

59. $\frac{2}{3}x = 18$

60. $\frac{3}{4}x = 24$

61. $5x - 3 = 12$

62. $4x + 3 = -13$

63. $7x + 8 = 3x$

64. $3 - 5x = -17$

65. $3x - 7 = x$

66. $2 - 4x = 5$

67. $\frac{x}{3} - 5 = 1$

68. $\frac{3}{4}x - 2 = 7$

69. $6x - 5 = 3x + 13$

70. $3x + 7 = x - 9$

71. $7x + 4 = 2x + 6$

72. $9x - 8 = 7x - 3$

73. $2x + 7 = 4x - 5$

74. $3x - 15 = 7x - 10$

75. $\frac{10}{3}x - 5 = \frac{4}{3}x + 7$

76. $\frac{11}{4}x - 15 = 5 - \frac{5}{4}x$

77. $3.7x + 8 = 1.7x + 16$

78. $5.4x - 3 = 8.4x + 9$

79. $3x - 2 + 5x = 7 + 2x + 21$

80. $8x + 3 - 2x + 5 = 3 - 4x$

81. $5(3x - 1) - 6x = 3x - 2$

82. $5x + 2(3x - 4) = 14x + 7$

self-test 11

Name ____________

Section ________ Date ________

The purpose of this self-test is to help you assess your progress so that you can find concepts that you need to review before the next exam. Allow yourself about an hour to take this test. At the end of that hour, check your answers against those given in the back of this text. If you miss any, go back to the appropriate section to reread the examples until you have mastered that particular concept.

Rewrite each phrase as an algebraic expression.

1. 5 less than a

2. The product of 6 and m

3. 4 times the sum of m and n

4. The sum of 4 times m and n

Determine whether the number in parentheses is a solution to the given equation.

5. $7x - 3 = 25 \quad (5)$

6. $8x - 3 = 5x + 9 \quad (4)$

Solve each equation and check your result.

7. $x - 7 = 4$

8. $7x = 49$

9. $7x - 5 = 16$

10. $10 - 3x = -2$

11. $\frac{x}{4} = -3$

12. $7x - 12 = 6x$

13. $9x - 2 = 8x + 5$

14. $\frac{4}{5}x = 20$

15. $7x - 3 = 4x - 5$

16. $2x - 7 = 5x + 8$

Evaluate each expression for $a = -2$, $b = 6$, and $c = -4$.

17. $4a - c$

18. $5c^2$

19. $6(2b - 3c)$

20. $\frac{3a - 4b}{a + c}$

Simplify each expression.

21. $8a + 7a$

22. $8x^2y - 5x^2y$

23. $10x + 8y + 9x - 3y$

24. $3m^2 + 7m - 5 - 9m + m^2$

Solve the application.

25. A coffee shop earns a (marginal) profit of $3.45 on each double latte it sells. However, the fixed costs associated with double lattes amount to $200 per day. How many lattes must the shop sell in order to break even? *Hint:* The break-even point is the number of lattes that the shop needs to sell in order to avoid a loss.

Answers

1. ________ **2.** ________

3. ________ **4.** ________

5. ________ **6.** ________

7. ________ **8.** ________

9. ________ **10.** ________

11. ________________

12. ________________

13. ________________

14. ________________

15. ________________

16. ________________

17. ________________

18. ________________

19. ________________

20. ________________

21. ________________

22. ________________

23. ________________

24. ________________

25. ________________

cumulative review chapters 1-11

Name ______________________

Section __________ Date __________

Answers

1. ______________________

2. ______________________

3. ______________________

4. ______________________

5. ______________________

6. ______________________

7. ______________________

8. ______________________

9. ______________________

10. ______________________

11. ______________________

12. ______________________

13. ______________________

14. ______________________

The following exercises are presented to help you review concepts from earlier chapters. This is meant as review material and not as a comprehensive exam. The answers are presented in the back of the text. Beside each answer is a section reference for the concept. If you have difficulty with any of these exercises, be certain to at least read through the summary related to that section.

Name the property illustrated.

1. $(7 + 3) + 8 = 7 + (3 + 8)$

2. $6 \times 7 = 7 \times 6$

3. $5 \cdot (2 + 4) = 5 \cdot 2 + 5 \cdot 4$

Round the numbers to the indicated place value.

4. 5,873 to the nearest hundred

5. 953,150 to the nearest ten-thousand

6. Evaluate $2 + 8 \times 3 \div 4$.

7. Write the prime factorization of 264.

8. Find the least common multiple (LCM) of 6, 15, and 45.

9. Convert to a mixed number: $\frac{22}{7}$

10. Convert to an improper fraction: $6\frac{5}{8}$

Perform the indicated operations.

11. $\frac{2}{3} \times 1\frac{4}{5} \times \frac{5}{8}$

12. $2\frac{2}{7} \div 1\frac{11}{21}$

13. $4\frac{7}{8} + 3\frac{1}{6}$

14. $9 + \left(-5\frac{3}{8}\right)$

The Streeter/Hutchison Series in Mathematics Basic Mathematical Skills with Geometry

15. **CONSTRUCTION** A $6\frac{1}{2}$-in. bolt is placed through a wall that is $5\frac{7}{8}$ in. thick. How far does the bolt extend beyond the wall?

16. **BUSINESS AND FINANCE** You pay for purchases of \$13.99, \$18.75, \$9.20, and \$5 with a \$50 bill. How much cash will you have left?

17. **GEOMETRY** Find the area of a circle whose diameter is 3.2 ft. Use 3.14 for π and round the result to the nearest hundredth.

18. **CONSTRUCTION** A 14-acre piece of land is being developed into home lots. If 2.8 acres of land will be used for roads, and each home site is to be 0.35 acre, how many lots can be formed?

19. Write the decimal equivalent of $\frac{8}{11}$. Use bar notation.

20. Solve for the unknown: $\frac{5}{m} = \frac{0.4}{9}$

21. **BUSINESS AND FINANCE** You are using a photocopy machine to reduce an advertisement that is 14 in. wide by 21 in. long. If the new width is to be 8 in., what will the new length be?

In exercises 22 to 24, write as percents.

22. 0.003

23. $\frac{5}{8}$

24. $3\frac{1}{2}$

25. 120% of what number is 180?

26. 72 is 12% of what number?

27. **BUSINESS AND FINANCE** Luisa works on an 8% commission basis. If she wishes to earn \$2,200 in commissions in 1 month, how much must she sell during that period?

28. Complete the statement: 300 mg = _______ g

Answers

15. ________________

16. ________________

17. ________________

18. ________________

19. ________________

20. ________________

21. ________________

22. ________________

23. ________________

24. ________________

25. ________________

26. ________________

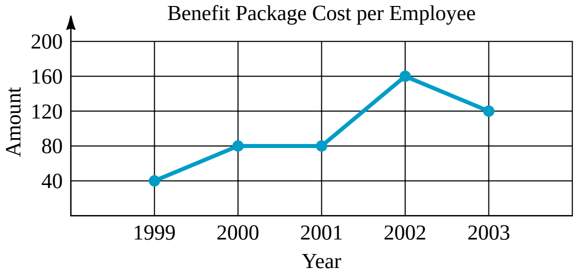
27. ________________

28. ________________

Answers

29. __________
30. __________
31. __________
32. __________
33. __________
34. __________
35. __________
36. __________
37. __________
38. __________
39. __________
40. __________
41. __________
42. __________
43. __________
44. __________
45. __________
46. __________
47. __________
48. __________
49. __________

29. **Business and Finance** According to the line graph, what is the difference in benefits between 2000 and 2002?



30. The opposite of 8 is ________.

31. The absolute value of -20 is ________.

In exercises 32 to 37, evaluate.

32. $-(-12)$

33. $|-5|$

34. $-12 + (-6)$

35. $-8 - (-4)$

36. $(-6)(15)$

37. $48 \div (-12)$

Write, using symbols.

38. 3 times the sum of x and y

39. The quotient when 5 less than n is divided by 3

In exercises 40 to 43, evaluate each expression if $a = 5$, $b = -3$, $c = 4$, and $d = -2$.

40. $6ad$

41. $3b^2$

42. $3(c - 2d)$

43. $\dfrac{2a - 7d}{a - b}$

Combine like terms.

44. $10a^2 + 5a + 2a^2 - 2a$

45. $5x - 3y + 2x + y - 7x$

Solve the following equations, and check your results.

46. $9x - 5 = 8x$

47. $-\frac{3}{4}x = 18$

48. $2x + 3 = 7x + 5$

49. $\frac{4}{3}x - 6 = 4 - \frac{2}{3}x$

Answers to Prerequisite Tests, Reading Your Text, Summary Exercises, Self-Tests, and Cumulative Reviews

Reading Your Text

Section 1.1 **(a)** decimal; **(b)** digit; **(c)** comma; **(d)** million
Section 1.2 **(a)** natural; **(b)** addition; **(c)** order; **(d)** read
Section 1.3 **(a)** difference; **(b)** less; **(c)** read; **(d)** borrowing
Section 1.4 **(a)** rounding; **(b)** place value; **(c)** rounding; **(d)** less
Section 1.5 **(a)** reasonable; **(b)** product; **(c)** powers; **(d)** rectangle
Section 1.6 **(a)** quotient; **(b)** remainder; **(c)** zero; **(d)** subtraction
Section 1.7 **(a)** first; **(b)** exponent; **(c)** inside; **(d)** one

Summary Exercises for Chapter 1

1. Hundreds
3. Twenty-seven thousand, four hundred twenty-eight
5. 37,583 **7.** Commutative property of addition **9.** 1,416
11. 4,801 **13.** **(a)** 27; **(b)** 11; **(c)** 15 **15.** $540
17. 18,800 **19.** 2,574 **21.** $536 **23.** 7,000
25. 550,000 **27.** $>$ **29.** 22 in.
31. Distributive property of multiplication over addition
33. 1,856 **35.** 154,602 **37.** 144 in.^{3} **39.** 30,960
41. 0 **43.** 308 r 5 **45.** 497 r 1 **47.** 28 mi/gal
49. 21 **51.** 1,000 **53.** 4 **55.** 36 **57.** 33 **59.** 0

Self-Test for Chapter 1

1. 1,918 **2.** 13,103 **3.** 4,984 **4.** 55,414
5. Three hundred two thousand, five hundred twenty-five
6. 12 in. **7.** $>$ **8.** $<$ **9.** Hundred thousands
10. Distributive property of multiplication over addition
11. Associative property of addition
12. Associative property of multiplication
13. Commutative property of addition **14.** 76 r 7
15. 209 r 145 **16.** 2,430,000 **17.** 21,696 **18.** $558,750
19. $223 **20.** 12 in.^{2} **21.** 280 ft^3 **22.** 55,978
23. 235 **24.** 40,555 **25.** 30,770 **26.** 12,220
27. 72 lb **28.** 39 **29.** 15 **30.** 7,700

Prerequisite Test for Chapter 2

1. No **2.** No **3.** Yes **4.** No **5.** Yes **6.** 1, 3, 9
7. 1, 2, 5, 10 **8.** 1, 17 **9.** 1, 2, 3, 4, 6, 8, 12, 16, 24, 48
10. 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60

Reading Your Text

Section 2.1 **(a)** One (1); **(b)** prime; **(c)** composite; **(d)** even
Section 2.2 **(a)** commutative; **(b)** prime; **(c)** common; **(d)** greatest
Section 2.3 **(a)** denominator; **(b)** proper; **(c)** mixed; **(d)** numerator
Section 2.4 **(a)** cross; **(b)** equivalent; **(c)** Fundamental; **(d)** common
Section 2.5 **(a)** simplest; **(b)** simplify; **(c)** reasonableness; **(d)** reasonable
Section 2.6 **(a)** reciprocal; **(b)** complex; **(c)** divisor; **(d)** units

Summary Exercises for Chapter 2

1. 1, 2, 4, 13, 26, 52 **3.** Prime: 2, 5, 7, 11, 17, 23, 43; composite: 14, 21, 27, 39 **5.** None **7.** $2 \times 2 \times 3 \times 5 \times 7$
9. $2 \times 3 \times 3 \times 5 \times 5 \times 5$ **11.** 1 **13.** 13 **15.** 11
17. Numerator: 17; denominator: 23
19. Fraction: $\frac{5}{6}$; numerator: 5; denominator: 6 **21.** $6\frac{5}{6}$
23. $7\frac{2}{3}$ **25.** $\frac{61}{8}$ **27.** $\frac{37}{7}$ **29.** No **31.** $\frac{2}{3}$ **33.** $\frac{7}{9}$
35. Yes **37.** $\frac{1}{9}$ **39.** $1\frac{1}{2}$ **41.** $9\frac{3}{5}$ **43.** 8 **45.** $204
47. $45 **49.** 408 mi **51.** $\frac{5}{8}$ **53.** $\frac{2}{3}$ **55.** $\frac{3}{16}$
57. $\frac{3}{7}$ **59.** $\frac{3}{4}$ ft **61.** 56 mi/h **63.** 48 lots

Self-Test for Chapter 2

1. 12 **2.** 8 **3.** $4\frac{1}{4}$ **4.** $\frac{7}{9}$ **5.** $\frac{3}{7}$ **6.** $\frac{8}{23}$ **7.** $\frac{10}{21}$
8. 4 **9.** $\frac{5}{8}$ **10.** $3\frac{3}{7}$ **11.** $\frac{4}{15}$ **12.** $2\frac{2}{3}$ **13.** $\frac{9}{16}$
14. $1\frac{1}{7}$ **15.** $9\frac{1}{5}$ **16.** $1\frac{3}{11}$
17. Prime: 5, 13, 17, 31; composite: 9, 22, 27, 45
18. $\frac{5}{6}$; numerator: 5; denominator: 6
19. $\frac{5}{8}$; numerator: 5; denominator: 8
20. $\frac{3}{5}$; numerator: 3; denominator: 5 **21.** $1.32
22. 48 books **23.** $4\frac{1}{4}$ **24.** 15 **25.** $9\frac{1}{4}$ **26.** 3
27. $2 \times 2 \times 2 \times 3 \times 11$
28. Proper: $\frac{10}{11}, \frac{1}{8}$; improper: $\frac{9}{5}, \frac{7}{7}, \frac{8}{1}$; mixed: $2\frac{3}{5}$ **29.** 47 homes
30. 20 yd^2 **31.** 190 mi **32.** 2, 3 **33.** Yes **34.** Yes
35. No **36.** $\frac{37}{7}$ **37.** $\frac{35}{8}$ **38.** $\frac{74}{9}$

Cumulative Review Chapters 1–2

[1.1] **1.** Hundred thousands
2. Three hundred two thousand, five hundred twenty-five
3. 2,430,000 **[1.2]** **4.** Commutative property of addition
5. Additive identity **6.** Associative property of addition
7. 966 **8.** 23,351 **[1.4]** **9.** 5,900 **10.** 950,000
11. 7,700 **12.** $>$ **13.** $<$ **[1.3]** **14.** 3,861 **15.** 17,465
[1.2] **16.** 905 **17.** $17,579
[1.5] **18.** Associative property of multiplication
19. Commutative property of multiplication
20. Distributive property **21.** 378,214 **22.** 686,000
23. $1,008 **[1.6]** **24.** 67 r 43 **25.** 103 r 176
[1.7] **26.** 38 **27.** 56 **28.** 36 **29.** 8 **30.** $58
[2.1] **31.** Prime: 5, 13, 17, 31; composite: 9, 22, 27, 45
32. 2 and 3 **[2.2]** **33.** $2 \times 2 \times 2 \times 3 \times 11$ **34.** 12 **35.** 8
[2.3] **36.** Proper: $\frac{7}{12}, \frac{3}{7}$; improper: $\frac{10}{8}, \frac{9}{9}, \frac{7}{1}$; mixed numbers: $3\frac{1}{5}, 2\frac{2}{3}$
[2.4] **37.** $2\frac{4}{5}$ **38.** 4 **39.** $\frac{13}{3}$ **40.** $\frac{63}{8}$ **41.** Yes
42. No **43.** $\frac{2}{3}$ **44.** $\frac{3}{8}$ **[2.5]** **45.** $\frac{8}{27}$ **46.** $\frac{4}{15}$
47. $5\frac{2}{5}$ **48.** $22\frac{2}{3}$ **49.** $\frac{3}{4}$ **[2.6]** **50.** $1\frac{1}{3}$ **51.** $4\frac{1}{2}$
52. $\frac{5}{6}$ **53.** $1\frac{1}{2}$ **54.** $540 **55.** 88 sheets

Prerequisite Test for Chapter 3

1. 2 × 2 × 2 × 3 **2.** 2 × 2 × 3 × 3 **3.** 2 × 3 × 3 × 5
4. $\frac{14}{3}$ **5.** $\frac{32}{5}$ **6.** $\frac{91}{10}$ **7.** $7\frac{1}{4}$ **8.** 7 **9.** 1 **10.** 27

Reading Your Text

Section 3.1 **(a)** like; **(b)** numerators; **(c)** simplify; **(d)** difference
Section 3.2 **(a)** multiples; **(b)** smallest; **(c)** denominator; **(d)** greater
Section 3.3 **(a)** LCM; **(b)** unlike; **(c)** equivalent; **(d)** numerator
Section 3.4 **(a)** LCD; **(b)** regrouping; **(c)** like; **(d)** smaller
Section 3.5 **(a)** grouping; **(b)** exponents; **(c)** invert; **(d)** improper
Section 3.6 **(a)** dual; **(b)** estimation; **(c)** approximations; **(d)** estimation

Summary Exercises for Chapter 3

1. $\frac{2}{3}$ **3.** $1\frac{2}{13}$ **5.** $1\frac{1}{3}$ **7.** $1\frac{2}{9}$ **9.** 12 **11.** 72 **13.** 60
15. 72 **17.** $\frac{7}{12}, \frac{5}{8}$ **19.** > **21.** < **23.** $\frac{36}{120}, \frac{75}{120}, \frac{70}{120}$
25. 36 **27.** 200 **29.** 132 **31.** 24 **33.** $\frac{19}{24}$ **35.** $\frac{7}{12}$
37. $1\frac{17}{90}$ **39.** $\frac{7}{8}$ **41.** $1\frac{13}{72}$ **43.** $\frac{5}{9}$ **45.** $\frac{1}{2}$ **47.** $\frac{5}{24}$
49. $\frac{7}{18}$ **51.** $\frac{11}{24}$ **53.** $\frac{13}{42}$ **55.** $\frac{1}{3}$ **57.** $\frac{1}{4}$ **59.** $\frac{7}{60}$
61. $\frac{35}{72}$ **63.** $9\frac{37}{60}$ **65.** $4\frac{1}{3}$ **67.** $2\frac{19}{24}$ **69.** $\frac{5}{12}$ cup
71. $19\frac{9}{16}$ in. **73.** $53\frac{9}{16}$ in. **75.** $3\frac{7}{12}$ gal **77.** $1\frac{3}{4}$ yd
79. $\frac{37}{96}$ **81.** $\frac{1}{48}$ **83.** $1\frac{1}{2}$ **85.** $2,500 **87.** 24

Self-Test for Chapter 3

1. 60 **2.** 36 **3.** $10\frac{1}{4}$ **4.** $1\frac{11}{18}$ **5.** $\frac{9}{10}$ **6.** $\frac{25}{42}$
7. $\frac{23}{30}$ **8.** $9\frac{1}{7}$ **9.** $7\frac{11}{12}$ **10.** $\frac{2}{3}$ **11.** $13\frac{11}{20}$ **12.** $1\frac{11}{60}$
13. $\frac{1}{4}$ h **14.** $1\frac{5}{12}$ c **15.** 72 **16.** $\frac{1}{12}$ **17.** $\frac{299}{24}$ or $12\frac{11}{24}$
18. $\frac{4}{5}$ **19.** $1\frac{3}{4}$ **20.** $1\frac{8}{15}$ **21.** $\frac{1}{9}$ **22.** $\frac{19}{24}$ **23.** $3\frac{23}{24}$
24. $\frac{1}{3}$ **25.** $1\frac{23}{40}$ **26.** $7\frac{7}{10}$ **27.** $\frac{7}{12}$ **28.** $12\frac{3}{40}$
29. 39,000 c **30.** $5\frac{3}{4}$ h

Cumulative Review Chapters 1–3

[1.1–1.2] **1.** 7,173 **2.** 1,918 **3.** 2,731 **4.** 13,103
[1.3] **5.** 235 **6.** 12,220 **7.** 429 **8.** 3,239
[1.5] **9.** 174 **10.** 1,911 **11.** 4,984 **12.** 55,414
[1.6] **13.** 24 r191 **14.** 22 r21 **15.** 209 r145
[1.7] **16.** 5 **17.** 7 **18.** 3 **19.** 16 **20.** 20 **21.** 3
[2.3] **22.** Proper: $\frac{5}{7}, \frac{2}{5}$; improper: $\frac{15}{9}, \frac{8}{8}, \frac{11}{1}$; mixed numbers: $4\frac{5}{6}, 3\frac{5}{6}$
23. $1\frac{7}{9}$ **24.** $7\frac{1}{5}$ **25.** $\frac{23}{4}$ **26.** $\frac{55}{9}$ **[2.4]** **27.** Yes
28. No **[2.5]** **29.** $\frac{1}{9}$ **30.** $\frac{1}{6}$ **31.** $1\frac{1}{2}$ **32.** $2\frac{1}{8}$ **33.** $9\frac{3}{5}$
34. $11\frac{1}{3}$ **35.** 8 **[2.6]** **36.** $\frac{2}{3}$ **37.** $\frac{5}{6}$ **38.** $\frac{3}{16}$
[3.1] **39.** $\frac{4}{5}$ **40.** $\frac{61}{75}$ **41.** $1\frac{31}{40}$ **[3.3]** **42.** $\frac{1}{2}$ **43.** $\frac{5}{36}$
44. $\frac{5}{9}$ **[3.4]** **45.** $6\frac{2}{7}$ **46.** $8\frac{1}{24}$ **47.** $4\frac{5}{9}$ **48.** $4\frac{1}{24}$
49. $3\frac{5}{8}$ **50.** $3\frac{13}{24}$ **51.** $14\frac{19}{30}$ h **52.** $\frac{5}{8}$ in. **53.** $1\frac{11}{12}$ h

Prerequisite Test for Chapter 4

1. Three and seven tenths **2.** Six and twenty-nine hundredths
3. Seventeen and eighty-nine thousandths **4.** 257 **5.** 164
6. 6,390 **7.** 79 r 2 **8.** 7 r 92 **9.** 136 ft^2 **10.** 225 in.^2

Reading Your Text

Section 4.1 **(a)** decimal; **(b)** places; **(c)** exact; **(d)** ten-thousandths
Section 4.2 **(a)** divide; **(b)** bar; **(c)** terminating; **(d)** places
Section 4.3 **(a)** decimal points; **(b)** value; **(c)** Perimeter; **(d)** following
Section 4.4 **(a)** add; **(b)** product; **(c)** zeros; **(d)** right
Section 4.5 **(a)** above; **(b)** past; **(c)** whole number; **(d)** left

Summary Exercises for Chapter 4

1. Hundredths **3.** 0.37 **5.** Seventy-one thousandths **7.** 4.5
9. > **11.** < **13.** 5.84 **15.** 4.876 **17.** $21\frac{857}{1{,}000}$
19. 0.429 **21.** 3.75 **23.** $\frac{21}{250}$ **25.** 3.47 **27.** 37.728
29. 23.32 **31.** 1.075 **33.** 28.02 cm **35.** 6.15 cm
37. $18.93 **39.** 0.000261 **41.** 0.0012275 **43.** 450
45. $287.50 **47.** $5,742 **49.** 4.65 **51.** 2.664 **53.** 1.273
55. 0.76 **57.** 0.0457 **59.** 39.3 mi/gal **61.** 29.8 mi/gal

Self-Test for Chapter 4

1. < **2.** > **3.** 0.4375 **4.** 0.429 **5.** $0.\overline{63}$
6. 0.004983 **7.** 0.00523 **8.** $\frac{9}{125}$ **9.** $4\frac{11}{25}$
10. Ten-thousandths **11.** Two and fifty-three hundredths
12. 0.598 **13.** 23.57 **14.** 36,000 **15.** 0.049
16. 12.017 **17.** 16.64 **18.** 10.54 **19.** 2.55
20. 1.4575 **21.** 2.35 **22.** 47.253 **23.** 17.437
24. 24.375 **25.** $543 **26.** 50.2 gal **27.** 32 lots
28. > **29.** 12.803 **30.** 7.525 in.^2 **31.** 735
32. 12,570 **33.** $573.40 **34.** $3.06 **35.** 3.888
36. 0.465 **37.** 0.02793 **38.** 2.385 **39.** $\frac{229}{500}$ **40.** 7.35
41. 0.051 **42.** 0.067

Cumulative Review Chapters 1–4

[1.1] **1.** Two hundred eighty-six thousand, five hundred forty-three
2. Hundreds **[1.2]** **3.** 34,594 **[1.3]** **4.** 48,888
[1.5] **5.** 5,063 **6.** 70,455 **[1.6]** **7.** 17 **8.** 35 r11
[1.7] **9.** 29 **[1.4]** **10.** 4,000 **[1.8]** **11.** P: 24 ft; A: 35 ft^2
[2.4] **12.** $\frac{5}{17}$ **[2.5]** **13.** $\frac{3}{4}$ **14.** $2\frac{6}{7}$ **[2.7]** **15.** $\frac{9}{17}$
[3.1] **16.** $\frac{5}{7}$ **[3.3]** **17.** $\frac{7}{30}$ **[3.4]** **18.** $3\frac{9}{10}$ **[4.3]** **19.** 12.468
[4.5] **20.** 3.9 **21.** 0.005238 **[4.4]** **22.** 1.1385 **23.** 15,300
[4.2] **24.** $\frac{43}{100}$ **25.** **(a)** 0.625; **(b)** 0.39 **[4.5]** **26.** 0.429
[4.4] **27.** 17.21 **[4.5]** **28.** 39.829

Prerequisite Test for Chapter 5

1. $\frac{3}{4}$ **2.** $\frac{8}{15}$ **3.** $\frac{3}{4}$ **4.** $\frac{28}{5}$ **5.** $\frac{3}{4}$ **6.** 525 **7.** 35
8. 5.56 **9.** 7.22 **10.** No **11.** Yes

Reading Your Text

Section 5.1 **(a)** fraction; **(b)** like; **(c)** simplest; **(d)** mixed
Section 5.2 **(a)** like; **(b)** rate; **(c)** Mixed; **(d)** Unit
Section 5.3 **(a)** equal; **(b)** variable; **(c)** proportion; **(d)** rates
Section 5.4 **(a)** equation; **(b)** coefficient; **(c)** read; **(d)** proportional

Summary Exercises for Chapter 5

1. $\frac{4}{17}$ **3.** $\frac{5}{8}$ **5.** $\frac{4}{9}$ **7.** $\frac{7}{36}$ **9.** $100\frac{\text{mi}}{\text{h}}$ **11.** $50\frac{\text{cal}}{\text{oz}}$
13. $200\frac{\text{ft}}{\text{s}}$ **15.** $6\frac{1}{2}\frac{\text{hits}}{\text{game}}$ **17.** Marisa **19.** $\frac{9¢}{\text{oz}}$
21. $9.5\frac{\text{cents}}{\text{oz}}$ **23.** $14.95\frac{\text{dollars}}{\text{CD}}$ **25.** $\frac{4}{9} = \frac{20}{45}$
27. $\frac{110\text{ mi}}{2\text{ h}} = \frac{385\text{ mi}}{7\text{ h}}$ **29.** No **31.** Yes **33.** Yes **35.** Yes
37. $m = 2$ **39.** $t = 4$ **41.** $w = 180$ **43.** $x = 100$
45. \$135 **47.** 15 in. **49.** 28 parts **51.** 140 g

Self-Test for Chapter 5

1. $4.8\frac{\text{mi}}{\text{gal}}$ **2.** $8.25\frac{\text{dollars}}{\text{h}}$ **3.** $x = 20$ **4.** $a = 18$
5. $\frac{\$2.56}{\text{gal}}$ **6.** $\frac{26}{33}; \frac{26}{7}$ **7.** $p = 3$ **8.** $m = 16$ **9.** $\frac{7}{19}$
10. $\frac{5}{3}$ **11.** $\frac{2}{3}$ **12.** $\frac{1}{12}$ **13.** \$2.28 **14.** 576 miles
15. 600 mufflers **16.** 24 tsp **17.** Yes **18.** No
19. Yes **20.** No

Cumulative Review Chapters 1–5

[1.1] **1.** Forty-five thousand, seven hundred eighty-nine
2. Ten thousands **[1.2]** **3.** 26,304 **[1.3]** **4.** 47,806
[1.5] **5.** 4,408 **[1.6]** **6.** 78 r67 **[1.3]** **7.** \$568
[1.7] **8.** 3 **[1.8]** **9.** P: 16 ft; A: 12 ft^2 **10.** \$1,104
[2.2] **11.** $2 \times 2 \times 3 \times 7 \times 11$ **12.** 14 **[2.4]** **13.** $\frac{1}{4}$
[2.5] **14.** $1\frac{1}{5}$ **15.** 10 **[2.7]** **16.** $4\frac{1}{2}$ **17.** $1\frac{9}{13}$
[3.3] **18.** $\frac{25}{44}$ **[3.4]** **19.** $7\frac{7}{12}$ **20.** $4\frac{1}{2}$ **[3.2]** **21.** 180
[2.6] **22.** 176 mi **[2.7]** **23.** $48\frac{\text{mi}}{\text{h}}$ **[4.2]** **24.** 7.828
[4.5] **25.** 1.23 **[4.3]** **26.** 6.6015 **[4.7]** **27.** $\frac{9}{25}$
[4.6] **28.** 0.32 **[4.2]** **29.** 14.06 m **[4.4]** **30.** 50.24 cm^2
[5.1] **31.** $\frac{6}{13}$ **32.** $\frac{10}{3}$ **33.** $\frac{4}{5}$ **[5.3]** **34.** Yes **35.** No
[5.4] **36.** $x = 2$ **37.** $x = 15$ **[5.2]** **38.** $\frac{24.4¢}{\text{oz}}$
[5.4] **39.** 600 km **40.** 50

Prerequisite Test for Chapter 6

1. $\frac{11}{25}$ **2.** 0.3125 **3.** 41.6 **4.** 20 **5.** 120 **6.** 52.5
7. 48 **8.** $58\frac{1}{3}$ **9.** 70 **10.** 8,000

Reading Your Text

Section 6.1 **(a)** hundred; **(b)** fraction; **(c)** left; **(d)** greater
Section 6.2 **(a)** right; **(b)** decimal; **(c)** percent; **(d)** zeros
Section 6.3 **(a)** base; **(b)** rate; **(c)** rate; **(d)** amount
Section 6.4 **(a)** greater; **(b)** amount; **(c)** markup; **(d)** principal

Summary Exercises for Chapter 6

1. 75% **3.** $\frac{1}{5}$ **5.** $1\frac{1}{2}$ **7.** 3 **9.** 0.04 **11.** 0.135
13. 2.25 **15.** 37.5% **17.** 700% **19.** 0.5% **21.** 70%
23. 125% **25.** 27.3% **27.** 140% **29.** 12.5% **31.** 75
33. 75 **35.** 175% **37.** \$1,800 **39.** 7.5% **41.** \$102
43. 720 students **45.** \$3,157.50 **47.** 500 s (8 min 20 s)

Self-Test for Chapter 6

1. 3% **2.** 4.2% **3.** 40% **4.** 62.5% **5.** 11.25 **6.** 20%
7. 500 **8.** 750 **9.** \$4.96 **10.** 60 questions **11.** \$70.20
12. 12% **13.** 80% **14.** 0.42 **15.** 0.06 **16.** 1.6
17. A: 50; R: 25%; B: 200 **18.** A: Unknown; R: 8%; B: 500
19. R: 6%; A: \$30; B: amount of purchase **20.** 175% **21.** 800
22. 300 **23.** 7.5% **24.** 8% **25.** \$18,000
26. 6,400 students **27.** \$18,500 **28.** 24% **29.** $\frac{7}{100}$
30. $\frac{18}{25}$

Cumulative Review Chapters 1–6

[1.1] **1.** Thousands **[1.5]** **2.** 11,368 **[1.6]** **3.** 89
[1.7] **4.** 5 **5.** 9 **6.** 42 **[2.1]** **7.** 53, 59, 61, 67
[2.2] **8.** $2 \times 2 \times 5 \times 13$ **9.** 28 **[3.2]** **10.** 180
[2.5] **11.** $8\frac{1}{2}$ **[2.7]** **12.** $1\frac{1}{3}$ **[3.4]** **13.** $8\frac{7}{12}$ **14.** $4\frac{19}{24}$
[2.6] **15.** \$286 **[2.7]** **16.** $54\frac{\text{mi}}{\text{h}}$ **[3.4]** **17.** $41\frac{1}{4}$ in.
[4.1] **18.** Hundredths **19.** Ten-thousandths **20.** <
21. = **[4.7]** **22.** $\frac{9}{25}$ **23.** $5\frac{1}{8}$ **[4.3]** **24.** 11.284
[4.5] **25.** 17.04 **[4.3]** **26.** \$108.05 **[4.4]** **27.** 6.29 m^2
28. 44.064 ft^3 **[5.1]** **29.** $\frac{2}{3}$ **30.** $\frac{17}{12}$ **[5.4]** **31.** $x = 18\frac{2}{3}$
32. $y = 0.4$ **33.** 350 mi **34.** \$140 **[6.1]** **35.** 0.34; $\frac{17}{50}$
[6.2] **36.** 0.55; 55% **[6.4]** **37.** 45 **38.** 500
39. 125 employees **40.** 8.5%

Prerequisite Test for Chapter 7

1. 428,400 **2.** 0.004284 **3.** $\frac{4}{3}$ **4.** $\frac{5}{12}$ **5.** $\frac{5}{2}$ **6.** $\frac{3}{5}$
7. \$10.75/h **8.** $50\frac{\text{mi}}{\text{h}}$ **9.** \$0.0514/fl oz **10.** \$0.45/DVD-R
11. 54 in. **12.** 500 mm

Reading Your Text

Section 7.1 **(a)** metric; **(b)** volume; **(c)** one; **(d)** like
Section 7.2 **(a)** metric; **(b)** yard; **(c)** centi-; **(d)** Kilometers
Section 7.3 **(a)** gram; **(b)** milligram; **(c)** liter; **(d)** milliliter
Section 7.4 **(a)** substitution; **(b)** unit-ratio; **(c)** gravity; **(d)** Celsius

Summary Exercises for Chapter 7

1. 132 **3.** 24 **5.** 64 **7.** 4 **9.** 4 ft 11 in.
11. 9 lb 3 oz **13.** 5 ft 7 in. **15.** 4 h 15 min **17.** 33 h 35 min
19. **(a)** **21.** **(c)** **23.** mm **25.** cm **27.** 30
29. 8,000 **31.** 0.008 **33.** **(b)** **35.** **(b)** **37.** kg
39. 5,000 **41.** 5,000 **43.** **(c)** **45.** **(b)** **47.** L
49. mL or cm^3 **51.** 5,000 **53.** 9,000 **55.** 326.77
57. 6.75 **59.** 5.51 **61.** 62.6 **63.** 37 **65.** 15 **67.** 41

Self-Test for Chapter 7

1. 96 **2.** 48 **3.** 5,000 **4.** 3,000 **5.** 3 **6.** 101.6
7. 3.2 **8.** 55.1 **9.** 67.5 **10.** 30.5 **11.** 14.4

12. 75.2 **13.** 6 ft 9 in. **14.** 4 days 12 min **15.** 11 ft 5 in.
16. 2 lb 9 oz **17.** 15 h 20 min **18.** 4 lb 6 oz **19.** **(b)**
20. **(b)** **21.** **(b)** **22.** **(b)** **23.** $1,050 **24.** 8.6 lb
25. $0.0013/mL

Cumulative Review Chapters 1–7

[1.5] 1. $896 **[1.6] 2.** $62 **[1.7] 3.** 16
[2.2] 4. $2 \times 2 \times 2 \times 3 \times 7$ **5.** 4 **[2.4] 6.** $\frac{3}{5}, \frac{5}{8}, \frac{2}{3}$
[2.5] 7. $\frac{3}{4}$ **[2.7] 8.** $\frac{5}{12}$ **[3.2] 9.** 60 **[3.3] 10.** $1\frac{1}{30}$
[3.4] 11. $3\frac{13}{24}$ **[4.2] 12.** $18.30 **[4.3] 13.** 27.84 cm^2
[4.2] 14. 21.5 cm **[4.6] 15.** 0.5625 **16.** 0.538
[5.4] 17. $x = 24$ **[5.5] 18.** 400 mi **19.** 450 mi
[6.2] 20. 37.5% **[6.1] 21.** $\frac{1}{8}$ **[6.4] 22.** 3,526
23. 225% **24.** 150 **[6.5] 25.** $142,500 **[7.1] 26.** 120
27. 1 min 35 s **28.** 21 lb 11 oz **29.** 2 ft 9 in.
30. 21 h 20 min **[7.2] 31.** 0.43 **[7.3] 32.** 62,000
[7.2] 33. 74 **[7.3] 34.** 14,000 **35.** 0.5 **[7.4] 36.** 13.3
37. 149.6 **38.** 29.4 **39.** 48.2

Prerequisite Test for Chapter 8

1. 25 **2.** 169 **3.** 78.5 **4.** 36 **5.** 17.4 **6.** $\frac{45}{2}$
7. 19.63 **8.** 7.0 in. **9.** 4.4 m **10.** 118 ft **11.** 144 mm^2
12. 173.25 in.^2

Reading Your Text

Section 8.1 **(a)** Earth; **(b)** perpendicular; **(c)** obtuse; **(d)** complementary
Section 8.2 **(a)** triangle; **(b)** regular; **(c)** sum; **(d)** nine
Section 8.3 **(a)** circumference; **(b)** radius; **(c)** composite; **(d)** area
Section 8.4 **(a)** equilateral; **(b)** isosceles; **(c)** similar; **(d)** corresponding
Section 8.5 **(a)** radical; **(b)** hypotenuse; **(c)** Pythagorean; **(d)** whole

Summary Exercises for Chapter 8

1. $\angle AOB$; acute; 70° **3.** $\angle AOC$; obtuse; 100°
5. $\angle XYZ$; straight; 180° **7.** 135° **9.** 47° **11.** 73°
13. 79° **15.** 67° **17.** $A = 750 \text{ ft}^2$
19. $P = 60$ in.; $A = 216 \text{ in.}^2$ **21.** $P = 93$ ft; $A = 457.5 \text{ ft}^2$
23. 180 ft^2; 20 yd^2 **25.** $C = 37.7$ in.; $A = 113.0 \text{ in.}^2$
27. $P = 56$ ft; $A = 128 \text{ ft}^2$ **29.** 60°; equilateral
31. 45°; isosceles **33.** 14.3 **35.** 32 m **37.** 28
39. 9.54 **41.** 15

Self-Test for Chapter 8

1. 60 mm **2.** 12 in. **3.** 256 yd **4.** 62 m **5.** 94.2 ft
6. 91.7 ft **7.** 189 mm^2 **8.** 7 in.^2 **9.** 2,800 yd^2
10. 54 m^2 **11.** 706.5 ft^2 **12.** 332.8 ft^2
13. **(a)** Parallel; **(b)** neither; **(c)** perpendicular; **(d)** neither
14. **(a)** Straight; **(b)** obtuse; **(c)** right **15.** 50° **16.** 135°
17. 137° **18.** 66° **19.** Obtuse **20.** Right **21.** 67°
22. 53.9° **23.** 21 **24.** 65 m **25.** 112 mm

Cumulative Review Chapters 1–8

[1.1] 1. Ten-thousands **[1.7] 2.** 64 **[2.1] 3.** $2 \cdot 3 \cdot 3 \cdot 5 \cdot 7$
4. 4 **5.** 120 **[2.5] 6.** $\frac{3}{4}$ **[2.6] 7.** $4\frac{1}{2}$ **8.** $578
9. $60 \frac{\text{mi}}{\text{h}}$ **[3.3] 10.** $1\frac{13}{30}$ **[3.4] 11.** $3\frac{13}{24}$ **12.** $6\frac{7}{12}$ mi
[4.1] 13. Hundredths **14.** Ten–thousandths

[4.4] 15. $P = 24.2$ ft; $A = 35.04 \text{ ft}^2$ **[4.2] 16.** $\frac{1}{8}$
17. 0.3125 **[5.2] 18.** $20,250/yr **[5.4] 19.** $w = 4$
20. 1,125 ft^2 **[6.1] 21.** 0.085 **22.** $\frac{3}{8}$ **[6.2] 23.** 67.5%
[6.4] 24. 51.2 **25.** 1,500 **26.** 1,350
[7.1–7.4] 27. 5,280 **28.** 0.25 **29.** 5,800
[8.1] 30. 45° **31.** 49.6 ft **32.** 342 cm^2
33. 1,256 m^2 **34.** 60.63 ft^2 **[8.3] 35.** 73°; isosceles
[8.4] 36. $x = 40$ **37.** 17 ft

Prerequisite Test for Chapter 9

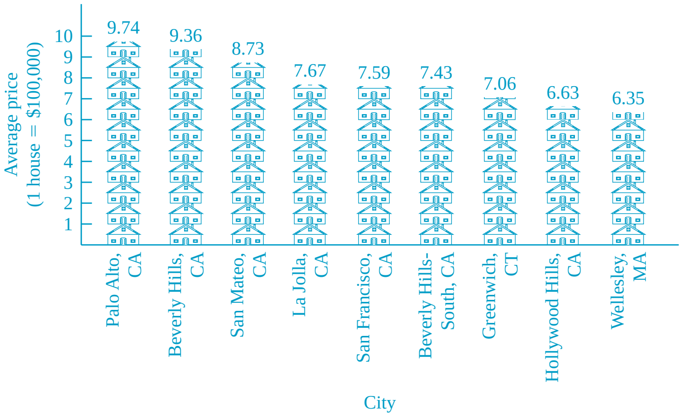
1. $\frac{4}{5}$ **2.** $\frac{3}{4}$ **3.** 6 **4.** $\frac{17}{2}$ **5.** 27 **6.** 77 **7.** 90
8. 300 **9.** 62.5% **10.** $16.\overline{6}\%$ **11.** 0, 1, 2, 5, 7, 10, 11, 13
12. 7, 12, 21, 50, 55, 56, 81, 123 **13.** $1\frac{1}{8}$ in. **14.** 120°

Reading Your Text

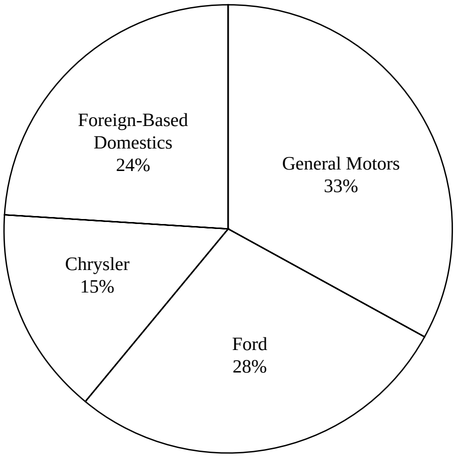
Section 9.1 **(a)** mean; **(b)** median; **(c)** mode; **(d)** bimodal
Section 9.2 **(a)** rows; **(b)** cell; **(c)** pictograph; **(d)** legend
Section 9.3 **(a)** line; **(b)** future; **(c)** extrapolation; **(d)** predict
Section 9.4 **(a)** table; **(b)** approximation; **(c)** pie; **(d)** circle
Section 9.5 **(a)** quartiles; **(b)** three; **(c)** summary; **(d)** box-and-whisker

Summary Exercises for Chapter 9

1. 6 **3.** 120 **5.** 85 **7.** 19; 20 **9.** 29; 28 **11.** 92
13. 32,000 vehicles; 10,145,000 vehicles **15.** 60%
17. 17.7% **19.** 60%
21. 1 house = $100,000

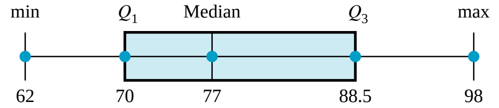


23. 5,000 students **25.** 250,000 computers
27. About 400,000 computers
29.



31. 47% **33.** 21, 24, 28, 30, 35

35. The data do not exhibit any striking features (the data set is mildly skewed right).

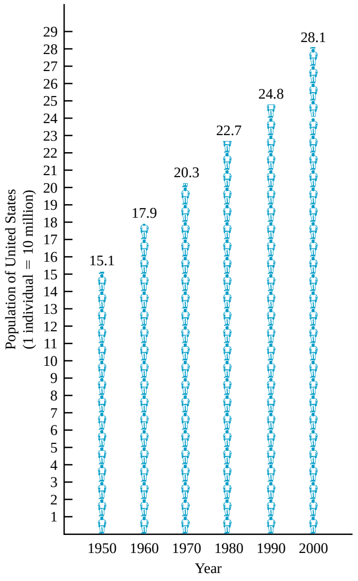


Self-Test for Chapter 9

1. 15 **2.** 8 **3.** 11 **4.** 6 **5.** 204 **6.** 93

7. Brown **8.** 31,162 **9.** 41,378 **10.** 324% **11.** 124%

12.

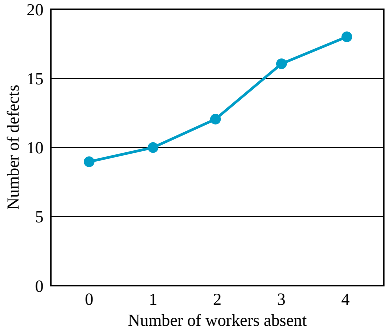


13. 16,000 people **14.** 30,000 people **15.** 10,000 people

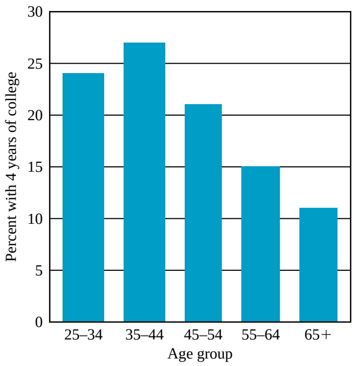
16. 16,000 people **17.** 2000 to 2001 **18.** December

19. August and September

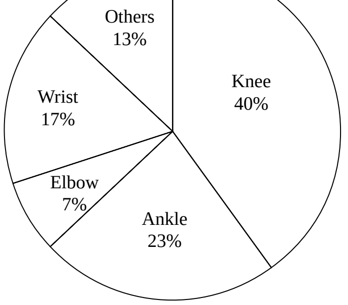
20. about 21 defects



21.

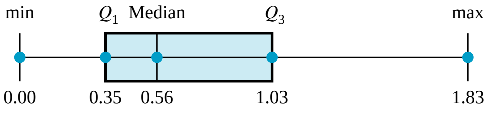


22.



23. 45% **24.** 85%

25.



Cumulative Review Chapters 1–9

[1.1] 1. Thousands **[1.2] 2.** 32,278 **[1.3] 3.** 39,288

[1.5] 4. 26,230 **[1.6] 5.** 308 r 6 **[1.3] 6.** 48,588

[4.3] 7. 75.215 **[2.5] 8.** $\frac{2}{3}$ **[2.7] 9.** $\frac{6}{11}$ **[3.4] 10.** $7\frac{1}{12}$

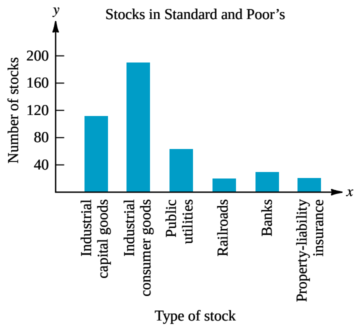
[5.4] 11. $x = 14$ **12.** $x = 9$ **[6.1] 13.** 0.18; $\frac{9}{50}$

14. 0.425; 42.5% **[7.1] 15.** 11 lb 5 oz **16.** 1 min 35 s

[7.2] 17. 8,000 **[7.3] 18.** 3 **19.** 5 **20.** 250

[9.3] 21. 2002 and 2003

[9.4] 22.



[9.1] 23. Mean: 9; median: 9; mode: 11 **[5.2] 24.** 70 gal

25. $88 \frac{\text{ft}}{\text{s}}$ **26.** $19 \frac{\text{pitches}}{\text{inning}}$ **[1.8] 27.** $408 **28.** $48 \frac{\text{mi}}{\text{h}}$

29. $28\frac{3}{5}$ cm **30.** $55\frac{1}{3}$ ft **[6.4] 31.** 133 **32.** 0.2%

33. 185 **[2.5] 34.** $4\frac{1}{3}$ **[6.5] 35.** 2,800 students

Prerequisite Test for Chapter 10

1. 21 **2.** 79 **3.** 368 **4.** $15\frac{1}{4}$ **5.** 187 **6.** 32

7. Commutative property of addition

8. Distributive property of multiplication over addition

9. Associative property of multiplication

10. Distributive property of multiplication over addition

11. 33.91 mm **12.** 91.56 mm^2

Reading Your Text

Section 10.1 **(a)** zero; **(b)** negative; **(c)** ascending; **(d)** absolute value

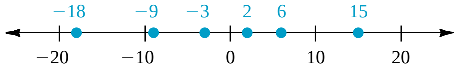
Section 10.2 **(a)** negative; **(b)** negative; **(c)** absolute; **(d)** zero

Section 10.3 **(a)** addition; **(b)** subtraction; **(c)** opposite; **(d)** positive

Section 10.4 **(a)** negative; **(b)** positive; **(c)** identity; **(d)** reciprocal
Section 10.5 **(a)** negative; **(b)** positive; **(c)** Division; **(d)** positive

Summary Exercises for Chapter 10

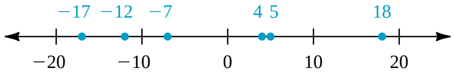
1.



3. $-\frac{4}{5}, -\frac{2}{3}, -\frac{1}{2}, \frac{3}{5}, \frac{7}{10}, \frac{5}{6}$ **5.** Max: 8; min: −9 **7.** 9
9. −9 **11.** −4 **13.** −4 **15.** 6 **17.** −32 **19.** −1
21. −14 **23.** −17 **25.** −4 **27.** 2 **29.** 16
31. −4 **33.** −4 **35.** −2 **37.** −1 **39.** −70
41. 45 **43.** 0 **45.** $-\frac{3}{2}$ **47.** 80 **49.** 10 **51.** −2
53. −60 **55.** −9 **57.** 0 **59.** Undefined **61.** −1

Self-Test for Chapter 10

[10.1] 1.



2. $-6, -3, -2, 0, \frac{1}{2}, \frac{3}{4}, 2, 4, 5$ **3.** Max: 6; min: −5 **4.** 7
5. 7 **6.** −13 **7.** −3 **8.** −6 **9.** −24 **10.** −40
11. 63 **12.** −25 **13.** 3 **14.** 11 **15.** 11 **16.** −21
17. 1 **18.** 9 **19.** 0 **20.** –27 **21.** −24 **22.** −5
23. Undefined **24.** −24 **25.** −10

Cumulative Review Chapters 1–10

[1.1] 1. Ten thousands **[1.2] 2.** 142,231 **[1.3] 3.** 29,573
[1.5] 4. 53,445 **[1.6] 5.** 402 r 28 **[4.2] 6.** 13.687
[4.3] 7. 1,837.353 **[2.5] 8.** $\frac{3}{7}$ **[2.7] 9.** 1
[3.4] 10. $8\frac{19}{54}$ **[5.4] 11.** $x = 4$ **[6.1] 12.** 0.58; $\frac{29}{50}$
[6.2] 13. 0.48; 48% **[7.1] 14.** 8 ft 10 in. **15.** 9 lb 4 oz
16. 12 ft 6 in. **17.** 2 lb 14 oz **18.** 10 h 30 min
19. 2 min 9 s **20.** 14 ft 8 in.
[5.2] 21. The three smaller bottles **[4.4] 22.** 49.02 cm^2
23. $72\frac{1}{4}$ in.^2 **24.** 25.7 ft **[6.4] 25.** 80 **[6.5] 26.** 7.5%
[7.3] 27. 0.017 **[7.2] 28.** 820 **[7.5] 29.** 160°
[9.2] 30. 5,000 students **[9.1] 31.** Mean: 17; median: 17;
mode: 17 **[10.2] 32.** 4 **[10.3] 33.** 20 **34.** −32
35. −15 **36.** 31 **[10.4] 37.** 108 **[10.5] 38.** 4
39. 41 **40.** 72

Prerequisite Test for Chapter 11

1. $10 - 8 = 2$ **2.** $3 + 5 \times 6 = 33$ **3.** $-\frac{1}{12}$ **4.** $\frac{8}{37}$
5. 1 **6.** 1 **7.** 1 **8.** 23 **9.** −64 **10.** 64
11. $14,117.65 **12.** $1.55

Reading Your Text

Section 11.1 **(a)** variables; **(b)** sum; **(c)** multiplication; **(d)** expression
Section 11.2 **(a)** evaluating; **(b)** variable; **(c)** operations; **(d)** positive
Section 11.3 **(a)** term; **(b)** number; **(c)** like; **(d)** distributive
Section 11.4 **(a)** equation; **(b)** true; **(c)** solution(s); **(d)** sentence
Section 11.5 **(a)** equivalent; **(b)** original; **(c)** dividing; **(d)** reciprocal
Section 11.6 **(a)** isolate; **(b)** addition; **(c)** original; **(d)** solved

Summary Exercises for Chapter 11

1. $y + 5$ **3.** $8a$ **5.** $5mn$ **7.** $17x + 3$ **9.** −7
11. 15 **13.** 19 **15.** 25 **17.** 1 **19.** −3
21. $4a^3, -3a^2$ **23.** $5m^2, -4m^2, m^2$ **25.** $12c$ **27.** $2a$
29. $3xy$ **31.** $19a + b$ **33.** $3x^3 + 9x^2$ **35.** $10a^3$
37. Yes **39.** Yes **41.** No **43.** $x = 2$ **45.** $x = -5$
47. $x = 5$ **49.** $x = 1$ **51.** $x = -7$ **53.** $x = 7$
55. $x = -4$ **57.** $x = 32$ **59.** $x = 27$ **61.** $x = 3$
63. $x = -2$ **65.** $x = \frac{7}{2}$ **67.** $x = 18$ **69.** $x = 6$
71. $x = \frac{2}{5}$ **73.** $x = 6$ **75.** $x = 6$ **77.** $x = 4$
79. $x = 5$ **81.** $x = \frac{1}{2}$

Self-Test for Chapter 11

1. $a - 5$ **2.** $6m$ **3.** $4(m + n)$ **4.** $4m + n$ **5.** No
6. Yes **7.** 11 **8.** 7 **9.** 3 **10.** 4 **11.** −12
12. 12 **13.** 7 **14.** 25 **15.** $-\frac{2}{3}$ **16.** −5 **17.** −4
18. 80 **19.** 144 **20.** 5 **21.** $15a$ **22.** $3x^2y$
23. $19x + 5y$ **24.** $4m^2 - 2m - 5$ **25.** 58 double lattes

Cumulative Review Chapters 1–11

[1.2] 1. Associative property of addition
[1.5] 2. Commutative property of multiplication
3. Distributive property **[1.4] 4.** 5,900 **5.** 950,000
[1.7] 6. 8 **[2.2] 7.** $2 \times 2 \times 2 \times 3 \times 11$ **[3.2] 8.** 90
[2.3] 9. $3\frac{1}{7}$ **10.** $\frac{53}{8}$ **[2.5] 11.** $\frac{3}{4}$ **[2.6] 12.** $1\frac{1}{2}$
[3.4] 13. $8\frac{1}{24}$ **14.** $3\frac{5}{8}$ **15.** $\frac{5}{8}$ in. **[4.3] 16.** $3.06
[8.1] 17. 8.04 ft^2 **[4.5] 18.** 32 **[4.2] 19.** $0.\overline{72}$
[5.4] 20. $m = 112.5$ **[5.5] 21.** 12 in. **[6.2] 22.** 0.3%
23. 62.5% **24.** 350% **[6.4] 25.** 150 **26.** 600
27. $27,500 **[7.3] 28.** 0.3 **[9.3] 29.** 80
[10.2] 30. −8 **[10.1] 31.** 20 **32.** 12 **33.** 5
[10.2] 34. −18 **[10.3] 35.** −4 **[10.4] 36.** −90
37. −4 **[11.1] 38.** $3(x + y)$ **39.** $\frac{n - 5}{3}$
[11.2] 40. −60 **41.** 27 **42.** 24 **43.** 3
[11.3] 44. $12a^2 + 3a$ **45.** $-2y$ **[11.4] 46.** 5
[11.5] 47. −24 **[11.6] 48.** $-\frac{2}{5}$ **49.** 5

index

- Absolute value, 648, 694
- Absolute value expressions, 648
- Acute angles, 485–486, 550
- Acute triangles, 526, 553
- Addends, 11
- Addition, 10–25, 90
  - in algebra, 703, 764
  - applications of, 14–15
    - with decimals, 269–270, 273
    - with like fractions, 185
    - with real numbers, 658
    - with unlike fractions, 202–203
  - associative property of, 12–13, 90
  - basic facts of, 11–13, 90
  - on calculators, 22–23
  - commutative property of, 12, 90
  - of decimals, 268–270, 273–280, 305
    - applications of, 269–270, 273
  - definition of, 10
  - of denominate numbers, 28, 429, 471
  - of digits of same place value, 13–14
  - distributive property of multiplication over, 50, 91
  - estimating, 41–42
  - in expanded form, 15
  - of fractions
    - with like denominators, 183–185, 187–190, 237
    - with unlike denominators, 200–203, 206–211, 238
  - identity property of, 13, 90
  - of like fractions, 183–185, 187–190, 237
    - applications of, 185
    - mixed numbers from, 184–185
  - of mixed numbers, 213–214, 219–223, 238
  - multiplication and, 48
  - on the number line, 11
  - in order of operations, 81, 82
    - with decimals, 297
    - with fractions, 225
    - with real numbers, 684
  - of real numbers, 654–662, 694
    - additive identity property of, 657
    - additive inverse property of, 657
    - applications of, 658
    - with different signs, 656
    - with same sign, 655
  - regrouping in, 15–16
  - in short form, 15–16
  - subtraction and, 26
  - symbol for, 11
  - of unlike fractions, 200–203, 206–211, 238
    - applications of, 202–203
  - of whole numbers, 10–19
  - words for, 14
  - of zero, 657
- Addition property of equality, 732
  - in solving equations, 732–735, 754–763, 765
- Additive identity property, 13, 90, 657
- Additive inverse property, 657
- Aerobic training zone, 236
- Algebra. *See also* Equations
  - addition in, 703, 764
  - applications of, 705–706, 759
  - division in, 705, 764
  - multiplication in, 704, 764
  - for percent problems, 746–749, 765
  - substitution in, 764
  - subtraction in, 703
  - variables in, 703
- Algebraic expressions, 704–705
  - applications of, 715, 724
  - evaluating, 711–719, 764
  - like terms in, 722–724, 764
  - numerical coefficients in, 722
  - order of operations for, 712–713
  - for perimeter, 721
  - simplifying, 721–728, 764
  - terms in, 721–722, 764
- Alternate interior angles, 489–490
- Amount, in percent problems, 389–398, 400, 414–415
- Angles, 485–490, 550
  - acute, 485–486, 550
  - alternate interior, 489–490
  - complementary, 488, 550
  - corresponding, 490
  - definition of, 484
  - measuring, 486–489
  - naming, 485
  - obtuse, 485–486, 550
  - of polygons, 497
  - right, 485, 550
  - straight, 485–486, 550
  - supplementary, 488, 550
  - of triangles
    - measuring, 528
    - sum of, 527–528
  - vertical, 489, 550
- Applications
  - of addition, 14–15
    - of decimals, 269–270, 273
    - of like fractions, 185
    - of real numbers, 658
    - of unlike fractions, 202–203
  - of algebra, 705–706, 759
  - of algebraic expressions, 715, 724
  - of circles, 519–520
  - of decimals
    - addition of, 269–270, 273
    - division of, 292–293, 294–295, 296–297
    - multiplication of, 282–283
    - subtraction of, 272–273
  - of denominate numbers, 431–432
  - of division, 66, 72
    - of decimals, 292–293, 294–295, 296–297
    - of mixed numbers, 157–159
  - of equations, 736, 748–749
  - estimating in, 42
  - of estimation, 231–235, 238
  - of fractions
    - like, 185
    - multiplication of, 144–147
    - order of operations in, 226–227
    - unlike, 202–203, 204–205
  - of like fractions, addition of, 185
  - of mean, 573–574
  - of metric length, 443–444
  - of metric volume, 452
  - of mixed numbers
    - division of, 157–159
    - multiplication of, 145–147
    - subtraction of, 217–218
  - of multiplication, 51, 55
    - of decimals, 282–283
    - of fractions, 144–147
    - of mixed numbers, 145–147
    - by powers of 10, 283–285
    - of real numbers, 676
  - order of operations in, for fractions, 226–227
  - of percents, 370, 381, 400–412, 414–415
    - base, amount, and rate, 391, 400–401, 414–415
  - of proportions, 337
    - solving, 345–347
  - of real numbers, 648–649
    - addition of, 658
    - multiplication, 676
    - subtraction, 666
  - of subtraction, 28–29, 31
    - of decimals, 272–273
    - of mixed numbers, 217–218
    - of real numbers, 666
    - of unlike fractions, 204–205
  - of temperature conversion, 464
  - of unit conversion, 462–463
  - of unit-ratio method, 431–432
  - of unlike fractions
    - addition of, 202–203
    - subtraction of, 204–205
- Approximately (≈), 515
- Approximate numbers, 252
- Archimedes, 81
- Area, 56, 91, 501–506, 551
  - of circle, 516–517
  - of composite figures, 518–519
  - of parallelogram, 503
  - of rectangle, 56, 502
  - of square, 56–57, 502
  - of trapezoid, 505
  - of triangle, 504
  - units of, conversion factors for, 506, 552
- Arithmetic, fundamental theorem of, 109
- Arithmetic sequences, 22
- Ascending order, 646
- Associative property
  - of addition, 12–13, 90
  - of multiplication, 53, 91
- Average. *See* Mean; Median; Mode

- Bar graphs, 586–587, 630
  - creating, 604–605, 630
- Bar notation, 260
- Base
  - in exponential notation, 79, 91
  - in percent problems, 389–398, 401, 414–415
- Basic addition facts, 11–13, 90
- Basic multiplication facts, 49, 91
- Bimodal data set, 573
- Bit, 425
- Borrowing, 29–30
  - in subtraction of decimals, 271
  - in subtraction of mixed numbers, 215–216
- Box-and-whisker plot, 618–619, 631
- Byte, 425

- Calculators, 22
  - addition on, 22–23
    - of decimals, 279
    - of fractions, 209
    - of mixed numbers, 221–222
  - algebraic expressions on, 714–715, 719
  - decimal equivalents on, 265
  - decimals on
    - addition of, 279
    - division of, 301
    - multiplication of, 288
    - subtraction of, 279
  - division on
    - of decimals, 301
    - of fractions, 163
  - fractions on
    - addition of, 209
    - division of, 163
    - multiplication of, 150–151
    - simplification of, 135
    - subtraction of, 209
  - mean on, 578
  - mixed numbers on
    - addition of, 221–222
    - subtraction of, 221–222
  - multiplication on
    - of decimals, 288
    - of fractions, 150–151
  - negative numbers on, 669, 690–691
  - parentheses on, 567
  - percent on, 410–411
  - proportions on, 353
  - scientific notation on, 289
  - square roots on, 546
  - statistical functions on, 578
  - subtraction on, 34–35

Calculators—*Cont.*
    of decimals, 279
    of fractions, 209
    of mixed numbers, 221–222
Carrying, 15–16
  in addition of decimals, 268–269
Cells, 583
Celsius, 463–464
Census, 1
Centi-, 442
Centimeters (cm), 439–440
Centimeters, cubic (cm^3), 451
Chain, 437
Circles, 515–517
  applications of, 519–520
  area of, 516–517
  circumference of, 515–516, 525
Circumference, 515–516, 525
Classmates, 99
Coefficient, in proportion, 342, 344
Commas, in numbers, 4
Commissions, 401–403
Common denominator, 193
Common factors, 110–111
  in simplification of fractions, 131
Common factors, greatest (GCFs), 111–112, 167–168
  in simplification of fractions, 130
Common fractions. *See* Fractions
Common multiples, 191–199, 237
Common multiples, least (LCMs), 192–193, 237
Commutative properties
  of addition, 12, 90
  of multiplication, 48–49, 109
Complementary angles, 488, 550
Complex fractions, 153–154
Composite figures, 517–519, 539
  area of, 518–519
  definition of, 517
  perimeter of, 517–518
Composite numbers, 100, 101, 167
  factorization of, 108–109
  prime factors of, 108–110
Compound interest, 406–407
Conditional equations, 730
Continued division, 110
Cord, 425
Corresponding angles, 490
Counting numbers. *See* Natural numbers
Covalent bonding number, 692
Cross products, 128–129, 169
Cubic centimeters (cm^3), 451
Cubic units, 58
Cubit, 425
Cup (c), 425

Data analysis
  graphs, 630
    bar graphs, 586–587, 604–605, 630
    box-and-whisker plot, 618–619, 631
    legends of, 586–587
    line graphs, 596–603, 630
    pictographs, 584–585
    pie charts, 605–609, 630
  interquartile range, 625
  mean, 567–568, 616, 629
    applications of, 573–574
    with extreme value, 568
    median and, 571–572
    not typical, 568
  median, 568–571, 616, 629
    mean and, 571–572
  mode, 572–573, 616, 629
  outliers, 625–626
  predictions, 597–599
  ranges, 625
  tables, 583–584, 629
Data sets
  bimodal, 573
  describing, 616–619, 630–631
  extreme values of, 646–647
  max of, 617–618, 646–647
  mean of, 567–568, 616, 629
    applications of, 573–574
    with extreme value, 568
    median and, 571–572
    not typical, 568
  median of, 568–571, 616, 629
    mean and, 571–572
  min of, 617–618, 646–647
  mode of, 572–573, 616, 629
  quartiles of, 617–618, 630
  summarizing, 617–618, 630–631
Day, 425
Deci-, 2, 442
Deciles, 617
Decimal(s) (decimal fractions)
  addition of, 268–270, 273–280, 305
    applications of, 269–270, 273
  applications
    of addition, 269–270, 273
    of division, 292–293, 294–295, 296–297
    of multiplication, 282–283
    percents as, 370
    of subtraction, 272–273
  comparing, 251–252
  definition of, 249, 304
  division of, 291–302, 305
    applications of, 292–293, 294–295, 296–297
    by decimals, 293–294
    by powers of ten, 295–297, 305
    by whole numbers, 291–292
  fractions compared to, 262
  fractions converted from, 261, 304
  fractions converted to, 258–260, 304
  as mixed numbers, 251
  multiplication of, 281–289, 305
    applications of, 282–283
    estimation of, 282
    by powers of ten, 283–285, 305
  order of operations with, 297
  as percents, 377–378, 414
  percents as, 369
    applications of, 370
  place values in, 250
  rates as, 326
  repeating, 259–260, 262, 267
  rounding, 252–253, 304
  subtraction of, 270–280, 305
    applications of, 272–273
  terminating, 258, 267, 304
  in words, 250–251, 304
Decimal equivalents, 258–266
  on calculators, 265
  for percents, 378, 414
Decimal place values, 249–257, 304
  adding digits of same, 13–14
  in decimals, 250
  definition of, 90
  diagram of, 3
  identifying, 3–4, 249, 250
  rounding to, 252–253
Decimal place-value system, 2–9, 90
  definition of, 2
  diagram of, 3
Decimal point, 249, 251
Degrees, 485
Deka-, 442
Demographics, in census, 1
Denominate numbers. *See also* Units
  addition of, 28, 429, 471
  applications of, 431–432
  division of, 66, 144, 430–431, 472
  fractional, 144
    conversion of, 158
    reciprocals of, 231
  multiplication of, 51, 55, 144, 430, 472
  percents and, 379
  rates of, 324–333
  ratios of, 315–318
  simplifying, 429
  subtraction of, 28, 429–430
Denominator, 117–118, 168
  common, 193
  in multiplication of fractions, 139
  one as, 122
  in simplification, 129–130
  zero and, 68, 683
Descartes, René, 79
Diameter, of circle, 515
Difference, 26, 90
Digits
  definition of, 2, 90
  in numbers, 2–3
    in expanded form, 3
    in standard form, 3
Discounts, 403
Distributive property
  of multiplication over addition, 50, 91
  with negative numbers, 686
  in solving equations, 735–736
Dividend, 65, 91
  trial, 69
Divisibility
  exact, 67
  tests for, 101–104, 167
Division, 65–77, 91
  in algebra, 705, 764
  applications of, 66, 72
    of decimals, 292–293, 294–295, 296–297
    of mixed numbers, 157–159
  continued, 110
  of decimals, 291–302, 305
    applications of, 292–293, 294–295, 296–297
    by decimals, 293–294
    by powers of ten, 295–297, 305
    by whole numbers, 291–292
  of denominate numbers, 66, 144, 430–431, 472
  estimating, 72
  factorization by, 109–110
  as fractions, 118
  of fractions, 153–165, 170
    on calculators, 163
    rewriting, 154
  long, 68–72
  of mixed numbers, 156, 170
    applications of, 157–159
  multiplication checked with, 67
  in order of operations, 81, 82
    with decimals, 297
    with fractions, 225
    with real numbers, 684
  of real numbers, 682–684, 695
  by subtraction, 65, 67
  zero and, 68, 683
Division statement, 66
Divisor, 65, 91, 99. *See also* Common factors
  trial, 69

Egyptian numerals, 2
Ellipsis, 646
English system of measurement. *See* U.S. customary system of measurement
Equations
  algebraic expressions and, 731
  applications of, 736, 748–749
  conditional, 730
  definition of, 342, 730, 765
  equivalent, 732
  graphing, 742
  linear, 731
    solving, 759, 765
  with parentheses, 758–759
  for percent problems, 746–749, 765
  for proportions, 342
  simplifying, 735
  solving, 730–731, 765
    with addition property of equality, 730–741, 754–763, 765
    applications of, 736
    by combining like terms, 735, 746–747
    distributive property in, 735–736
    with multiplication property of equality, 743–753, 754–763, 765
    with reciprocals, 745–746
  writing, 729
Equilateral triangles, 527, 553
Equivalent equations, 732

- Equivalent fractions, 128–129, 169
  - in adding unlike fractions, 200
- Estimation, 41–42, 90
  - applications of, 231–235, 238
  - of division, 72
  - of metric length, 439–441
  - of products, 55
    - of decimals, 282
    - of mixed numbers, 143
  - of sums, 41–42
- Even numbers, 101
- Exact fractions, 380–381
- Expanded form
  - addition in, 15
  - numbers in, 3
- Exponent
  - in exponential notation, 79–80, 91
  - in order of operations, 82
    - with decimals, 297
    - with fractions, 225
  - in square units, 56
- Exponent notation, 79–81, 91
- Expressions
  - absolute value in, 648
  - algebraic, 704–705
    - applications of, 715, 724
    - equations and, 731
    - evaluating, 711–719, 764
    - like terms in, 722–724, 764
    - numerical coefficients in, 722
    - order of operations for, 712–713
    - for perimeter, 721
    - simplifying, 721–728, 764
    - terms in, 721–722, 764
  - evaluating, 225–226
- Extrapolation, 597
- Extreme outliers, 626
- Extreme values, 646–647

- Factor(s)
  - common, 110–111
    - in fraction simplification, 131
  - finding, 99–100
  - greatest common, 111–112, 167–168
    - in simplification of fractions, 130
  - in multiplication, 48, 90
  - prime
    - of composite numbers, 108–110, 167
    - in fraction simplification, 130–131
    - in least common denominators, 200
    - in least common multiples, 192
  - of whole numbers, 99–100, 108–116
- Factorization
  - definition of, 108
  - by division, 109–110
  - of whole numbers, 108–116, 167–168
- Fahrenheit, 463–464
- Fibonacci sequences, 22
- First-degree equations. *See* Linear equations
- First power, 80
- Five-number summary, 617–618, 631
- Fluid ounce, 425
- Focal length, 715
- Foot (ft), 425
- Formulas
  - for area of circle, 516
  - for area of rectangle, 56
  - for area of square, 56
  - for circumference, 515, 516
  - definition of, 17
  - for interest, 399
  - for perimeter, 17–18, 500
  - for volume of solid, 58
- Fraction(s), 117–127, 168–169
  - addition of
    - with like denominators, 183–185, 187–190
    - with unlike denominators, 200–203, 206–211
  - applications
    - of like fractions, 185
    - multiplication of, 144–147
    - order of operations with, 226–227
    - percents as, 370
    - of unlike fractions, 202–203, 204–205
  - comparing, 194–195
  - comparing to decimals, 262
  - complex, 153–154
  - cross products of, 128–129, 169
  - decimal (*See* Decimals)
  - decimal equivalents of, 258–266
  - decimals converted from, 258–260, 304
  - decimals converted to, 261, 304
  - definition of, 117, 168
  - with denominator of 1, 122
  - division as, 118
  - division of, 153–165, 170
    - on calculators, 163
    - rewriting, 154
  - equivalent, 128–129, 169
    - in adding unlike fractions, 200
  - exact, 380–381
  - fundamental principle of, 129, 169, 193
  - improper, 119, 168
    - converting to mixed numbers, 120–121, 169
    - from mixed numbers, 121–122, 169
    - ratios as, 316
    - in subtracting mixed numbers, 216
  - inequality symbols with, 194–195
  - like
    - addition of, 183–185, 187–190, 237
    - applications of, 185
    - simplifying, 184
    - subtraction of, 185–190, 237
  - in mixed numbers, 119–120
  - multiplication of, 139–151, 170
    - applications of, 144–147
    - on calculators, 150–151
    - denominator in, 139
    - by mixed numbers, 141–142
    - numerator in, 139
    - simplification before, 142–143
    - by whole numbers, 140–141
  - with negative signs, simplifying, 684
  - order of operations with, 225–230, 238
    - applications of, 226–227
  - as percents, 378–381, 414
    - proportion method for, 380–381, 414
  - percents as, 368
    - applications of, 370
  - proper, 119, 168
  - proportional, 335, 336–337
  - ratios as, 315
  - reciprocal of, 153
  - simplifying, 128–137, 169–170
    - on calculator, 135
    - common factors in, 131
    - greatest common factor in, 130
    - before multiplication, 142–143
    - with negative signs, 684
    - by prime factorization, 130–131
  - subtraction of
    - with like denominators, 185–190
    - with unlike denominators, 203–204
  - unlike
    - addition of, 200–203, 206–211, 238
    - applications of, 202–203, 204–205
    - simplifying, 201–202
    - subtraction of, 203–204, 238
- Fraction bar, 685, 705, 712, 714
- Fundamental frequency, 198
- Fundamental principle of fractions, 129, 169, 193
- Fundamental theorem of arithmetic, 109

- Gallon (g), 425
- Geometry
  - angles, 485–490
  - area in, 501–506
  - circles, 515–517
  - composite figures, 517–519, 539
  - foundation of, 483
  - lines, 483–485
  - perimeter in, 499–501
  - triangles, 526–538
- Googol, 9
- Gram (g), 449
- Graph(s), 630
  - bar graphs, 586–587, 630
    - creating, 604–605, 630
  - box-and-whisker plot, 618–619, 631
  - legends of, 586–587
  - line graphs, 596–603, 630
  - pictographs, 584–585
  - pie charts, 630
    - creating, 607–609
    - reading, 605–607
- Graphing calculator
  - box-and-whisker plots on, 618
  - five-number summaries on, 618
  - fraction division on, 163
  - fraction multiplication on, 150
  - fraction simplification on, 135
  - mixed number addition on, 221–222
  - mixed number subtraction on, 221–222
- Greater than (>)
  - with fractions, 194–195
  - in ordering, 42–43
- Greatest common factors (GCFs), 111–112, 167–168
  - in simplification of fractions, 130
- Greek numerals, 2
- Grouping symbols. *See also* Parentheses
  - in order of operations, with real numbers, 684

- Hecto-, 442
- Hexagon
  - angles of, 497
  - definition of, 498
- Hieroglyphics, 2
- Hindu-Arabic numeration system, 2
- Homework, 26
- Hour (h), 425
- Hundreds, rounding to, 39, 40
- Hundreds digits, 3
- Hundredth, rounding to, 253
- Hypotenuse, of right triangles, 540–541

- Identity property, of addition, 13
- Improper fractions, 119, 168
  - mixed numbers converted from, 120–121, 169
  - mixed numbers converted to, 121–122, 169
  - ratios as, 316
  - in subtracting mixed numbers, 216
- Inequality symbols
  - with fractions, 194–195
  - in ordering, 42–43
- Integers
  - definition of, 646, 694
  - real numbers as, 647
- Interest
  - compound, 406–407
  - formula for, 399
  - simple, 405–406
- Interest rate, 405
- Interquartile range, 625
- Irrational numbers, 647
- Isosceles trapezoid, 504
- Isosceles triangle, 527, 553

- Kilo-, 442
- Kilogram (kg), 449
- Kilometer (km), 441

Leap year, 425
Least common denominators (LCDs), 200
  in addition
    of fractions, 201, 237
    of mixed numbers, 213
  in subtraction
    of fractions, 203, 237
    of mixed numbers, 214
Least common multiples (LCMs), 192–193, 199, 237
Legends, of bar graphs, 586–587
Legs, of right triangles, 540
Length
  converting between systems, 459–461, 473
  metric units of, 439–448, 472
  U.S. customary units of, 425
Less than (<)
  with fractions, 194–195
  in ordering, 42–43
Like fractions
  addition of, 183–185, 187–190, 237
    applications of, 185
  simplifying, 184
  subtraction of, 185–190, 237
Like terms, 722–723, 764
  combining, 723–724, 764
    to solve equations, 735, 746–747
Linear equations, 731
  solving, 759, 765
Line graphs, 596–603, 630
Lines, 483–485, 550
  definition of, 483
  parallel, 484, 550
    and transversal, 489–490, 551
  perpendicular, 484, 550
Line segments, 483, 550
Liters (L), 451
Long division, 68–72
Lowest terms, 130

Magic square, 24
Marginal cost, 163
Markups, 403
Mass
  converting units of, 461–462, 474
  U.S. customary units of, 425
Maximum (max), 617–618, 646–647
Mean, 567–568, 616, 629
  applications of, 573–574
  with extreme value, 568
  median and, 571–572
  not typical, 568
Measurement, units of. *See* Units of measure
Median, 568–571, 616, 629
  mean and, 571–572
Meters (m), 439–440
Metric system
  applications
    of conversion, 462–463
    of length, 443–444
    of volume, 452
  conversion of, 459–469, 472, 473–474
    applications of, 462–463
  prefixes in, 442
  square units in, 506, 552
  units of length in, 439–448, 472
    applications of, 443–444
    converting, 441–443
    estimating, 439–441
  units of temperature in, 463–464
  units of volume in, 450–452, 473
    applications of, 452
  units of weight in, 449–450, 473
    conversion of, 450
Metric ton (t), 450
Mild outliers, 626
Mile (mi), 425
Milli-, 442
Milligram (mg), 449
Milliliters (mL), 451
Millimeters (mm), 440–441
Minimum (min), 617–618, 646–647
Minuend, 26, 90
Minus sign (−), 26, 645
Minute (min), 425
Mixed numbers, 119–120, 168
  addition of, 213–214, 219–223, 238
  from addition of like fractions, 184–185
  applications
    of division, 157–159
    of multiplication, 145–147
    of subtraction, 217–218
  in decimal form, 249–250
  decimals converted from, 260–261
  decimals converted to, 251, 261–262
  division of, 156, 170
    applications of, 157–159
  improper fractions converted from, 121–122, 169
  improper fractions converted to, 120–121, 169
  multiplication of, 141–142, 170
    applications of, 145–147
    estimating, 143
    by fractions, 141–142
    simplification before, 142–143
  negative, 665
  as percents, 379
  percents as, 368
  rates as, 326
  ratios and, 316
  subtraction of, 214–223, 238
    applications of, 217–218
Mode, 572–573, 616, 629
Month, 425
Multiples
  common, 191–199
  definition of, 191
  least common, 192–193, 199
Multiplication, 48–64, 90–91
  addition and, 48, 91
  in algebra, 704, 764
  applications, 51, 55
    of decimals, 282–283
    of fractions, 144–147
    of mixed numbers, 145–147
    of real numbers, 676
  associative property of, 53
  basic facts of, 49, 91
  checking division with, 67
  commutative property of, 48–49, 91, 109
  of decimals, 281–289, 305
    applications of, 282–283
    estimation of, 282
    by powers of ten, 283–285, 305
  of denominate numbers, 51, 55, 144, 430, 472
  distributive property of, over addition, 50, 91
  estimating, 55
  of fractions, 139–151, 170
    applications of, 144–147
    on calculators, 150–151
    denominator in, 139
    by mixed numbers, 141–142
    numerator in, 139
    simplification before, 142–143
    by whole numbers, 140–141
  identity property of, 673
  of mixed numbers, 141–142, 170
    applications of, 145–147
    estimating, 143
    by fractions, 141–142
    simplification before, 142–143
  by numbers ending in zero, 54–55
  in order of operations, 81, 82
    with decimals, 297
    with fractions, 225
    with real numbers, 684
  powers as, 79
  of real numbers, 671–681, 695
    applications of, 676
    with different signs, 672
    negative, 677
    with same sign, 673, 677
  regrouping in, 50
  of single-digit numbers, 48–50
  by ten, 54
  of three-digit numbers, 52
  of two-digit numbers, 52
Multiplication property of equality, solving equations with, 743–753, 754–763, 765
Multiplicative identity property, 673
Multiplicative inverse property, 674
Multiplicative property of zero, 673

Natural numbers, 10, 645
  perfect, 116
Negative numbers, 645, 694. *See also* Signed numbers
Number line
  addition on, 11
    of real numbers, 654
  for graphing equations, 742
  integers on, 646
  order on, 42
  origin of, 11
  rounding on, 39
Numbers. *See also* Denominate numbers; Mixed numbers; Natural numbers; Real numbers; Whole numbers
  approximate, 252
  composite, 100, 101, 167
    factorization of, 108–109
    prime factors of, 108–110
  digits in, 2–3
  even, 101
  in expanded form, 3
  irrational, 647
  natural, 10, 645
    perfect, 116
  prime, 100–101, 167
    twin, 106–107
  rational, 647
  standard form for, 3
  translating words into, 4
  writing in words, 4
Number systems, development of, 2
Numerator, 117–118, 168
  in multiplication of fractions, 139
  in simplification, 129–130
Numerical coefficient, 722

Obtuse angles, 485–486, 550
Obtuse triangles, 526, 553
Octagon, definition of, 498
One, prime numbers and, 101, 167
Ones digits, 3
Opposites, 657, 694
Optics, 715
Order, 42–43, 90
Ordering
  ascending, 646
  of real numbers, 646
Order of operations, 81–83, 91
  for algebraic expressions, 712–713
  with decimals, 297
  with fractions, 225–230, 238
    applications of, 226–227
  with real numbers, 684–686, 695
Origin
  of circle, 515
  of number line, 11
  zero as, 645, 648
Ounce, 425
Outliers, 625–626
  extreme, 626
  mild, 626

Parallel lines, 484, 550
  and transversal, 489–490, 551
Parallelogram
  area of, 503, 551
  definition of, 499, 502
  perimeter of, 503, 551
Parentheses
  for negative numbers, 686
  in order of operations, 82
    with decimals, 297
    with fractions, 225
Pentagon
  angles of, 497
  definition of, 498

Percent(s), 414
  applications of, 370, 381, 400–412, 414–415
    base, amount, and rate, 389–398, 400–401, 414–415
  as decimals, 369, 414
  decimals as, 377–378
  definition of, 367, 414
  equations for, 746–749, 765
  as fractions, 368, 414
  fractions as, 378–381
    proportion method for, 380–381, 414
  frequently used, 382
  as mixed numbers, 368
  mixed numbers as, 379
Percent decrease, 404
Percentiles, 617
Percent increase, 404–405
Percent notation, 367–368
Percent proportion, 392–394, 415
Perfect number, 116
Perfect triples, 541
Perimeter, 16–18, 90, 499–501, 551
  algebraic expression for, 721
  of composite figures, 517–518
  formulas for, 500
  of parallelogram, 503
  of rectangle, 500
  of regular polygon, 500
  of square, 500
  of trapezoid, 505
  of triangle, 504
Perpendicular lines, 484, 550
Pi (π), 515, 647
Pictographs, 584–585
Pie charts, 630
  creating, 607–609
  reading, 605–607
Pint (pt), 425
Place-value system. *See* Decimal place-value system
Plus sign (+), 11, 645
Point, 483
Polygons
  angles of, 497
  area of, 501–506
  definition of, 498
  perimeter of, 499–501
  regular, 499
    perimeter of, 500
  types of, 498–499
Positive numbers, 645, 694. *See also* Signed numbers
Pound (lb), 425
Powers
  in exponential notation, 79
  first, 80
  as multiplication, 79
  in square units, 56
  of ten, 81
    decimals divided by, 295–297, 305
    decimals multiplied by, 283–285, 305
  of whole numbers, 79
  zero, 80
Prime factors
  of composite numbers, 108–110, 167
  in fraction simplification, 130–131
  in least common denominators, 200
  in least common multiples, 192
Prime numbers, 100–101, 167
  relatively, 112
  twin, 106–107
Principal, 399
Products, 48
  estimating, 55
  reciprocal, 153
Proper fractions, 119, 168
Proportion method, for fractions as percents, 380–381, 414
Proportion rule, 336, 356
  in solving proportions, 342
Proportions, 335–341, 356
  applications of, 337
    solving, 345–347
  definition of, 335
  solving, 342–354, 356–357
  verifying, 336
  writing, 335
Protractor, 486
Pythagorean theorem, 541–543, 549, 553

Quadrilateral, 497
  definition of, 499
Quart (qt), 425
Quartiles, 617–618, 630
Quotient, 65, 91, 118

Radical sign, 540
Radius, of circle, 515, 525
Range, 625
Rates, 324–333, 356
  converting between systems of measure, 461
  as decimals, 326
  definition of, 324
  as mixed numbers, 326
  in percent problems, 389–398, 401, 414–415
  proportional, 335, 337
Rational numbers, 647
Ratios, 315–322, 356
  conversion of measures by, in U.S. customary system, 427–428
  definition of, 315
  of denominate numbers, 315–318
  as fractions, 315
  simplifying, 316–318
Real numbers
  addition of, 654–662, 694
    additive identity property of, 657
    additive inverse property of, 657
    applications of, 658
    with different signs, 656
    with same signs, 655
  applications of, 648–649
    addition, 658
    multiplication, 676
    subtraction, 666
  definition of, 646, 694
  distributive property with, 686
  division of, 682–684, 695
  as integers, 647
  multiplication of, 671–681, 695
    applications of, 676
    with different signs, 672
    negative, 677
    with same sign, 673, 677
  negative, simplifying, 675
  ordering, 646
  order of operations with, 684–686, 695
  reciprocals of, 674
  subtraction of, 664–670, 694
    applications of, 666
Reciprocal
  of denominate numbers, 231
  of fractions, 153
  products of, 153
  of real numbers, 674
  solving equations with, 745–746
Rectangle
  area of, 56, 502, 551
  definition of, 499
  perimeter of, 18, 500, 551
Rectangular array, 49, 66
Rectangular solids, 58
Reduction. *See* Simplification
Regrouping
  in addition, 15–16
  in multiplication, 50
  in subtraction, 29–30
Regular polygon, 499
  perimeter of, 500
Relatively prime numbers, 112
Remainder, 67, 91
Repeating decimals, 259–260, 262, 267
Right angles, 485, 550
Right trapezoid, 504
Right triangles, 526, 553
  hypotenuse of, 540–541
  legs of, 540
  length of sides of, 541–543
Roman numerals, 2
Rounding, 39–41, 90
  of decimals, 252–253, 304
  in estimating products, 55
  of quotients from decimals, 292, 294

Scalene triangle, 527, 553
Scientific calculator
  fraction division on, 163
  fraction multiplication on, 150
  mixed number addition on, 221
  mixed number subtraction on, 221
  simplification of fractions on, 135
Scientific notation, on calculators, 289
Sequences, arithmetic, 22
Short form, addition in, 15–16
Sieve of Eratosthenes, 100, 106
Signed numbers
  addition of, 654–662, 694
    additive identity property of, 657
    additive inverse property of, 657
    applications of, 658
    with different signs, 656
    with same signs, 655
  applications, 648–649
    of addition, 658
  definition of, 646, 694
  distributive property with, 686
  division of, 682–684, 695
  as integers, 647
  multiplication of, 671–681, 695
    with different signs, 672
    negative, 677
    with same sign, 673, 677
  ordering, 646
  order of operations with, 684–686, 695
  reciprocals of, 674
  simplifying, 675
  subtraction of, 664–670, 694
Similar triangles, 528–529, 553
Simple interest, 405–406
Simplification
  of absolute value expressions, 648
  of algebraic expressions, 721–728, 764
  of denominate numbers, 429
  of equations, 735
  of fractions, 128–137, 169–170
    on calculator, 135
    common factors in, 131
    greatest common factor in, 130
    like, 184
    before multiplication, 142–143
    with negative signs, 684
    by prime factorization, 130–131
    unlike, 201–202
  of like fractions, before adding, 184
  of ratios, 316–318
  of real numbers with negative signs, 675
  of unlike fractions, before adding, 201–202
Solid
  definition of, 58
  rectangular, 58
  volume of, 57–58
Solutions, to equations, 730–731, 765
Spreadsheets, 584, 701
Square
  area of, 56–57, 502, 551
  definition of, 499
  perimeter of, 500, 551
Square inches, 56
Square roots, 540, 553
  approximating, 543
Square units, 56
  conversion of, 506, 552
Standard form, for numbers, 3
Straight angles, 485–486, 550

Substitution
  in algebra, 764
  conversion of measures by
    between systems, 459–460
    in U.S. customary system, 425–427, 471
Subtraction, 26–37, 90
  addition and, 26
  in algebra, 703
  applications, 28–29, 31
    of decimals, 272–273
    of mixed numbers, 217–218
    of real numbers, 666
    of unlike fractions, 204–205
  on calculators, 34–35
  of decimals, 270–280, 305
    applications of, 272–273
  of denominate numbers, 28, 429–430
  of digits of same place value, 27
  division by, 65, 67
  of fractions
    with like denominators, 185–190, 237
    with unlike denominators, 203–204, 238
  of like fractions, 185–190, 237
  of mixed numbers, 214–223, 238
    applications of, 217–218
  in order of operations, 81, 82
    with decimals, 297
    with fractions, 225
    with real numbers, 684
  of real numbers, 664–670, 694
    applications of, 666
  regrouping in, 29–30
  symbol for, 26
  of unlike fractions, 203–204, 238
    applications of, 204–205
  of whole numbers, 27
  words for, 27
Subtrahend, 26, 90
Sum, 11. *See also* Addition
Supplementary angles, 488, 550
Syllabus, familiarity with, 10

Tables, 583–584, 629
Tablespoon, 425
Taxes, 403
Teaspoon, 425
Temperature, units of, 463–464, 474
Ten, powers of, 81
  decimals multiplied by, 283–285
Tens, rounding to, 40, 41
Tens digits, 3
Tenths, rounding to, 252
Term(s)
  in algebraic expressions, 721–722, 764
  like, 722–723, 764
    combining, 723–724, 735, 746–747, 764
Terminating decimals, 258, 267, 304
Test preparation, 79
Test taking, 83
Textbook, familiarity with, 2
Thousands, rounding to, 39
Time, measures of, 425
Ton, 425
Ton, metric (t), 450
Transit, 484
Transversal, and parallel lines, 489–490, 551
Trapezoid
  area of, 505, 551
  definition of, 504
  isosceles, 504
  perimeter of, 505, 551
  right, 504
Trial dividend, 69
Trial divisor, 69
Triangles, 526–538, 553
  acute, 526, 553
  angles of
    measuring of, 528
    sum of, 527–528
  area of, 504, 551
  definition of, 499, 503
  equilateral, 527, 553
  isosceles, 527, 553
  obtuse, 526, 553
  perimeter of, 504, 551
  right, 526, 553
    hypotenuse of, 540–541
    legs of, 540
    length of sides of, 541–543
  scalene, 527, 553
  similar, 528–529, 553
Twin primes, 106–107
Typical member, 572

Unit prices, 327–333, 356
Unit rates, 325–326, 356
Unit-ratio method, for conversion of measures
  applications of, 431–432
  between systems, 460–461
  in U.S. customary system, 427–428, 471
Units. *See also* Denominate numbers
  adding or subtracting in, 28
  cubic, 58
  dividing by, 66, 144
  fractional, 144
    conversion of, 158
    reciprocal of, 231
  multiplying in, 51, 55, 144
  as percents, 379
  square, 56
Units of measure, 425
  for area, conversion factors for, 552
  conversion of, 459–469, 473–474
    applications of, 462–463
  metric system
    applications of, 443–444
    conversion of, 459–469, 472
    length, 439–448, 472
    volume, 450–452, 473
    weight, 449–450, 473
  square, conversion of, 506, 552
  for temperature, 463–464, 474
  for time, 425
  U.S. customary system, 425–438, 471
    applications of, 431–432
    conversion by substitution, 425–427, 471
    conversion by unit-ratio method, 427–428, 471
    conversion to metric, 459–469
    length, 425
    mass, 425
    temperature, 463–464
    volume, 425
    weight, 425
Unlike fractions
  addition of, 200–203, 206–211
    applications of, 202–203
  applications of, 202–203, 204–205
  simplifying, 201–202
  subtraction of, 203–204
    applications of, 204–205
U.S. Census Bureau, 1
U.S. customary system of measurement, 425–438, 471
  conversion of, 459–469, 471, 473–474
    applications of, 462–463
    by substitution, 425–427, 471
    by unit-ratio method, 427–428, 471
  square units in, 506, 552
  units of length in, 425
  units of mass in, 425
  units of temperature in, 463–464
  units of volume in, 425
  units of weight in, 425

Valence number, 692
Variables
  in algebra, 703
  in proportion, 335
Vertical angles, 489, 550
Volume
  converting units of, 462, 474
  metric units of, 450–452, 473
    applications of, 452
  of solid, 57–58, 91
  U.S. customary units of, 425

Week, 425
Weight
  converting units of, 461–462, 474
  metric units of, 449–450, 473
    conversion of, 450
  U.S. customary units of, 425
Whole numbers
  addition of, 10–19
  composite, 100, 101
  decimals divided by, 291–292
  definition of, 10, 645
  factorization of, 108–116, 167–168
  factors of, 99–100, 108–116
  in mixed numbers, 119–120
  multiplied by fractions, 140–141
  ordering, 42–43
  powers of, 79–81
  prime, 100–101
  rounding, 39–41
  subtraction of, 27
Word problems. *See* Applications
Words
  for addition, 14
  for decimals, 250–251, 304
  for numbers, 4
  for operations in algebra, 706
  for subtraction, 27

Yard (yd), 425

Zero
  addition of, 657
  division and, 68, 683
  as identity for addition, 13
  multiplicative property of, 673
  prime numbers and, 101, 167
  in whole numbers, 645
Zero power, 80

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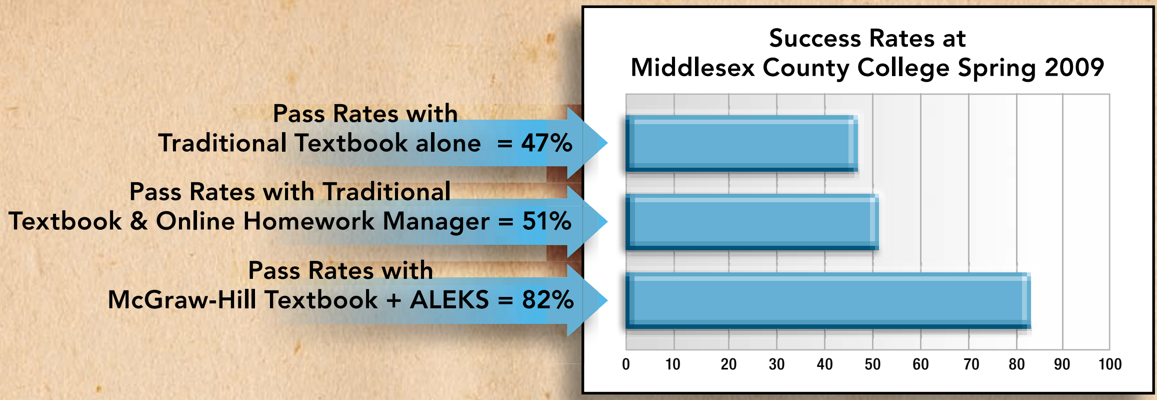

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